1. $\frac{3}{2}(\theta - \sin\theta) = \pi$ $\qquad f(\theta) = \frac{3}{2}(\theta - \sin\theta) - \pi = 0$

a) $x_{i+1} = x_i - \dfrac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$

$\theta_{-1} = 1 \quad \theta_0 = 1.5$

Iteration 1 (Example calcs)

$\theta_{i+1} = 1.5 - \dfrac{f(1.5)(0.5)}{f(1.5) - f(1)}$

$\theta_1 = 3.81$

$\epsilon_1 = \left|\dfrac{x_{i+1} - x_i}{x_i}\right| \cdot 100 = 60.67\%$

Iteration 2

$\theta_{i-1} = 1.5 \quad \theta_i = 3.81$

$\therefore \theta_{i+1} = 2.44$

$\epsilon = 56.55\%$

Iteration 3

$\theta_{i-1} = 3.81 \quad \theta_i = 2.44$

$\therefore \theta_{i+1} = 2.60$

$\epsilon = 6.14\%$

Iteration 4

$\theta_{i-1} = 2.44 \quad \theta_i = 2.60$

$\theta_{i+1} = 2.61$

$\epsilon = 0.38\%$

b) interval $[1, 3]$ $\quad f(\theta) = \frac{3}{2}(\theta - \sin\theta) - \pi$

Iteration 1

$x_l = 1 \quad x_u = 3$

$x_r = \dfrac{1+3}{2} = 2$ $\qquad$ error $= \left|\dfrac{x_u - x_l}{x_u + x_l}\right| \cdot 100 = 50\%$

$f(x_l) \cdot f(x_r) = -2.904 \cdot (-1.506) = 4.372$

$f(x_u) \cdot f(x_r) = 1.147 \cdot (-1.506) = -1.73 < 0 \quad \therefore x_l = x_r$

Iteration 2

$\epsilon = 20\%$

$x_l = 2 \quad x_u = 3 \qquad x_r = \dfrac{2+3}{2} = 2.5$

$f(x_l) \cdot f(x_r) =$

$f(x_u) \cdot f(x_r) = -0.332 < 0 \qquad x_l = x_r$

Iteration 3

$x_l = 2.5 \quad x_u = 3 \quad x_r = 2.75 \quad \epsilon = 9.09\%$

$f(x_l) \cdot f(x_r) < 0 \quad \therefore x_u = x_r$

Iteration 4

$x_l = 2.5$ $x_u = 2.75$ $x_r = 2.625$ $\epsilon = 4.76\%$

$f(x_l) \cdot f(x_r) = -0.016 < 0 \therefore x_u = x_r$

2.

a) estimate $\sqrt{2}$ $\rightarrow$ $f(x) = x^2 - 2$ $\sqrt{2} = 1.414214$

$f'(x) = 2x$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

guess $x_i = 1.5$

I1

$$x_{i+1} = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{0.25}{3} = 1.41\dot{6}$$

I2

$$x_{i+1} = 1.416 - \frac{f(1.416)}{f'(1.416)} = 1.41348$$

I3

$$x_{i+1} = 1.41348 - \frac{f(1.41348)}{f'(1.41348)} = 1.414214$$ ✓

b) estimate $e^5 = 148.4132$

$f(x) = \ln(x) - 5$

$f'(x) = \frac{1}{x}$

guess 150 for x_i

$\underline{I1}$

$$x_{i+1} = 150 - \frac{f(150)}{f'(150)} = 148.4047$$

$\underline{I2}$

$$x_{i+1} = 148.4047 - \frac{f(148.4047)}{f'(148.4047)} = 148.4132 \checkmark$$

3. $\min_x \; x^4 - 14x^3 + 60x^2 - 70x$

a) $f'(x) = 4x^3 - 42x^2 + 120x - 70$

roots $= 0.78088,\ 3.76192,\ 5.95719$ (Solved graphically using DESMOS)

b)

$f(0.781) = -24.37$ $f(3.762) = 40.725$ $f(5.96) = 11.958$

$f(0.780) = -24.367$ $f(3.75) = 40.723$ $f(5.95) = 11.960$

$f(0.782) = -24.367$ $f(3.77) = 40.723$ $f(5.97) = 11.960$

↑

local maxima @ 0.781

c) $f(0.781) = -24.37$

$f(0) = 0$

$f(-5) = 4225$

$f(6) = 12$

$f(10) = 1300$

d) $x_0 = 0$ $x_{k+1} = x_0 - \frac{F'(x_0)}{F''(x_0)}$ $\boxed{0.7809}$

$F'(x) = 4x^3 - 42x^2 + 120x - 70$

$F''(x) = 12x^2 - 84x + 120$

I1

$$x_{k+1} = 0 - \frac{F'(0)}{F''(0)} = 0.5833$$

I2

$$x_{k+1} = 0.5833 - \frac{F'(0.5833)}{F''(0.5833)} = 0.7631$$

I3

$$x_{k+1} = 0.7631 - \frac{F'(0.7631)}{F''(0.7631)} = 0.7807$$

I4

$$x_{k+1} = 0.7809$$ ✓

e) $[0 \quad 2]$, $x_l = 0$, $x_u = 2$

$$d = 0.668(x_u - x_l)$$

$$d = 0.668(2) = 1.336$$

$$-f(x) = x^4 - 14x^3 + 60x^2 - 70x$$

$$f(x) = -x^4 + 14x^3 - 60x^2 + 70x$$

$x_1 = x_l + d \qquad x_2 = x_u - d$

$x_1 = 1.336 \qquad x_2 = 0.664$

$F(x_1) = 16.63$

$F(x_2) = 23.93$

$F(x_1) < F(x_2) \therefore x_l = x_l$

$x_u = x_1$

$[0 \quad 1.336] \qquad d = 0.8924$

$x_1 = 0.8924$

$x_2 = 0.4436$

$F(x_1) = 24.001$

$F(x_2) = 20.4285$

$F(x_1) > F(x_2) \therefore x_l = x_2$

$x_u = x_u$

$[0.4436 \quad 1.336] \qquad d = 0.5961$

$x_1 = 1.0397 \qquad x_2 = 0.7399$

$F(x_1) = 22.4864 \qquad F(x_2) = 24.3170$

$F(x_1) < F(x_2) \therefore \quad x_\ell = x_\ell$

$x_u = x_1$

4. $f(x,y) = 2\pi x^2 + y^2 \cdot (2 + \sin(\pi x)) \qquad x_0 = 1 \quad y_0 = 2$

a) $\nabla f(x,y) = (\underbrace{4\pi x + \pi y^2 \cos(\pi x)}_{\frac{\delta f}{\delta x}}, \underbrace{2y(2+\sin\pi x)}_{\frac{\delta f}{\delta y}})$

b) $(x_{n+1}, y_{n+1}) = (x_n, y_n) - \cdot \nabla f(x_n, y_n)$

$(x_{n+1}, y_{n+1}) = (1, 2) - \quad (4\pi + \pi 4\cos(\pi), 4(2 + \sin\pi))$

$x_1 = 1 - \alpha(4\pi + \pi 4\cos(\pi)) = 1 - \alpha(0) = 1$

$y_1 = 2 - \alpha(4(2 + \sin(\pi)) = 2 - \alpha(8)$

$F(x_1, y_1) = 2\pi(1)^2 + (2 - 8\alpha)^2 \cdot 2$

$= 2\pi + 8 - 64\alpha + 128\alpha^2$

$\alpha = 0.25$

$x_1 = 1$

$y_1 = 2 - (0.25) \cdot 8 = 0$

I 2

$x_2 = x_1 - \alpha_2 \frac{\partial f}{\partial x_1} \qquad x_2 = 1 - \alpha_2 4\pi$

$y_2 = y_1 - \alpha_2 \frac{\partial f}{} \qquad y_2 = 0 - \alpha_2(0) = 0$

∂y_1

$$f(x_2, y_2) = 2\pi(1-4\pi\alpha_2)^2 = 2\pi(1-8\pi\alpha_2 + 16\pi^2\alpha^2)$$

$$\alpha_2 = 1/4$$

$$x_2 = 1 - \tfrac{1}{4}\cdot 4\pi = 1-\pi$$

$$y_2 = 0$$

c) As the value of y has remained stationary, this signifies that y has been found. X is still changing, therefore, a maximum has not yet been found.

5. $f(x) = \sin(x_1) + \cos(x_2) - \frac{1}{2}(x_1^2 + x_2^2)$

$$f(x_1) = \cos(x_1) - x_1$$

$$f'(x_1) = -\sin(x_1) - 1$$

$$x_0 = 0.5$$

$$x_1 = 0.5 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0.5 - \frac{0.1224}{1.479}$$

$$x_1 = 0.4172$$

$$f(x_2) = -\sin(x_2) - x_2$$

$$f'(x_2) = -\cos(x_2) - 1$$

$$x_0 = 0$$

$$x_2 = 0$$

$$x_2^* = 0$$

6. From the Matlab code, four roots are found, all potentially representing a local maxima.