one, i.e., $q(\phi) \sim 1$, and thus $\phi \leq h/\delta p_F \ll 1$ (δ is the characteristic dimension of the microscopic surface roughnesses, and h/p_F is the de Broglie wavelength of the electron). According to (7), this means $\xi \lesssim (h/\delta p_F) \ll 1$. At such extremely small ξ, the here-employed concepts of path time and mean free path, and for the same reason also $q(\phi)$, have an exact meaning [9] (see also [8]). Further, since the region $\phi \ll 1$ is of importance in (3), the integration in (3) can be formally extended to infinity. Relation (3) then takes the form of the Wiener-Hopf equation for the function $\phi = \phi(\Phi)$, so that this function can be determined from the known function $\phi(\xi)$. Such a more exact analysis, however, is hardly meaningful under our assumption that the dispersion is isotropic.

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AMPLIFICATION OF ELECTROMAGNETIC FIELD IN TOTAL INTERNAL REFLECTION FROM A REGION OF INVERTED POPULATION

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The solution of the problem of passage and reflection of electromagnetic waves at an interface of two media, each having its own dielectric constant $\varepsilon_m = \varepsilon'_m + i\varepsilon''_m$ ($m = 1, 2$), is determined by Maxwell's equation together with the conditions of continuity on the interface and the radiation condition at infinity.

It is convenient to represent the general solution of the problem in the form of a superposition of plane waves, the reflection and transmission coefficients of which are given by particularly simple expressions in the case of a plane boundary. But the imaginary part of the dielectric constant enters quadratically in these expressions, and it may therefore turn out that the coefficient of reflection from the interface is independent of whether the reflecting medium is absorbing ($\varepsilon' > 0$) or inverted ($\varepsilon' < 0$). This is indeed the case if the wave incidence angle is smaller than the angle of total internal reflection. In the case of "total internal reflection" from an inverted-population region, however, the reflection coefficient turns out to be larger than unith! This is the consequence of selecting the type of the solution of Maxwell's equation as a function of the sign of ε''.

Let us assume for simplicity that the plane wave is incident from a transparent medium ($\varepsilon'_1 = \varepsilon_0$, $\varepsilon''_1 = 0$) on a plane boundary of an inverted medium

($\varepsilon_2 = \varepsilon' + i\varepsilon''$, $\varepsilon'' < 0$). Then the solution of Maxwell's equation $\nabla^2\vec{E} + k^2(\varepsilon'_m + i\varepsilon''_m)\vec{E} = 0$, $k = \omega/c$, in conjunction with the conditions for continuity of the electric and magnetic fields on the boundary, yields for the case when the electric vector of the incident wave is perpendicular to the plane of incidence, the following expression for the reflection coefficient (θ is the incidence angle):

$$|R|^2 = 1 - \frac{4\nu'\cos\theta}{(\nu' + \cos\theta)^2 + (\nu'')^2}. \tag{1}$$

Here $k\nu'\sqrt{\varepsilon_0}$ and $k\nu''\sqrt{\varepsilon_0}$ are the projections of the complex wave vector on the axis, which is directed from the first medium into the second and is perpendicular to the interface. By virtue of the boundary conditions, ν' and ν'' are connected by the relation

$$\nu' + i\nu'' = \sqrt{(\varepsilon'/\varepsilon_0) - \sin^2\theta + [i(\varepsilon''/\varepsilon_0)]}. \tag{2}$$

It follows from (1) that if $\nu' < 0$, then the modulus of the reflection coefficient is larger than unity. The necessary condition for this is total internal reflection in the interface between the media; the latter is possible if $(\varepsilon'/\varepsilon_0) < 1$. Indeed, in the case of total internal reflection ($(\varepsilon'/\varepsilon_0) - \sin^2\theta < 0$) the vector $\varepsilon = (\varepsilon'/\varepsilon_0) - \sin^2\theta + i(\varepsilon''/\varepsilon_0)$ lies in the third quadrant of the complex plane, and the root of ε lies in the second. Consequently $\nu' < 0$, $\nu'' > 0$, and the reflection coefficient is larger than unity. The conjugate root corresponds to a solution that increases without limit at infinity, and must be discarded. To demonstrate this, it suffices to solve Maxwell's equation for the case when the inverted medium borders not only on the first transparent medium (plane $z = 0$) but also on an absorbing medium (plane $z = d$), and to let d tend to infinity. Then, out of the two possible solutions, we are left with the one that satisfies exactly the indicated properties of the root and is continuous in ε'', so that the solution for the inverted medium is obtained in continuous fashion from the solution for the absorbing medium as ε'' changes from positive to negative values.

It follows from the presented method of choosing the type of solution that the reflection coefficient of an electromagnetic wave incident from a medium of lower optical density into an optically denser medium ($(\varepsilon'/\varepsilon_0) > 1$), is a smooth continuous function of the incidence angle, with a modulus smaller than unity, both when $\varepsilon'' > 0$ and $\varepsilon'' < 0$.

If the wave is incident from an optically denser to an optically less dense absorbing medium ($\varepsilon'' > 0$), then the reflection coefficient does not exceed unity and remains a continuous function in the entire range of variation of the angle of incidence. When the wave is incident on an optically less dense non-absorbing medium, the reflection coefficient likewise does not exceed unity, but has a kink (a discontinuous derivative) at a total-internal-reflection angle θ_0 determined by the condition $\sin^2\theta_0 = \varepsilon'/\varepsilon_0$. On the other hand, in the case of an inverted medium, it is the reflection coefficient that is discontinuous at the angle of total internal reflection, with $|R| < 0$ at $\theta = \theta_0$ and $|R| < 1$ at $\theta = \theta_0 + 0$.

The gain of the incident wave in the case of total internal reflection from an inverted medium has a simple physical explanation: since the z-component of the Poynting vector, averaged over the coordinate, is given by

$$\langle S_z \rangle \sim \frac{2\nu'\cos^2\theta}{(\nu' + \cos\theta)^2 + (\nu'')^2}, \tag{3}$$

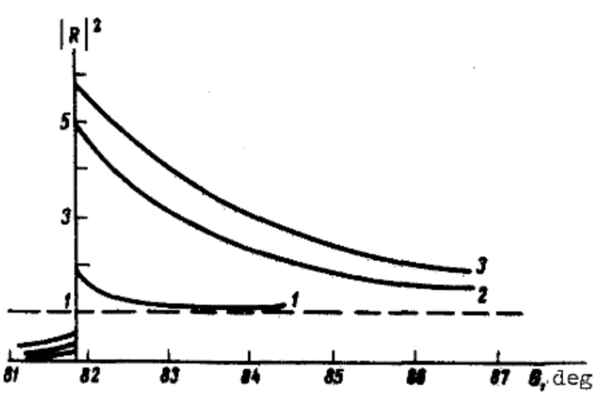

Fig. 1

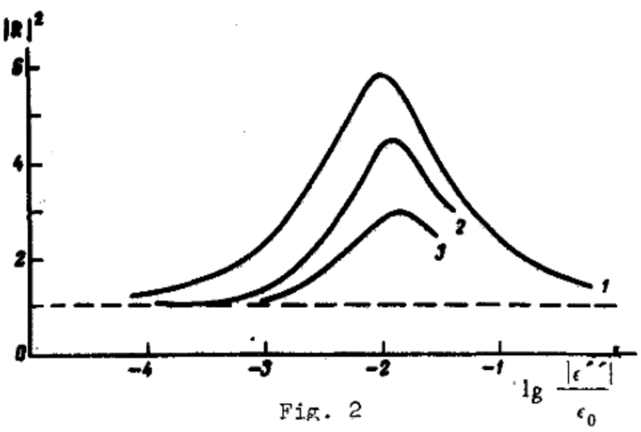

Fig. 2

Fig. 1. Square of the modulus of the reflection coefficient ($|R|^2$) vs. the incidence angle θ at a fixed value $\varepsilon'/\varepsilon_0 = 0.98$ and at various values of the ratio $\varepsilon''/\varepsilon_0$: 1) 10^{-3}, 2) 10^{-2}, 3) 2×10^{-2}.

Fig. 2. Square of the modulus of the reflection coefficient ($|R|^2$) vs. $\log(|\varepsilon''|/\varepsilon_0)$ at a fixed $\varepsilon'/\varepsilon_0 = 0.99$ and different incidence angles: 1) incidence at the angle of total internal reflection $\theta_0 \simeq 84°17'$, 2) $\theta = 84°50'$, 3) $\theta = 85°57'$.

it follows that the condition $\nu' < 0$ (at which $|R| > 1$) means flow of energy from the second inverted medium into the first. A comparison of the expressions for $|R|^2$ and $\langle S_z \rangle$ allows us to conclude that the incident (one might say priming) flux, corresponding to unity in the expression for $|R|^2$, gives rise to stimulated transitions in the inverted medium, amplifying by the same token the reflected wave. The maximum value of the reflection coefficient for arbitrary $\varepsilon''/\varepsilon_0$ and $\varepsilon'/\varepsilon_0$ is attained near the total-internal-reflection angle $\theta = \theta_0 + 0$ (Fig. 1). But an unbounded increase of the inverted population does not mean an unbounded increase of the reflection coefficient, for when the so-called effective refractive index $n = (\varepsilon' + |\varepsilon''|)/\varepsilon_0$ is equal to unity, $|R|^2$ reaches an absolute maximum value approximately equal to e^2 near the angle $\theta = \theta_0 + 0$ (Fig. 2). This is explained by the fact that in the case of total internal reflection the depth d of penetration of the field into the inverted medium decreases with increasing inverted population. For example, $d \lesssim 30\lambda$ at $|\varepsilon''|/\varepsilon_0 = 10^{-4}$ and $d \lesssim 3\lambda$ at $|\varepsilon''|/\varepsilon_0 = 10^{-2}$. Similarly, in the geometrical-optics approximation, a large linear gain K of the inverted medium corresponds to a smaller path ℓ transversed in it by the incident beam, and the upper bound of the product $K\ell$ has the same value for all $\varepsilon'/\varepsilon_0$ and $\varepsilon''/\varepsilon_0$, so that $|R|^2 \lesssim e^{K\ell} \lesssim e^2$. These results are valid for both polarizations of the incident wave.

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POSSIBILITY OF GENERATING ULTRASHORT LASER PULSES ON COMBINATION VIBRATIONAL-ROTATIONAL TRANSITIONS OF MOLECULAR HYDROGEN

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1. We consider here the possibility of obtaining short laser-radiation pulses on vibrational-rotational transitions of the molecules of hydrogen and