# *Lecture Notes – The Julia Set*

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## *Preliminaries*

As a quick recap of what we saw last week, recall the following facts. First, for the quadratic map $Q_0(z) = z^2$ we saw chaotic behavior only on $S^1$ by angle doubling ($\theta \to 2\theta$). We saw that $Q_0(z) \to \infty$ for all $|z| > 1$ and $Q_0(z) \to 0$ for all $|z| < 1$.

We also defined the Julia Set $J_c$ as the boundary of the Filled Julia Set $K_c$. We could alternatively define $J_c$ as the closure of the set of repelling points of $Q_c$ (in fact, this definition isn't limited to the quadratic map; any polynomial will do, and is denoted $J(P)$ for some $P : \mathbb{C} \to \mathbb{C}$).

The *stable set* of a a complex polynomial $P$, denoted $S(P)$, is the complement of $J(C)$.

Finally, the previous group discussed how the points of $S^1$ were *supersensitive*. That is, any open ball around $z \in S^1$ has the property that $\bigcup_{n=0}^{\infty} Q_0^{\circ n}(z) = \mathbb{C} \setminus \{p\}$ for at most one point $p$.

## *16.3 – The Julia Set as a Cantor Set*

For this section of the talk, we will consider the family of quadratic functions $Q_c(z) = z^2 + c$ when $|c| > 2$.

The first big Theorem we will discuss, not in complete detail, is the following.

**Theorem 1.** *Suppose $|c| > 2$. Then $J_c = K_c$ is a Cantor set. Moreover, the quadratic map $Q_c$, when restricted to $J_c$, is conjugate to the shift map on two symbols.*

**Theorem 2** (The Escape Criterion)**.**

**Corollary 1.**

**Corollary 2.**

**Corollary 3.**

## *16.4 – Computing the Filled Julia Set*

## *16.5 – Computing the Julia Set Another Way*