Analysis of 2014 Brazil Tuberculosis Data

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1 Introduction

1.1 Problem Statement

Analysis of tuberculosis (TB) data originating from Brazil using Generalized Additive Models (GAMs). Brazil is divided into 557 administrative microregions, and the available data comprises counts of TB cases in each microregion for each of the years from 2012 to 2014.

2 Exploratory Analysis of Data and Problem

The TB data from Brazil includes 1,671 entries or samples with 14 columns of numeric data types that specify the characteristics of each sample. The columns are selfexplanatory because they are called Indigenous, Illiteracy, Urbanisation, Density, Poverty, Poor Sanitation, Unemployment, Timeliness, Year, TB, Population, Region, lon, and lat. TB stands for tuberculosis, whereas lon and lat stand for longitude and latitude. The dataset has no missing values in the technical sense, but it contains some abnormalities, which increases the amount of pre-processing needed. The region is stored as a continuous variable despite being a factor variable. Nonetheless, changing it depends on the task at hand. Moreover, the collection includes coordinates that describe the precise geospatial locations of the micro-regions listed in the column. The next part gives a detailed exploration of the data.

2.1 Data Exploration

An in-depth analysis of the datasets reveals that the mean and median values for the indigenous population are low, but the maximum value is 50, which suggests that there are individual areas where the indigenous population is concentrated and that these areas may be areas of potential poverty and poor sanitation and should be areas where there are more cases of tuberculosis. The mean and median illiteracy rates are only 14 and 11, respectively, indicating that illiteracy is not widespread. However, the maximum value of 41 suggests that there are specific backward areas with significant populations lacking access to education, which also suggests that the area seems poor and has poor sanitation. There are still some places that are less urbanised, where there may be more occurrences of tuberculosis, but the mean, median, and minimum values for urbanisation are 70 and 22, respectively, suggesting that most areas are highly urbanised. Based on the population density data, most locations can fit one person in a room, but the highest value of 1.6 highlights the existence of some places with very high population densities, which sharply raise the rate of TB transmission. The distribution of the poverty data suggests that each district has

different poverty levels, with only a limited number of districts where poverty is not a significant issue. Although the general results on inadequate sanitation are low, the maximum score of 58 indicates that some districts have poor sanitation and substantial disease risk. Although average unemployment rates are low, a maximum value of 20 indicates that some isolated regions may experience severe economic hardship, protracted social unrest, and potentially significant morbidity rates. With a minimum value of 0, notification timeliness data has a fairly wide range.

Data from the actual world follow a normal distribution. However, some of the columns indicate otherwise. Timeliness, Unemployment, and Urbanization are approximations of a normal distribution with few extremes, whilst the remainder is multimodal normal distributions. The above suggests that employing semi-parametric or nonparametric models to demonstrate the relationship between the target and predictors would be advantageous. The target variable in this study is a risk, defined as 'TB/Population',' whereas the remaining variables are possible predictors. As demonstrated in the table below, most features in the dataset exhibit some connection. As some features are correlated, basic regression cannot be used because it would yield false results; rather, models that account for the connection can be used. It is vital to note that some characteristics are anticipated to have positive correlations (tuberculosis versus population, density versus poverty) and vice versa. Specifically, population density, poverty, health conditions, unemployment, and notification timeliness are likely high due to the high population density, the low economic share per capita, the high poverty rate, and the high jobless rate.

We want to model the count of cases TB_i by actually modelling ρ_i using

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \ TB_i$$
 indep.
 $log(\lambda_i) = log(z_i) + log(\rho_i)$

where TB_i is the count of TB cases, z_i is the total population. Model $log(\rho_i)$ as

$$log(\rho_i) = \sum_{j=1}^{8} f_j(x_{i,j})$$
$$f(x_i) = \sum_{k=1}^{q} \beta_k b_k(x_i)$$

where $x_{i,j}$ is the jth covariate (out of 8 socio-economic covariates) for the ith instance/datum in the dataset and $f(\cdot)$ is a smooth function of said covariate. Hence, the model

boils down to

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \ TB_i \text{ indep.}$$

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^8 \sum_{k=1}^q \beta_{j,k} b_{j,k}(x_{i,j})$$

Looking at the distribution of the residuals of the model, we can see that the data is clearly far too overdispersed to be modelled by a Poisson, which has a fixed dispersion parameter. Even with 80 knots per smooth term the model doesn't seem to have enough flexibility which may be another indicator that a Poisson model is unsuitable for such overdispersed data. We propose the conventional alternative to the Poisson - the Negative Binomial model. Doing so, leads to a drop in the AIC. So the model distribution is changed to Negative Binomial with the same parameterisation except for the feature that the count of TB cases is now Negative Binomial distributed with mean as described above. See Table 1 appendix for a showcase of different model configurations and their associated AIC.

$$TB_i \sim NB(\lambda_i, \sigma_i^2) \ TB_i \ \mathrm{indep}.$$

$$\lambda_i = z_i \rho_i; \ \sigma_i^2 = \lambda_i + \frac{\lambda_i^2}{k}$$

$$log(\lambda_i) = log(z_i) + \sum_{i=1}^8 \sum_{k=1}^q \beta_{j,k} b_{j,k}(x_{i,j})$$

where k is a dispersion parameter, later estimated by the gam function in R.

When having a look at the relationship between the squared residuals and the fitted values one sees that the relation is not exactly quadratic, but rather close to 0, which would reflect the relation between model variance and the expected value in a Gaussian Distribution Model (additional evidence is provided by the Residuals vs. Fitted plot). However, fitting a Gaussian model leads to very skewed residuals, indicating that the data is apparently not Gaussian. So the model distribution is changed to Negative Binomial with the same parameterization except for the feature that the count of TB cases is now Negative Binomial distributed with mean λ_i as described above.

Given this base model, we investigate whether all given socio-economic variables are needed to explain the response or whether there exists a model with fewer parameters. The p-value for the smooth term of Illiteracy points towards it not being statistically significant. Poverty, although not statistically insignificant, has the second-largest p-value. These terms are sequentially dropped and the resulting model checked against the original model via a Likelihood Ratio Test (conducted using the anova function in R). We find that leaving out Illiteracy does not alter the model at a 5%-level of significance, whereas taking out both Poverty and Illiteracy does. So, in the following, we use a model with all of the socio-economic variables except Illiteracy. Note that this converts our linear predictor to

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j})$$

This leaves us with a model with AIC = 14,391.19 and deviations on high as well as low quantiles. Finally, we cre-43.9% of deviance explained. Running gam.check() lets at a spatio-temporal model, including both Year as well

us analyse the residual plots (see Figure) and examine the basis functions for the model. The QQ plot tells us that the model fails to predict well on the upper and lower ends of the response variable. Increasing the knots to 20 per covariate leads to marginal improvement with 44.9% deviance explained. More efficient extensions can be to add 1) spatial, 2) temporal and 3) spatio-temporal covariates.

First, we will try adding spatial terms. The spatial model adds a smoothed term which is function of the longitude and the latitude. A bivariate function is used because it makes sense to assume that there are more cases at certain locations (defined by the interaction between latitude and longitude) than others, rather than there being more cases at locations with a certain longitude for any latitude, or the other way round. Hence, our linear predictor is now

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Using this model with the regular s smoother function from the mgcv package leads to a model that can explain 56.4% of the deviance and has a slightly lower AIC of 14,013.13. The QQ plot still points to the upper and lower tails being incorrectly predicted. At the cost of significantly more computation, using a tensor product smooth te on the bivariate spatial term with 20 knots allows us to make a decent improvement on this. See Appendix for different numbers of knots that were tested. This gets us to 69.9% deviance explained. The QQ plot looks considerably better with only a few problematic instances at the top and bottom quantiles.

We contest this with an extension on the model with only socio-economic covariates, but instead of adding spatial terms, we add the temporal dimension Year. The linear predictor becomes

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} f_{2012,j}(x_{i,j}) \times x_{2012} + \sum_{j=1}^{7} f_{2013,j}(x_{i,j}) \times x_{2013} + \sum_{j=1}^{7} f_{2013,j}(x_{i,j}) \times x_{2013}$$

where the new terms $x_{2012}, x_{2013}, x_{2014}$ are indicator variables equating to 1 if Year is respectively 2012,2013,2014 and zero otherwise. Exercising some shorthand, it can be expressed as

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \sum_{j=1}^{7} f_{t,j}(x_{i,j}) \times x_t$$

where x_t is now the indicator variable for Year. A slightly separate approach can be tested with Year as a covariate instead of a grouping variable. In that case, the linear predictor would be

$$log(\lambda_i) = log(z_i) + \sum_{i=1}^{7} f_{t,j}(x_{i,j}) + \sum_{t=2012}^{2014} \beta_t x_t$$

Neither of the temporal formulations show much increase in deviance explained (the one with year as grouping variable actually shows a decrease to 41.5%!). Their QQ plots are also much worse than the spatial model, showing gross deviations on high as well as low quantiles. Finally, we create a spatio-temporal model, including both Year as well

as lon, lat. Its linear predictor is formulated as below

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \left(\sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k (lon_{iot} a_{ii}) \right)$$
 does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do does not seem as accurate. But the highest residuals do do do does not seem as accurate. But the highest residuals do do do do does not seem as accurate. But the highest residuals do do do does not seem as accurate. But the highest residuals do do do do does

This is a model which includes the term for the location and estimates a functional relation for each year and each explaining variable. The AIC of this model does not drop compared to the spatial model, so the spatial model (given that it is simpler) is the model we choose to best explain the ratio of TB cases per capita. To recall, it is formulated as

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Let us now have a closer look at the fit of the spatial model: It fits well even though the largest residuals are higher than expected from the model distribution. For districts that have a high number of cases, the predictor does not seem as accurate. But the highest residuals do not seem as accurate. But the highest residuals do not seem as accurate. But the highest residuals do not seem as accurate. But the highest residuals do not seem as accurate. But the highest residuals do not seem as accurate. But the highest residuals do not seem as accurate. But the highest residuals of TB cases is high (see residuals vs. response). The variance of the model still seems too low for those values given that there are some predicted values in that high segment of response values (absolute number of TB cases) where the prediction for the response value is lower than the actual value is higher than the actual value.

Using this model, we predict the rate of TB per 100,000 inhabitants. See Figure 1

3 Critical Review and Conclusion

Drawbacks of the Model:

- 1. Predictions do not cover full range of data, as evinced by deviations in the QQ plot
- 2 ?7

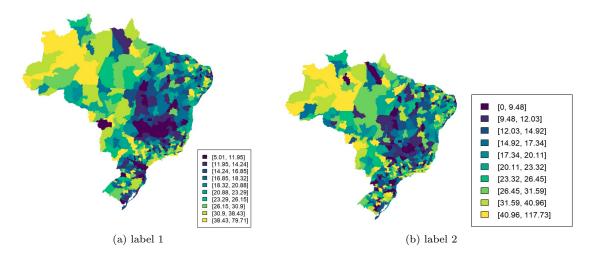


Figure 1: Predicted (a) and True(b) rates of TB per 100k inhabitants

4 References

Wood, S. N. (2017). Generalized Additive Models: An Introduction with R (2nd ed.). CRC Press.

5 Appendix

5.1 Figures

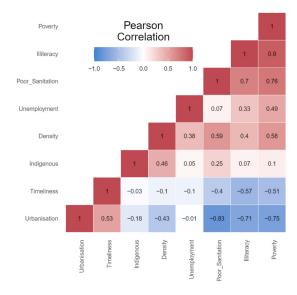


Figure 2: Correlogram shows covariates with highest positive and negative correlations.

5.2 Tables

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

5.3 Code

```
# Import Libraries
     library(mgcv) # required for GAM
     library(tidyverse)
     library(ggplot2) # required for plotting
     library(dplyr) # required for filtering dataset
     library(fields) # required for maps
     library(maps) # required for maps
     library(reshape2) # only required for melt in corr plot
     library(car) # only required for VIF
10
11
12
     load("C:/Users/soura/Documents/COMM511/group_coursework/datasets_project.RData")
13
14
     # Investigate correlation
15
     ### Resource -> http://www.sthda.com/english/wiki/ggplot2-quick-
16
     \textit{### correlation-matrix-heatmap-r-software-and-data-visualization}
17
     cormat <- cor(TBdata[,c(1,2,3,4,5,6,7,8)])</pre>
18
19
     # Reorder
20
     reorder_cormat <- function(cormat){</pre>
21
     # Use correlation between variables as distance
     dd <- as.dist((1-cormat)/2)</pre>
22
    hc <- hclust(dd)
24
     cormat <-cormat[hc$order, hc$order]</pre>
25
     # Reorder the correlation matrix
     cormat <- reorder_cormat(cormat)</pre>
     # Get lower triangular matrix
29
```

```
cormat[lower.tri(cormat)] <- NA</pre>
30
31
     melted_cormat <- melt(cormat , na.rm = TRUE)</pre>
32
     melted_cormat$value = round(melted_cormat$value, 2)
33
34
     # Create a ggheatmap
35
     ggheatmap <- ggplot(melted_cormat, aes(Var2, Var1, fill = value))+</pre>
36
     geom_tile(color = "white")+
     scale_fill_gradient2(low = "#1a85d6", high = "#cf3e4f", mid = "white",
     midpoint = 0, limit = c(-1,1), space = "Lab",
     name="Pearson\nCorrelation") +
41
     theme_minimal()+ # minimal theme
     theme(axis.text.x = element_text(angle = 90, vjust = 1,
42
     size = 12, hjust = 1))+
43
     coord_fixed()
44
45
     # Add correlation coefficients
46
     ggheatmap +
47
     geom_text(aes(Var2, Var1, label = value), color = "black", size = 2) +
48
49
     axis.text.x = element_text(size = 6),
50
     axis.text.y = element_text(size = 6),
51
     axis.title.x = element_blank(),
52
     axis.title.y = element_blank(),
53
     panel.grid.major = element_blank(),
54
     panel.border = element_blank(),
55
     panel.background = element_blank(),
56
57
     axis.ticks = element_blank(),
     legend.justification = c(1, 0),
     legend.position = c(0.6, 0.7),
     legend.direction = "horizontal"
     legend.text = element_text(size = 6)
61
62
     guides(fill = guide_colorbar(barwidth = 7, barheight = 1,
63
     title.position = "top", title.hjust = 0.5))
64
65
     #### Illiteracy is highly correlated with Poverty
66
     #### Carry out a Variance Inflation Test
67
     model_all <- lm(TB ~ . , data = select(TBdata, 'Indigenous' , 'Illiteracy' ,</pre>
68
      'Urbanisation', 'Density', 'Poverty', 'Unemployment', 'Timeliness', 'Year',
69
     'TB' , 'Population')) # with all the independent variables
70
71
72
     vif(model_all) # Several variables are highly correlated
73
74
     model_no_illiteracy <- lm(TB ~ . , data = select(TBdata, 'Indigenous',</pre>
75
     'Urbanisation', 'Density', 'Poverty', 'Unemployment', 'Timeliness', 'Year',
76
     'TB' , 'Population')) # with all the independent variables
77
     vif(model_no_illiteracy) # Poverty and Unemployment still seem highly correlated
78
79
     model_no_illiteracy_no_poverty <- lm(TB ~ . , data = select(TBdata, 'Indigenous',</pre>
80
     'Urbanisation', 'Density', 'Unemployment', 'Timeliness', 'Year',
81
     'TB' , 'Population')) # with all the independent variables
82
83
     vif(model_no_illiteracy_no_poverty) # almost no variable is highly correlated
84
85
     ## More formal tests are conducted to confirm the dropping of Illiteracy.
86
     ## Check to see if Poverty should be dropped as well
87
     prelim.model.1 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +</pre>
88
     s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
89
     + s(Unemployment) + s(Timeliness),
90
     data = TBdata ,
91
     family = nb(link = 'log')
92
93
94
     # Show summary
95
     summary(prelim.model.1)
     par(mfrow=c(2,2))
96
```

```
gam.check(prelim.model.1)
97
98
      ### Only the effect of illiteracy cannot be reliably stated to be non-zero
99
      prelim.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
100
      + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
101
      + s(Unemployment) + s(Timeliness),
      data = TBdata ,
103
      family = nb(link = 'log')
105
      # Show summary
106
      summary(prelim.model.2)
107
108
109
      # Show summary
110
      summary(prelim.model.2)
      par(mfrow=c(2,2))
111
      gam.check(prelim.model.2)
112
113
      # Likelihood ratio test
114
      anova(prelim.model.1 , prelim.model.2 , test = 'F') # p-value is over 0.05
115
      # The models are statistically indistinguishable
116
117
      \#\#\# Only the effect of illiteracy cannot be reliably stated to be non-zero
118
      prelim.model.3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
119
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
120
      + s(Timeliness),
121
      data = TBdata,
122
      family = nb(link = 'log')
123
124
125
      # Show summary
      summary(prelim.model.3)
      # Likelihood ratio test
128
      anova(prelim.model.2 , prelim.model.3 , test = 'F') # p-value is less than 0.05
129
      # The models are statistically different. Poverty should not be excluded.
130
131
      ### Model chosen (with social covariates) is the negative binomial without
132
      ### Illiteracy
133
      summary(prelim.model.2) # Only 44% of the deviance is explained. Adding temporal
134
      # and spatial covariates may improve this
135
      par(mfrow=c(2,2))
136
      gam.check(prelim.model.2)
137
138
      par(mfrow = c(1,1))
139
      \#\#\# Only the effect of illiteracy cannot be reliably stated to be non-zero
140
141
      prelim.model.4 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous , k = 20)</pre>
      + s(Urbanisation , k = 20) + s(Density , k = 20) + s(Poverty , k = 20)
142
143
      + s(Poor_Sanitation, k = 20) + s(Unemployment, k = 20) + s(Timeliness, k = 20),
      data = TBdata ,
144
      family = nb(link = 'log')
145
146
      # Show summary
147
      summary(prelim.model.4)
148
149
      par(mfrow=c(2,2))
150
      gam.check(prelim.model.4)
151
152
      ### Adding spatial covariates
153
      spatial.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
154
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) +s(Poverty)
155
      + s(Timeliness) + s(lon , lat),
156
      data = TBdata ,
157
      family = nb(link = 'log')
158
159
      # Check summary
160
161
      summary(spatial.model)
      # Check the smooth functions of the covars
162
163
      plot(spatial.model)
```

```
par(mfrow=c(2,2))
164
165
      gam.check(spatial.model)
166
      par(mfrow = c(1,1))
167
      spatial.model$aic
168
      ### Using separate smoothers
169
      spatial.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
170
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
      + s(Timeliness) + te(lon , lat , k = 20),
      data = TBdata ,
      family = nb(link = 'log')
174
175
176
      # Check summary
177
      summary(spatial.model.2)
      # Check the smooth functions of the covariates
178
      plot(spatial.model.2)
179
      par(mfrow=c(2,2))
180
      gam.check(spatial.model.2 , pch = 20)
181
      par(mfrow = c(1,1))
182
183
      ### PREDICTIONS
184
      fitted_nb <- predict(spatial.model.2, newdata = TBdata , type = 'response')</pre>
185
186
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
187
      par(mfrow = c(1,1))
      plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
192
      TBdata$Year.asFactor <- factor(TBdata$Year)</pre>
      #### Temporal covariates
194
      temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
195
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
196
      + s(Timeliness) + Year.asFactor,
197
      data = TBdata ,
198
      family = nb(link = 'log')
199
200
      # Check summary
201
      summary(temporal.model) # Temporal alone doesn't add much to explaining the variance
202
      # Check the smooth functions of the covariates
203
      plot(temporal.model)
204
      par(mfrow=c(2,2))
205
206
      gam.check(temporal.model)
207
      par(mfrow = c(1,1))
208
209
      #### Temporal covariates
      temporal.model.2 <- gam(formula = TB ~ offset(log(Population))</pre>
210
      + s(Indigenous , by=Year.asFactor) + s(Urbanisation , by=Year.asFactor)
^{211}
      + s(Density , by=Year.asFactor) + s(Poor_Sanitation , by=Year.asFactor)
212
      + s(Unemployment , by=Year.asFactor) + s(Poverty , by=Year.asFactor)
213
      + s(Timeliness , by=Year.asFactor) ,
214
215
      data = TBdata ,
      family = nb(link = 'log')
216
217
218
      # Check summary
      summary(temporal.model.2) # Temporal alone doesn't add much to explaining the variance
219
      # Check the smooth functions of the covariates
220
      plot(temporal.model.2)
221
      par(mfrow=c(2,2))
222
      gam.check(temporal.model.2)
223
      par(mfrow = c(1,1))
224
225
      ### Spatio-temporal model
226
      spatio.temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
227
228
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
229
230
      data = TBdata,
```

```
family = nb(link = 'log')
231
232
233
      # Check summary
      summary(spatio.temporal.model) # Temporal alone doesn't add much to explaining the variance
234
      # Check the smooth functions of the covariates
235
      plot(spatio.temporal.model)
      par(mfrow=c(2,2))
237
      gam.check(spatio.temporal.model , pch = 20)
238
      par(mfrow = c(1,1))
      # PLOT
242
      fitted_nb <- predict(spatio.temporal.model, newdata = TBdata , type = 'response')</pre>
243
      TBdata$pred_rate <- fitted_nb/TBdata$Population*100000</pre>
244
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
245
      par(mfrow = c(1,1))
246
      plot.map(TBdata$pred_rate , n.levels = 10)
247
248
      ### WIHTOUT POVERTY
249
      ### Spatio-temporal model - Poverty
250
      spatio.temporal.model.wo.pov <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
251
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
252
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
253
      data = TBdata ,
254
      family = nb(link = 'log')
      # Check summary
      summary(spatio.temporal.model.wo.pov) # Temporal alone doesn't add much to explaining the variance
      # Check the smooth functions of the covariates
      plot(spatio.temporal.model.wo.pov)
      par(mfrow=c(2,2))
262
      gam.check(spatio.temporal.model.wo.pov , pch = 20)
263
      par(mfrow = c(1,1))
264
265
266
      anova(spatio.temporal.model.wo.pov , spatio.temporal.model , test = 'F')
267
268
      ### Without Indigenous
269
      spatio.temporal.model.wo.indig <- gam(formula = TB ~ offset(log(Population))</pre>
270
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
271
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
272
      data = TBdata ,
273
      family = nb(link = 'log')
274
275
276
      # Check summary
^{277}
      summary(spatio.temporal.model.wo.indig) # Temporal alone doesn't add much to explaining the variance
278
      # Check the smooth functions of the covariates
279
      plot(spatio.temporal.model.wo.indig)
280
      par(mfrow=c(2,2))
      gam.check(spatio.temporal.model.wo.indig , pch = 20)
282
283
      par(mfrow = c(1,1))
284
      # Anova test
285
      anova(spatio.temporal.model.wo.indig , spatio.temporal.model , test = 'F')
286
      # NOT ENOUGH EVIDENCE TO REMOVE INDIGENOUS
287
288
      ## Spatio-temporal model
289
      spatio.temporal.model.true <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
290
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
291
      + s(Timeliness) + te(lon , lat , Year , k = 3),
292
      data = TBdata ,
293
      family = nb(link = 'log')
294
295
296
297
      # Check summary
```

```
summary(spatio.temporal.model.true) # Temporal alone doesn't add much to explaining the variance
298
          # Check the smooth functions of the covariates
299
          plot(spatio.temporal.model.true)
300
          par(mfrow=c(2,2))
301
          gam.check(spatio.temporal.model.true , pch = 20)
          par(mfrow = c(1,1))
303
          spatio.temporal.model$aic
          spatio.temporal.model.true$aic
306
          spatio.temporal.model.wo.indig$aic
307
308
309
310
          ### Spatio-temporal model - no poor sanitation , indigenous
311
          spatio.temporal.model.wo.indig.poor_s <- gam(formula = TB ~ offset(log(Population))</pre>
312
          + s(Urbanisation) + s(Density) + s(Unemployment) + s(Poverty)
313
          + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
314
          data = TBdata ,
315
         family = nb(link = 'log')
316
317
          # Check summary
318
          summary(spatio.temporal.model.wo.indig.poor_s) # Temporal alone doesn't add much to explaining the variance
319
          # Check the smooth functions of the covariates
320
          plot(spatio.temporal.model.wo.indig.poor_s)
321
          par(mfrow=c(2,2))
          gam.check(spatio.temporal.model.wo.indig.poor_s , pch = 20)
          par(mfrow = c(1,1))
          anova(spatio.temporal.model , spatio.temporal.model.wo.indig , rest = 'F')
          library (mgcv)
329
330
331
          par(mfrow = c(2,2))
332
          #fit poisson model with socio-economic variables
333
          model_poisson < -gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Illiteracy) + s(Urbanisation) + s(Density) + s(Population) +
334
          summary(model_poisson)
335
336
          model_poisson$aic
          par(mfrow = c(2,2), pch = 20)
337
338
          gam.check(model_poisson)
339
          # #add flexibility
         model_poisson <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, k = 80) + s(Illiteracy, k = 80) + s(Urbanisation, k =
340
341
          gam.check(model_poisson)
          summary(model_poisson) # SIGNS OF OVERFIT
342
          plot(model_poisson) # SIGNS OF OVERFIT
343
          #fit negative binomial model with socioeconomic
344
          model_nb <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty)
345
          summary(model_nb)
          model_nb$aic
          #fit a linear relation between squured residuals and prediction to see whether another model describes the variance-fitted values re
          summary(lm(log(model_nb$residuals^2) ~ log(predict(model_nb, type = 'response'))))
349
350
          #drop Illiteracy
          model_nb_2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poverty) + s(Poverty)
351
352
353
          #LRT
354
          anova.gam(model_nb_2, model_nb, test = 'LRT')
355
          #Null hypothesis not rejected -> drop poverty
356
          model_nb_3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Urbanisation)
357
358
          anova.gam(model_nb_3, model_nb_2, test = 'LRT')
359
          # NULL hypothesis rejected -> drop only illiteracy, not poverty
360
361
         model_nb_final <- model_nb_2</pre>
362
363
          summary(model_nb_final)
364
          gam.check(model_nb_final)
```

```
fitted_nb <- predict(model_nb_final, type = 'response')</pre>
365
366
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
367
368
      par(mfrow = c(1,2))
      plot.map(log(fitted_nb) - log(TBdata$Population) , n.levels = 10)
      plot.map(log(TBdata$TB) - log(TBdata$Population))
      #temporal model
372
      par(mfrow = c(2,2), pch = 20)
373
      model_nb_time <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, by = Year) + s(Urbanisation, by = Year) + s(Density, by
374
      summary(model_nb)
375
376
      model_nb$aic
377
      #spatial model
378
      model_nb_space <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_S
379
      summary(model_nb_space)
380
      model_nb_space$aic
381
      gam.check(model_nb_space)
382
      anova.gam(model_nb_space, model_nb_final, test = 'LRT')
383
384
      #spatio-temporal model
385
      model_nb_time_and_space <- gam(formula = TB ~ offset(log(Population)) + s(Urbanisation, by = Year) + s(Density, by = Year) + s(Pove
      summary(model_nb_time_and_space)
      {\tt model\_nb\_time\_and\_space\$aic}
      gam.check(model_nb_time_and_space)
      # PLOT
      fitted_nb <- predict(model_nb_time_and_space, newdata = TBdata , type = 'response')</pre>
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
394
      par(mfrow = c(1,1))
395
      plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
396
```