1. a)

1	8	7
s	c	f
	$= e + 127$	

$c = 0 \to$ zero

$c = 255 \to \infty$

$e = c - 127$

$e_{min} = -126$

$e_{max} = 127$

Smallest

$= (-1)^0 \cdot 2^{e_{min}} (1 + f)$

$= 2^{-126} \cdot 1$

$= 1.17 \times 10^{-38}$

Largest

$= 2^{128} - 2^{120}$

$= 3.38 \times 10^{38}$

b) $= 2^{1-t}$ $(t = 8)$

$= 2^{-7} = 0.0078125$

c) $[-100, 100]$

$s = 1$ bit

$e = 5$ bits $\to [-15, 15]$ after

$f = 16 - 1 - 5 = 10$

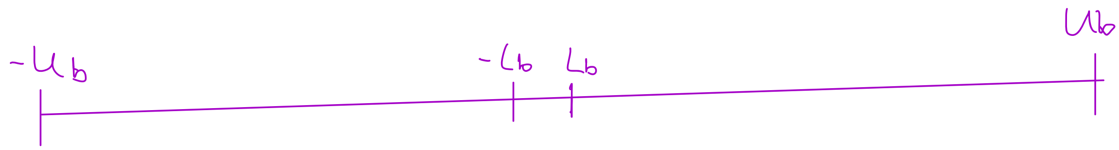
s	e	f
1	5	10

this allocation allows for more precision in the interval $[-100, 100]$

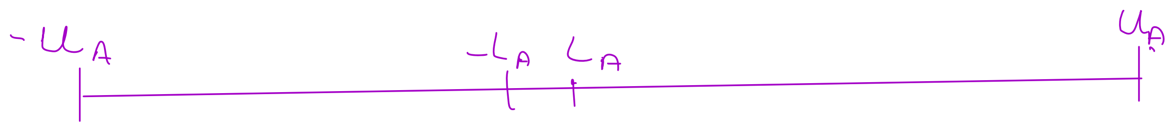
given considering that more bits are allocated to the floating point values

before

d)



after



$U_b = 3.38 \times 10^{38}$

$L_b = 1.17 \times 10^{-38}$

$U_A = 131071$

$L_A = 5.96 \times 10^{-8}$

Problem 2 (Bounds $1.755 \times 10^{-38} \leftrightarrow 3.4028 \times 10^{38}$)

a) 5.3×10^{200}, overflow as number is too large

b) 127.0125 Fine

c) 1/127 Fine

d) 2.625×2^{-125}, Underflow, exists between lower bounds

e) $1 + \underbrace{2.625 \times 10^{-125}}_{0}$ Fine

Problem 3: See attached file

Problem 4:

a)

	@x=0
$f(x) = \cos x$	1
$f'(x) = -\sin x$	0
$f''(x) = -\cos x$	-1
$f'''(x) = \sin x$	0
$f^{(4)}(x) = \cos x$	1

repeating

$$\cos x = \sum_{0}^{\infty} \frac{f^n(0)}{n!} x^n = \underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots}$$

Can be expressed as

$$(-1)^k \cdot \frac{x^{2k}}{(2k)!}$$

$$\therefore \cos x = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k}}{(2k)!}$$

b) After running the attached matlab file for Problem 4 (q4.m) one can see the McLaurin estimation is correct within the alloted error after 7 trials for the value $\pi/3$, and 5 trials for value $\pi/6$.

Problem 5

$f(x) = x^3 - 2\ln(x)$; @ $x=1$ $= 1$

$f'(x) = 3x^2 - \frac{2}{x}$ $= 1$

$f''(x) = 6x + \frac{2}{x^2}$ $= 8$

$f'''(x) = 6 - \frac{4}{x^3}$ $= 2$

$f^{(4)}(x) = \frac{12}{x^4}$ $= 12$

$$f(3) = f(1) + \frac{f'(1)\,2^1}{1!} + \frac{f''(1)\,2^2}{2!} + \frac{f'''(1)\,2^3}{3!} + \frac{f^{(4)}(1)\,2^4}{4!}$$

$$= 1 + 2 + 16 + \frac{16}{6} + 8 = 29.6667$$ 4th order

Real $f(3) = 3^3 - 2\ln(3) = 24.8028$

Error calc $= \left|\frac{\text{real} - \text{approx}}{\text{real}}\right| \cdot 100\%$ $e_4 = \left|\frac{24.8028 - 29.6667}{24.8028}\right| \cdot 100$

$= 19.610\%$

Order Approx (from Matlab)

0 order :	1	95.97%
1st order :	3	87.90%
2nd order :	19	23.39%
3rd order :	21.667	12.64%
4th order :	29.667	19.61%

From the values obtained, and the plot in Matlab, we can see the most correct approximation was the 3rd order approximation.