Analysis of 2012–2014 Brazil Tuberculosis Data

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1 Introduction

Problem Statement

Analysis of tuberculosis (TB) data originating from Brazil using Generalized Additive Models (GAMs). Brazil is divided into 557 administrative microregions, and the available data comprises counts of TB cases in each microregion for each of the years from 2012 to 2014.

2 Exploratory Analysis of Data and Problem

The TB data from Brazil includes 1,671 entries or samples with 14 columns of numeric data types that specify the characteristics of each sample. The columns are self-explanatory because they are called Indigenous, Illiteracy, Urbanisation, Density, Poverty, Poor Sanitation, Unemployment, Timeliness, Year, TB, Population, Region, Ion, and Iat. TB stands for tuberculosis, whereas Ion and Iat stand for longitude and latitude. The dataset has no missing values in the technical sense, but it contains some abnormalities, which increases the amount of pre-processing needed. The region is stored as a continuous variable despite being a factor variable. Nonetheless, changing it depends on the task at hand. Moreover, the collection includes coordinates that describe the precise geospatial locations of the micro-regions listed in the column. The next part gives a detailed exploration of the data.

Data Exploration

An in-depth analysis of the datasets reveals that the mean and median values for the indigenous population are low, but the maximum value is 50, which suggests that there are individual areas where the indigenous population is concentrated and that these areas may be areas of potential poverty and poor sanitation and should be areas where there are more cases of tuberculosis. The mean and median illiteracy rates are only 14 and 11, respectively, indicating that illiteracy is not widespread. However, the maximum value of 41 suggests that there are specific backward areas with significant populations lacking access to education, which also suggests that the area seems poor and has poor sanitation. There are still some places that are less urbanised, where there may be more occurrences of tuberculosis, but the mean, median, and minimum values for urbanisation are 70 and 22, respectively, suggesting that most areas are highly urbanised. Based on the population density data, most locations can fit one person in a room, but the highest value of 1.6 highlights the existence of some places with very high population densities, which sharply raise the rate of TB transmission. The distribution of the poverty data suggests that each district has different poverty levels, with only a limited number of districts where poverty is not a significant issue. Although the general results on inadequate sanitation are low, the maximum score of 58 indicates that some districts have poor sanitation and substantial disease risk. Although average unemployment rates are low, a maximum value of 20 indicates that some isolated regions may experience severe economic hardship, protracted social unrest, and potentially significant morbidity rates. With a minimum value of 0, notification timeliness data has a fairly wide range.

Data from the actual world follow a normal distribution. However, some of the columns indicate otherwise. Timeliness, Unemployment, and Urbanization are approximations of a normal distribution with few extremes, whilst the remainder is multimodal normal distributions. The above suggests that employing semi-parametric or non-parametric models to demonstrate the relationship between the target and predictors would be advantageous. The target variable in this study is a risk, defined as 'TB/Population',' whereas the remaining variables are possible predictors. As demonstrated in the table below, most features in the dataset exhibit some connection. As some features are correlated, basic regression cannot be used because it would yield false results; rather, models that account for the connection can be used. It is vital to note that some characteristics are anticipated to have positive correlations (tuberculosis versus population, density versus poverty) and vice versa. Specifically, population density, poverty, health conditions, unemployment, and notification timeliness are likely high due to the high population density, the low economic share per capita, the high poverty rate, and the high jobless rate.

3 Model

We want to model the count of cases TB_i by actually modelling ρ_i using

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \ TB_i$$
 indep.
 $log(\lambda_i) = log(z_i) + log(\rho_i)$

where TB_i is the count of TB cases, z_i is the total population. Model $log(\rho_i)$ as

$$log(\rho_i) = \sum_{j=1}^{8} f_j(x_{i,j})$$
$$f(x_i) = \sum_{k=1}^{q} \beta_k b_k(x_i)$$

where $x_{i,j}$ is the jth covariate (out of 8 socio-economic covariates) for the ith instance/datum in the dataset and $f(\cdot)$ is a smooth function of said covariate. Hence, the model boils down to

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \ TB_i \text{ indep.}$$

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^8 \sum_{k=1}^q \beta_{j,k} b_{j,k}(x_{i,j})$$

Looking at the distribution of the residuals of the model, we can see that the data is clearly far too overdispersed to be modelled by a Poisson, which has a fixed dispersion parameter. Even with 80 knots per smooth term the model doesn't seem to have enough flexibility which may be another indicator that a Poisson model is unsuitable for such overdispersed data. We propose the conventional alternative to the Poisson - the Negative Binomial model. Doing so, leads to a drop in the AIC. So the model distribution is changed to Negative Binomial with the same parameterisation except for the feature that the count of TB cases is now Negative Binomial distributed with mean as described above. See Table ?? appendix for a showcase of different model configurations and their associated AIC.

$$TB_i \sim NB(\lambda_i, \sigma_i^2) \ TB_i \ \mathrm{indep.}$$

$$\lambda_i = z_i \rho_i; \ \sigma_i^2 = \lambda_i + \frac{\lambda_i^2}{k}$$

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^8 \sum_{k=1}^q \beta_{j,k} b_{j,k}(x_{i,j})$$

where k is a dispersion parameter, later estimated by the gam function in R.

When having a look at the relationship between the squared residuals and the fitted values one sees that the relation is not exactly quadratic, but rather close to 0, which would reflect the relation between model variance and the expected value in a Gaussian Distribution Model (additional evidence is provided by the Residuals vs. Fitted plot). However, fitting a Gaussian model leads to very skewed residuals, indicating that the data is apparently not Gaussian. So the model distribution is changed to Negative Binomial with the same parameterization except for the feature that the count of TB cases is now Negative Binomial distributed with mean λ_i as described above.

Given this base model, we investigate whether all given socio-economic variables are needed to explain the response or whether there exists a model with fewer parameters. The p-value for the smooth term of Illiteracy points towards it not being statistically significant. Poverty, although not statistically insignificant, has the second-largest p-value. These terms are sequentially dropped and the resulting model checked against the original model via a Likelihood Ratio Test (conducted using the anova function in R). We find that leaving out Illiteracy does not alter the model at a 5%-level of significance, whereas taking out both Poverty and Illiteracy does. So, in the following, we use a model with all of the socio-economic variables except Illiteracy. Note that this converts our linear predictor to

$$log(\lambda_i) = log(z_i) + \sum_{i=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j})$$

This leaves us with a model with AIC = 14,391.19 and 43.9% of deviance explained. Running gam.check() lets us analyse the residual plots (see Figure) and examine the basis functions for the model. The QQ plot tells us that the model fails to predict well on the upper and lower ends of the response variable. Increasing the knots to 20 per covariate leads to marginal improvement with 44.9% deviance explained. More efficient extensions can be to add 1) spatial, 2)

temporal and 3) spatio-temporal covariates.

First, we will try adding spatial terms. The spatial model adds a smoothed term which is function of the longitude and the latitude. A bivariate function is used because it makes sense to assume that there are more cases at certain locations (defined by the interaction between latitude and longitude) than others, rather than there being more cases at locations with a certain longitude for any latitude, or the other way round. Hence, our linear predictor is now

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Using this model with the regular s smoother function from the mgcv package leads to a model that can explain 56.4% of the deviance and has a slightly lower AIC of 14,013.13. The QQ plot still points to the upper and lower tails being incorrectly predicted. At the cost of significantly more computation, using a tensor product smooth te on the bivariate spatial term with 20 knots allows us to make a decent improvement on this. See Appendix for different numbers of knots that were tested. This gets us to 69.9% deviance explained. The QQ plot looks considerably better with only a few problematic instances at the top and bottom quantiles.

We contest this with an extension on the model with only socio-economic covariates, but instead of adding spatial terms, we add the temporal dimension Year. The linear predictor becomes

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} f_{2012,j}(x_{i,j}) \times x_{2012} + \sum_{j=1}^{7} f_{2013,j}(x_{i,j}) \times x_{2013} + \sum_{j=1}^{7} f_{2014,j}(x_{i,j}) \times x_{2014}$$

where the new terms $x_{2012}, x_{2013}, x_{2014}$ are indicator variables equating to 1 if Year is respectively 2012,2013,2014 and zero otherwise. Exercising some shorthand, it can be expressed as

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \sum_{i=1}^{7} f_{t,j}(x_{i,j}) \times x_t$$

where x_t is now the indicator variable for Year. A slightly separate approach can be tested with Year as a covariate instead of a grouping variable. In that case, the linear predictor would be

$$log(\lambda_i) = log(z_i) + \sum_{i=1}^{7} f_{t,j}(x_{i,j}) + \sum_{t=2012}^{2014} \beta_t x_t$$

Neither of the temporal formulations show much increase in deviance explained (the one with year as grouping variable actually shows a decrease to 41.5%!). Their QQ plots are also much worse than the spatial model, showing gross deviations on high as well as low quantiles. Finally, we create a spatio-temporal model, including both Year as well as lon,lat. Its linear predictor is formulated as below

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \left(\sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i) \right) \times x_t$$

This is a model which includes the term for the location and estimates a functional relation for each year and each explaining variable. The AIC of this model does not drop compared to the spatial model, so the spatial model (given that it is simpler) is the model we choose to best explain the ratio of TB cases per capita. To recall, it is formulated as

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Let us now have a closer look at the fit of the spatial model: It fits well even though the largest residuals are higher than expected from the model distribution. For districts that have a high number of cases, the predictor does not seem as accurate. But the highest residuals do not arise when the ratio of TB cases per capita is extraordinarily high, but rather when the absolute number of TB cases is high (see residuals vs. response). The variance of the model still seems too low for those values given that there are some predicted values in that high segment of response values (absolute number of TB cases) where the prediction for the response value is lower than the actual value, and some where the prediction of the actual value is higher than the actual value.

Using this model, we predict the rate of TB per 100,000 inhabitants. See Figure ??

4 Critical Review and Conclusion

Drawbacks of the Model:

- 1. Predictions do not cover full range of data, as evinced by deviations in the QQ plot
- 2. ??

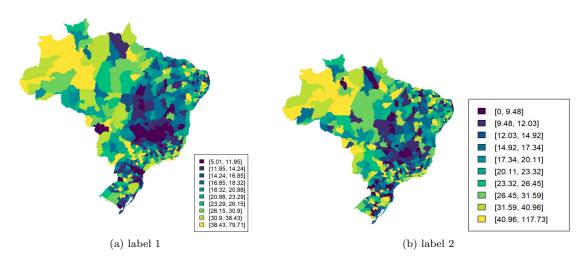


Figure 1: Predicted (a) and True(b) rates of TB per 100k inhabitants

5 References

Wood, S. N. (2017). Generalized Additive Models: An Introduction with R (2nd ed.). CRC Press.

6 Appendix

6.1 Figures

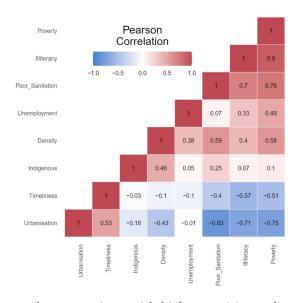


Figure 2: Correlogram shows covariates with highest positive and negative correlations.

6.2 Tables

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

6.3 Code

```
# Import Libraries
    library(mgcv) # required for GAM
    library(tidyverse)
    library(ggplot2) # required for plotting
    library(dplyr) # required for filtering dataset
    library(fields) # required for maps
    library(maps) # required for maps
    library(reshape2) # only required for melt in corr plot
10
    library(car) # only required for VIF
11
     # Load Data
12
    load("C:/Users/soura/Documents/COMM511/group_coursework/datasets_project.RData")
13
14
     # Investigate correlation
15
    ### Resource -> http://www.sthda.com/english/wiki/ggplot2-quick-
16
     ### correlation-matrix-heatmap-r-software-and-data-visualization
17
     cormat <- cor(TBdata[,c(1,2,3,4,5,6,7,8)])</pre>
18
     # Reorder
19
     reorder_cormat <- function(cormat){</pre>
20
     # Use correlation between variables as distance
21
     dd <- as.dist((1-cormat)/2)</pre>
22
     hc <- hclust(dd)
23
     cormat <-cormat[hc$order, hc$order]</pre>
25
     # Reorder the correlation matrix
27
     cormat <- reorder_cormat(cormat)</pre>
28
29
     # Get lower triangular matrix
     cormat[lower.tri(cormat)] <- NA</pre>
30
31
     melted_cormat <- melt(cormat , na.rm = TRUE)</pre>
32
     melted_cormat$value = round(melted_cormat$value, 2)
33
34
     # Create a ggheatmap
35
     ggheatmap <- ggplot(melted_cormat, aes(Var2, Var1, fill = value))+</pre>
36
     geom_tile(color = "white")+
37
     scale_fill_gradient2(low = "#1a85d6", high = "#cf3e4f", mid = "white",
38
     midpoint = 0, limit = c(-1,1), space = "Lab",
39
     name="Pearson\nCorrelation") +
40
41
     theme_minimal()+ # minimal theme
42
     theme(axis.text.x = element_text(angle = 90, vjust = 1,
43
     size = 12, hjust = 1))+
     coord_fixed()
44
45
     # Add correlation coefficients
46
     ggheatmap +
     geom_text(aes(Var2, Var1, label = value), color = "black", size = 2) +
48
     axis.text.x = element_text(size = 6),
     axis.text.y = element_text(size = 6),
51
    axis.title.x = element_blank(),
52
    axis.title.y = element_blank(),
53
    panel.grid.major = element_blank(),
54
     panel.border = element_blank(),
55
     panel.background = element_blank(),
56
     axis.ticks = element_blank(),
57
     legend.justification = c(1, 0),
58
     legend.position = c(0.6, 0.7),
59
     legend.direction = "horizontal",
60
     legend.text = element_text(size = 6)
61
62
     guides(fill = guide_colorbar(barwidth = 7, barheight = 1,
63
     title.position = "top", title.hjust = 0.5))
64
65
     #### Illiteracy is highly correlated with Poverty
66
```

```
#### Carry out a Variance Inflation Test
      model_all <- lm(TB ~ . , data = select(TBdata, 'Indigenous' , 'Illiteracy' ,</pre>
68
      'Urbanisation', 'Density', 'Poverty', 'Unemployment', 'Timeliness', 'Year',
69
      'TB' , 'Population')) # with all the independent variables
70
71
      vif(model_all) # Several variables are highly correlated
72
73
      model_no_illiteracy <- lm(TB ~ . , data = select(TBdata, 'Indigenous',</pre>
74
      'Urbanisation', 'Density', 'Poverty', 'Unemployment', 'Timeliness', 'Year',
75
      'TB' , 'Population')) # with all the independent variables
76
77
      vif(model_no_illiteracy) # Poverty and Unemployment still seem highly correlated
79
      model_no_illiteracy_no_poverty <- lm(TB ~ . , data = select(TBdata, 'Indigenous',</pre>
80
      'Urbanisation', 'Density', 'Unemployment', 'Timeliness', 'Year',
      'TB' , 'Population')) # with all the independent variables
      vif(model_no_illiteracy_no_poverty) # almost no variable is highly correlated
85
      ## More formal tests are conducted to confirm the dropping of Illiteracy.
86
      ## Check to see if Poverty should be dropped as well
      prelim.model.1 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +</pre>
      s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
89
      + s(Unemployment) + s(Timeliness),
90
      data = TBdata ,
      family = nb(link = 'log')
92
93
      # Show summaru
94
      summary(prelim.model.1)
95
      par(mfrow=c(2,2))
96
      gam.check(prelim.model.1)
97
98
      ### Only the effect of illiteracy cannot be reliably stated to be non-zero
     prelim.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
100
      + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
101
      + s(Unemployment) + s(Timeliness),
      data = TBdata ,
     family = nb(link = 'log')
105
      # Show summary
106
107
      summary(prelim.model.2)
108
      # Show summaru
109
      summary(prelim.model.2)
110
     par(mfrow=c(2,2))
111
      gam.check(prelim.model.2)
112
113
      # Likelihood ratio test
114
      anova(prelim.model.1 , prelim.model.2 , test = 'F') # p-value is over 0.05
115
      # The models are statistically indistinguishable
116
117
      ### Only the effect of illiteracy cannot be reliably stated to be non-zero
118
      prelim.model.3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
119
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
120
121
      + s(Timeliness),
      data = TBdata ,
122
     family = nb(link = 'log')
123
124
      # Show summary
      summary(prelim.model.3)
      # Likelihood ratio test
128
      anova(prelim.model.2 , prelim.model.3 , test = 'F') # p-value is less than 0.05
129
      # The models are statistically different. Poverty should not be excluded.
130
131
      ### Model chosen (with social covariates) is the negative binomial without
132
      ### Illiteracy
133
```

```
summary(prelim.model.2) # Only 44% of the deviance is explained. Adding temporal
134
      # and spatial covariates may improve this
135
      par(mfrow=c(2,2))
136
      gam.check(prelim.model.2)
137
      par(mfrow = c(1,1))
138
139
      ### Only the effect of illiteracy cannot be reliably stated to be non-zero
140
       prelim.model.4 <- \ gam(formula = TB \ \tilde{\ } offset(log(Population)) \ + \ s(Indigenous \ , \ k = 20) 
141
      + s(Urbanisation , k = 20) + s(Density , k = 20) + s(Poverty , k = 20)
142
      + s(Poor_Sanitation , k = 20) + s(Unemployment , k = 20) + <math>s(Timeliness , k = 20),
      data = TBdata ,
      family = nb(link = 'log')
146
      # Show summary
      summary(prelim.model.4)
148
      par(mfrow=c(2,2))
149
      gam.check(prelim.model.4)
150
151
152
      ### Adding spatial covariates
153
      spatial.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
154
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) +s(Poverty)
155
      + s(Timeliness) + s(lon , lat),
156
      data = TBdata ,
157
      family = nb(link = 'log')
158
159
160
      # Check summary
161
      summary(spatial.model)
      # Check the smooth functions of the covars
162
      plot(spatial.model)
163
      par(mfrow=c(2,2))
164
      gam.check(spatial.model)
165
      par(mfrow = c(1,1))
166
      spatial.model$aic
167
168
      ### Using separate smoothers
169
      spatial.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
170
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
171
      + s(Timeliness) + te(lon , lat , k = 20),
172
      data = TBdata ,
173
      family = nb(link = 'log')
174
175
      # Check summary
176
      summary(spatial.model.2)
177
      # Check the smooth functions of the covariates
178
      plot(spatial.model.2)
179
      par(mfrow=c(2,2))
180
      gam.check(spatial.model.2 , pch = 20)
181
      par(mfrow = c(1,1))
182
      ### PREDICTIONS
184
      fitted_nb <- predict(spatial.model.2, newdata = TBdata , type = 'response')</pre>
185
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
187
      par(mfrow = c(1,1))
188
      plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
189
190
191
      TBdata$Year.asFactor <- factor(TBdata$Year)
192
193
      #### Temporal covariates
194
      temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
195
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
196
      + s(Timeliness) + Year.asFactor,
197
      data = TBdata ,
198
      family = nb(link = 'log')
199
200
      )
```

```
# Check summary
201
      summary(temporal.model) # Temporal alone doesn't add much to explaining the variance
202
      # Check the smooth functions of the covariates
203
      plot(temporal.model)
204
      par(mfrow=c(2,2))
205
      gam.check(temporal.model)
206
      par(mfrow = c(1,1))
207
208
209
      #### Temporal covariates
      temporal.model.2 <- gam(formula = TB ~ offset(log(Population))</pre>
210
211
      + s(Indigenous , by=Year.asFactor) + s(Urbanisation , by=Year.asFactor)
212
      + s(Density , by=Year.asFactor) + s(Poor_Sanitation , by=Year.asFactor)
213
      + s(Unemployment , by=Year.asFactor) + s(Poverty , by=Year.asFactor)
214
      + s(Timeliness , by=Year.asFactor) ,
215
      data = TBdata ,
      family = nb(link = 'log')
      # Check summary
      summary(temporal.model.2) # Temporal alone doesn't add much to explaining the variance
219
      # Check the smooth functions of the covariates
220
      plot(temporal.model.2)
221
      par(mfrow=c(2,2))
222
      gam.check(temporal.model.2)
223
      par(mfrow = c(1,1))
224
225
      ### Spatio-temporal model
226
      spatio.temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
227
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
228
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
229
      data = TBdata ,
230
     family = nb(link = 'log')
231
232
      # Check summary
233
      summary(spatio.temporal.model) # Temporal alone doesn't add much to explaining the variance
234
      # Check the smooth functions of the covariates
235
      plot(spatio.temporal.model)
      par(mfrow=c(2,2))
      gam.check(spatio.temporal.model , pch = 20)
     par(mfrow = c(1,1))
239
240
      # PLOT
241
      fitted_nb <- predict(spatio.temporal.model, newdata = TBdata , type = 'response')</pre>
242
      TBdata$pred_rate <- fitted_nb/TBdata$Population*100000</pre>
243
244
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
245
      par(mfrow = c(1,1))
246
      plot.map(TBdata$pred_rate , n.levels = 10)
247
248
      ### WIHTOUT POVERTY
249
      ### Spatio-temporal model - Poverty
250
      spatio.temporal.model.wo.pov <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
251
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
252
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
253
      data = TBdata ,
254
      family = nb(link = 'log')
255
256
      # Check summary
      summary(spatio.temporal.model.wo.pov) # Temporal alone doesn't add much to explaining the variance
259
      # Check the smooth functions of the covariates
260
      plot(spatio.temporal.model.wo.pov)
261
      par(mfrow=c(2,2))
262
      gam.check(spatio.temporal.model.wo.pov , pch = 20)
263
      par(mfrow = c(1,1))
264
265
      # Anova test
266
      anova(spatio.temporal.model.wo.pov , spatio.temporal.model , test = 'F')
267
```

```
268
           ### Without Indigenous
269
          spatio.temporal.model.wo.indig <- gam(formula = TB ~ offset(log(Population))</pre>
270
           + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
271
           + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
272
          data = TBdata ,
273
          family = nb(link = 'log')
275
276
           # Check summary
277
          summary(spatio.temporal.model.wo.indig) # Temporal alone doesn't add much to explaining the variance
278
           # Check the smooth functions of the covariates
279
          plot(spatio.temporal.model.wo.indig)
280
          par(mfrow=c(2,2))
281
           gam.check(spatio.temporal.model.wo.indig , pch = 20)
282
          par(mfrow = c(1,1))
283
284
           # Anova test
285
          anova(spatio.temporal.model.wo.indig , spatio.temporal.model , test = 'F')
286
           # NOT ENOUGH EVIDENCE TO REMOVE INDIGENOUS
287
288
          ## Spatio-temporal model
289
          spatio.temporal.model.true <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
290
           + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
291
           + s(Timeliness) + te(lon , lat , Year , k = 3),
292
293
          data = TBdata ,
          family = nb(link = 'log')
           # Check summary
          summary(spatio.temporal.model.true) # Temporal alone doesn't add much to explaining the variance
           # Check the smooth functions of the covariates
          plot(spatio.temporal.model.true)
300
          par(mfrow=c(2,2))
301
           gam.check(spatio.temporal.model.true , pch = 20)
302
          par(mfrow = c(1,1))
303
304
          spatio.temporal.model$aic
305
          spatio.temporal.model.true$aic
306
           spatio.temporal.model.wo.indig$aic
307
308
309
310
           \textit{### Spatio-temporal model - no poor sanitation , indigenous}
311
          spatio.temporal.model.wo.indig.poor_s <- gam(formula = TB ~ offset(log(Population))</pre>
312
           + s(Urbanisation) + s(Density) + s(Unemployment) + s(Poverty)
313
           + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
          data = TBdata ,
          family = nb(link = 'log')
317
318
           # Check summary
          summary(spatio.temporal.model.wo.indig.poor_s) # Temporal alone doesn't add much to explaining the variance
           # Check the smooth functions of the covariates
          plot(spatio.temporal.model.wo.indig.poor_s)
321
          par(mfrow=c(2,2))
322
          gam.check(spatio.temporal.model.wo.indig.poor_s , pch = 20)
323
          par(mfrow = c(1,1))
324
325
          anova(spatio.temporal.model , spatio.temporal.model.wo.indig.poor_s , spatio.temporal.model.wo.indig , test = 'F')
326
327
           328
          library (mgcv)
329
330
331
           par(mfrow = c(2,2))
332
333
          {\it \#fit\ poisson\ model\ with\ socio-economic\ variables}
334
           model_poisson \leftarrow gam(formula = TB \sim offset(log(Population)) + s(Indigenous) + s(Illiteracy) + s(Urbanisation) + s(Density) + s(Population) +
```

```
summary(model_poisson)
335
336
      model_poisson$aic
      par(mfrow = c(2,2), pch = 20)
337
      gam.check(model_poisson)
338
      # #add flexibility
339
      model_poisson <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, k = 80) + s(Illiteracy, k = 80) + s(Urbanisation, k =
340
      gam.check(model_poisson)
      summary(model_poisson) # SIGNS OF OVERFIT
      plot(model_poisson) # SIGNS OF OVERFIT
      #fit negative binomial model with socioeconomic
      model_nb \leftarrow gam(formula = TB \sim offset(log(Population)) + s(Indigenous) + s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty)
      summary(model_nb)
      model_nb$aic
      #fit a linear relation between squired residuals and prediction to see whether another model describes the variance-fitted values re
348
      summary(lm(log(model_nb$residuals^2) ~ log(predict(model_nb, type = 'response'))))
349
      #drop Illiteracy
350
      model_nb_2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sani
351
352
353
      #LRT
354
      anova.gam(model_nb_2, model_nb, test = 'LRT')
355
      #Null hypothesis not rejected -> drop poverty
356
      model_nb_3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Urbanisation)
357
358
      anova.gam(model_nb_3, model_nb_2, test = 'LRT')
359
360
      \# NULL hypothesis rejected \rightarrow drop only illiteracy, not poverty
361
      model_nb_final <- model_nb_2</pre>
362
      summary(model_nb_final)
363
      gam.check(model_nb_final)
      fitted_nb <- predict(model_nb_final, type = 'response')</pre>
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
367
368
      par(mfrow = c(1,2))
      plot.map(log(fitted_nb) - log(TBdata$Population) , n.levels = 10)
369
      plot.map(log(TBdata$TB) - log(TBdata$Population))
370
371
      #temporal model
372
      par(mfrow = c(2,2), pch = 20)
373
      model_nb_time <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, by = Year) + s(Urbanisation, by = Year) + s(Density, by
374
      summary(model_nb)
375
      model_nb$aic
376
377
      #spatial model
378
      model_nb_space <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_S)</pre>
379
      summary(model_nb_space)
380
      model_nb_space$aic
      gam.check(model_nb_space)
382
      anova.gam(model_nb_space, model_nb_final, test = 'LRT')
383
      \#spatio-temporal\ model
      model_nb_time_and_space <- gam(formula = TB ~ offset(log(Population)) + s(Urbanisation, by = Year) + s(Density, by = Year) + s(Pove
      summary(model_nb_time_and_space)
      model_nb_time_and_space$aic
      gam.check(model_nb_time_and_space)
389
391
      fitted_nb <- predict(model_nb_time_and_space, newdata = TBdata , type = 'response')</pre>
392
393
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
394
      par(mfrow = c(1,1))
395
      plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
396
```