Assignment: Linear Systems Control

16th March 2021

For this assignment, you will need the template code `assignment.m`. Download this from the module's KEATS page and save it to your computer. When you are ready to submit your assignment, you will need to upload this file so bear this in mind while completing the assignment (further instructions on how to submit your assignment are given below). Open the file using Matlab, and complete the following exercises.

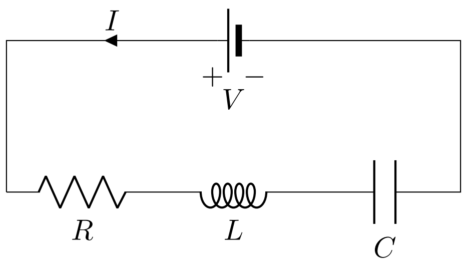

Figure 1: The schematic diagram of an R-L-C circuit.

A series resistor-inductor-capacitor circuit (see Figure 1) can be described as a linear system, in whichthe current across the components follows the equation

$$\frac{d^2}{dt^2}I(t) + \frac{R}{L}\frac{d}{dt}I(t) + \frac{1}{LC}I(t) = \frac{1}{L}\frac{dV}{dt} \tag{1}$$

where I is the current, R the resistance, L the inductance, C the capacitance and dV/dt the rate of change of the voltage at the power source.

1. Write (1) in state space formulation, as a continuous time, linear time invariant system. You may assume that the rate of change of the voltage is the control input (*i.e.*, $u = dV/dt$) and the system state is the current and its first time derivative[1] (*i.e.*, $\mathbf{x} = (I, dI/dt)^\top$). Using the template code implement the matrices $\mathbf{A}$ and $\mathbf{B}$ assuming that $L = 20\,H$, $C = 0.1\,F$, $R = 4\,\Omega$.

   [5 marks]

2. Derive the equations for the system in discrete time, such that you can compute $\mathbf{x}_{t+1}$ as a function of $\mathbf{x}_t$ and u_t. Using the template file, implement a simulation of the system, such that you can compute the current for $0 \le t \le 20\,s$ if the voltage increases at a constant rate of $1\,V/s$. Assume that the current is zero and constant at $t = 0\,s$ and use sampling rate $\delta t = 2\,ms$.

   [10 marks]

[1] Throughout the assignment, treat the state and its derivatives as a *column* vector.

3. Consider the case that the circuit is equipped with a multimeter that enables measurement of current at the same sampling rate. Using the template file, implement the observer matrix $\mathbf{C}$ and the observability matrix $\mathbf{H}$ using your answer to Question 2. Derive the transfer function for this system and solve for the poles to four decimal places. Implement a vector $\mathbf{z}$ in the template file that contains the poles.

[15 marks]

Completed assignments should be submitted to KEATS on 5pm, 5th April 2021.

To submit your assignment, please follow the following steps:

1. Complete the following lines of the source code by adding your name and student number:

```
% Please complete the following with your details
firstname ='';
surname   ='';
number    =''; % this should be your 'k' number, e.g., 'k1234567'
```

2. Upload the resultant source code as a single `.m` file on KEATS.

Important notes on submitting:

1. Please **ensure that your code runs cleanly prior to submission**. Marks will be deducted from submissions that do not run without errors or warnings.

2. Please **do not change the variable names in the template**.

This assignment is worth 15% of the module mark.