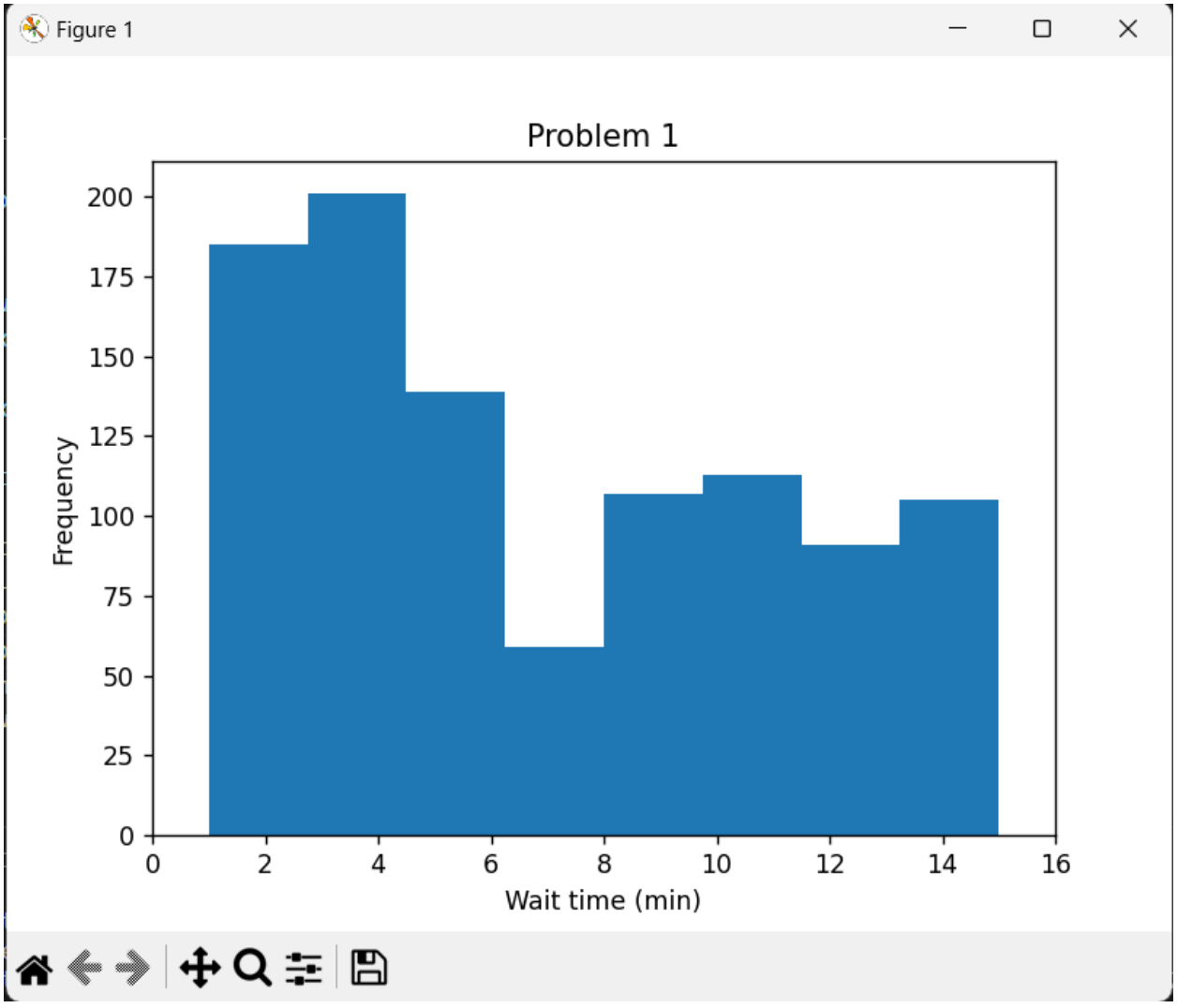
I Joshua Patton, declare that I have completed this assignment in accordance with the UAB Academic Integrity Code and the UAB CS Honor Code. I have read the UAB Academic Integrity Code and understand that any breach of the Code may result in severe penalties.

Student signature/initials: JDP

Date: 4/12/23

1.



#1 $7:10 \le x \le 7:30$ $7:30 - 7:10 = 20$ min.

$$f_x(x) = \begin{cases} 1/20 & 7:10 \le x \le 7:30 \\ 0 & \text{otherwise} \end{cases}$$

$A = (7:10 \le x \le 7:15)$ $B = (7:10 \le x \le 7:30)$

$$f_{Y|A}(y) = \begin{cases} 1/5 & 0 \le y \le 5 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{Y|B}(y) = \begin{cases} 1/15 & 0 \le y \le 15 \\ 0 & \text{otherwise} \end{cases}$$

$P(A) = 1/4$ $P(B) = 3/4$

$f_Y(y) = 1/4 \cdot 1/5 + 3/4 \cdot 1/15 = 1/20$ $5 \le y \le 15$

2.

#2

$$f_X(x) = \begin{cases} x/4 & 1 < x \leq 3 \\ 0 & \text{otherwise} \end{cases} \qquad A = (X \geq 2)$$

a) $E[x] = \int_1^3 x(x/4)\,dx = \frac{x^3}{12}\Big]_1^3 = \frac{27}{12} - \frac{1}{12} = \frac{26}{12}$

$P(A) = \int_2^3 x/4\,dx = \frac{x^2}{8}\Big]_2^3 = \frac{9}{8} - \frac{4}{8} = \frac{5}{8}$

$f_{X|A}(x) = \frac{f(x, A)}{f(A)}$

a) $E[X|A] = \int_x x f_{X|A}(x)\,dx$

B) $Y = X^2$

Answer

$E[Y] = E[X^2] = \int_1^3 x^2(\frac{x}{4})\,dx = \frac{x^4}{16}\Big]_1^3 = \frac{81}{16} - \frac{1}{16} = \frac{80}{16}$

$V(Y) = E[Y^2] - E[Y]^2 = E[x^4] - (E[x^2])^2$

$= \int_1^3 x^4(\frac{x}{4})\,dx = \frac{x^6}{24}\Big]_1^3 = \frac{729}{24} - \frac{1}{24}$ $\frac{728}{24} = E[Y^2]$

$E[Y]^2 = (\frac{80}{16})^2 = 25$

$V(Y) = 25 - \frac{728}{24} = \frac{-16}{3}$ or -5.333 Answer

3.

#3 a)

$$f(x) = \begin{cases} cx^{-2}, & \text{if } 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

$$\int_1^2 cx^{-2}\,dx = c\int_1^2 \frac{x^{-2+1}}{-1} = c\left[-\frac{1}{x}\right]_1^2$$

$$= \left(\left(-\tfrac{1}{2}\right) - \left(-\tfrac{1}{1}\right)\right)c = \tfrac{1}{2}c \qquad \tfrac{2}{1}\cdot\tfrac{1}{2}c = \tfrac{2}{1} \qquad \boxed{c=2}$$

B) $P(A) = (P > 1.5)$

$$= \int_{1.5}^2 2\cdot x^{-2}\,dx = 2\left[-\tfrac{1}{x}\right]_{1.5}^2 = 2\left[\left(-\tfrac{1}{2}\right) - \left(-\tfrac{1}{1.5}\right)\right] = \boxed{\tfrac{1}{3}}$$

$$f_{X|P(A)} = \begin{cases} \frac{2\cdot x^{-2}}{1/3} & 1 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

C) $Y = X^2$ find $E[Y]$ and $Var(Y)$

$$E[Y] = E[X^2] = \int_1^2 x^2(2x^{-2})\,dx = 2x\Big]_1^2 = 4 - 2 = \boxed{2}$$

$$Var(Y) = E[Y^2] - E[Y]^2 = \int_1^2 x^4(2x^{-2})\,dx - 2^2$$

$$= 2x^3\Big]_1^2 = 16 - 2 = 14$$

$$Var(Y) = 14 - 4 = \boxed{10}$$