Markov Chain Monte Carlo using Hamiltonian Dynamics

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Performance of MCMC

- MCMC is used to simulate s dependent sample from intractable distributions
- Performance as dimension grows is important with the rise of Big Data
- How far can we move with each proposal while keeping a optimal acceptance rate

Motivation for HMC

Algorithm	Scaling	Optimal Acceptance Rate ($d \to \infty$)
RWM	$d^{-1/2}$	23.4% (Roberts & Rosenthal 2001)
MALA	$d^{-1/3}$	57.4% (Roberts & Rosenthal 2001)
HMC	$d^{-1/4}$	64% (Beskos et. al. 2010)

Extended Parameter Space

- A ball rolling around a surface
- At any time the ball has displacement x and momentum p
- Also has a mass m

The Hamiltonian

$$U(x) = -\log\{\pi(x)\}$$

$$K(p; m) = \frac{1}{2}\frac{p^2}{m}$$

$$H(x, p; m) = U(x) + K(p; m)$$

Hamiltonian Equations

$$\frac{dx}{dt} = \frac{p}{m}$$

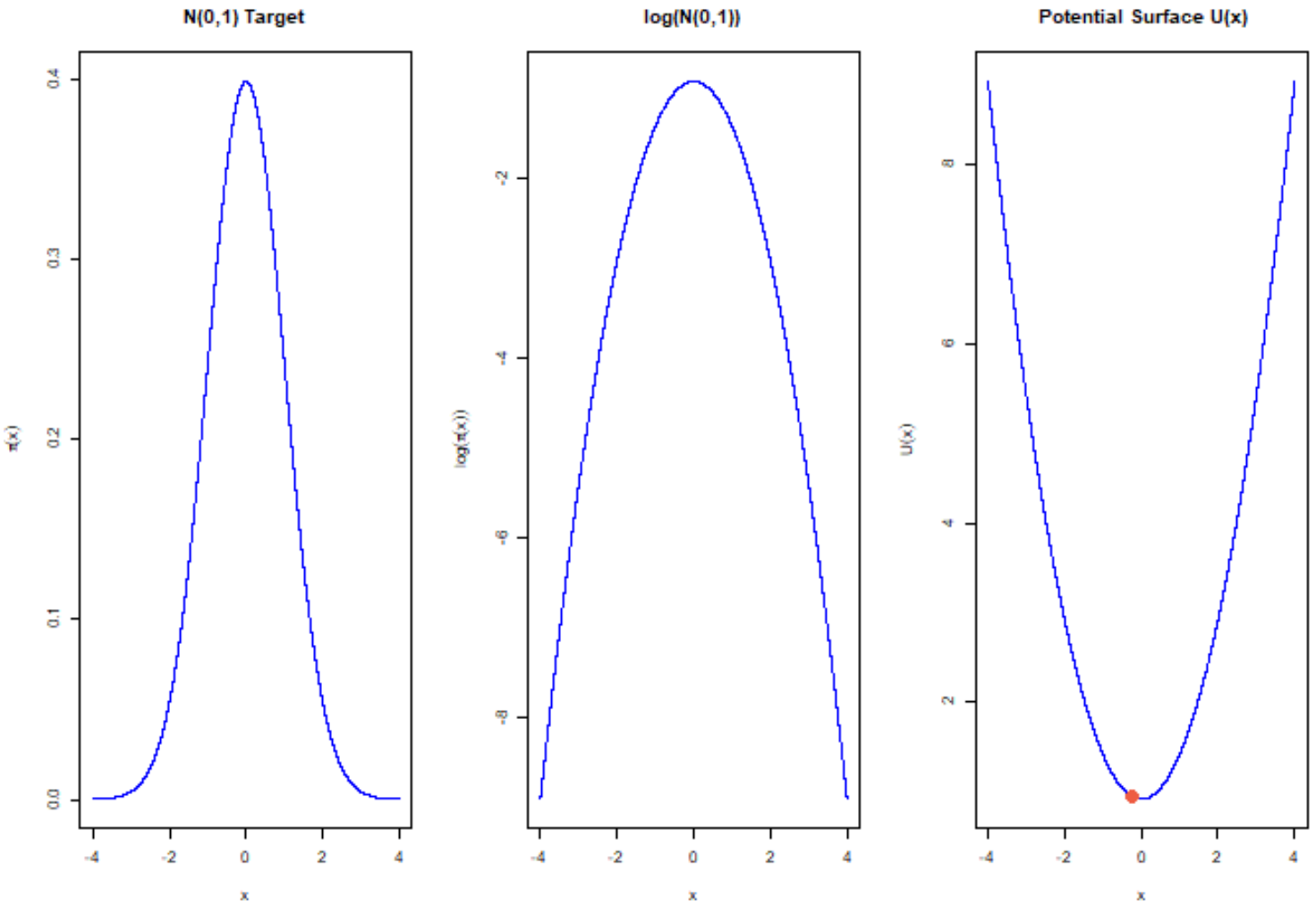
$$\frac{dp}{dt} = -\frac{dU}{dx}$$

Can be generalised to higher dimensions

$$\frac{dx}{dt} = M^{-1}p$$

$$\frac{dp}{dt} = -\nabla U(x)$$

1-dimensional Gaussian Example



Why Hamiltonian Dynamics

- Reversibility
- If Hamiltonian is conserved, we have acceptance probability of 1
- Volume preservation

Störmer-Verlet Approximation (Leapfrog)

- A method of approximating Hamiltonian Dynamics
- Preserves the desirable properties of Hamiltonian Dynamics
- Approximately conserves the Hamiltonian

Störmer-Verlet Approximation (Leapfrog)

- T, Integration Time
- ε, stepsize
- L, Leapfrog steps

$$T = L\varepsilon \tag{1}$$

Report Writing and Research

- Gained experience in report writing throughout my University Career
- Logical structure and clear presentation
- Many of these reports included a Literature review which required research

Communication and Group Work

- I am able to convey statistical/mathematical ideas to non-academics
- I can work effectively as part of a team

Computing Experience

- My experience is mostly in Object-orientated computer languages
- Model fitting and writing Simulation Algorithms
- Experienced in using LaTeX for reports and presentations

Questions

Thank you for listening.

Any Questions?