Julia Sets

Theory and Algorithms

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MATH/CAAM 435

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Definition

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Remark

The points of S^1 are supersensitive under Q_0. That is, any open ball around $z \in S^1$ has the property that $\bigcup_{n=0}^{\infty} Q_0^{\circ n}(z) = \mathbb{C} \setminus \{p\}$ for at most one point p.

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For the quadratic map $Q_0(z) = z^2$ we saw chaotic behavior only on S^1 by angle doubling ($\theta \to 2\theta$). We also saw that $|Q_0(z)| \to \infty$ for all $|z| > 1$ and $|Q_0(z)| \to 0$ for all $|z| < 1$.

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Remark

Finally, we learned that for $Q_{-2}(z) = z^2 - 2$, we have $J_2 = K_2 = [-2, 2]$, so any $z \in \mathbb{C}\,[-2, 2] \to \infty$ as we compose $Q_{-2}(z)$ infinitely many times.

- Goal: discuss and prove parts of:

Theorem

If $|c|$ is sufficiently large, Λ, the set of points whose entire forward orbits lie within the circle $|z| = |c|$, is a Cantor set on which Q_c is topologically conjugate to the shift map on two symbols. All points in $\mathbb{C} - \Lambda$ tend to ∞ under iteration of Q_c. Hence, $J_c = K_c$.