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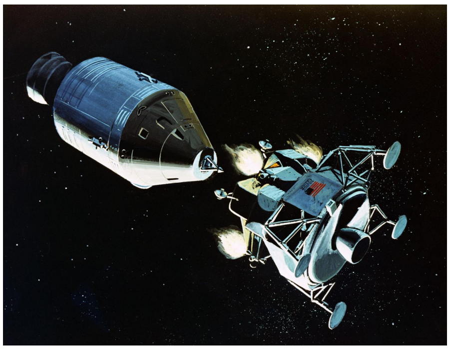

Guidance Navigation and Control System for a Rendezvous and Docking Mission

Design Report

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Guidance Navigation and Control

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Contents

Chapter 1

Introduction

One of the most complex and demanding maneuvers in an Apollo mission was the rendezvous and docking of the Lunar Module (LM) ascent stage with the Command and Service Module (CSM). The mission design principles were largely shaped by experience from the Gemini program, which not only refined rendezvous techniques but also influenced the design and capabilities of the Lunar Module. Two primary methods emerged from this experience: the coelliptic and direct rendezvous approaches. Both relied on the same fundamental principles, assuming that a single "active" vehicle would perform all the necessary maneuvers. This vehicle would be placed in a lower orbit to catch up with its target, executing precise engine burns to adjust its orbital plane (if needed) and then raising its orbit to intercept the passive vehicle.

During the development of the Gemini rendezvous strategy, it became clear that standardizing the terminal phase was crucial for mission success. Defining specific vehicle positions, closure rates, and lighting conditions helped simplify the most critical phases of rendezvous and docking, significantly reducing both crew training requirements and in-flight workload. Mathematical models, simulations, and Gemini flight data established that the optimal altitude difference between the two spacecraft should be 15 miles, with the active vehicle positioned below the passive one. A transfer angle of 130 degrees was chosen to ensure that the active vehicle would traverse the required distance while ascending to the target orbit. Additionally, lighting conditions were carefully planned so that the Sun would remain behind the active vehicle during the braking phase. With these parameters in place, mission planners worked backward to design ascent trajectories and orbital profiles that would precisely set up the final approach phase.

For Apollo 11, the coelliptic rendezvous method was selected. To simplify the simulation and maintain a two-dimensional approach, the rendezvous maneuvers in this project assume that the LM starts in an 83 km circular orbit, while the passive CSM is positioned in a 110 km coplanar circular orbit. This setup ensures a planar rendezvous, where the LM performs ascent burns and orbital adjustments to match the CSM's position and velocity. By focusing on a 2D model, the analysis remains centered on radial and along-track dynamics in lunar orbit, avoiding the complexities of simulating plane changes [1].

1.1 Document Outline

The following chapters will outline the project's objectives, starting with the minimum required goal and progressing toward the ideal outcome. This will involve transitioning from a simplified, constrained version of the previously introduced problem to a more general case that better reflects real-world conditions. Next, an overview will be provided on the approach taken to achieve these objectives, beginning with the fundamental equations that define the problem and the methodology used to solve them. Finally, a detailed examination of the vehicles involved in the mission will be presented, with particular emphasis on the active element—the Lunar Module—as well as its sensors and actuators, which play a crucial role in the mission's success.

Chapter 2

Goal

2.1 General Idea

Within this project the general goal is to design, implement and simulate a GNC system for a rendezvous and docking mission of a chasing spacecraft to a target spacecraft. In the following sections, these spacecrafts are referred to as chaser and target. To be able to accomplish the desired goal, which is described in section 2.5, the problem will be approached from a very simple scenario which will then be extended incrementally towards a 2D orbit docking scenario. In the following, the minimum, the intermediate and the desired goal are presented in more detail after the constraints are listed.

2.2 Constraints

The constraints for this project are that the docking will be simulated in a simplified 2D environment, reducing complexity while still capturing the essential dynamics of the docking process. The spacecraft are modeled as point masses with inertia, which means that their rotational and translational motions are considered in a realistic manner. Docking is considered successful within a certain range of speed, position, and angle. Initially, no disturbance forces or torques are introduced in the simulation, allowing for a baseline analysis of the control algorithm. Once the minimum and intermediate goals are achieved, disturbances such as environmental forces or system noise may be introduced. Similarly, for the desired final goal, orbit disturbances are initially neglected but could be implemented in future iterations to enhance the realism of the model.

The simulation will be implemented in MATLAB Simulink, where both the GNC system and the physical system will be modeled. Additionally, actuators and sensors will be simulated to capture the full behavior of the docking process, enabling a comprehensive analysis of the system's performance.

2.3 Minimum Goal

The starting scenario is depicted in Fig.2.1 consists of a the target located at a fixed position in space $p_{target} = (0, 0)$ and of the chaser initially located at a certain distance d with respect to the target $p_{chaser} = (0, -d)$ and

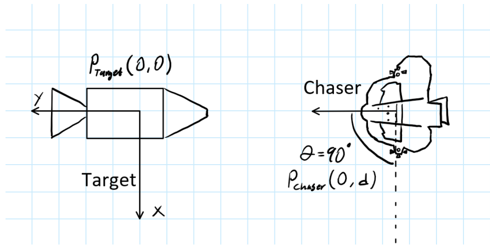

Figure 2.1: Minimum goal

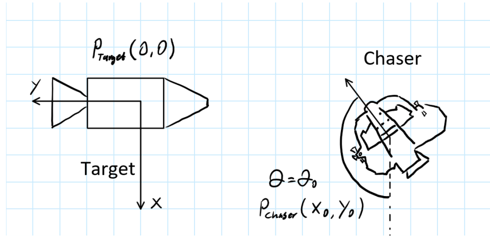

Figure 2.2: Intermediate goal

with no initial relative velocity or angular velocity. No forces are acting on the two spacecrafts except of the ones generated by the actuators of the chaser. The initial attitude of the chaser is chosen so that the docking port of the chaser and of the target are aligned. Using the actuators (specified in Chapter 4), the chaser's velocity will be controlled so that a successful docking will take place. The Apollo 11 used a probe and drogue docking system while more modern spacecrafts use the soft dock hard dock system. These systems allow minor alignment errors, which will be included in this simulation as tolerances in the angle and position when docking. During this phase, when the distance between the chaser and the target drops below 10m, the controller limits the relative approach speed of the chaser to 0.03 m/s to ensure a smooth and controlled docking process. [2]. Successful docking is considered when the position of the two docking ports is less than 10cm in both directions. The absolute attitude has to be controlled to $\leq 1\,\mathrm{deg}$, mostly because of the necessary alignment of thrusters for trajectory control [3]. To implement and validate a GNC system for this scenario is considered as the minimum goal of this project.

2.4 Intermediate Goal

The first extension consists of a random initialization of the chaser's attitude and position $p_{chaser} = (x_{c,0}, y_{c,0})$ as well as an initial angle θ_0 (see Fig. 2.2). For this scenario the attitude control must be taken into account and will be implemented within the GNC system. Initially, no forces are acting on the two spacecrafts except of the ones generated by the actuators of the chaser. The same success criteria as defined in Section 2.3 are used to validate the GNC system. One idea of approaching this problem would be to first set an intermediate target position and attitude of $p_1 = (0, -d_1)$ and $\theta_1 = 90\,\mathrm{deg}$. After successful moving the chaser to that position and attitude, the controller developed in the first task can be used. If the intermediate goal is achieved, initial relative velocity and angular velocity will be introduced as an extension to the initial conditions.

2.5 Desired Goal

The desired goal of this project is to implement a GNC system for a 2D orbit docking. In this scenario, the target is in a circular lunar orbit with a certain radius r_{target}. The chaser spacecraft is positioned in the same orbital plane, but with a different true anomaly and an initially random attitude. The objective of the GNC system is to rendezvous and dock with the target using a v-bar approach, meaning the chaser approaches the target from behind along the velocity vector direction. To achieve this, the actuators are used to control both position and attitude throughout the manoeuver.

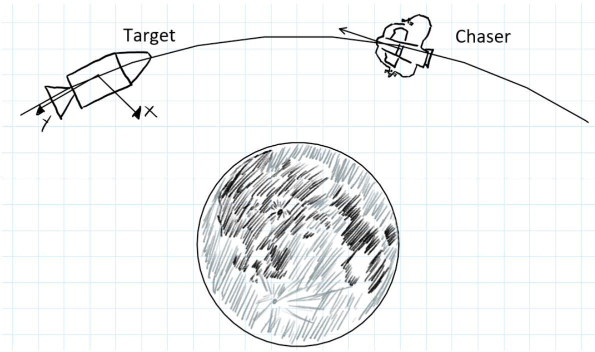

Figure 2.3: Desired goal

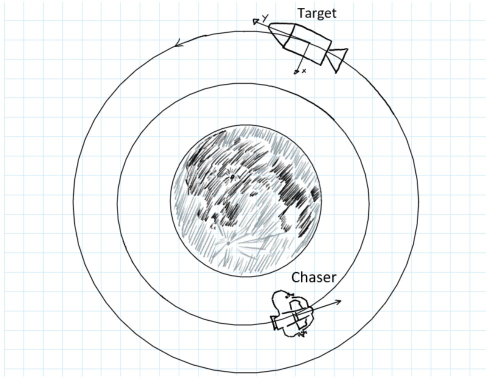

Figure 2.4: Optional extension

2.6 Optional Extension

If the desired goal can be achieved early and additional time is available, the scenario mentioned in Section 2.5 can be extended by starting the mission of the chaser in a lower circular lunar orbit with radius r_{chaser} (see Fig. 2.4). The orbital plane is still the same as the one of the target. Then, first a Hohmann transfer must be performed to get the chaser into the same orbit as the target. Afterwards, a phasing manoeuvrer has to be performed to reduce the difference of the true anomaly of the chaser and the target. When these manoeuvres are performed successfully by the GNC system, the remaining docking can be performed by the system developed in section 2.5.

Chapter 3

Methodology

3.1 General Approach

To build and validate the proposed approaches to the problem, a closed-loop simulation will be developed. A block diagram of that simulation is shown in Fig.3.1.

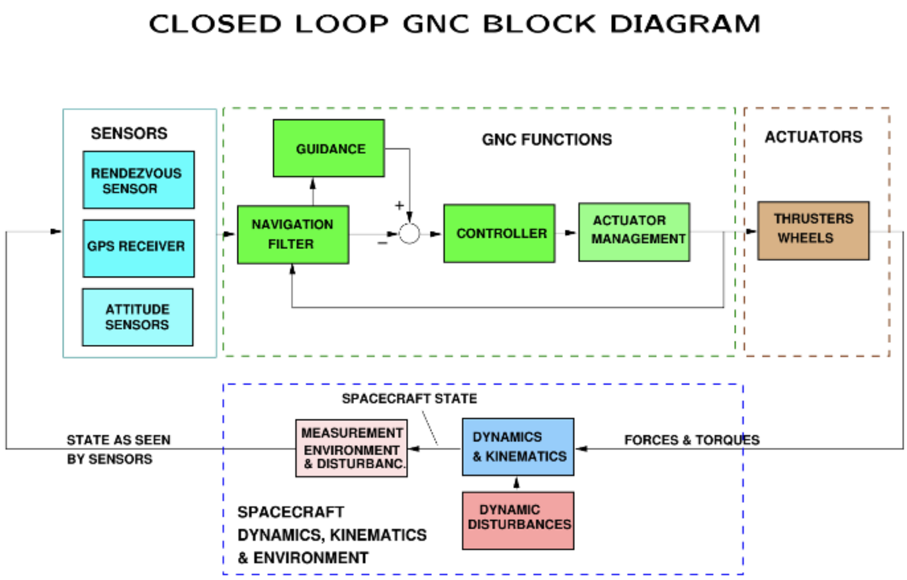

Figure 3.1: Block-diagram for the GNC system simulation [3].

3.2 Non-Orbit Scenario

3.2.1 Translational Dynamics (Position and Velocity)

For the non-orbit scenario (referring to Section 2.3 and 2.4), an inertial 2D coordinate frame is defined and in which the target is fixed at the origin and the position of the chaser is given by $p_{chaser}(t) = \begin{bmatrix} x_c(t) \ y_c(t) \end{bmatrix}^T$.

The motion of the chaser relative to the target can be described by Newton's law:

$$F_x = m_c \cdot \ddot{x}_c \tag{3.1}$$

$$F_y = m_c \cdot \ddot{y}_c \tag{3.2}$$

where:

- $\ddot{x}, \ddot{y}$: Relative velocity of the chaser.
- F_x, F_y: Control forces from actuators.
- m_c: Mass of the chaser spacecraft.

A solution of these differential equations can be written in closed-form if the control forces are assumed to be constant within the considered time interval:

$$x_c(t) = x_{c,0} + v_{c,x_0} t + \frac{1}{2} a_x t^2, \tag{3.3}$$

$$y_c(t) = y_{c,0} + v_{c,y_0} t + \frac{1}{2} a_y t^2, \tag{3.4}$$

where:

- $x_{c,0}$, $y_{c,0}$: Initial position of chaser.
- v_{c,x_0}, v_{c,y_0}: Initial velocity of chaser.
- $a_x = \frac{F_x}{m_c}$, $a_y = \frac{F_y}{m_c}$: Acceleration generated by control force.

3.2.2 Rotational Dynamics (Attitude Motion)

The attitude dynamics of the chaser are described by Euler's equation in 2D:

$$I_c \ddot{\theta} = \tau, \tag{3.5}$$

where:

- I_c: Moment of inertia of the chaser about the z-axis.
- θ: Attitude angle relative to the LVLH ((Local Vertical, Local Horizontal) frame.
- $\dot{\theta}$: Angular velocity.
- τ: Control torque applied to rotate the chaser.

A solution of this differential equations can be written in closed-form if the control torque is assumed to be constant within the considered time interval:

$$\theta(t) = \theta_{c,0} + \omega_{c,0} t + \frac{1}{2} \alpha t^2 \tag{3.6}$$

where:

- $\theta_{c,0}$: Initial orientation of chaser.
- $\omega_{c,0}$: Initial angular velocity of chaser.
- $\alpha = \frac{\tau}{I_c}$: Angular acceleration generated by control torque.

3.2.3 Control Inputs (Forces and Torques)

Control inputs are applied to maneuver the chaser:

- **Translational Control:** Adjusting thrusters to modify F_x and F_y.
- **Rotational Control:** Using reaction wheels or thrusters to control τ.

3.2.4 Full State Space Model

A combined state vector is defined that includes both translational and rotational dynamics:

$$\mathbf{x} = \begin{bmatrix} x \ y \ \dot{x} \ \dot{y} \ \theta \ \dot{\theta} \end{bmatrix}^T, \quad \dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \ \dot{y} \ \ddot{x} \ \ddot{y} \ \dot{\theta} \ \ddot{\theta} \end{bmatrix}^T \tag{3.7}$$

With the following expressions for accelerations:

$$\ddot{x} = \frac{F_x}{m_c} \tag{3.8}$$

$$\ddot{y} = \frac{F_y}{m_c} \tag{3.9}$$

$$\ddot{\theta} = \frac{\tau}{I_c}. \tag{3.10}$$

The state evolution equations can be written in matrix form as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{3.11}$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_c} & 0 & 0 \\ 0 & \frac{1}{m_c} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_c} \end{bmatrix} \tag{3.12}$$

and:

$$\mathbf{u}(t) = \begin{bmatrix} F_x(t) \\ F_y(t) \\ \tau(t) \end{bmatrix} \tag{3.13}$$

3.3 In-Orbit Scenario

3.3.1 Translational Dynamics (Position and Velocity)

For modelling the scenario in orbit, the 2D LVLH frame is considered which is centred on the target spacecraft and rotating with its orbit:

- x axis: Radial (towards the Moon).
- y axis: Along-track (in the direction of velocity vector).

The motion of the chaser relative to the target in a circular orbit is governed by the 2D Hill's Equation equations [4]:

$$\ddot{x} - 3n^2 x + 2n\dot{y} = \frac{F_x}{m_c}, \tag{3.14}$$

$$\ddot{y} - 2n\dot{x} = \frac{F_y}{m_c}, \tag{3.15}$$

where:

- x, y: Relative position of the chaser in the LVLH frame.
- $\dot{x}, \dot{y}$: Relative velocity.
- F_x, F_y: Control forces from actuators.
- m_c: Mass of the chaser spacecraft.
- $n = \sqrt{\mu/a^3}$: Mean motion of the target's orbit.

Assuming pulses (instantaneous changes of velocity) and for constant forces a closed-form solution can be derived by the Clohessy-Wiltshire equations [4]. However, as they are only approximations, a numerical integration of the Hill's equation could be required in order to obtain exact results.

3.3.2 Rotational Dynamics (Attitude Motion)

The attitude dynamics of the chaser are described by Euler's equation in 2D:

$$I_c\ddot{\theta} = \tau, \tag{3.16}$$

where:

- I_c: Moment of inertia of the chaser about the z-axis.
- θ: Attitude angle relative to the LVLH frame.
- $\dot{\theta}$: Angular velocity.
- τ: Control torque applied to rotate the chaser.

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A combined state vector is defined that includes both translational and rotational dynamics:

$$\mathbf{x} = \begin{bmatrix} x \ y \ \dot{x} \ \dot{y} \ \theta \ \dot{\theta} \end{bmatrix}^T, \quad \dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \ \dot{y} \ \ddot{x} \ \ddot{y} \ \dot{\theta} \ \ddot{\theta} \end{bmatrix}^T \tag{3.17}$$

With the following expressions for accelerations:

$$\ddot{x} = 3n^2 x - 2n\dot{y} + \frac{F_x}{m_c}, \tag{3.18}$$

$$\ddot{y} = -2n\dot{x} + \frac{F_y}{m_c}, \tag{3.19}$$

$$\ddot{\theta} = \frac{\tau}{I_c}. \tag{3.20}$$

The state evolution equations can be written in matrix form as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{3.21}$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -2n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_c} & 0 & 0 \\ 0 & \frac{1}{m_c} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_c} \end{bmatrix} \tag{3.22}$$

and:

$$\mathbf{u}(t) = \begin{bmatrix} F_x(t) \\ F_y(t) \\ \tau(t) \end{bmatrix} \tag{3.23}$$

Chapter 4

Vehicle Description

4.1 Vehicles to be considered

Since the intent of this project is to simulate a rendezvous and docking situation, it is necessary to model both the chaser and the target chosen.

4.1.1 Lunar Module (Ascent Stage)

The Lunar Module (also known as *Eagle*) will be the chaser considered for this project. In particular, its ascent stage will be the only considered (the one that left the moon with the two astronauts).

From here emerges the necessity of modulating this stage in a 2D model. The following figures illustrates the real model and the 2D design chosen:

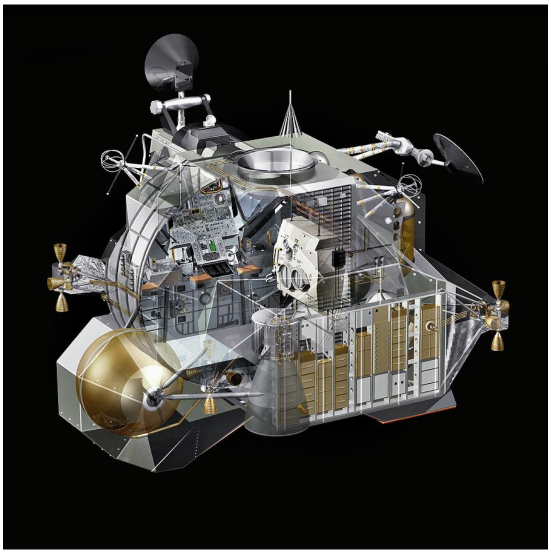

Figure 4.1: Lunar Module Ascent Stage Realistic Model [5]

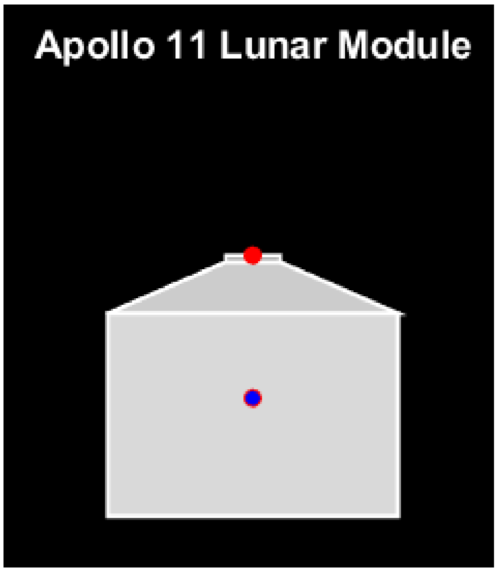

Figure 4.2: Lunar Module Ascent Stage 2D Model Design

Considering figure 4.2, the docking point of contact is represented with a red dot, while the centroid coincides with the blue dot.

To elaborate the 2D design, [6] was used as a reference. With this, in the 2D model, the larger rectangle is considered to have $4.29m$ width and $3.00m$ height. The trapezoid is isosceles with an height of $0.76m$, a small base of $0.81m$ and a big base of $4.29m$. Finally, the docking port is represented with a small rectangle with $0.81m$ width and $0.1m$ height.

The parameters of this complex shape are presented below:

```
LM Centroid Coordinates (x,y): (0.0000, 0.2460)
LM polar moments (Ix,Iy,Iz): (15.6815, 30.2416, 45.9231)
Docking Point Coordinates: (0.00, 2.36)
```

Figure 4.3: Lunar Module Ascent Stage 2D Model Geometric Parameters

The centroid and docking point coordinates in Fig.4.3 were obtained considering the origin of the axis coincident with the centroid of the larger rectangle. The x-axis is considered positive to the right and the y-axis upwards.

4.1.2 Command and Service Modules

The Command and Service Modules will be the chased considered for this project. Just like the *Eagle*, they were modulated in 2D:

Figure 4.4: Command and Service Module Realistic Model [7]

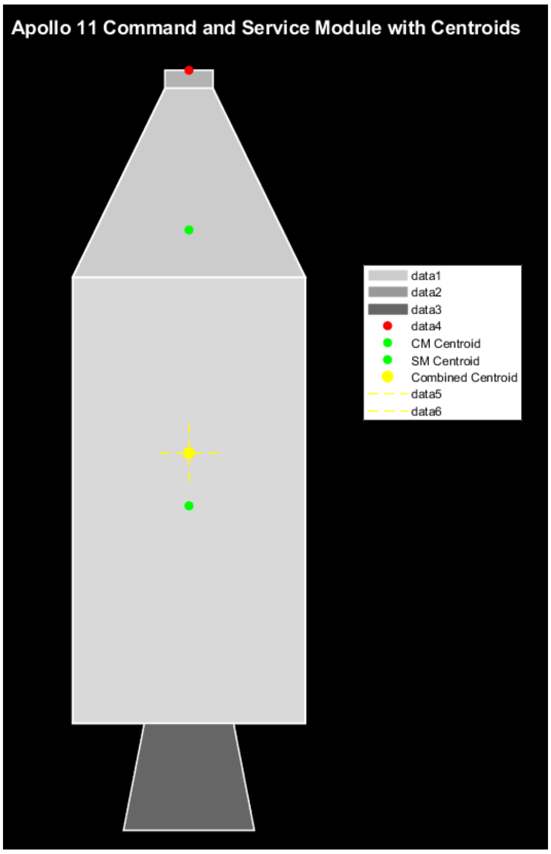

Figure 4.5: Command and Service Module 2D Model Design

The Command and the Service Module were designed as 2 separate figures joined together. The Command

Module is a cone shape with a height of $3.18m$, a base diameter of $3.91m$, and a top width of $0.8m$. A curve was added to the bottom to represent the heat shield, but it is not shown in the combination of the 2 spacecraft. The Service Module is a cylinder with a length of $7.5m$ and a diameter of $3.91m$ with a trapezoidal nozzle at the bottom with a top width of $1.5m$, a bottom width of $2.2m$ and a height of $1.8m$. The axis system is set so that the origin is in the center of the Service Module. The centroid and some relevant coordinates are then presented:

```
>> apollo11_command_module
Docking Point Coordinates: (0.00, 7.23)
Command Module Centroid: (0.00, 4.54) m
Service Module Centroid: (0.00, -0.09) m
Combined CM-SM Centroid: (0.00, 0.79) m
```

Figure 4.6: Command Module 2D Model Geometric Parameters

4.2 Sensors

The selection of sensors for the 2D simulation prioritizes essential functions for rendezvous and docking while ensuring practicality for the simulation.

Key parameters to measure in this type of mission include the relative position and velocity (range and closing speed) between the chaser and target vehicle, as well as the orientation and attitude of the active vehicle to ensure proper alignment before docking. To meet these requirements, the following sensors are included in the simulation:

- **Docking Radar** – Tracks the distance and closing speed between the Lunar Module (LM) and the Command/Service Module (CSM). It operates by sending radio signals from one spacecraft and receiving the reflected signals from the other, allowing a precise measurement of the range, the range rate, and, in a more advanced setup, the angle to the target.

- **Inertial Measurement Unit (IMU)** - Consisting of a accelerometer and a gyroscope, it measures acceleration and angular velocity and is mainly used for LM attitude determination.

- **Altimeter** - Measures the altitude above the surface of the plant around the spacecraft is orbiting. This can be helpful to determine the current orbit and also the relative position (relative true anomaly) for the in-orbit scenarios.

- **Star Tracker (optional)** - For high accurate absolute attitude estimation, in most space missions a star tracker is used. However, as this sensor is hard to model in a simulation, it is an optional extension to inlcude it for the desired goal.

Visual guidance and final approach sensors are not considered. Instead, docking is assumed to occur instantly once specific conditions are met: the spacecraft must be within a predefined distance threshold, moving at nearly the same speed, and aligned within an acceptable attitude tolerance. Based on typical rendezvous and docking requirements, attitude measurement accuracy should be within 0.1°, while relative attitude accuracy should be within 1–2° to ensure proper alignment with the docking axis [3]. Since real docking mechanisms engage automatically when these constraints are satisfied, this assumption serves as a practical and efficient simplification for the simulation.

4.3 Actuators

For generating control forces and torques on the chaser, two main actuators will be considered. The position will be controlled using a reaction control system (RCS). In general, the RCS enables the control of translation and attitude using small thrusters. However, for this mission the RCS is only used for generating forces enabling precise position control which is crucial for the docking mission.

In addition, as in the simulation only a 2D scenario is assumed, there is only 1 DOF (Degree of freedom) regarding the orientation. Thus, one reaction wheel is implemented as it will be sufficient to control the attitude of the spacecraft. As the moon doesn't have a magnetic field to interact with, mangnetorquers - another typical attitude control actuator - cannot be used.

Chapter 5

Literature

For developing the GNC system for the rendezvous and docking, different sources were found that could be helpful for the further development. They are shortly described in the following section.

In [2], a detailed *Matlab* simulation is presented that covers the rendezvous and docking of a chaser and a target spacecraft in low earth orbit. Celestini [8] presents different algorithms for the navigation and guidance for docking missions in Earth orbit. Furthermore, general methods and concepts regarding all three system components - namely guidance, navigation and control subsystem - are presented in [4].

Bibliography

[1] F. O'Brien. "Lunar Orbit Rendezvous". https://www.nasa.gov/history/afj/loressay.html. Accessed: 2025-03-06.

[2] Mathworks. "Space Rendezvous and Docking Simulation". https://de.mathworks.com/help/aeroblks/spacecraft-rendezvous-and-docking.html. Accessed: 2025-03-06.

[3] European Space Agency. "Rendezvous and Docking of Spacecraft". Technical report, European Space Agency. URL https://www.yumpu.com/en/document/read/51264483/rendezvous-and-docking-of-spacecraft-esa. Accessed: 2025-03-08.

[4] Y. Xie, C. Chen, T. Liu, and M. Wang. *"Guidance, navigation, and control for spacecraft rendezvous and docking: theory and methods"*. Springer Nature, 2021.

[5] PIXELS. "Apollo Lunar Module Ascent Stage". https://pixels.com/featured/apollo-lunar-module-ascent-stage-carlos-clarivanscience-photo-library.html. Accessed: 2025-03-10.

[6] NASA. "LUNAR MODULE QUICK REFERENCE DATA". https://www.nasa.gov/wp-content/uploads/static/history/alsj/lm04_lunar_module_pplv1-17.pdf. Accessed: 2025-03-05.

[7] S. P. Library. "Apollo Command Service Module 7". https://sciencephotogallery.com/featured/7-apollo-command-service-module-carlos-clarivanscience-photo-library.html. Accessed: 2025-03-10.

[8] D. Celestini. "Navigation and Guidance Algorithms for In-Orbit servicing Rendezvous Mission". Master's thesis, Politecnico di Torino, 2021.