Analysis of 2012–2014 Brazil Tuberculosis Data

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1 Introduction

1.1 Problem Statement

Analysis of tuberculosis (TB) data originating from Brazil using Generalized Additive Models (GAMs). Brazil is divided into 557 administrative microregions, and the available data comprises counts of TB cases in each microregion for each of the years from 2012 to 2014.

2 Exploratory Analysis of Data and Problem

The TB data from Brazil includes 1,671 entries or samples with 14 columns of numeric data types that specify the characteristics of each sample. The columns are self-explanatory because they are called Indigenous, Illiteracy, Urbanisation, Density, Poverty, Poor Sanitation, Unemployment, Timeliness, Year, TB, Population, Region, lon, and lat. TB stands for tuberculosis, whereas lon and lat stand for longitude and latitude. The dataset has no missing values in the technical sense, but it contains some abnormalities, which increases the amount of pre-processing needed. The region is stored as a continuous variable despite being a factor variable. Nonetheless, changing it depends on the task at hand. Moreover, the collection includes coordinates that describe the precise geospatial locations of the micro-regions listed in the column. The next part gives a detailed exploration of the data.

2.1 Data Exploration

An in-depth analysis of the datasets reveals that the mean and median values for the indigenous population are low, but the maximum value is 50, which suggests that there are individual areas where the indigenous population is concentrated and that these areas may be areas of potential poverty and poor sanitation and should be areas where there are more cases of tuberculosis. The mean and median illiteracy rates are only 14 and 11, respectively, indicating that illiteracy is not widespread. However, the maximum value of 41 suggests that there are specific backward areas with significant populations lacking access to education, which also suggests that the area seems poor and has poor sanitation. There are still some places that are less urbanised, where there may be more occurrences of tuberculosis, but the mean, median, and minimum values for urbanisation are 70 and 22, respectively, suggesting that most areas are highly urbanised. Based on the population density data, most locations can fit one person in a room, but the highest value of 1.6 highlights the existence of some places with very high population densities, which sharply raise the rate of TB transmission. The distribution of the poverty data suggests that each district has different poverty levels, with only a limited number of districts where poverty is not a significant issue. Although the general results on inadequate sanitation are low, the maximum score of 58 indicates that some districts have poor sanitation and substantial disease risk. Although average unemployment rates are low, a maximum value of 20 indicates that some isolated regions may experience severe economic hardship, protracted social unrest, and potentially significant morbidity rates. With a minimum value of 0, notification timeliness data has a fairly wide range.

Data from the actual world follow a normal distribution. However, some of the columns indicate otherwise. Timeliness, Unemployment, and Urbanization are approximations of a normal distribution with few extremes, whilst the remainder is multimodal normal distributions. The above suggests that employing semi-parametric or non-parametric models to demonstrate the relationship between the target and predictors would be advantageous. The target variable in this study is a risk, defined as 'TB/Population',' whereas the remaining variables are possible predictors. As demonstrated in the table below, most features in the dataset exhibit some connection. As some features are correlated, basic regression cannot be used because it would yield false results; rather, models that account for the connection can be used. It is vital to note that some characteristics are anticipated to have positive correlations (tuberculosis versus population, density versus poverty) and vice versa. Specifically, population density, poverty, health conditions, unemployment, and notification timeliness are likely high due to the high population density, the low economic share per capita, the high poverty rate, and the high jobless rate.

We want to model the count of cases TB_i by actually modelling ρ_i using

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \, TB_i \text{ indep.}$$
$$log(\lambda_i) = log(z_i) + log(\rho_i)$$

where TB_i is the count of TB cases, z_i is the total population. Model $log(\rho_i)$ as

$$log(\rho_i) = \sum_{j=1}^{8} f_j(x_{i,j})$$

$$f(x_i) = \sum_{k=1}^{q} \beta_k b_k(x_i)$$

where $x_{i,j}$ is the jth covariate (out of 8 socio-economic covariates) for the ith instance/datum in the dataset and $f(\cdot)$ is a smooth function of said covariate. Hence, the model boils down to

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \ TB_i \text{ indep.}$$

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{8} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j})$$

Looking at the distribution of the residuals of the model, we can see that the data is clearly far too overdispersed to be modelled by a Poisson, which has a fixed dispersion parameter. Even with 80 knots per smooth term the model doesn't seem to have enough flexibility which may be another indicator that a Poisson model is unsuitable for such overdispersed data. We propose the conventional alternative to the Poisson - the Negative Binomial model. Doing so, leads to a drop in the AIC. So the model distribution is changed to Negative Binomial with the same parameterisation except for the feature that the count of TB cases is now Negative Binomial distributed with mean as described above. See Table 1 appendix for a showcase of different model configurations and their associated AIC.

$$TB_i \sim NB(\lambda_i, \sigma_i^2) \ TB_i \text{ indep.}$$

$$\lambda_i = z_i \rho_i; \ \sigma_i^2 = \lambda_i + \frac{\lambda_i^2}{k}$$

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{8} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j})$$

where k is a dispersion parameter, later estimated by the `gam` function in R.

When having a look at the relationship between the squared residuals and the fitted values one sees that the relation is not exactly quadratic, but rather close to 0, which would reflect the relation between model variance and the expected value in a Gaussian Distribution Model (additional evidence is provided by the Residuals vs. Fitted plot). However, fitting a Gaussian model leads to very skewed residuals, indicating that the data is apparently not Gaussian. So the model distribution is changed to Negative Binomial with the same parameterization except for the feature that the count of TB cases is now Negative Binomial distributed with mean λ_i as described above.

Given this base model, we investigate whether all given socio-economic variables are needed to explain the response or whether there exists a model with fewer parameters. The p-value for the smooth term of Illiteracy points towards it not being statistically significant. Poverty, although not statistically insignificant, has the second-largest p-value. These terms are sequentially dropped and the resulting model checked against the original model via a Likelihood Ratio Test (conducted using the `anova` function in R). We find that leaving out Illiteracy does not alter the model at a 5%-level of significance, whereas taking out both Poverty and Illiteracy does. So, in the following, we use a model with all of the socio-economic variables except Illiteracy. Note that this converts our linear predictor to

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j})$$

This leaves us with a model with AIC = 14,391.19 and 43.9% of deviance explained. Running `gam.check()` lets us analyse the residual plots (see Figure) and examine the basis functions for the model. The QQ plot tells us that the model fails to predict well on the upper and lower ends of the response variable. Increasing the knots to 20 per covariate leads to marginal improvement with 44.9% deviance explained. More efficient extensions can be to add 1) spatial, 2) temporal and 3) spatio-temporal covariates.

First, we will try adding spatial terms. The spatial model adds a smoothed term which is function of the longitude and the latitude. A bivariate function is used because it makes sense to assume that there are more cases at certain locations (defined by the interaction between latitude and longitude) than others, rather than there being more cases at locations with a certain longitude for any latitude, or the other way round. Hence, our linear predictor is now

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Using this model with the regular `s` smoother function from the `mgcv` package leads to a model that can explain 56.4% of the deviance and has a slightly lower AIC of 14,013.13. The QQ plot still points to the upper and lower tails being incorrectly predicted. At the cost of significantly more computation, using a tensor product smooth `te` on the bivariate spatial term with 20 knots allows us to make a decent improvement on this. See Appendix for different numbers of knots that were tested. This gets us to 69.9% deviance explained. The QQ plot looks considerably better with only a few problematic instances at the top and bottom quantiles.

We contest this with an extension on the model with only socio-economic covariates, but instead of adding spatial terms, we add the temporal dimension `Year`. The linear predictor becomes

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} f_{2012,j}(x_{i,j}) \times x_{2012} + \sum_{j=1}^{7} f_{2013,j}(x_{i,j}) \times x_{2013} + \sum_{j=1}^{7} f_{2014,j}(x_{i,j}) \times x_{2014}$$

where the new terms $x_{2012}, x_{2013}, x_{2014}$ are indicator variables equating to 1 if `Year` is respectively 2012,2013,2014 and zero otherwise. Exercising some shorthand, it can be expressed as

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \sum_{j=1}^{7} f_{t,j}(x_{i,j}) \times x_t$$

where x_t is now the indicator variable for `Year`. A slightly separate approach can be tested with `Year` as a covariate instead of a grouping variable. In that case, the linear predictor would be

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} f_{t,j}(x_{i,j}) + \sum_{t=2012}^{2014} \beta_t x_t$$

Neither of the temporal formulations show much increase in deviance explained (the one with year as grouping variable actually shows a decrease to 41.5%!). Their QQ plots are also much worse than the spatial model, showing gross deviations on high as well as low quantiles. Finally, we create a spatio-temporal model, including both `Year` as well as `lon,lat`. Its linear predictor is formulated as below

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \left(\sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i) \right) \times x_t$$

This is a model which includes the term for the location and estimates a functional relation for each year and each explaining variable. The AIC of this model does not drop compared to the spatial model, so the spatial model (given that it is simpler) is the model we choose to best explain the ratio of TB cases per capita. To recall, it is formulated as

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Let us now have a closer look at the fit of the spatial model: It fits well even though the largest residuals are higher than expected from the model distribution. For districts that have a high number of cases, the predictor does not seem as accurate. But the highest residuals do not arise when the ratio of TB cases per capita is extraordinarily high, but rather when the absolute number of TB cases is high (see residuals vs. response). The variance of the model still seems too low for those values given that there are some predicted values in that high segment of response values (absolute number of TB cases) where the prediction for the response value is lower than the actual value, and some where the prediction of the actual value is higher than the actual value.

Using this model, we predict the rate of TB per 100,000 inhabitants. See Figure 1

3 Critical Review and Conclusion

Drawbacks of the Model:
1. Predictions do not cover full range of data, as evinced by deviations in the QQ plot
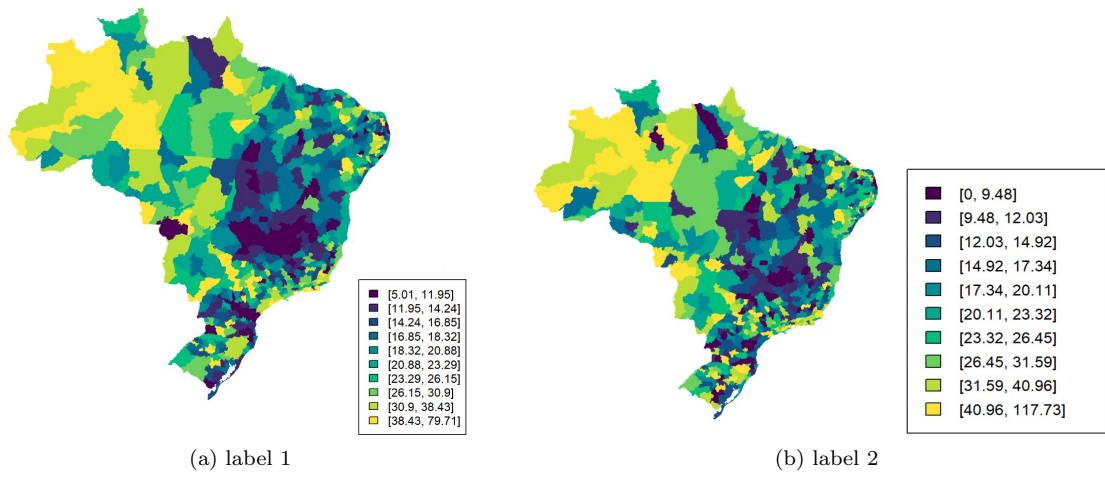2. ??

(a) label 1   (b) label 2

Figure 1: Predicted (a) and True(b) rates of TB per 100k inhabitants

4 References

Wood, S. N. (2017). Generalized Additive Models: An Introduction with R (2nd ed.). CRC Press.
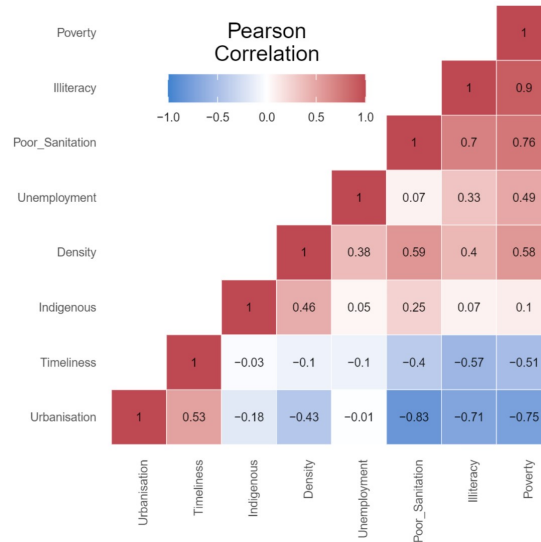
5 Appendix

5.1 Figures

Figure 2: Correlogram shows covariates with highest positive and negative correlations.

5.2 Tables

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

5.3 Code

```
1   #####################SS CODE#####################
2   # Import Libraries
3   library(mgcv) # required for GAM
4   library(tidyverse)
5   library(ggplot2) # required for plotting
6   library(dplyr) # required for filtering dataset
7   library(fields) # required for maps
```

```r
library(maps) # required for maps
library(reshape2) # only required for melt in corr plot
library(car) # only required for VIF

# Load Data
load("C:/Users/soura/Documents/COMM511/group_coursework/datasets_project.RData")

# Investigate correlation
### Resource -> http://www.sthda.com/english/wiki/ggplot2-quick-
### correlation-matrix-heatmap-r-software-and-data-visualization
cormat <- cor(TBdata[,c(1,2,3,4,5,6,7,8)])
# Reorder
reorder_cormat <- function(cormat){
# Use correlation between variables as distance
dd <- as.dist((1-cormat)/2)
hc <- hclust(dd)
cormat <-cormat[hc$order, hc$order]
}

# Reorder the correlation matrix
cormat <- reorder_cormat(cormat)
# Get lower triangular matrix
cormat[lower.tri(cormat)] <- NA

melted_cormat <- melt(cormat , na.rm = TRUE)
melted_cormat$value = round(melted_cormat$value, 2)

# Create a ggheatmap
ggheatmap <- ggplot(melted_cormat, aes(Var2, Var1, fill = value))+
geom_tile(color = "white")+
scale_fill_gradient2(low = "#1a85d6", high = "#cf3e4f", mid = "white",
midpoint = 0, limit = c(-1,1), space = "Lab",
name="Pearson\nCorrelation") +
theme_minimal()+ # minimal theme
theme(axis.text.x = element_text(angle = 90, vjust = 1,
size = 12, hjust = 1))+
coord_fixed()

# Add correlation coefficients
ggheatmap +
geom_text(aes(Var2, Var1, label = value), color = "black", size = 2) +
theme(
axis.text.x = element_text(size = 6),
axis.text.y = element_text(size = 6),
axis.title.x = element_blank(),
axis.title.y = element_blank(),
panel.grid.major = element_blank(),
panel.border = element_blank(),
panel.background = element_blank(),
axis.ticks = element_blank(),
legend.justification = c(1, 0),
legend.position = c(0.6, 0.7),
legend.direction = "horizontal",
legend.text = element_text(size = 6)
) +
guides(fill = guide_colorbar(barwidth = 7, barheight = 1,
title.position = "top", title.hjust = 0.5))

#### Illiteracy is highly correlated with Poverty
#### Carry out a Variance Inflation Test
model_all <- lm(TB ~ . , data = select(TBdata, 'Indigenous' , 'Illiteracy' ,
'Urbanisation' , 'Density' , 'Poverty', 'Unemployment' , 'Timeliness' , 'Year' ,
'TB' , 'Population'))  # with all the independent variables

vif(model_all) # Several variables are highly correlated

model_no_illiteracy <- lm(TB ~ . , data = select(TBdata, 'Indigenous',
```

```
75    'Urbanisation' , 'Density' , 'Poverty', 'Unemployment' , 'Timeliness' , 'Year' ,
76    'TB' , 'Population'))  # with all the independent variables
77
78    vif(model_no_illiteracy) # Poverty and Unemployment still seem highly correlated
79
80    model_no_illiteracy_no_poverty <- lm(TB ~ . , data = select(TBdata, 'Indigenous',
81    'Urbanisation' , 'Density', 'Unemployment' , 'Timeliness' , 'Year' ,
82    'TB' , 'Population'))  # with all the independent variables
83
84    vif(model_no_illiteracy_no_poverty) # almost no variable is highly correlated
85
86    ## More formal tests are conducted to confirm the dropping of Illiteracy.
87    ## Check to see if Poverty should be dropped as well
88    prelim.model.1 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +
89    s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
90    + s(Unemployment) + s(Timeliness),
91    data = TBdata ,
92    family = nb(link = 'log')
93    )
94    # Show summary
95    summary(prelim.model.1)
96    par(mfrow=c(2,2))
97    gam.check(prelim.model.1)
98
99    ### Only the effect of illiteracy cannot be reliably stated to be non-zero
100   prelim.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
101   + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
102   + s(Unemployment) + s(Timeliness),
103   data = TBdata ,
104   family = nb(link = 'log')
105   )
106   # Show summary
107   summary(prelim.model.2)
108
109   # Show summary
110   summary(prelim.model.2)
111   par(mfrow=c(2,2))
112   gam.check(prelim.model.2)
113
114   # Likelihood ratio test
115   anova(prelim.model.1 , prelim.model.2 , test = 'F') # p-value is over 0.05
116   # The models are statistically indistinguishable
117
118   ### Only the effect of illiteracy cannot be reliably stated to be non-zero
119   prelim.model.3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
120   + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
121   + s(Timeliness),
122   data = TBdata ,
123   family = nb(link = 'log')
124   )
125   # Show summary
126   summary(prelim.model.3)
127
128   # Likelihood ratio test
129   anova(prelim.model.2 , prelim.model.3 , test = 'F') # p-value is less than 0.05
130   # The models are statistically different. Poverty should not be excluded.
131
132   ### Model chosen (with social covariates) is the negative binomial without
133   ### Illiteracy
134   summary(prelim.model.2) # Only 44% of the deviance is explained. Adding temporal
135   # and spatial covariates may improve this
136   par(mfrow=c(2,2))
137   gam.check(prelim.model.2)
138   par(mfrow = c(1,1))
139
140   ### Only the effect of illiteracy cannot be reliably stated to be non-zero
141   prelim.model.4 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous , k = 20)
```

```r
142    + s(Urbanisation , k = 20) + s(Density , k = 20) + s(Poverty , k = 20)
143    + s(Poor_Sanitation , k = 20) + s(Unemployment , k = 20) + s(Timeliness , k = 20),
144    data = TBdata ,
145    family = nb(link = 'log')
146    )
147    # Show summary
148    summary(prelim.model.4)
149    par(mfrow=c(2,2))
150    gam.check(prelim.model.4)
151
152
153    ### Adding spatial covariates
154    spatial.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
155    + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) +s(Poverty)
156    + s(Timeliness) + s(lon , lat),
157    data = TBdata ,
158    family = nb(link = 'log')
159    )
160    # Check summary
161    summary(spatial.model)
162    # Check the smooth functions of the covars
163    plot(spatial.model)
164    par(mfrow=c(2,2))
165    gam.check(spatial.model)
166    par(mfrow = c(1,1))
167    spatial.model$aic
168
169    ### Using separate smoothers
170    spatial.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
171    + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
172    + s(Timeliness) + te(lon , lat , k = 20),
173    data = TBdata ,
174    family = nb(link = 'log')
175    )
176    # Check summary
177    summary(spatial.model.2)
178    # Check the smooth functions of the covariates
179    plot(spatial.model.2)
180    par(mfrow=c(2,2))
181    gam.check(spatial.model.2 , pch = 20)
182    par(mfrow = c(1,1))
183
184    ### PREDICTIONS
185    fitted_nb <- predict(spatial.model.2, newdata = TBdata , type = 'response')
186
187    # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
188    par(mfrow = c(1,1))
189    plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
190
191
192    TBdata$Year.asFactor <- factor(TBdata$Year)
193
194    #### Temporal covariates
195    temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
196    + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
197    + s(Timeliness) + Year.asFactor,
198    data = TBdata ,
199    family = nb(link = 'log')
200    )
201    # Check summary
202    summary(temporal.model) # Temporal alone doesn't add much to explaining the variance
203    # Check the smooth functions of the covariates
204    plot(temporal.model)
205    par(mfrow=c(2,2))
206    gam.check(temporal.model)
207    par(mfrow = c(1,1))
208
```

```
209    #### Temporal covariates
210    temporal.model.2 <- gam(formula = TB ~ offset(log(Population))
211    + s(Indigenous , by=Year.asFactor) + s(Urbanisation , by=Year.asFactor)
212    + s(Density , by=Year.asFactor) + s(Poor_Sanitation , by=Year.asFactor)
213    + s(Unemployment , by=Year.asFactor) + s(Poverty , by=Year.asFactor)
214    + s(Timeliness , by=Year.asFactor) ,
215    data = TBdata ,
216    family = nb(link = 'log')
217    )
218    # Check summary
219    summary(temporal.model.2) # Temporal alone doesn't add much to explaining the variance
220    # Check the smooth functions of the covariates
221    plot(temporal.model.2)
222    par(mfrow=c(2,2))
223    gam.check(temporal.model.2)
224    par(mfrow = c(1,1))
225
226    ### Spatio-temporal model
227    spatio.temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
228    + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
229    + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
230    data = TBdata ,
231    family = nb(link = 'log')
232    )
233    # Check summary
234    summary(spatio.temporal.model) # Temporal alone doesn't add much to explaining the variance
235    # Check the smooth functions of the covariates
236    plot(spatio.temporal.model)
237    par(mfrow=c(2,2))
238    gam.check(spatio.temporal.model , pch = 20)
239    par(mfrow = c(1,1))
240
241    # PLOT
242    fitted_nb <- predict(spatio.temporal.model, newdata = TBdata , type = 'response')
243    TBdata$pred_rate <- fitted_nb/TBdata$Population*100000
244
245    # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
246    par(mfrow = c(1,1))
247    plot.map(TBdata$pred_rate , n.levels = 10)
248
249    ### WIHTOUT POVERTY
250    ### Spatio-temporal model - Poverty
251    spatio.temporal.model.wo.pov <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
252    + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
253    + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
254    data = TBdata ,
255    family = nb(link = 'log')
256    )
257
258    # Check summary
259    summary(spatio.temporal.model.wo.pov) # Temporal alone doesn't add much to explaining the variance
260    # Check the smooth functions of the covariates
261    plot(spatio.temporal.model.wo.pov)
262    par(mfrow=c(2,2))
263    gam.check(spatio.temporal.model.wo.pov , pch = 20)
264    par(mfrow = c(1,1))
265
266    # Anova test
267    anova(spatio.temporal.model.wo.pov , spatio.temporal.model , test = 'F')
268
269    ### Without Indigenous
270    spatio.temporal.model.wo.indig <- gam(formula = TB ~ offset(log(Population))
271    + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
272    + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
273    data = TBdata ,
274    family = nb(link = 'log')
275    )
```

```
276
277    # Check summary
278    summary(spatio.temporal.model.wo.indig) # Temporal alone doesn't add much to explaining the variance
279    # Check the smooth functions of the covariates
280    plot(spatio.temporal.model.wo.indig)
281    par(mfrow=c(2,2))
282    gam.check(spatio.temporal.model.wo.indig , pch = 20)
283    par(mfrow = c(1,1))
284
285    # Anova test
286    anova(spatio.temporal.model.wo.indig , spatio.temporal.model , test = 'F')
287    # NOT ENOUGH EVIDENCE TO REMOVE INDIGENOUS
288
289    ## Spatio-temporal model
290    spatio.temporal.model.true <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)
291    + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
292    + s(Timeliness) + te(lon , lat , Year , k = 3),
293    data = TBdata ,
294    family = nb(link = 'log')
295    )
296
297    # Check summary
298    summary(spatio.temporal.model.true) # Temporal alone doesn't add much to explaining the variance
299    # Check the smooth functions of the covariates
300    plot(spatio.temporal.model.true)
301    par(mfrow=c(2,2))
302    gam.check(spatio.temporal.model.true , pch = 20)
303    par(mfrow = c(1,1))
304
305    spatio.temporal.model$aic
306    spatio.temporal.model.true$aic
307    spatio.temporal.model.wo.indig$aic
308
309
310
311    ### Spatio-temporal model - no poor sanitation , indigenous
312    spatio.temporal.model.wo.indig.poor_s <- gam(formula = TB ~ offset(log(Population))
313    + s(Urbanisation) + s(Density) + s(Unemployment) + s(Poverty)
314    + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
315    data = TBdata ,
316    family = nb(link = 'log')
317    )
318    # Check summary
319    summary(spatio.temporal.model.wo.indig.poor_s) # Temporal alone doesn't add much to explaining the variance
320    # Check the smooth functions of the covariates
321    plot(spatio.temporal.model.wo.indig.poor_s)
322    par(mfrow=c(2,2))
323    gam.check(spatio.temporal.model.wo.indig.poor_s , pch = 20)
324    par(mfrow = c(1,1))
325
326    anova(spatio.temporal.model , spatio.temporal.model.wo.indig.poor_s , spatio.temporal.model.wo.indig , test = 'F')
327
328    ##############LH CODE#########################################
329    library (mgcv)
330
331
332    par(mfrow = c(2,2))
333    #fit poisson model with socio-economic variables
334    model_poisson <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +  s(Illiteracy) +  s(Urbanisation) + s(Density) + s(Po
335    summary(model_poisson)
336    model_poisson$aic
337    par(mfrow = c(2,2),pch = 20)
338    gam.check(model_poisson)
339    # #add flexibility
340    model_poisson <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, k = 80) +  s(Illiteracy, k = 80) +  s(Urbanisation, k =
341    gam.check(model_poisson)
342    summary(model_poisson) # SIGNS OF OVERFIT
```

```
343    plot(model_poisson) # SIGNS OF OVERFIT
344    #fit negative binomial model with  socioeconomic
345    model_nb <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +  s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty)
346    summary(model_nb)
347    model_nb$aic
348    #fit a linear relation between sqaured residuals and prediction to see whether another model describes the variance-fitted values re
349    summary(lm(log(model_nb$residuals^2) ~ log(predict(model_nb, type = 'response'))))
350    #drop Illiteracy
351    model_nb_2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +  s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sani
352
353
354    #LRT
355    anova.gam(model_nb_2, model_nb, test = 'LRT')
356    #Null hypothesis not rejected -> drop poverty
357    model_nb_3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +  s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(U
358    #LRT
359    anova.gam(model_nb_3, model_nb_2, test = 'LRT')
360    # NULL hypothesis rejected -> drop only illiteracy, not poverty
361
362    model_nb_final <- model_nb_2
363    summary(model_nb_final)
364    gam.check(model_nb_final)
365    fitted_nb <- predict(model_nb_final, type = 'response')
366
367    # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
368    par(mfrow = c(1,2))
369    plot.map(log(fitted_nb) - log(TBdata$Population) , n.levels = 10)
370    plot.map(log(TBdata$TB) - log(TBdata$Population))
371
372    #temporal model
373    par(mfrow = c(2,2), pch = 20)
374    model_nb_time <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, by = Year) + s(Urbanisation, by = Year) + s(Density, by
375    summary(model_nb)
376    model_nb$aic
377
378    #spatial model
379    model_nb_space <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_S
380    summary(model_nb_space)
381    model_nb_space$aic
382    gam.check(model_nb_space)
383    anova.gam(model_nb_space, model_nb_final, test = 'LRT')
384
385    #spatio-temporal model
386    model_nb_time_and_space <- gam(formula = TB ~ offset(log(Population)) + s(Urbanisation, by = Year) + s(Density, by = Year) + s(Pove
387    summary(model_nb_time_and_space)
388    model_nb_time_and_space$aic
389    gam.check(model_nb_time_and_space)
390
391    # PLOT
392    fitted_nb <- predict(model_nb_time_and_space, newdata = TBdata , type = 'response')
393
394    # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
395    par(mfrow = c(1,1))
396    plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
```