# Analysis of 2012–2014 Brazil Tuberculosis Data

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#### 1 Introduction

#### Problem Statement

Analysis of tuberculosis (TB) data originating from Brazil using Generalized Additive Models (GAMs). Brazil is divided into 557 administrative microregions, and the available data comprises counts of TB cases in each microregion for each of the years from 2012 to 2014.

### 2 Exploratory Analysis of Data and Problem

The TB data from Brazil includes 1,671 entries or samples with 14 columns of numeric data types that specify the characteristics of each sample. The columns are self-explanatory because they are called Indigenous, Illiteracy, Urbanisation, Density, Poverty, Poor Sanitation, Unemployment, Timeliness, Year, TB, Population, Region, Ion, and Iat. TB stands for tuberculosis, whereas Ion and Iat stand for longitude and latitude. The dataset has no missing values in the technical sense, but it contains some abnormalities, which increases the amount of pre-processing needed. The region is stored as a continuous variable despite being a factor variable. Nonetheless, changing it depends on the task at hand. Moreover, the collection includes coordinates that describe the precise geospatial locations of the micro-regions listed in the column. The next part gives a detailed exploration of the data.

#### **Data Exploration**

An in-depth analysis of the datasets reveals that the mean and median values for the indigenous population are low, but the maximum value is 50, which suggests that there are individual areas where the indigenous population is concentrated and that these areas may be areas of potential poverty and poor sanitation and should be areas where there are more cases of tuberculosis. The mean and median illiteracy rates are only 14 and 11, respectively, indicating that illiteracy is not widespread. However, the maximum value of 41 suggests that there are specific backward areas with significant populations lacking access to education, which also suggests that the area seems poor and has poor sanitation. There are still some places that are less urbanised, where there may be more occurrences of tuberculosis, but the mean, median, and minimum values for urbanisation are 70 and 22, respectively, suggesting that most areas are highly urbanised. Based on the population density data, most locations can fit one person in a room, but the highest value of 1.6 highlights the existence of some places with very high population densities, which sharply raise the rate of TB transmission. The distribution of the poverty data suggests that each district has different poverty levels, with only a limited number of districts where poverty is not a significant issue. Although the general results on inadequate sanitation are low, the maximum score of 58 indicates that some districts have poor sanitation and substantial disease risk. Although average unemployment rates are low, a maximum value of 20 indicates that some isolated regions may experience severe economic hardship, protracted social unrest, and potentially significant morbidity rates. With a minimum value of 0, notification timeliness data has a fairly wide range.

Data from the actual world follow a normal distribution. However, some of the columns indicate otherwise. Timeliness, Unemployment, and Urbanization are approximations of a normal distribution with few extremes, whilst the remainder is multimodal normal distributions. The above suggests that employing semi-parametric or non-parametric models to demonstrate the relationship between the target and predictors would be advantageous. The target variable in this study is a risk, defined as 'TB/Population',' whereas the remaining variables are possible predictors. As demonstrated in the table below, most features in the dataset exhibit some connection. As some features are correlated, basic regression cannot be used because it would yield false results; rather, models that account for the connection can be used. It is vital to note that some characteristics are anticipated to have positive correlations (tuberculosis versus population, density versus poverty) and vice versa. Specifically, population density, poverty, health conditions, unemployment, and notification timeliness are likely high due to the high population density, the low economic share per capita, the high poverty rate, and the high jobless rate.

#### 3 Model

We want to model the count of cases  $TB_i$  by actually modelling  $\rho_i$  using

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \ TB_i$$
 indep.  
 $log(\lambda_i) = log(z_i) + log(\rho_i)$ 

where  $TB_i$  is the count of TB cases,  $z_i$  is the total population. Model  $log(\rho_i)$  as

$$log(\rho_i) = \sum_{j=1}^{8} f_j(x_{i,j})$$
$$f(x_i) = \sum_{k=1}^{q} \beta_k b_k(x_i)$$

where  $x_{i,j}$  is the jth covariate (out of 8 socio-economic covariates) for the ith instance/datum in the dataset and  $f(\cdot)$  is a smooth function of said covariate. Hence, the model boils down to

$$TB_i \sim Pois(\lambda_i = z_i \rho_i) \ TB_i \ \text{indep.}$$
 
$$log(\lambda_i) = log(z_i) + \sum_{j=1}^8 \sum_{k=1}^q \beta_{j,k} b_{j,k}(x_{i,j})$$

Looking at the distribution of the residuals of the model, we can see that the data is clearly far too overdispersed to be modelled by a Poisson, which has a fixed dispersion parameter. Even with 80 knots per smooth term the model doesn't seem to have enough flexibility which may be another indicator that a Poisson model is unsuitable for such overdispersed data. We propose the conventional alternative to the Poisson - the Negative Binomial model. Doing so, leads to a drop in the AIC. So the model distribution is changed to Negative Binomial with the same parameterisation except for the feature that the count of TB cases is now Negative Binomial distributed with mean as described above. See Table ?? appendix for a showcase of different model configurations and their associated AIC.

$$TB_i \sim NB(\lambda_i, \sigma_i^2) \ TB_i \ \text{indep.}$$

$$\lambda_i = z_i \rho_i; \ \sigma_i^2 = \lambda_i + \frac{\lambda_i^2}{k}$$

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^8 \sum_{k=1}^q \beta_{j,k} b_{j,k}(x_{i,j})$$

where k is a dispersion parameter, later estimated by the gam function in R.

When having a look at the relationship between the squared residuals and the fitted values one sees that the relation is not exactly quadratic, but rather close to 0, which would reflect the relation between model variance and the expected value in a Gaussian Distribution Model (additional evidence is provided by the Residuals vs. Fitted plot). However, fitting a Gaussian model leads to very skewed residuals, indicating that the data is apparently not Gaussian. So the model distribution is changed to Negative Binomial with the same parameterization except for the feature that the count of TB cases is now Negative Binomial distributed with mean  $\lambda_i$  as described above.

Given this base model, we investigate whether all given socio-economic variables are needed to explain the response or whether there exists a model with fewer parameters. The p-value for the smooth term of Illiteracy points towards it not being statistically significant. Poverty, although not statistically insignificant, has the second-largest p-value. These terms are sequentially dropped and the resulting model checked against the original model via a Likelihood Ratio Test (conducted using the anova function in R). We find that leaving out Illiteracy does not alter the model at a 5%-level of significance, whereas taking out both Poverty and Illiteracy does. So, in the following, we use a model with all of the socio-economic variables except Illiteracy. Note that this converts our linear predictor to

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j})$$

This leaves us with a model with AIC = 14,391.19 and 43.9% of deviance explained. Running gam.check() lets us analyse the residual plots (see Figure) and examine the basis functions for the model. The QQ plot tells us that the model fails to predict well on the upper and lower ends of the response variable. Increasing the knots to 20 per covariate leads to marginal improvement with 44.9% deviance explained. More efficient extensions can be to add 1) spatial, 2) temporal and 3) spatio-temporal covariates.

First, we will try adding spatial terms. The spatial model adds a smoothed term which is function of the longitude and the latitude. A bivariate function is used because it makes sense to assume that there are more cases at certain locations

(defined by the interaction between latitude and longitude) than others, rather than there being more cases at locations with a certain longitude for any latitude, or the other way round. Hence, our linear predictor is now

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Using this model with the regular s smoother function from the mgcv package leads to a model that can explain 56.4% of the deviance and has a slightly lower AIC of 14,013.13. The QQ plot still points to the upper and lower tails being incorrectly predicted. At the cost of significantly more computation, using a tensor product smooth te on the bivariate spatial term with 20 knots allows us to make a decent improvement on this. See Appendix for different numbers of knots that were tested. This gets us to 69.9% deviance explained. The QQ plot looks considerably better with only a few problematic instances at the top and bottom quantiles.

We contest this with an extension on the model with only socio-economic covariates, but instead of adding spatial terms, we add the temporal dimension Year. The linear predictor becomes

$$log(\lambda_i) = log(z_i) + \sum_{j=1}^{7} f_{2012,j}(x_{i,j}) \times x_{2012} + \sum_{j=1}^{7} f_{2013,j}(x_{i,j}) \times x_{2013} + \sum_{j=1}^{7} f_{2014,j}(x_{i,j}) \times x_{2014}$$

where the new terms  $x_{2012}, x_{2013}, x_{2014}$  are indicator variables equating to 1 if Year is respectively 2012,2013,2014 and zero otherwise. Exercising some shorthand, it can be expressed as

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \sum_{i=1}^{7} f_{t,j}(x_{i,j}) \times x_t$$

where  $x_t$  is now the indicator variable for Year. A slightly separate approach can be tested with Year as a covariate instead of a grouping variable. In that case, the linear predictor would be

$$log(\lambda_i) = log(z_i) + \sum_{i=1}^{7} f_{t,j}(x_{i,j}) + \sum_{t=2012}^{2014} \beta_t x_t$$

Neither of the temporal formulations show much increase in deviance explained (the one with year as grouping variable actually shows a decrease to 41.5%!). Their QQ plots are also much worse than the spatial model, showing gross deviations on high as well as low quantiles. Finally, we create a spatio-temporal model, including both Year as well as lon,lat. Its linear predictor is formulated as below

$$log(\lambda_i) = log(z_i) + \sum_{t=2012}^{2014} \left( \sum_{j=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i) \right) \times x_t$$

This is a model which includes the term for the location and estimates a functional relation for each year and each explaining variable. The AIC of this model does not drop compared to the spatial model, so the spatial model (given that it is simpler) is the model we choose to best explain the ratio of TB cases per capita. To recall, it is formulated as

$$log(\lambda_i) = log(z_i) + \sum_{i=1}^{7} \sum_{k=1}^{q} \beta_{j,k} b_{j,k}(x_{i,j}) + \sum_{k=1}^{q} \beta_k b_k(lon_i, lat_i)$$

Let us now have a closer look at the fit of the spatial model: It fits well even though the largest residuals are higher than expected from the model distribution. For districts that have a high number of cases, the predictor does not seem as accurate. But the highest residuals do not arise when the ratio of TB cases per capita is extraordinarily high, but rather when the absolute number of TB cases is high (see residuals vs. response). The variance of the model still seems too low for those values given that there are some predicted values in that high segment of response values (absolute number of TB cases) where the prediction for the response value is lower than the actual value, and some where the prediction of the actual value is higher than the actual value. Using this model, we predict the rate of TB per 100,000 inhabitants. See Figure ??

#### 4 Critical Review and Conclusion

Drawbacks of the Model:

- 1. Predictions do not cover full range of data, as evinced by deviations in the QQ plot
- 2. ??

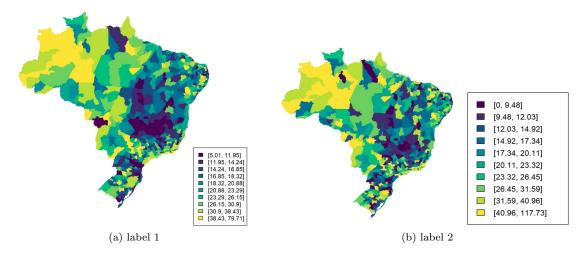


Figure 1: Predicted (a) and True(b) rates of TB per 100k inhabitants

### 5 References

Wood, S. N. (2017). Generalized Additive Models: An Introduction with R (2nd ed.). CRC Press.

## 6 Appendix

### **Figures**

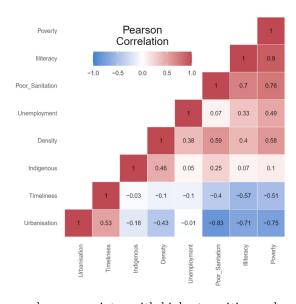


Figure 2: Correlogram shows covariates with highest positive and negative correlations.

#### **Tables**

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

#### Code

```
library(maps) # required for maps
     library(reshape2) # only required for melt in corr plot
     library(car) # only required for VIF
10
11
     # I.oad Data
12
     load("C:/Users/soura/Documents/COMM511/group_coursework/datasets_project.RData")
13
14
     # Investigate correlation
15
     ### Resource -> http://www.sthda.com/english/wiki/ggplot2-quick-
16
     \textit{##\# correlation-matrix-heatmap-r-software-and-data-visualization}
     cormat <- cor(TBdata[,c(1,2,3,4,5,6,7,8)])</pre>
     # Reorder
     reorder_cormat <- function(cormat){</pre>
20
     # Use correlation between variables as distance
     dd <- as.dist((1-cormat)/2)</pre>
22
     hc <- hclust(dd)
23
     cormat <-cormat[hc$order, hc$order]</pre>
24
25
26
     # Reorder the correlation matrix
27
     cormat <- reorder_cormat(cormat)</pre>
28
     # Get lower triangular matrix
29
     cormat[lower.tri(cormat)] <- NA</pre>
30
31
     melted_cormat <- melt(cormat , na.rm = TRUE)</pre>
32
     melted_cormat$value = round(melted_cormat$value, 2)
33
34
35
     # Create a ggheatmap
     ggheatmap <- ggplot(melted_cormat, aes(Var2, Var1, fill = value))+</pre>
36
     geom_tile(color = "white")+
37
     scale_fill_gradient2(low = "#1a85d6", high = "#cf3e4f", mid = "white",
     midpoint = 0, limit = c(-1,1), space = "Lab",
     name="Pearson\nCorrelation") +
     theme_minimal()+ # minimal theme
     theme(axis.text.x = element_text(angle = 90, vjust = 1,
42
     size = 12, hjust = 1))+
43
44
     coord_fixed()
45
     # Add correlation coefficients
46
     ggheatmap +
47
     geom_text(aes(Var2, Var1, label = value), color = "black", size = 2) +
48
     theme(
49
     axis.text.x = element_text(size = 6),
50
     axis.text.y = element_text(size = 6),
51
     axis.title.x = element_blank(),
52
     axis.title.y = element_blank(),
53
     panel.grid.major = element_blank(),
54
     panel.border = element_blank(),
55
     panel.background = element_blank(),
56
     axis.ticks = element_blank(),
57
     legend.justification = c(1, 0),
     legend.position = c(0.6, 0.7),
     legend.direction = "horizontal",
     legend.text = element_text(size = 6)
62
     guides(fill = guide_colorbar(barwidth = 7, barheight = 1,
63
     title.position = "top", title.hjust = 0.5))
64
65
     #### Illiteracy is highly correlated with Poverty
66
     #### Carry out a Variance Inflation Test
67
     model_all <- lm(TB ~ . , data = select(TBdata, 'Indigenous' , 'Illiteracy' ,</pre>
68
      'Urbanisation', 'Density', 'Poverty', 'Unemployment', 'Timeliness', 'Year',
69
     'TB' , 'Population')) # with all the independent variables
70
71
     vif(model_all) # Several variables are highly correlated
72
73
     model_no_illiteracy <- lm(TB ~ . , data = select(TBdata, 'Indigenous',</pre>
74
```

```
'Urbanisation', 'Density', 'Poverty', 'Unemployment', 'Timeliness', 'Year',
75
      'TB' , 'Population')) # with all the independent variables
76
77
      vif(model_no_illiteracy) # Poverty and Unemployment still seem highly correlated
78
79
      model_no_illiteracy_no_poverty <- lm(TB ~ . , data = select(TBdata, 'Indigenous',</pre>
80
      'Urbanisation', 'Density', 'Unemployment', 'Timeliness', 'Year',
81
      'TB' , 'Population')) # with all the independent variables
82
83
84
      vif(model_no_illiteracy_no_poverty) # almost no variable is highly correlated
85
      ## More formal tests are conducted to confirm the dropping of Illiteracy.
      ## Check to see if Poverty should be dropped as well
      prelim.model.1 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) +</pre>
      s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
      + s(Unemployment) + s(Timeliness),
      data = TBdata ,
     family = nb(link = 'log')
93
      # Show summary
94
      summary(prelim.model.1)
95
      par(mfrow=c(2,2))
96
      gam.check(prelim.model.1)
97
98
      ### Only the effect of illiteracy cannot be reliably stated to be non-zero
99
     prelim.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
100
      + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanitation)
101
      + s(Unemployment) + s(Timeliness),
102
      data = TBdata ,
103
     family = nb(link = 'log')
104
105
      # Show summaru
106
      summary(prelim.model.2)
107
108
      # Show summary
109
      summary(prelim.model.2)
110
      par(mfrow=c(2,2))
111
112
      gam.check(prelim.model.2)
113
114
      # Likelihood ratio test
      anova(prelim.model.1 , prelim.model.2 , test = 'F') # p-value is over 0.05
115
      # The models are statistically indistinguishable
116
117
      ### Only the effect of illiteracy cannot be reliably stated to be non-zero
118
     prelim.model.3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
119
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
120
      + s(Timeliness),
121
      data = TBdata ,
122
     family = nb(link = 'log')
123
124
      # Show summaru
125
      summary(prelim.model.3)
126
127
      # Likelihood ratio test
128
      anova(prelim.model.2 , prelim.model.3 , test = 'F') # p-value is less than 0.05
129
      # The models are statistically different. Poverty should not be excluded.
130
      ### Model chosen (with social covariates) is the negative binomial without
      summary(prelim.model.2) # Only 44% of the deviance is explained. Adding temporal
134
      # and spatial covariates may improve this
135
      par(mfrow=c(2,2))
136
      gam.check(prelim.model.2)
137
      par(mfrow = c(1,1))
138
139
      ### Only the effect of illiteracy cannot be reliably stated to be non-zero
140
       prelim.model.4 <- \ gam(formula = TB \ \tilde{\ } offset(log(Population)) \ + \ s(Indigenous \ , \ k = 20) 
141
```

```
+ s(Urbanisation , k = 20) + <math>s(Density , k = 20) + s(Poverty , k = 20)
142
      + s(Poor_Sanitation , k = 20) + <math>s(Unemployment , k = 20) + s(Timeliness , k = 20),
143
      data = TBdata .
144
      family = nb(link = 'log')
145
146
147
      # Show summary
      summary(prelim.model.4)
      par(mfrow=c(2,2))
149
      gam.check(prelim.model.4)
150
      ### Adding spatial covariates
      spatial.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
154
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) +s(Poverty)
155
      + s(Timeliness) + s(lon , lat),
156
      data = TBdata ,
      family = nb(link = 'log')
158
159
      # Check summary
160
      summary(spatial.model)
161
      # Check the smooth functions of the covars
162
      plot(spatial.model)
163
      par(mfrow=c(2,2))
164
      gam.check(spatial.model)
165
      par(mfrow = c(1,1))
166
      spatial.model$aic
167
168
169
      ### Using separate smoothers
      spatial.model.2 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
170
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
171
      + s(Timeliness) + te(lon , lat , k = 20),
172
      data = TBdata ,
      family = nb(link = 'log')
174
175
      # Check summary
176
      summary(spatial.model.2)
177
      # Check the smooth functions of the covariates
178
      plot(spatial.model.2)
179
      par(mfrow=c(2,2))
180
      gam.check(spatial.model.2 , pch = 20)
181
      par(mfrow = c(1,1))
182
183
      ### PREDICTIONS
184
      fitted_nb <- predict(spatial.model.2, newdata = TBdata , type = 'response')</pre>
185
186
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
187
      par(mfrow = c(1,1))
188
      plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
189
190
191
      TBdata$Year.asFactor <- factor(TBdata$Year)</pre>
192
      #### Temporal covariates
194
      temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
      + s(Timeliness) + Year.asFactor,
      data = TBdata ,
198
      family = nb(link = 'log')
199
200
      # Check summary
201
      summary(temporal.model) # Temporal alone doesn't add much to explaining the variance
202
      # Check the smooth functions of the covariates
203
      plot(temporal.model)
204
      par(mfrow=c(2,2))
205
      gam.check(temporal.model)
206
      par(mfrow = c(1,1))
207
208
```

```
#### Temporal covariates
209
     temporal.model.2 <- gam(formula = TB ~ offset(log(Population))</pre>
210
      + s(Indigenous , by=Year.asFactor) + s(Urbanisation , by=Year.asFactor)
211
      + s(Density , by=Year.asFactor) + s(Poor_Sanitation , by=Year.asFactor)
212
      + s(Unemployment , by=Year.asFactor) + s(Poverty , by=Year.asFactor)
213
      + s(Timeliness , by=Year.asFactor) ,
214
      data = TBdata ,
215
      family = nb(link = 'log')
216
217
      # Check summary
218
      {\color{red} \textbf{summary(temporal.model.2)}} \ \textit{\# Temporal alone doesn't add much to explaining the variance}
219
220
      # Check the smooth functions of the covariates
221
      plot(temporal.model.2)
222
      par(mfrow=c(2,2))
223
      gam.check(temporal.model.2)
      par(mfrow = c(1,1))
225
      ### Spatio-temporal model
      spatio.temporal.model <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
228
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
229
      data = TBdata ,
230
      family = nb(link = 'log')
231
232
      # Check summary
233
      summary(spatio.temporal.model) # Temporal alone doesn't add much to explaining the variance
234
      # Check the smooth functions of the covariates
235
      plot(spatio.temporal.model)
236
237
      par(mfrow=c(2,2))
      gam.check(spatio.temporal.model , pch = 20)
238
239
      par(mfrow = c(1,1))
240
      # PI.NT
241
      fitted_nb <- predict(spatio.temporal.model, newdata = TBdata , type = 'response')</pre>
^{242}
      TBdata$pred_rate <- fitted_nb/TBdata$Population*100000</pre>
243
      # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
245
      par(mfrow = c(1,1))
247
     plot.map(TBdata$pred_rate , n.levels = 10)
248
      ### WIHTOUT POVERTY
249
     ### Spatio-temporal model - Poverty
250
     spatio.temporal.model.wo.pov <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
251
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment)
252
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
253
      data = TBdata ,
254
      family = nb(link = 'log')
255
256
257
      # Check summary
258
      summary(spatio.temporal.model.wo.pov) # Temporal alone doesn't add much to explaining the variance
259
      # Check the smooth functions of the covariates
260
261
      plot(spatio.temporal.model.wo.pov)
      par(mfrow=c(2,2))
262
      gam.check(spatio.temporal.model.wo.pov , pch = 20)
263
      par(mfrow = c(1,1))
264
      # Anova test
266
      anova(spatio.temporal.model.wo.pov , spatio.temporal.model , test = 'F')
267
268
      ### Without Indigenous
269
      spatio.temporal.model.wo.indig <- gam(formula = TB ~ offset(log(Population))</pre>
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
271
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
272
      data = TBdata ,
273
      family = nb(link = 'log')
274
275
```

```
276
277
      # Check summary
     summary(spatio.temporal.model.wo.indig) # Temporal alone doesn't add much to explaining the variance
278
      # Check the smooth functions of the covariates
279
      plot(spatio.temporal.model.wo.indig)
280
      par(mfrow=c(2,2))
281
      gam.check(spatio.temporal.model.wo.indig , pch = 20)
282
     par(mfrow = c(1,1))
283
284
      # Anova test
285
      anova(spatio.temporal.model.wo.indig , spatio.temporal.model , test = 'F')
286
      # NOT ENOUGH EVIDENCE TO REMOVE INDIGENOUS
287
288
      ## Spatio-temporal model
289
     spatio.temporal.model.true <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous)</pre>
290
      + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Unemployment) + s(Poverty)
291
      + s(Timeliness) + te(lon , lat , Year , k = 3),
292
     data = TBdata ,
293
     family = nb(link = 'log')
294
295
296
297
      # Check summary
     {\color{red}\textbf{summary}} (\textbf{spatio.temporal.model.true}) \ \textit{\# Temporal alone doesn't add much to explaining the variance}
298
      # Check the smooth functions of the covariates
     plot(spatio.temporal.model.true)
      par(mfrow=c(2,2))
      gam.check(spatio.temporal.model.true , pch = 20)
      par(mfrow = c(1,1))
      spatio.temporal.model$aic
      spatio.temporal.model.true$aic
      spatio.temporal.model.wo.indig$aic
308
309
310
      ### Spatio-temporal model - no poor sanitation , indigenous
311
     spatio.temporal.model.wo.indig.poor_s <- gam(formula = TB ~ offset(log(Population))</pre>
312
      + s(Urbanisation) + s(Density) + s(Unemployment) + s(Poverty)
313
      + s(Timeliness) + te(lon , lat , k = 20) + Year.asFactor,
314
     data = TBdata ,
315
     family = nb(link = 'log')
316
317
318
      # Check summary
     summary(spatio.temporal.model.wo.indig.poor_s) # Temporal alone doesn't add much to explaining the variance
319
      # Check the smooth functions of the covariates
320
     plot(spatio.temporal.model.wo.indig.poor_s)
321
     par(mfrow=c(2,2))
      gam.check(spatio.temporal.model.wo.indig.poor_s , pch = 20)
     par(mfrow = c(1,1))
      anova(spatio.temporal.model , spatio.temporal.model.wo.indig.poor_s , spatio.temporal.model.wo.indig , test = 'F')
      library (mgcv)
329
330
331
     par(mfrow = c(2,2))
332
      #fit poisson model with socio-economic variables
333
     model_poisson <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Illiteracy) + s(Urbanisation) + s(Density) + s(Po
334
     summary(model_poisson)
335
     model_poisson$aic
336
      par(mfrow = c(2,2), pch = 20)
337
      gam.check(model_poisson)
338
      # #add flexibility
339
     model_poisson <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, k = 80) + s(Illiteracy, k = 80) + s(Urbanisation, k =
340
341
      gam.check(model_poisson)
342
      summary(model_poisson) # SIGNS OF OVERFIT
```

```
plot(model_poisson) # SIGNS OF OVERFIT
343
                #fit negative binomial model with socioeconomic
344
               \verb|model_nb| <- \verb|gam|(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Illiteracy) + s(Urbanisation) + s(Density) + s(Poverty) + s(Illiteracy) + s(Urbanisation) + s(Density) +
345
               summary(model_nb)
346
               model nb$aic
347
                #fit a linear relation between squired residuals and prediction to see whether another model describes the variance-fitted values re
348
                summary(lm(log(model_nb$residuals^2) ~ log(predict(model_nb, type = 'response'))))
                model_nb_2 < -gam(formula = TB \circ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_Sanisation) + s(Poor_Sanisat
                anova.gam(model_nb_2, model_nb, test = 'LRT')
355
                #Null hypothesis not rejected -> drop poverty
356
                model_nb_3 <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poor_Sanitation) + s(Urbanisation)
357
358
                anova.gam(model_nb_3, model_nb_2, test = 'LRT')
359
                # NULL hypothesis rejected -> drop only illiteracy, not poverty
360
361
               model_nb_final <- model_nb_2
362
                summary(model_nb_final)
363
                gam.check(model_nb_final)
364
               fitted_nb <- predict(model_nb_final, type = 'response')</pre>
365
366
                # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
367
368
                par(mfrow = c(1,2))
               plot.map(log(fitted_nb) - log(TBdata$Population) , n.levels = 10)
369
               plot.map(log(TBdata$TB) - log(TBdata$Population))
370
371
                #temporal model
372
                par(mfrow = c(2,2), pch = 20)
                model_nb_time <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous, by = Year) + s(Urbanisation, by = Year) + s(Density, by
375
                summary(model_nb)
               model_nb\$aic
376
377
378
               model_nb_space <- gam(formula = TB ~ offset(log(Population)) + s(Indigenous) + s(Urbanisation) + s(Density) + s(Poverty) + s(Poor_S
379
               summary(model_nb_space)
380
               model_nb_space$aic
381
                gam.check(model_nb_space)
382
                anova.gam(model_nb_space, model_nb_final, test = 'LRT')
383
384
                #spatio-temporal model
385
               model_nb_time_and_space <- gam(formula = TB ~ offset(log(Population)) + s(Urbanisation, by = Year) + s(Density, by = Year) + s(Pove
386
                summary(model_nb_time_and_space)
387
               model_nb_time_and_space$aic
388
                gam.check(model_nb_time_and_space)
389
391
                fitted_nb <- predict(model_nb_time_and_space, newdata = TBdata , type = 'response')</pre>
                # PLOTTING STUFF - ERROR WHILE COMPILING - WILL FIX LATER
                par(mfrow = c(1,1))
                plot.map(exp(log(fitted_nb) - log(TBdata$Population))*100000 , n.levels = 10)
```