

RSA Encryption Example

$$p = 11$$

$$q = 13$$

$$n = p \times q = 143$$

$$\text{The totient of } n, \phi(n) = (p - 1) \cdot (q - 1) = 120$$

For the public key, a random prime number that has a greatest common divisor (gcd) of 1 with $\phi(n)$ and is less than $\phi(n)$ is chosen. Let's choose 7.

$$e = 7$$

d = the secret key

To find d , we need to find the inverse of 7 with $\phi(n)$:

$$e \cdot d = 1 \pmod{\phi(n)}$$

$$d = 103$$

******(where m is the message, e is the public key and c is the cipher.)

Encryption formula: $F(m,e) = m^{**}e \pmod n = c$

Decryption formula: $F(c,d) = c^{**}d \pmod n = m$

Example use:

Let's choose our plaintext message, m to be 9.

Encryption:

$$m^{**}e \pmod n = 9^{**}7 \pmod{143} = 48 = c$$

Decryption:

$$c^{**}d \pmod n = 48^{**}103 \pmod{143} = 9 = m$$