Bounding Inter-Signal Alignment, Stability, and Drift

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***Abstract*—This paper presents a mathematical model to validate a method for determining the amplitude and phase alignment, stability, and drift between two signals. The proposed method feeds two signals into a power combiner and uses the relative amplitude of the output signal to determine the amplitude and phase deltas between the input signals. From these deltas, the inter-signal alignment, stability, and drift can be determined.**

The proposed method is limited however, by the interdependencies between these characteristics; It is only possible to bound each characteristic to some degree when nothing is known about the input signals. This problem can be mitigated if the amplitude or phase of one or both of the input signals can be controlled. Under the right circumstances, it is possible to use this method in place of more expensive or unavailable equipment.

***Index Terms*—Inter-Signal, Phase, Power, Amplitude, Alignment, Stability, Drift.**

I. Introduction

Current technology allows the usage of a Vector Network Analyzer (VNA) to measure amplitude and phase alignment, stability, and drift. A VNA is not always readily available though, and depending on the precision required, it may come with a prohibitive price tag. Without access to a VNA, measuring these characteristics becomes difficult. The proposed method in this paper is an alternative approach to using a VNA; it is much simpler and a more cost effective solution for measuring these inter-signal characteristics.

II. Proposed Method

The proposed method for bounding the inter-signal characteristics between two signals is to feed them into a power combiner and then measure the relative amplitude of the output signal (See Figure (1)). The closer the signals are to being anti-aligned (180° out of phase of each other), the more sensitive the measurement will be and the more accurate the inter-signal characteristics can be determined.

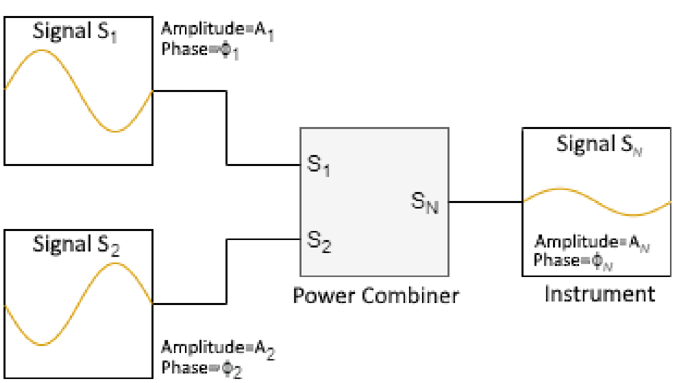

Fig. 1: Proposed Method Block Diagram

Conceptually, if two anti-aligned signals of equal amplitude are combined, there will be no signal at the output of the combiner. Conversely, if two aligned signals are combined, the amplitude of the output signal will be the sum of the amplitudes of the inputs.

More definitively, if two arbitrary signals are combined, the amplitude of the output signal will be between these two extremes (see Figure (2) as an example). By measuring the amplitude of the output, information about the input signals can be determined.

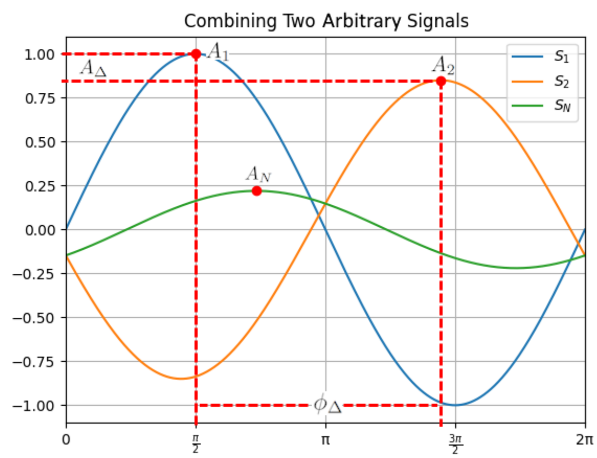

Fig. 2: Result of Combining Two Arbitrary Signals

III. Creating a Model

A. Signal Notation

A mathematical model is required to quantify and validate this relationship between the amplitude of the output signal to the characteristics of the input signals. For the purpose of this paper, the input signals shall be denoted as S_1 and S_2, and the output signal shall be denoted as S_N. The signals can thus be written as:

$$S_1 = A_1 \sin(\omega t + \phi_1) \tag{1}$$

$$S_2 = A_2 \sin(\omega t + \phi_2) \tag{2}$$

$$S_N = A_N \sin(\omega t + \phi_N) \tag{3}$$

If S_1 is used as a point of reference for the phase such that $\phi_1 = 0$, the inputs can be rewritten as follows:

$$S_1 = A_1 \sin(\omega t) \tag{4}$$

$$S_2 = A_2 \sin(\omega t + \phi_\Delta) \tag{5}$$

Where ϕ_Δ is the phase difference between S_1 and S_2:

$$\phi_\Delta = \phi_2 - \phi_1 = \phi_2 \tag{6}$$

B. Amplitude of Output Signal

The first step in quantifying this relationship is to determine the amplitude of the output in terms of the amplitudes of the inputs. Ideally, the amplitude of the output will be the sum of the amplitudes of the input signals:

$$S_N = S_1 + S_2$$

$$A_N \sin(\omega t + \phi_N) = A_1 \sin(\omega t) + A_2 \sin(\omega t + \phi_\Delta) \tag{7}$$

When $\omega t = 0$, $\sin(\phi_N)$ can be determined, and when $\omega t = \frac{\pi}{2}$, $\cos(\phi_N)$ can be determined:

$$\sin(\phi_N) = \frac{A_2 \sin(\phi_\Delta)}{A_N} \tag{8}$$

$$\cos(\phi_N) = \frac{A_1 + A_2 \cos(\phi_\Delta)}{A_N} \tag{9}$$

Equation (7) can now be simplified using equations (8) and (9) along with the trig identity $\cos^2(\phi_N) + \sin^2(\phi_N) = 1$:

$$\left[\frac{A_1 + A_2 \cos(\phi_\Delta)}{A_N}\right]^2 + \left[\frac{A_2 \sin(\phi_\Delta)}{A_N}\right]^2 = 1$$

$$A_N = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_\Delta)} \tag{10}$$

C. Relative Amplitude of Output Signal

There are two things that need to happen to relate the amplitude of the output signal to the amplitude and phase deltas of the input signals; The amplitude delta needs to be defined and the amplitude of the output signal (A_N) needs to be made relative to the amplitude of one of the input signals:

$$A_\Delta = A_2 - A_1 \tag{11}$$

$$A_{NR} = A_N - A_1 \tag{12}$$

By putting the amplitude delta and relative output amplitude on a logarithmic scale, they can be rewritten as follows:

$$A_\Delta = 20\log\left(\frac{A_2}{A_1}\right) \tag{13}$$

$$A_{NR} = 20\log\left(\frac{A_N}{A_1}\right) \tag{14}$$

A_{21} will be used to denote the ratio of A_2 to A_1. Using equation (13), it can be written as a function of A_Δ:

$$A_{21} = \frac{A_2}{A_1} = 10^{\frac{A_\Delta}{20}} \tag{15}$$

Using equations (10), (14), and (15), A_{NR} can now be written as a function of just A_Δ and ϕ_Δ:

$$A_{NR}(A_\Delta, \phi_\Delta) = 10\log\left(A_{21}^2 + 2A_{21}\cos(\phi_\Delta) + 1\right) \tag{16}$$

The power of the output signal relative to the power of S_1 has the same relationship (See appendix (A) for details) and can be written:

$$P_{NR}(P_\Delta, \phi_\Delta) = 10\log\left(A_{21}^2 + 2A_{21}\cos(\phi_\Delta) + 1\right) \tag{17}$$

D. Introducing Noise

So far only ideal, noiseless signals, have been considered. To introduce noise into the model, the amplitude and phase of each signal can be redefined to include any noise model:

$$A_1 = A_1 + A_{\sigma 1}, \phi_1 = \phi_1 + \phi_{\sigma 1}$$
$$A_2 = A_2 + A_{\sigma 2}, \phi_2 = \phi_2 + \phi_{\sigma 2}$$
$$A_N = A_N + A_{\sigma N}, \phi_N = \phi_N + \phi_{\sigma N}$$

By redefining the amplitude and phase of each signal in this manner, only A_Δ and ϕ_Δ need to change:

$$A_\Delta = (A_2 - A_1) + (A_{\sigma 2} - A_{\sigma 1}) \tag{18}$$

$$\phi_\Delta = \phi_2 + (\phi_{\sigma 2} - \phi_{\sigma 1}) \tag{19}$$

IV. Application

A. Terminology

With equation (16), which relates the amplitude of the output signal to the amplitude and phase deltas of the input signals, the inter-signal amplitude and phase, alignment and stability can be determined. The inter-signal amplitude and phase alignments can be defined as the absolute amplitude and phase differences between the input signals. The inter-signal amplitude and phase stabilities can be defined as the difference between the individual input signal amplitude and phase stabilities.

The amplitude and phase deltas consist of an alignment term and a stability term. From equations (18) and (19), the inter-signal amplitude and phase alignments can be defined as $(A_2 - A_1)$ and (ϕ_2) respectively. The inter-signal amplitude and phase stabilities can be defined as $(A_{\sigma 2} - A_{\sigma 1})$ and $(\phi_{\sigma 2} - \phi_{\sigma 1})$ respectively. This assumes similar, zero-mean noise models for both of the inputs.

B. Limitations

There are three major limitations to using this method that can be deduced from equations (16), (18), and (19).

1) Alignment and stability are inter-dependent: The first limitation makes separating alignment from stability difficult. This inter-dependency can be mitigated however, by controlling the alignment or by averaging A_{NR} over time. For example, if the input signals are made to be aligned in power and phase, then only the stability terms are left to be determined. Conversely, if the means of the noise models are 0 and an average measurement of A_{NR} is made over time, then just the alignment terms are left to be determined.

2) A_{NR} is dependent on both A_Δ and ϕ_Δ: The second limitation prevents the absolute determination of the amplitude and phase deltas from A_{NR}. Instead, it forces the amplitude and phase deltas to be bounded by each other. What this means is that, given a specific value for A_{NR}, an exact value for A_Δ and phi_Δ cannot be given, unless one or the other is known. This correlates to the first limitation in that none of the four inter-signal characteristics can be precisely determined without knowing the other three. Plotting equation (16) helps make these inter-dependencies a little clearer:

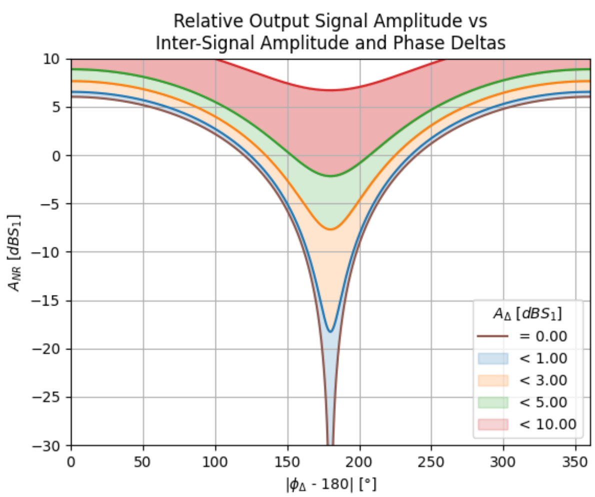

Fig. 3: Amplitude and Phase Inter-Dependency

3) A_{NR} depends on the sign of A_Δ: This third limitation is caused by the logarithmic scale of the amplitude delta in equation (16). It forces A_{NR} to behave differently when the amplitude delta is negative ($A_1 > A_2$) than from when it is positive ($A_1 < A_2$). Plotting various negative values for A_Δ makes this clear:

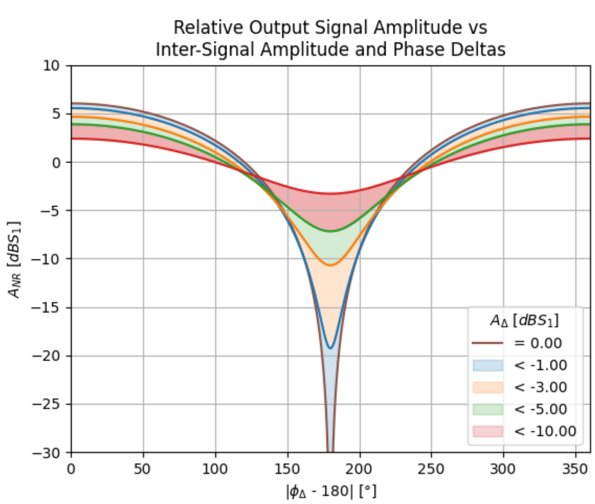

Fig. 4: Affect of Negative A_Δ Values

For certain values of $A_{A_{NR}}$ when A_Δ is negative, it becomes impossible to determine or bound the amplitude and phase deltas. This problem can be mitigated by reducing A_Δ by either matching the amplitude of the input signals, or by reducing their noise.

It should also be noted that these figures can be recreated with the relative output amplitude on the x-axis. See appendix (B) for additional information.

C. Implementation

This method is most useful when different approaches are used to measure inter-signal alignment, stability, and drift.

1) Alignment: When using this method to determine amplitude and phase alignment, it is advantageous to average A_{NR} measurements over time. This is because when the amplitude and phase both have zero-mean noise models, the A_{NR} measurements will average down to just the amplitude and phase alignment terms.

2) Stability: When using this method to determine amplitude and phase stability, it is advantageous to anti-align the input signals. This is because A_{NR} is the most sensitive when the input signals are anti-aligned. The more sensitive the A_{NR} measurement is, the more accurate the stability terms can be determined.

Stability is generally measured at a much lower scale and resolution than alignment, so it is nice to plot A_{NR} with ϕ_Δ on a logarithmic scale:

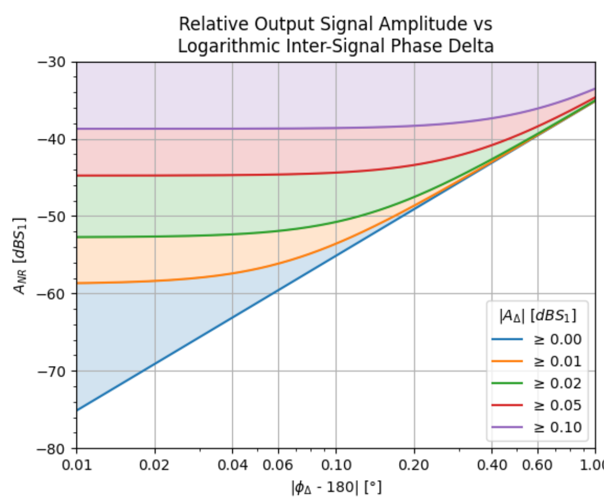

Fig. 5: Logarithmic ϕ_Δ Axis for Stability

3) Drift: Inter-signal amplitude and phase drift can be defined as the long-term change in amplitude and phase alignment. It should therefore be measured in a similar manner to the alignment approach, but multiple averaged A_{NR} measurements need to be taken over time instead.

Care should be taken when using this approach to measure inter-signal drift as the sensitivity of A_{NR} changes with ϕ_Δ. The drift is therefore not a linear fit of the A_{NR} over time, but an exponential one. Figure (6) shows a possible scenario with this effect:

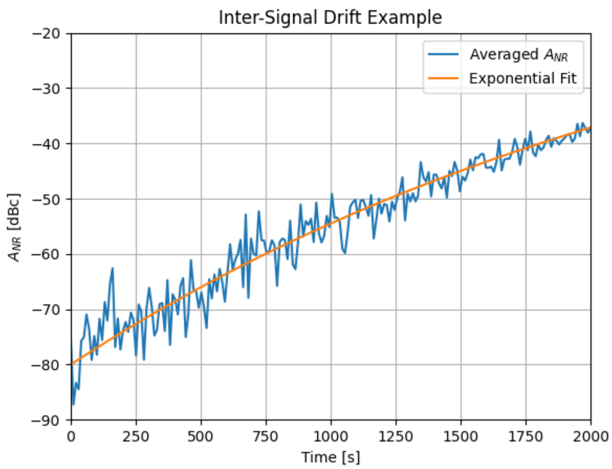

Fig. 6: Inter-Signal Amplitude and Phase Drift

V. Conclusion

The presented mathematical model validates the proposed method of combining two signals to find the inter-signal amplitude and phase alignment and stability. It is not without limitation however; Amplitude and phase, alignment and stability, are all tightly coupled. Some mitigation strategies were presented to uncouple these characteristics, but ultimately, the amplitude and phase, stability and alignment can generally only be bounded, not precisely determined. Despite it's limitations, this method may be useful in certain circumstances where expensive equipment cannot be afforded or where the exact determination of the inter-signal characteristics is not needed.

With additional research, it would probably be possible to use the relative phase of the output signal (instead of its relative amplitude) to determine the inter-signal characteristics of the input signals. This would be helpful if the equipment available was only capable of measuring the phase of the output relative to the phase of one of the input signals. It may also be possible to extrapolate this method to any number of input signals or to signals of differing frequency. These concepts would likely require additional information such as a time delta measurement or the usage of an external reference for the phase measurements.

Appendix A
Relative Power of Output Signal

Let P_Δ represent the power delta between the input signals and P_{NR} represent the relative power of the output signal:

$$P_\Delta = P_2 - P_1 \tag{20}$$

$$P_{NR} = P_N - P_1 \tag{21}$$

Following the same approach used for amplitude the amplitude terms, the power terms can be made logarithmic:

$$P_\Delta = 10\log\left(\frac{P_2}{P_1}\right) = 20\log\left(\frac{A_2}{A_1}\right) \tag{22}$$

$$P_{NR} = 10\log\left(\frac{P_N}{P_1}\right) = 20\log\left(\frac{A_N}{A_1}\right) \tag{23}$$

When done this way, the amplitude and power terms become synonymous ($P_\Delta = A_\Delta$ and $P_{NR} = A_{NR}$).

Appendix B
Alternative Plot

By re-arranging equation (16), ϕ_Δ can be written as a function of A_{NR} and A_Δ:

$$\phi_\Delta(A_{NR}, A_\Delta) = \arccos\left[\frac{10^{\frac{A_{NR}}{10}} - 1 - A_{21}^2}{2A_{21}^2}\right] \tag{24}$$

When plotted, it is essentially Figure (3) rotated so that A_{NR} is on the x-axis:

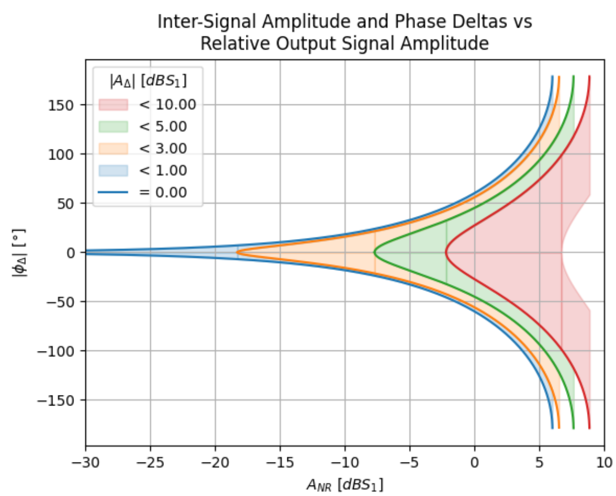

Fig. 7: Alternative Plot

Similarly to Figure (5), a logarithmic scale can once again be used for ϕ_Δ for a better stability plot:

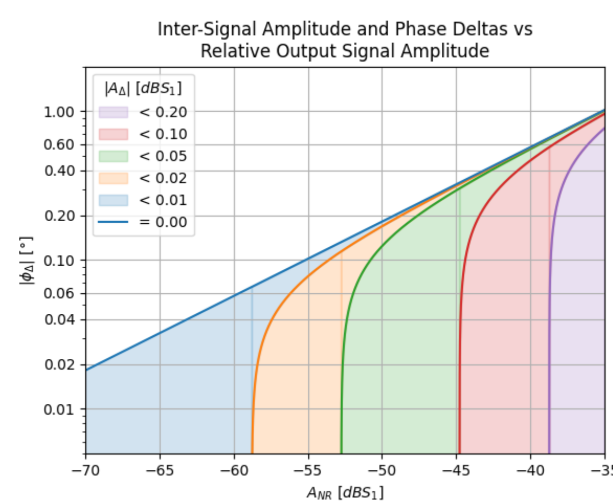

Fig. 8: Alternative Plot with Logarithmic ϕ_Δ Axis

ACKNOWLEDGMENT

The author would like to thank Cosette Thompson, Digital Signal Processing Engineer at Safran Federal Systems, for the countless inquisitive discussions, as well as for the testing and review of this methodology and paper. The author would also like to thank Tim Erbes, Technical Director and Senior Software Engineer at Safran Federal Systems, for the development and testing of the initial concept.

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[1] H. Haber. (2009). *How to add sine functions of different amplitude and phase* [Online]. Available: https://scipp.ucsc.edu/ haber/ph5B/addsine.pdf

Joshua Wilkins Received his B.S. degree in 2016 from the Rochester Institute of Technology in electrical engineering. From 2015 to 2017, he worked on advanced mobile x-ray machines and cone-beam computed tomography (CBCT) systems at Carestream Health. In 2017, he started working at Microsemi (now Microchip) under the Frequency and Timing Division. Here he worked in R&D to create a prototype for the next generation of atomic clocks. From there, he started working for Raytheon in 2020, redesigning legacy PCB modules with the latest in RF technologies. In 2023, he started working at Orolia Defense Systems (now Safran Federal Systems) as a Senior RF Engineer. He is still currently working here, characterizing and developing new ways to test wavefront systems.