Homework 1 – Due Tuesday, May 28th, 2024 <u>09:00 pm</u>

- Provide step-by-step explanations, not just answers. Answers without explanations will earn a small fraction of the points.

- Submit your solutions on Gradescope. Don't forget to include information about your collaborators (or say "Collaborators: none").

Problems

0. (**0 points**) The following steps are required to get you started in the course. Please complete them **today**.

   (a) Make sure you are signed up on Piazza at `https://piazza.com/bu/summer2024/cascs330s` using your BU email address.

   (b) Sign up on Gradescope using your BU email address and the code **PWDGPG**.

   (c) Read and sign the Collaboration and Honesty Policy and submit it on Gradescope. We will be able to grade your homework only after you complete this step.

   (d) (**Nameplate**) Please print out (or make by hand) a nameplate with your name and bring it to every lecture and lab. A template is available at the bottom of the course web page.

   (e) Check out the course webpage: `https://cs-people.bu.edu/januario/teaching/cs330/su24/index.html`;

   (f) Familiarize yourself with the homework template files at the bottom of the course webpage. Each problem must include a note about collaborators (even if you did the problem by yourself).

1. (**Tracing Algorithms, 10 points**)

   Consider the following algorithm:

```
// A is an array of integers, indexed from 0 to n-1.
mystery( int A[] ){
    int i = 1
    while(i < A.length){
        int j = i
        while(j > 0 && A[j-1] > A[j]) {
            swap(A[j],A[j - 1])
            j = j - 1
        }
        i = i + 1
    }
}
```

(a) Trace the run of the algorithm on the following inputs. You can draw a table showing the progress of the nested loop (the progress of the counters i, j) and the contents of the array A as the algorithm progresses. If the contents of A do not change, you can leave the corresponding cell blank.

   - $A = [1, 2, 3, 4, 5]$
   - $A = [4, 1, 5, 3, 2]$
   - $A = [5, 4, 3, 2, 1]$

   Solution:

   $A = [1, 2, 3, 4, 5]$:

i, j	4	3	2	1
1				
2				
3				
4				

   $A = [4, 1, 5, 3, 2]$:

i, j	4	3	2	1
1				
2				
3				
4				

   $A = [5, 4, 3, 2, 1]$:

i, j	4	3	2	1
1				
2				
3				
4				

(b) Briefly describe what the algorithm does in general. One word can be enough!

(c) For an input array A of length n, how many steps does the algorithm need to finish (in the worst case)? For simplicity, count the number of times the variable j is updated.

2. (**Proof Techniques: Contradiction and Contraposition, 15 points**) In this problem, let n be a positive integer. You will prove the following claim using two different proof techniques.

Claim 1. *If $3n + 2$ is odd, then n is odd.*

(a) Prove the above claim by **contradiction**.

(b) Rewrite the contrapositive equivalent of the claim.

(c) Prove the claim you wrote in part (b). Please do not use a proof by contradiction in this question item because we want you to train on both techniques.

3. (**Proofs by Induction, 15 points**) A *binary tree* is a rooted tree in which each node has at most two children. Let T be a binary tree with n nodes and let L denote the set of leaves and I the set of nodes with exactly two children. Prove by induction that it's always true that $|I| = |L| - 1$.

   Hint: Start by writing down the statement you want to prove formally. Start by proving the base case. Then, prove the inductive case by first identifying the inductive hypothesis.

   Proof. **Base case:**

   Inductive Step:

   Inductive Hypothesis:

   Conclusion:

   $\square$

   Hint 1: In the inductive step, consider deleting a leaf node.

   Hint 2: What happens if it has a sibling?
   When you delete the node, what happens if the deleted node is the only child?

4. (**Programming Assignment, 10 points**) Login to Vjudge and solve the programming assignment. You can choose either Pypy 3 or Python 3 as your language. No other programming language will be accepted.