Phenomenology of Indirect CP Violation within the D^0 system

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Abstract

Indirect CP violation within the D^0 meson is parameterised in terms of final state dependent dispersive and absorptive CP violating phases ϕ_f^M, ϕ_f^Γ and CP conserved mixing parameters x_{12}, y_{12}. Observables and asymmetries within this formalism are obtained. SCS decays to CP eigenstates depend on ϕ_f^M while SCS decays to CP non-eigenstates and CF/DCS decays to $K^\pm$X final state measurements are dominantly sensitive to ϕ_f^Γ. Through *approximate universality*, the difference of the intrinsic phases of the ϕ_2^M and ϕ_2^Γ and the final statement dependent phases is approximately negligible. These intrinsic phases are investigated through U-spin decomposition of mixing amplitudes, yielding a Standard Model estimate of ϕ_2^M, ϕ_2^Γ of $-\left(2.12 \times 10^{-3}\right)$ radians; similar in magnitude but different in sign to that of [1]. Misalignments of intrinsic and final state dependent phases are extremely suppressed within CF/DCS decays, but of $\mathcal{O}(0.1)$ in SCS decays, which in tandem provides sensitivity to the CKM angle γ.

1 Introduction

All fermionic matter has an associated antiparticle, a particle with the same mass, but different electric and colour charge. If the universe began with equal amounts of both antimatter and matter, it should then undergo matter-antimatter annihilation resulting in a radiation dominated universe today instead of the observed matter domination [2]. To satisfy this asymmetry between matter and antimatter, Soviet physicist Andrei Sakharov proposed three conditions; baryon asymmetry (through *baryogenesis*), charge conjugation(C) and CP violation and interactions departing from thermal equilibrium [3].

All these conditions are satisfied within the Standard Model; however each condition comes with its own caveat. Although CP violation enters within both the neutrino and quark sectors, all currently observed asymmetries are insufficient to explain the current abundance of matter [4]. Promising new sources of CP violation that may address this problem lie within the quark sector, particularly the D^0 meson, where direct CP violation was found in 2019; the study of this system is the focus of this report [5].

Charge-conjugation (C) transformations involve a change of particle $\rightarrow$ antiparticle. Parity (P) transformations reverse the spatial coordinate $\mathbf{x} \rightarrow -\mathbf{x}$. It was previously thought that C and P where good symmetries of nature, physical laws were invariant under such transformations, however these symmetries were violated in experiments of ^{60}Co atoms and pion decays to $\mu^{\pm}$ [6][7]. A similar violation was found for the combined CP transformation in neutral kaons [8]. CP violation occurs in the quark sector due to a complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix and requires three generations of quarks [9].

This report will cover the indirect CP violation of the D^0 system. "Indirect" encompasses all possible CP violation mechanisms within neutral meson mixing - an oscillation between D^0 and antiparticle $\overline{D}^0$, whilst direct CP violation occurs in decay only. A recently proposed minimum parameterisation for D^0 CP violation will be investigated for its asymmetries, observables for different decays and its relationship of its intrinsic and final state phases.

In *"Theoretical Background"*, elementary principles such as the CP operators and the CKM matrix are discussed, then in the next section the standard formalism is briefly introduced and its phenomenology discussed. The *"The dispersive/absorptive formalism"* section introduces and motivates the new formalism parameterised in terms of dispersive and absorptive phases and its relation to the old parameters.

In *"CP Violation asymmetries"*, indirect, direct and semileptonic CP asymmetries are derived in this formalism. Observables are then constructed for three different classes of hadronic decays and their associated phenomenology is discussed. The last chapter covers relationship between intrinsic and final state dependent CP violating phases through U-spin decomposition. The implementation of this relationship is discussed for different decay channels.

2 Theoretical background

2.1 General background

The CKM matrix, V_{ij} is a 3×3 unitary matrix which relates the quark flavour eigenstates to the physical quark mass eigenstates [9]. The modulus squared of any $|V_{ij}|^2$ is the probability of a j-flavored quark decaying to an i-th flavour and appear at vertices in Feynman diagrams. The CKM matrix contains 4 independent parameters; in the Wolfenstein parameterisation (A, ρ, η, λ), powers of λ indicate the degree of suppression in quark weak interactions [10].

The D^0 and $\overline{D}^0$ mesons are eigenstates of the flavour operator, with eigenvalues of ± 1 respectively. These mesons exhibit a phenomenon known as *neutral meson mixing*, due to the mass and decay width differences between their Hamiltonian mass eigenstates, given by the following Feynman diagrams,

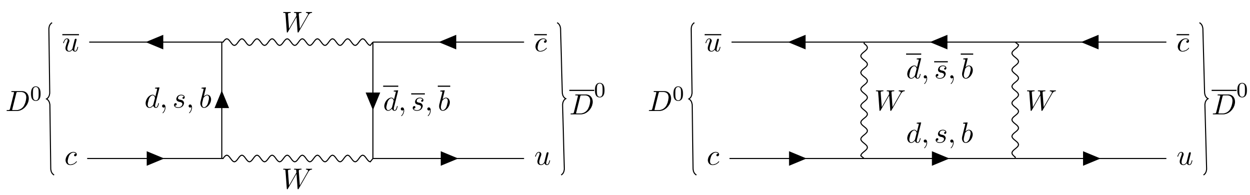

Figure 2.1.1: Box diagrams of $D^0 \leftrightarrow \overline{D}^0$ mixing

Charge-parity (CP) transformations are discrete, composite transformations, involving P, an inversion of spatial coordinates and C, a swapping of charge. In combination with T, the time reversal operator, the symmetry CPT holds for all Lorentz-invariant Hermitian quantum field theories and has never been experimentally violated [11]. The CP operator turns a particle into its antiparticle, so evidently the flavour eigenstates are not CP eigenstates, but can be linearly combined to form CP eigenstates, with eigenvalues of ± 1,

$$|D_{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|D^0\rangle \pm |\overline{D^0}\rangle\right) \tag{2.1.1}$$

2.2 Standard formalism for neutral meson mixing and decays

As D^0 and $\overline{D}^0$ mesons decay, the probability norm is not conserved and thus the Hamiltonian of the system is non-Hermitian. The dynamics of this system can be approximated effective 2×2 Hamiltonian decomposed in terms of the mass and decay matrices, $\mathbf{M}$ and $\mathbf{\Gamma}$, as [12],

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} \tag{2.2.1}$$

Both matrices are Hermitian hence $M_{21} = M_{12}^*$ and $\Gamma_{21} = \Gamma_{12}^*$ and CPT symmetry implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The off-diagonal elements M_{12} and Γ_{12} represent the dispersive (virtual) and absorptive (real) contributions to mixing respectively [13]. The eigenvalues of the Hamiltonian are,

$$\lambda_{1,2} = \left(M_{11} - \frac{i}{2}\Gamma_{11}\right) \pm \left(\Delta m - \frac{i}{2}\Delta\Gamma\right) \tag{2.2.2}$$

where Δm,$\Delta\Gamma$ satisfy,

$$(\Delta m)^2 - \left(\frac{\Delta\Gamma}{2}\right)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 \tag{2.2.3}$$

$$\Delta m \cdot \Delta\Gamma = 4Re(M_{12}^*\Gamma_{12}) \tag{2.2.4}$$

The Hamiltonian and flavour operators do not commute and do not share eigenstates because mixing changes flavour [12]. The dynamics of this system are only considered for times larger than typical strong interactions, the Weisskopf-Wigner approximation, and contributions to the mass eigenstates from decay products are ignored [14]. Thus they can be expressed as linear combinations of the flavour eigenstates as,

$$|D_{1,2}\rangle = p\,|D^0\rangle \pm q\,|\overline{D^0}\rangle \tag{2.2.5}$$

with $|p|^2 + |q|^2 = 1$ and

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \tag{2.2.6}$$

Unlike the flavour eigenstates, the mass eigenstates are not orthogonal, $\langle D_1|D_2\rangle \neq 0$. For no CP violation, the mass eigenstates are equal to the CP eigenstates from (2.1.1). Time evolution of the flavour eigenstates is found by rearranging (2.2.5) in terms of $|D_1\rangle, |D_2\rangle$ and propagating, giving equations for $|D^0(t)\rangle$ and $|\overline{D^0}(t)\rangle$ which are parameterised by two dimensionless mixing parameters,

$$x = \frac{\Delta m}{\Gamma} \tag{2.2.7}$$

$$y = \frac{\Delta\Gamma}{2\Gamma} \tag{2.2.8}$$

and Γ, the average decay width. The decay amplitudes are defined, $A_f = \langle f|\mathbf{H}|D^0\rangle$, $\overline{A}_f = \langle f|\mathbf{H}|\overline{D}^0\rangle$ and likewise for CP conjugate states $\left(f \to \overline{f}\right)$; the decay rates are the square modulo of the weighted sum of these amplitudes. CP violation in neutral mesons falls into the following three categories [16],

$$\left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| \neq 1 \qquad \text{in decay (direct)} \tag{2.2.9}$$

$$\left|\frac{q}{p}\right| \neq 1 \qquad \text{in mixing (indirect)} \tag{2.2.10}$$

$$\phi = \arg\left(\frac{q}{p}\right) \neq 0 \qquad \text{in interference (indirect)} \tag{2.2.11}$$

2.3 The dispersive/absorptive formalism

2.3.1 Motivation and formalism

CP violation (CPV) is reformulated in terms of dispersive and absorptive off-diagonal mixing elements M_{12} and Γ_{12} [1]. Mixing is described by two CP conserved parameters x_{12}, y_{12}, which to a good approximation are $|x|$ and $|y|$, and one final state independent weak mixing phase ϕ_{12}, defined as,

$$x_{12} = \frac{2\,|M_{12}|}{\Gamma} \tag{2.3.1}$$

$$y_{12} = \frac{|\Gamma_{12}|}{\Gamma} \tag{2.3.2}$$

$$\phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = \phi_2^M - \phi_2^\Gamma = \phi_f^M - \phi_f^\Gamma \tag{2.3.3}$$

Rearranging (2.3.3) suggests misalignments of final state and intrinsic mixing phases are common among both dispersive and absorptive phases. Dispersive and absorptive indirect CPV is described by two final state dependent CP violating weak phases ϕ_f^M and ϕ_f^Γ, yielding simplified expressions for CP asymmetries and a clear distinction between both types of mixing. A phase-based parameterisation has its basis in the U-spin decomposition of M_{12} and Γ_{12}, where it is more appropriate to consider the relative arguments of these amplitudes, rather than a complicated dependence on traditional indirect CPV parameters such as $\left|\frac{q}{p}\right|$ and ϕ.

Two intrinsic mixing phases ϕ_2^M and ϕ_2^Γ are also introduced that are physical, quark independent and accessible due to approximate universality with regard to the final state dependent phases. Two meson and quark dependent weak phases ϕ^M, ϕ^Γ are related to these intrinsic mixing phases and are defined,

$$\phi^M = \arg\left(M_{12}\right),\ \phi^\Gamma = \arg\left(\Gamma_{12}\right),\ \phi_{12} = \phi^M - \phi^\Gamma \tag{2.3.4}$$

These are weak phases are unphysical, but will be stand-ins for ϕ_2^M and ϕ_2^Γ until Section 3 wherein their relations and distinctions will need to be stressed in light of U-spin decomposition.

Phenomenological CPV in mixing and interference parameters (2.2.10)(2.2.11) are given by,

$$\left|\frac{q}{p}\right| = 1 + \frac{x_{12}y_{12}\sin(\phi_{12})}{x_{12}^2 + y_{12}^2} \tag{2.3.5}$$

$$\phi = -\frac{x_{12}^2\phi^M + y_{12}^2\phi^\Gamma}{x_{12}^2 + y_{12}^2} \tag{2.3.6}$$

More details on this translation are given here [17]. (2.3.6) is also valid for final state dependent ϕ_{λ_f} and intrinsic ϕ_2, with the necessary subscripts added.

3 CP Violation observables

3.1 CP asymmetries, the direct CP asymmetry and semileptonic decays

The underlying theoretical parameters discussed in the previous section are related to the behaviour of different decay channels of the D^0 through observables present within the time-dependent decay rates. The mixing parameters x_{12} and y_{12} are universal to all decays and have been measured to be greater than zero at $> 5\sigma$ [18]. The final state dependent phases are in general not the same for every decay, ie $\phi^{\Gamma}_{K^+\pi^-} \neq \phi^{\Gamma}_{K^+K^-}$, but are all related to the same intrinsic phase through approximate universality. The difference between phases has a negligible contribution to the world-average fits for the intrinsic phases[1].

It is useful to consider the widely used final state dependent CP asymmetries and CP conserving observables introduced originally for analysis of $K_S\pi^+\pi^-$ [19]. These asymmetries are, in general, quoted without subscripts, either when studying a single decay channel or in an average over multiple channels. Using the translations of (2.3.5) and (2.3.6) gives the following CP asymmetries,

$$\Delta X = \Delta x_f = -y_{12} \sin \phi^{\Gamma}_f \tag{3.1.1}$$

$$A_{\Gamma} = \Delta y_f = x_{12} \sin \phi^{M}_f \tag{3.1.2}$$

$$y_{CP} = y_{12} \cos \phi^{\Gamma}_f \tag{3.1.3}$$

$$x_{CP} = x_{12} \cos \phi^{M}_f \tag{3.1.4}$$

Later, these asymmetries will appear in a more generalised form for specific decay channels, but will have a recognisable form similar to that shown above.

Direct CP violation occurs when the inequality in (2.2.9) is satisfied. It is therefore useful to gauge the deviation from unity of this violation by defining the direct CP asymmetry a^d_f,

$$a^d_f = 1 - \left| \frac{\overline{A}_{\overline{f}}}{A_f} \right| \tag{3.1.5}$$

Later, it will shown that the misalignments between final state and intrinsic mixing phases is proportional to a^d_f. The difference of asymmetries between K^+K^- and $\pi^+\pi^-$ has been observed to be on the order of 0.2% [5].

The leptonic and rare decays such as $D^0 \to e^+e^-$, $D^0 \to \mu^+\mu^-$ and $D^0 \to \gamma\gamma$ are allowed but suppressed, whilst $D^0 \to \mu^{\pm}e^{\mp}$ is forbidden; searches for these decays are useful tests for new physics [20]. Semileptonic ($D^0 \to l^-X$) decays have been observed, so it

[1] Thank you to Cristina Alexe for correspondence on this point

would be useful to define a semileptonic asymmetry as the normalised difference of decay rates [21],

$$a_{SL} = \frac{\Gamma\left(D^0\left(t\right) \to l^- X\right) - \Gamma(\overline{D}^0\left(t\right) \to l^+ X)}{\Gamma\left(D^0\left(t\right) \to l^- X\right) + \Gamma(\overline{D}^0\left(t\right) \to l^+ X)} \tag{3.1.6}$$

Semileptonic amplitude factors cancel, giving the asymmetry,

$$a_{SL} = \frac{2x_{12}y_{12}\sin\phi_{12}}{x_{12}^2 + y_{12}^2} \tag{3.1.7}$$

3.2 SCS decays to CP eigenstates

The following three subsections will discuss D^0 hadronic decays and the sensitivity their observables and asymmetries provide for CP violation parameters such as ϕ_f^M, ϕ_f^Γ. These hadronic decays are classed by their suppression by the Wolfenstein parameter λ, for instance Singly Cabbibo Suppressed (SCS) decays are suppressed by λ^1. Singly Cabbibo Suppressed are split into two classes of decays; decays CP eigenstates and non-CP eigenstates. The first of these that will be studied is CP eigenstates, meaning that the final state products are eigenstates of the CP operator, e.g. decays such as $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$. In general, the decay amplitudes for all hadronic decays are defined,

$$\begin{aligned} A_f &= A_f^0 e^{+i\phi_f^0}\left[1 + r_f e^{i(\delta_f+\phi_f)}\right],\ A_{\overline{f}} = A_{\overline{f}}^0 e^{i(\Delta_f^0+\phi_f^0)}\left[1 + r_{\overline{f}} e^{i(\delta_{\overline{f}}+\phi_{\overline{f}})}\right] \\ \overline{A}_{\overline{f}} &= A_f^0 e^{-i\phi_f^0}\left[1 + r_f e^{i(\delta_f-\phi_f)}\right],\ \overline{A}_f = A_{\overline{f}}^0 e^{i(\Delta_f^0-\phi_f^0)}\left[1 + r_{\overline{f}} e^{i(\delta_{\overline{f}}-\phi_{\overline{f}})}\right] \end{aligned} \tag{3.2.1}$$

A_f^0 and $A_{\overline{f}}^0$ are the leading contributions to the amplitudes, whilst r_f and $r_{\overline{f}}$ are the subleading amplitudes of the decay (relative to the leading contributions). The phases ϕ_f and $\phi_{\overline{f}}$ are CP-odd weak phases while Δ_f^0, δ_f and $\delta_{\overline{f}}$ are CP-even strong phases. The phases ϕ_f^0 and $\phi_{\overline{f}}^0$ are CP-odd weak phases also, however are quark and meson phase dependent, so will cancel out in all observables. For SCS decays to CP eigenstates, obviously $f = \overline{f}$ for SCS decays to CP eigenstates, and $e^{i\Delta_f^0} = \eta_f^{CP} = \pm$, hence (3.2.1) becomes,

$$A_f = A_f^0 e^{+i\phi_f^0}\left[1 + r_f e^{i(\delta_f+\phi_f)}\right],\ \overline{A}_f = \eta_f^{CP} A_f^0 e^{-i\phi_f^0}\left[1 + r_f e^{i(\delta_f-\phi_f)}\right] \tag{3.2.2}$$

Two final state dependent CP observables are defined in the case of SCS decays, namely,

$$\begin{aligned} \lambda_f^M &= \frac{M_{12}}{|M_{12}|}\frac{A_f}{\overline{A}_f} = \eta_f^{CP}\left|\frac{A_f}{\overline{A}_f}\right| e^{i\phi_f^M} \\ \lambda_f^\Gamma &= \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_f}{\overline{A}_f} = \eta_f^{CP}\left|\frac{A_f}{\overline{A}_f}\right| e^{i\phi_f^\Gamma} \end{aligned} \tag{3.2.3}$$

Inserting the equations for A_f and $\overline{A}_f$ from (3.2.2) into (3.2.3) and equating exponents gives a relationship between the final state dependent phases and (2.3.4) of,

$$\phi_f^{M/\Gamma} = \phi^{M/\Gamma} + 2\phi_f^0 + 2r_f\cos(\delta_f)\sin(\phi_f) \tag{3.2.4}$$

The decay rates are given as follows,

$$\begin{aligned}\Gamma\left(D^0(t)\to f\right) &= e^{-\tau}\left|A_f\right|^2\left(1+c_f^+\tau+c_f^{\prime +}\tau^2\right),\\ \Gamma\left(\overline{D}^0(t)\to f\right) &= e^{-\tau}\left|\overline{A}_f\right|^2\left(1+c_f^-\tau+c_f^{\prime -}\tau^2\right)\end{aligned} \tag{3.2.5}$$

where

$$\begin{aligned}c_f^{\pm} &= \eta_f^{CP}\left[\mp x_{12}\sin\phi_f^M - y_{12}\cos\phi_f^\Gamma\left(1\mp a_f^d\right)\right]\\ c_f^{\prime\pm} &= \frac{1}{2}y_{12}^2 \pm \frac{1}{4}\left(y_{12}^2+x_{12}^2\right)\left(a_{SL}-2a_f^d\right)\end{aligned} \tag{3.2.6}$$

The observable $c_f^{\prime\pm}$ is suppressed in SCS decays therefore $c_f^{\prime\pm}$ may safely be approximated as $\frac{1}{2}y_{12}^2$, however is usually ignored altogether. In the latter case, the decay widths then become proportional to decaying exponentials with an effective decay rate parameter of,

$$\hat{\Gamma}_{D^0/\overline{D}^0\to f} = 1 - c_f^{\pm} \tag{3.2.7}$$

The known CP conserved observables and CP asymmetries of (3.1.2) and (3.1.3) can be defined from these effective decay rate parameters respectively as,

$$y_{CP}^f = \frac{\hat{\Gamma}_{D^0\to f}+\hat{\Gamma}_{\overline{D}^0\to f}}{2} - 1 \tag{3.2.8}$$

$$\Delta Y_f = \frac{\hat{\Gamma}_{D^0\to f}-\hat{\Gamma}_{\overline{D}^0\to f}}{2} \tag{3.2.9}$$

Giving,

$$y_{CP}^f = \eta_f^{CP} y_{12}\cos\phi_f^\Gamma \tag{3.2.10}$$

$$\Delta Y_f = \eta_f^{CP}\left(-x_{12}\sin\phi_f^M + a_f^d y_{12}\right) \tag{3.2.11}$$

Sensitivity to ϕ_f^M comes from ΔY_f, as this phase enters at first order, relative to the mixing parameters, as $\sin\phi_f^M$. This asymmetry also provides sensitivity to the direct CP asymmetry, which itself provides sensitivity to subleading parameters through the relation of $a_f^d = -2r_f\sin\phi_f\sin\delta_f$. The observable y_{CP}^f cannot be used to probe ϕ_f^Γ at the current level of sensitivity, as the contribution of the phase is second order; this ϕ_f^Γ dependence also vanishes when averaging over different decays, ie. K^+K^-, $\pi^+\pi^-$.

3.3 SCS decays to non-CP eigenstates

SCS decays to non-CP eigenstates include such decays as $D^0\to K^{*+}K^-$. The observables for this decay are defined as,

$$\begin{aligned}\lambda_f^M &= \left|\frac{A_f}{\overline{A}_f}\right| e^{i(\phi_f^M-\Delta_f)},\ \lambda_f^\Gamma = \left|\frac{A_f}{\overline{A}_f}\right| e^{i(\phi_f^\Gamma-\Delta_f)}\\ \lambda_{\overline{f}}^M &= \left|\frac{A_{\overline{f}}}{\overline{A}_{\overline{f}}}\right| e^{i(\phi_f^M+\Delta_f)},\ \lambda_{\overline{f}}^\Gamma = \left|\frac{A_{\overline{f}}}{\overline{A}_{\overline{f}}}\right| e^{i(\phi_f^\Gamma+\Delta_f)}\end{aligned} \tag{3.3.1}$$

Unlike decays to CP eigenstates, a CP-even strong phase, Δ_f is present within the observables and is the total phase difference between A_f and $\overline{A}_f$. Rearranging and substituting to remove the ϕ_f^Γ, ϕ_f^M phases and using (3.2.1) gives an expression for the total strong phase difference of,

$$\Delta_f = \Delta_f^0 + r_{\overline{f}} \sin(\delta_{\overline{f}}) \cos(\phi_{\overline{f}}) - r_f \sin(\delta_f) \cos(\phi_f) \tag{3.3.2}$$

Following this, the relation between final state dependent and intrinsic phases is given,

$$\phi_f^{M/\Gamma} = \phi^{M/\Gamma} + \phi_f^0 + \phi_{\overline{f}}^0 + r_f \cos(\delta_f) \sin(\phi_f) + r_{\overline{f}} \cos(\delta_{\overline{f}}) \sin(\phi_{\overline{f}}) \tag{3.3.3}$$

This is a more generalised expression encompassing all SCS decays, as in the limit of $\overline{f} \to f$, (3.3.3) reduces to (3.2.4). As before, the decay rates are given as,

$$\begin{aligned}
\Gamma\left(D^0(t) \to f\right) &= e^{-\tau} \left|A_f\right|^2 \left(1 + \sqrt{R_f} c_f^+ \tau + R_f c_f^{\prime +} \tau^2\right), \\
\Gamma\left(D^0(t) \to f\right) &= e^{-\tau} \left|\overline{A}_f\right|^2 \left(1 + \frac{1}{\sqrt{R_f}} c_f^- \tau + \frac{1}{R_f} c_f^{\prime -} \tau^2\right)
\end{aligned} \tag{3.3.4}$$

with the coefficients are given,

$$\begin{aligned}
c_f^{\pm} &= \mp x_{12} \sin\left(\phi_f^M - \Delta_f\right) - y_{12} \cos\left(\phi_f^\Gamma - \Delta_f\right) \\
c_f^{\prime \pm} &= \frac{1}{4} \left[R_f^{\mp 1} \left(y_{12}^2 - x_{12}^2\right) + \left(x_{12}^2 + y_{12}^2\right)\left(1 \pm a_{SL}\right)\right]
\end{aligned} \tag{3.3.5}$$

and where $R_f = \left|\frac{\overline{A}_f}{A_f}\right|^2$. As these are non-eigenstates, similar expressions exist for CP conjugate decay rates. As in CP eigenstates, the expression for $c_f^{\prime \pm}$ may be simplified by removing the a_{SL} term, as this is beyond the level of even Run 3 precision.
Two CPV asymmetries, ΔY_f and $\Delta Y_{\overline{f}}$ parameterise this decay process,

$$\Delta Y_f = \frac{\sqrt{R_f} c_f^+ - c_{\overline{f}}^- / \sqrt{R_{\overline{f}}}}{2} \tag{3.3.6}$$

With $\Delta Y_{\overline{f}}$ being the CP conjugate. To calculate this asymmetry, it is useful to calculate the product of R_f and $R_{\overline{f}}$ which is found to be,

$$R_f R_{\overline{f}} = 1 - 2a_f^d - 2a_{\overline{f}}^d \tag{3.3.7}$$

This gives the CPV asymmetry, ΔY_f, as

$$\Delta Y_f = \sqrt{R_f} \left[-x_{12} \sin\phi_f^M \cos\Delta_f - y_{12} \sin\phi_f^\Gamma \sin\Delta_f - \frac{1}{2}\left(a_f^d + a_{\overline{f}}^d\right)\left(x_{12} \sin\Delta_f - y_{12} \cos\Delta_f\right)\right] \tag{3.3.8}$$

The strong phase Δ_f is the same in both ΔY_f and $\Delta Y_{\overline{f}}$. This asymmetry is sensitive to ϕ_f^M and ϕ_f^Γ as they both enter through sines, however such sensitivity is limited by the size of Δ_f.

3.4 CF/DCS decays to $K^{\pm}X$

Cabbibo Favoured (CF) decays are decays which have no suppression, ie. their amplitudes are proportional to λ^0, whilst Doubly Cabbibo Suppressed (DCS) decays are suppressed by λ^2. CF/DCS decays are classed into two types of kaonic decay; $D^0 \to K^{\pm}X$ and $D^0 \to K^0 X$, with this report focusing on the former. CF/DCS decays are further split into two types; for an example, the decay $D^0 \longrightarrow K^+\pi^-$ is a wrong-sign (WS) decay, with decay amplitudes $A_f, \overline{A}_{\overline{f}}$ and contributions to this decay coming from a CF $D^0 \to K^+\pi^-$ decay and a DCS $D^0 \to \overline{D}^0 \to K^+\pi^-$ decay. Conversely $D^0 \longrightarrow K^-\pi^+$ is a right-sign (RS) decay and includes the amplitudes $A_f, \overline{A}_{\overline{f}}$.

The observables are parameterised by Δ_f which is a phase difference between CF and DCS amplitudes. The observables are given by,

$$\begin{aligned} \lambda_f^M &= -\left|\frac{A_f}{\overline{A}_f}\right| e^{i(\phi_f^M - \Delta_f)}, \ \lambda_f^\Gamma = -\left|\frac{A_f}{\overline{A}_f}\right| e^{i(\phi_f^\Gamma - \Delta_f)} \\ \lambda_{\overline{f}}^M &= -\left|\frac{A_{\overline{f}}}{\overline{A}_{\overline{f}}}\right| e^{i(\phi_f^M + \Delta_f)}, \ \lambda_{\overline{f}}^\Gamma = -\left|\frac{A_{\overline{f}}}{\overline{A}_{\overline{f}}}\right| e^{i(\phi_f^\Gamma + \Delta_f)} \end{aligned} \tag{3.4.1}$$

The overall minus sign is present in the observables due to CKM elements involved for tree-level decays [22]. The strong-phase Δ_f obeys the same relation as (3.3.2), however due to this minus sign, the relations for $\phi^f_{M/\Gamma}$ satisfy

$$\phi_f^{M/\Gamma} = \phi^{M/\Gamma} + \phi_f^0 + \phi_{\overline{f}}^0 + \pi + r_f \cos(\delta_f)\sin(\phi_f) + r_{\overline{f}}\cos(\delta_{\overline{f}})\sin(\phi_{\overline{f}}) \tag{3.4.2}$$

The decay rates have the same form as in SCS decays. It is to common to measure the ratio between WS and RS decay rates, which is parameterised by,

$$R^{\pm}(t) \approx R_f^{\pm} + \sqrt{R_f^{\pm}}\, y'^{\pm}(\Gamma t) + \frac{(x'^{\pm})^2 + (y'^{\pm})^2}{4}(\Gamma t)^2 \tag{3.4.3}$$

where $R_f^+ = \left|\frac{A_{\overline{f}}}{A_f}\right|^2$, $R_f^- = \left|\frac{\overline{A}_f}{\overline{A}_{\overline{f}}}\right|^2$ and where,

$$y'^{\pm} = y_{12}\cos\Delta_f + x_{12}\sin\Delta_f \pm x_{12}\cos\Delta_f \sin\phi_f^M \tag{3.4.4}$$

$$x'^{2\pm} = x_{12}^2 - 2x_{12}y_{12}\sin\Delta_f \mp 2x_{12}y_{12}\sin\phi_f^\Gamma \tag{3.4.5}$$

These observables give access to both phases. $y'^{\pm}$ has been measured more precisely than $x'^{2\pm}$ therefore sensitivity on ϕ_f^Γ is limited as in SCS decays to non-eigenstates by the strong phase Δ_f.

4 U-spin decomposition of the D^0 meson

4.1 Introduction

The strong interaction is approximately invariant under an SU(3) flavour symmetry between the three lightest quarks; the up, down and strange quark [23]. A subset of this symmetry is the SU(2) "U-spin" symmetry between the down and strange quark, much like isospin symmetry for up and down quarks [24]. This symmetry is valid due to difference in mass between the quarks being small compared to the energy scale of QCD[2].

It can be seen from the $D^0 \leftrightarrow \overline{D}^0$ mixing diagram in Figure 2.1.1 that the number of up and charm quark are not conserved whilst the number of down and strange quarks are. This makes U-spin a natural symmetry in which to investigate the D^0 system. The standard model off diagonal mixing elements, M_{12}^{SM} and Γ_{12}^{SM} can be written as,

$$\Gamma_{12}^{SM} = -\sum_{i,j=d,s} \lambda_i \lambda_j \Gamma_{ij},\; M_{12}^{SM} = -\sum_{i,j=d,s,b} \lambda_i \lambda_j M_{ij} \tag{4.1.1}$$

with λ_i corresponding to the CKM elements involving in each individual contribution to the mixing box diagrams, namely, $V_{ci}V_{ci}^*$. The terms M_{ij} and Γ_{ij} are the transition amplitudes for each individual box diagram. Recall that Γ_{12}^{SM} represents a real (or on-shell) process, hence there are no contributions from the bottom quarks due to its high mass. Conversely, M_{12}^{SM} has a sum over the bottom quarks, as it is a virtual (off-shell process). The CKM matrix is unitary, hence places the following constraint on this decomposition,

$$\lambda_b + \lambda_d + \lambda_s = 0 \tag{4.1.2}$$

From this and the symmetry of the CKM elements within the box diagram (ie. $\Gamma_{sd} = \Gamma_{ds}$, the U-spin decompositions of these mixing amplitudes can be written as,

$$\begin{aligned} \Gamma_{12}^{SM} &= -\frac{(\lambda_s - \lambda_d)^2}{4}\Gamma_2 + \frac{(\lambda_s - \lambda_d)\lambda_b}{2}\Gamma_1 - \frac{\lambda_b^2}{4}\Gamma_0 \\ M_{12}^{SM} &= -\frac{(\lambda_s - \lambda_d)^2}{4}M_2 + \frac{(\lambda_s - \lambda_d)\lambda_b}{2}M_1 - \frac{\lambda_b^2}{4}M_0 \end{aligned} \tag{4.1.3}$$

where $\Gamma_{2,1,0}$ and $M_{2,1,0}$ are the U-spin amplitudes, the third-component of the U-spin multiplet. These amplitudes are given respectively as,

$$\Gamma_2 = \Gamma_{ss} + \Gamma_{dd} - 2\Gamma_{sd} \tag{4.1.4}$$

$$\Gamma_1 = \Gamma_{ss} - \Gamma_{dd} \tag{4.1.5}$$

$$\Gamma_0 = \Gamma_{ss} + \Gamma_{dd} + 2\Gamma_{sd} \tag{4.1.6}$$

[2]Thank you to Dr. Stefan Schacht for correspondence on this point

$$
\begin{aligned}
M_2 &= M_{ss} + M_{dd} - 2M_{sd} && (4.1.7)\\
M_1 &= M_{ss} - M_{dd} + 2\left(M_{bd} - M_{bs}\right) && (4.1.8)\\
M_0 &= M_{ss} + M_{dd} + 2M_{sd} + 4\left(M_{bb} - M_{bd} - M_{bs}\right) && (4.1.9)
\end{aligned}
$$

The decompositions in (4.1.3) for Γ_{12}^{SM} are in disagreement with that found in [1], while the definitions of (4.1.4)-(4.1.6) are in agreement. However they are in agreement with that found in [25]. This will later be shown to have an impact on the sign of the intrinsic phases of ϕ_2^M and ϕ_2^Γ, but not their magnitudes. (4.1.7)-(4.1.9) are also in disagreement, even under the assumption that the decompositions in [1] are correct.

The subscripts of $\Gamma_{2,1,0}$ and $M_{2,1,0}$ are equal to their order of U-spin breaking, ie. $\Gamma_2 = \mathcal{O}\left(\epsilon^2\right)$, where ϵ is the U-spin breaking parameter, $\epsilon = \left(m_s - m_d\right)/\Lambda_{QCD}$ which is around 20-30%. The Γ_0 and M_0 terms are proportional to λ_b^2 so can be neglected. Γ_2 and M_2 are the dominant components, thus they are the components that contribute to mixing [26]. Then the Γ_1 and M_1 terms represent CPV in mixing. The U-spin amplitudes can be parameterised as follows,

$$
M_i = \eta_i^M \left|M_i\right| e^{2i\xi},\ \Gamma_i = \eta_i^\Gamma \left|\Gamma_i\right| e^{2i\xi} \tag{4.1.10}
$$

where $\eta_i^{M/\Gamma} = \pm$ and ξ is a meson phase convention, which cancels in all observables. Phases from the dominant U-spin contribution are consistent with +1 from experimental observations.

4.2 Intrinsic phases

Recall that the $\Delta U = 2$ component contributes to mixing, hence the CP violating intrinsic mixing phases are defined with respect to this component, as to essentially “divide out” the pure mixing contribution and focus only on the CP violating contributions. Thus the CP violating phases are defined as,

$$
\phi_2^M = \arg\left(-\frac{M_{12}}{\frac{1}{4}(\lambda_s-\lambda_d)^2 M_2}\right), \phi_2^\Gamma = \arg\left(-\frac{\Gamma_{12}}{\frac{1}{4}(\lambda_s-\lambda_d)^2 \Gamma_2}\right) \tag{4.2.1}
$$

The relation between these final state independent, meson independent intrinsic phases and the meson dependent phases defined in (2.3.4) is,

$$
\begin{aligned}
\phi^M &= \phi_2^M + 2\arg\left[\lambda_s - \lambda_d\right] + 2\xi + \frac{\pi}{2}\left(1-\eta_2^M\right)\\
\phi^\Gamma &= \phi_2^\Gamma + 2\arg\left[\lambda_s - \lambda_d\right] + 2\xi + \frac{\pi}{2}\left(1-\eta_2^\Gamma\right)
\end{aligned} \tag{4.2.2}
$$

The term proportional to meson dependent phase ξ differs from that found in [1] by a factor of i.

In general, the Γ_{12} and M_{12} in (4.2.1) can contain new physics, however these will be ignored when determined a standard model estimate. Replacing these general mixing amplitudes by their standard model counterparts from (4.1.3) give, for example,

$$\phi_2^\Gamma = \arg\left(1 - \frac{2\lambda_b}{\lambda_s - \lambda_d}\frac{\Gamma_1}{\Gamma_2}\right) \tag{4.2.3}$$

Measurements of the CKM hierarchy indicate that $\left|\frac{\lambda_b}{\lambda_s - \lambda_d}\right| \ll 1$, hence the phase can be expanded to give [27],

$$\phi_2^\Gamma = -\mathrm{Im}\left(\frac{2\lambda_b}{\lambda_s - \lambda_d}\frac{\Gamma_1}{\Gamma_2}\right) \tag{4.2.4}$$

Expanding the λ_i dependent term in terms of the Wolfenstein parameterisation of the CKM elements, and comparing to the Wolfenstein parameterisation of the Cabbibo angle $\theta_C = \lambda$, CKM angle $\gamma = \frac{\eta}{\rho}$ and $\lambda_b = A^2\lambda^5\left(\rho^2 + \eta^2\right)^{\frac{1}{2}}$ gives,

$$\phi_2^\Gamma = -\left|\frac{\lambda_b}{\theta_C}\right| \sin\gamma \times \frac{\Gamma_1}{\Gamma_2} \tag{4.2.5}$$

The ratio of $\frac{\Gamma_1}{\Gamma_2}$ is proportional to $\frac{\Gamma_1}{\Gamma_2}$. The sign of this ratio is not yet determined due to hadronic uncertainties, but may be assumed to be positive[3]. Hence the intrinsic phase becomes,

$$\phi_2^\Gamma = -\left|\frac{\lambda_b}{\theta_C}\right| \sin\gamma \times \frac{1}{\epsilon} \tag{4.2.6}$$

Using the most recent fits, this estimate determines (with negligible errors) that the phases are,

$$\phi_2^\Gamma \sim \phi_2^M \sim -\left(2.12 \times 10^{-3}\right) \times \left[\frac{0.3}{\epsilon}\right] \text{radians} \tag{4.2.7}$$

For most typical estimates of U-spin breaking, the magnitude of this phase is $\mathcal{O}\left(10^{-3}\right)$. This is equal in magnitude to what is found in [1] however opposite in sign. Current measurements of these phases are $\phi_2^M = 0.012 \pm 0.017$ and $\phi_2^\Gamma = 0.046 \pm 0.030$, meaning that for a reasonable estimate of $\epsilon = 30\%$, $\phi_2^\Gamma = -\left(2.12 \times 10^{-3}\right)$, which is a 1.6σ deviation from experiment [28]. This is not surprising given that the error on the observed phases is an order of magnitude bigger than the standard model estimate, so future measurements could fall more in line with what is estimated in (4.2.7).

More interestingly, projections for the upcoming precision era at the LHC say that errors will be on the order of the magnitude of the standard model estimates [29]. However, if the central value stays at approximately that of the current measurement, this would be a $> 3\sigma$ hint of physics beyond the standard model.

[3]Thank you to Prof. Alexander Kagan for correspondence on this point

4.3 Approximate universality and final state dependence

U-spin decomposition can be used to probe approximate universality for a variety of different decays. Firstly SCS decays, where SCS decays to CP eigenstates will be considered in detail and then generalised to SCS decays to non-CP eigenstates.
For a given final state, the misalignments are equal for both dispersive and absorptive phases and are defined,

$$\delta\phi_f = \phi_f^M - \phi_2^M = \phi_f^\Gamma - \phi_2^\Gamma \tag{4.3.1}$$

For SCS decays to eigenstates, the amplitudes are given,

$$A_f = \frac{1}{2}\left(\lambda_s^* - \lambda_d^*\right)\mathcal{A}_{f,1} + \lambda_b^*\mathcal{A}_{f,0} \tag{4.3.2}$$

$$\overline{A}_f = \frac{1}{2}\left(\lambda_s - \lambda_d\right)\overline{\mathcal{A}}_{f,1} + \lambda_b\overline{\mathcal{A}}_{f,0} \tag{4.3.3}$$

From (3.2.3), it can be seen that,

$$\phi_f^\Gamma - \phi^\Gamma = \phi_f^M - \phi^M = \arg\left(\eta_f^{CP}\frac{A_f}{\overline{A}_f}\right) \tag{4.3.4}$$

Using the definitions of (4.3.2) and (4.3.3), these expressions can be equated, giving the same form as (3.2.4), namely that

$$\phi_f^{M/\Gamma} = \phi^{M/\Gamma} + 2\phi_f^0 + 2r_f\cos(\delta_f)\sin(\phi_f) \tag{4.3.5}$$

where $\delta_f, \phi_f, \phi_f^0$ and r_f are defined,

$$\delta_f = \arg\left[\frac{\mathcal{A}_{f,0}}{\mathcal{A}_{f,1}}\right],\ \phi_f = \gamma, \tag{4.3.6}$$

$$r_f = \left|\frac{\lambda_b}{\theta_C}\frac{\mathcal{A}_{f,0}}{\mathcal{A}_{f,1}}\right|,\ \phi_f^0 = -\arg\left[\lambda_s - \lambda_d\right] - 2\xi \tag{4.3.7}$$

These are similar to that found in [1], however the weak phase ϕ_f is equal to $-\gamma$ in this case. Combining (4.3.5) and (4.2.2), using the fact that $\eta_2^{M/\Gamma} = +1$ and using the equation for the misalignments (4.3.1) gives the relation,

$$\delta\phi_f = 2r_f\cos\delta_f\sin\gamma = -a_f^d\cot\delta_f \tag{4.3.8}$$

This can be generalised to non-CP eigenstates, through similar procedures to give the result,

$$\delta\phi_f = -\frac{1}{2}\left(a_f^d\cot\delta_f + a_{\overline{f}}^d\cot\delta_{\overline{f}}\right) \tag{4.3.9}$$

It is interesting to note that for non-CP eigenstates, the misalignments do not depend on the strong phase Δ_f. The ratio between the the final state dependent phases and the misalignments is at the order of U-spin breaking.

Finally, it is useful to briefly consider CF/DCS decays to $K^{\pm}X$. By considering the unitarity of the CKM elements that enter into the CF/DCS decay amplitudes, the misalignment is calculated to be on the order of,

$$\delta\phi_f = \mathcal{O}\left(\frac{\lambda_b^2}{\lambda_d^2}\right) \tag{4.3.10}$$

which is $\mathcal{O}\left(10^{-6}\right)$. For all current and near-future projections, this is essentially perfect universality between the intrinsic phases. Measurements of CF/DCS decays to $K^{\pm}X$ in tandem with SCS decays give precise access to the intrinsic phases, meaning better constraints of the misalignments of SCS decays. From these misalignments, sensitivity to the direct CPV asymmetries a_f^d and $a_{\bar{f}}^d$ and the CKM angle γ can be achieved.

5 Conclusion and Outlook

In this report, the phenomenology of indirect CPV violation within the charm sector was parameterised within a recently proposed theoretical formalism. This formalism parameterised mixing in terms of two mixing parameters, x_{12}, y_{12}, and CP violation within this mixing process with two phases ϕ_f^M, ϕ_f^Γ.

The decay of the D^0 meson is particularly important in gaining access to these fundamental CPV parameters. The semileptonic decays and various general purpose asymmetries were defined, before moving onto various different classes of hadronic decays. Observables and asymmetries measured from hadronic SCS decays to CP eigenstates were shown to exhibit sensitivity towards ϕ_f^M, with a weak sensitivity to ϕ_f^Γ. Hadronic SCS decays to non-eigenstates and CF/DCS decays to $K^{\pm}X$ were shown to have good sensitivity to ϕ_f^Γ, however this sensitivity is limited by the strong phase difference, Δ_f.

The mixing of a neutral meson conserves U-spin, meaning that the decay process can be decomposed in terms of U-spin amplitudes; the dominant amplitude is responsible for mixing and the sub-leading amplitude is responsible for CP violation within this mixing. The intrinsic phases ϕ_2^M, ϕ_2^Γ are defined with respect to this dominant amplitude. Assuming no new physics, a standard model estimate of the phases, for a typical value of U-spin breaking, is given by $-\left(2.12 \times 10^{-3}\right)$ radians. This estimate is currently 1.6σ beyond the measured value, however future measurements could provide sensitivity to beyond 3σ.

Approximate universality, the relation of these aforementioned intrinsic phases to the final state dependent phases was investigated for all previously examined decay channels. The misalignment between the phases for CF/DCS is highly suppressed, giving incredible sensitivity to the intrinsic phases. Whilst misalignment between the phases for SCS decays to both eigenstates and non-eigenstates is not trivial, but when combined with CF/DCS decay measurements offers access to direct CP asymmetries and to the CKM angle γ.

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