STATICS:

Final Design Project

Monroe Community College
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ENR 251 : Prof. Christopher Kumar

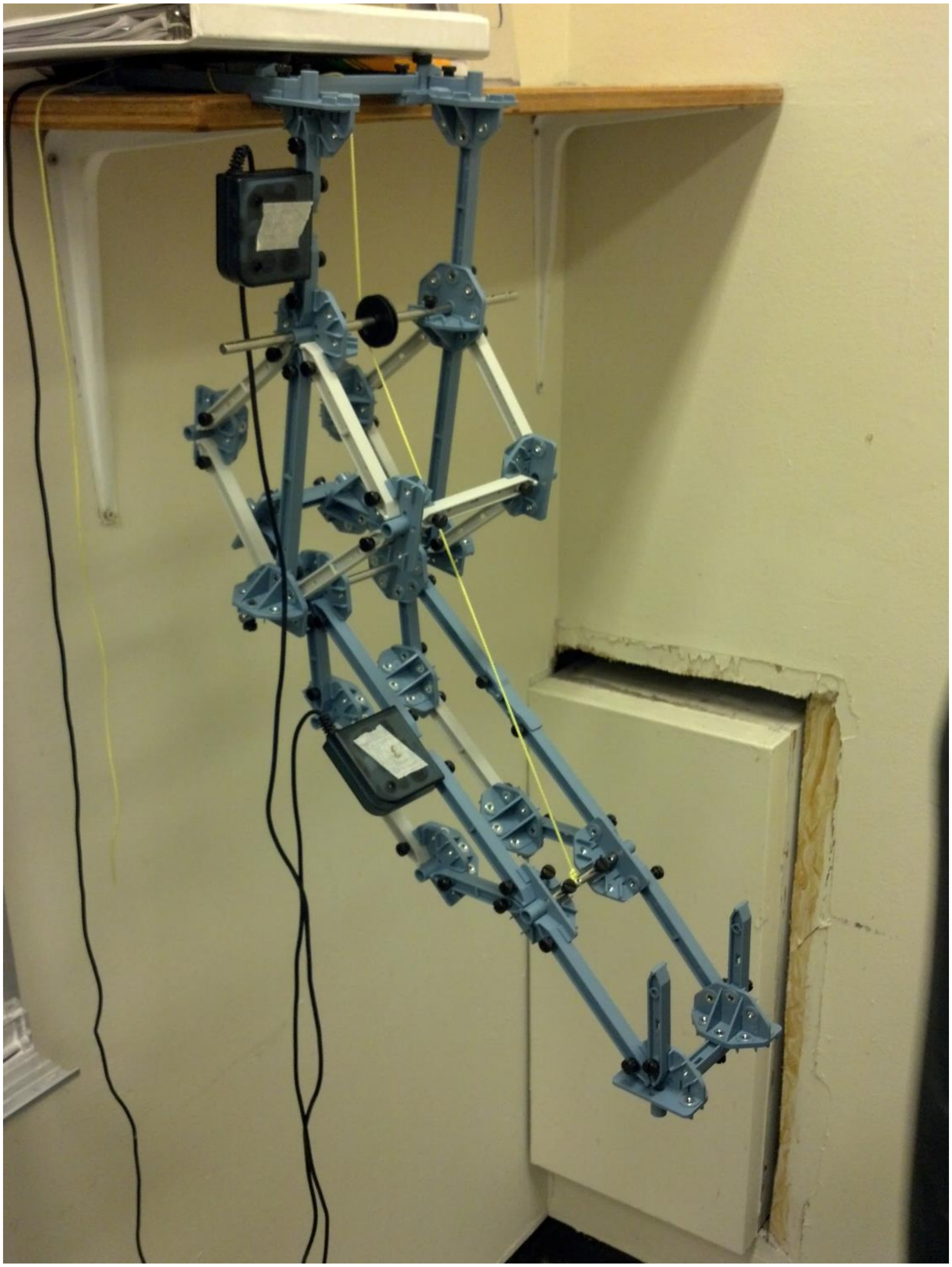

Figure 1: Full View of Project

Statics Model for our mechanical arm, once fixed to a position, it is used to collect stress data for many different angles and positions of the arm.

PROJECT SUMMARY

This semesters second Statics Design Project, was based on the stress of a spine and the floor from a leg while weights are lifted by a human. We incorporated Matlabs ability to quickly work with unknown variables to find these forces as the individual lifted the weight, for the entire lift.

For our third and final design project, we wanted to be able to raise the bar even farther, and designed an arm that would incorporate what we learned during and after our second project, into our analysis. We designed an arm, that would be capable of moving with the same flexibility as a human arm, and then collect forces on the bicep, forearm and the Humerus . To simulate the action of our model arm we made use of a Pasco simulation kit, and Pasco's Capstone Simulation Software, both of which were provided to our team through the Engineering Science Department at Monroe Community College. We also once more, used Matlab programing software to compute for the unknowns in our theoretical analysis, and then apply the unknowns to two multi-angular graphs. To find the Matlab though, we first had to hand compute for our unknown equations, which can be found to be very tedious. The three methods once again allowed our team to perform an in-depth analysis of the arm, allowing us to bring the results we gathered to a more in-depth and conceptual level which greatly benefited our learning experience. Due to our use of the Matlab software, we were able to compute for the arms forces as the angle changed, and this ability let us capture points of data that we would then be able to compare to our physical Capstone results. After comparing our three forms of data, we found our calculations to differ from our data collected in real world testing. First due to our arms weight being greater than zero, which differed to our Matlab values of zero for each members weight. This was fixed marginally by reducing the weight of our arm, but we still had issues with our structure flexing during testing. Next we altered the arm once more to incorporate les heavy support on the arm, and this also reduced our error we still had some difficulty in reaching the accuracy that we found in our two other design projects, but still managed to keep the values closer than we found them at the start of our testing. This project really stretched the level of accuracy and the physical capabilities of our Pasco simulation kit and Matlab as well.

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INTRODUCTION

The basis of all mathematics performed over the course of this project can be based on the fact that all forces and moments placed on a nonmoving object equate to zero. In the case of our final design project, the forces we are measuring are the forces in the Humerus, the forearm, and the bicep muscles, as well as the reactionary forces of the shoulder in the x axis. We are to split the components of our forces into x and y components since our model is symmetrical along the z axis, and produce equations that let us solve for our forces in terms of the angle our arm is bent at. This can be split into the equations shown in our theoretical analysis on page

$$\sum \vec{F} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + W$$

$$\sum F_y = 0 = A_y + B_y$$

$$\sum F_x = 0 = A_x + B_x + .5\sin(\Psi)$$

$$\sum F_z = 0 = A_z + B_z - F_4 - F_3\cos(\Psi)$$

$$\sum M_c = \langle 0|0|0\rangle = \langle -2F_3\cos(\Psi)\,|0|-2F_3\sin(\Psi)\rangle + \langle -4F_4|0|0\rangle + \langle 4A_z|0|2F_3\sin(\Psi)\rangle$$

These equations are the basis to our arms theoretical analysis, but are first clouded by a large amount of tricky trigonometry, as we solve for our arms dimensions in terms of theta, the angle between our forearm and Humerus, we can begin to produce a model that can be applied for many different applications. The applications of a mechanical arm are endless, and as we see more individuals every year losing appendages to disease, such as diabetes and cancer, and incidents such as war and work injuries, the ability to create a successful prosthetic limb prove very useful in the engineering world. Prosthetics and weight carrying exoskeletons are part of the future, and the tech will continue to flourish as many more engineers apply their skills to the subject.

Since we knew from the start that we would be calculating forces in the terms of many unsolved variables, we held a desire to move to Matlab as soon as possible for our calculations, designing the equations we would need on paper, and bringing them to life with the programing software given to us. This was due to the fact that our arms proportions change as it flexes, making standard solving procedures long and difficult. To allow for our Matlab and physical Calculations to mesh smoothly, we took advantage of Matlab’s ability to produce graphs, and then use Pasco capstone graphing tools to bring together two different variations of the same load and angle of the arm, and allow for a certain amount of readability between the two.

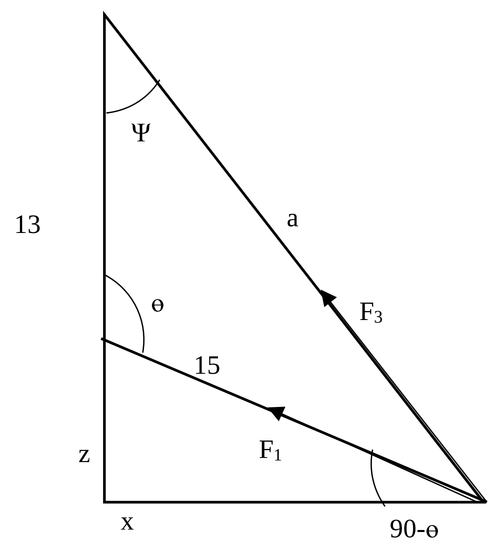
THEORETICAL ANALYSIS

Through symmetry we see that $F_1 = F_2$ and that the reactionary forces at points A and B are equal. By method of sections, we take and cut the arm in half with a plane and solve the two halves separately for the desired forces.

Considering the right half first and by using static equilibrium equations we can solve for $F_1, F_2, and\ F_3$.

$$\sum \vec{F} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + W$$

We must first consider what each force is in vector format through the observation and manipulation of angles.



$$\vec{F}_1 = \vec{F}_2 = \langle -F_1\cos(90-\theta)\,|0|F_1\sin(90-\theta)\rangle$$

By the law of Cosines, we can obtain a:

$$a^2 = 13^2 + 15^2 - 2(15)(13)\cos(\theta)$$

$$a = \sqrt{13^2 + 15^2 - 2(15)(13)\cos(\theta)}$$

By the law of Sines, we can obtain Ψ:

$$\frac{\sin(\theta)}{a} = \frac{\sin(\Psi)}{15}$$

$$\Psi = sin^{-1}\left(\frac{15\sin(\theta)}{a}\right)$$

The vector F_3 is now equal to its magnitude times its unit direction vector:

$$\vec{F}_3 = \langle -F_3\sin(\Psi)\,|0|F_3\cos(\Psi)\rangle$$

$$W = \langle 0|0|-W\rangle$$

Going back to the static equilibrium equation, we get:

$$\sum \vec{F} = 0 = 2\vec{F}_1 + \vec{F}_3 + W$$

Written in vector form:

$$\langle 0|0|W\rangle = -2\langle F_1 \cos(90-\theta)\ |0|F_1 \sin(90-\theta)\rangle + \langle -F_3 \sin(\Psi)\ |0|F_3 \cos(\Psi)\rangle$$

Which then can be solved for F_1 and F_3 with matrices:

$$\begin{vmatrix} -2\cos(90-\theta) & -\sin(\Psi) \\ 2\sin(90-\theta) & \cos(\Psi) \end{vmatrix} * \begin{vmatrix} F_1 \\ F_3 \end{vmatrix} = \begin{vmatrix} 0 \\ W \end{vmatrix}$$

$$\begin{vmatrix} F_1 \\ F_3 \end{vmatrix} = \begin{vmatrix} -2\cos(90-\theta) & -\sin(\Psi) \\ 2\sin(90-\theta) & 15\sin(90-\theta) \end{vmatrix}^{-1} * \begin{vmatrix} 0 \\ W \end{vmatrix}$$

Considering the other half now and by using static equilibrium equations we can solve for F_4. As stated earlier, the reactionary forces at A and B will be the same.

$$\sum F_y = 0 = A_y + B_y$$

Since $A_y = B_y$:

$$A_y = B_y = 0$$

$$\sum F_x = 0 = A_x + B_x + .5\sin(\Psi)$$

Since $A_x = B_x$:

$$A_x = B_x = -.5\sin(\Psi)$$

$$\sum F_z = 0 = A_z + B_z - F_4 - F_3\cos(\Psi)$$

Since $A_z = B_z$:

$$0 = F_4 + F_3\cos(\Psi) - 2A_z$$

This can then be carried into a moment equation, as seen below to then allow for solving of the unknowns, by placing the moment into one equation with one unknown.

$$\sum M_c = 0 = \langle F_3\sin(\Psi)\,|0|{-F_3}\cos(\Psi)\rangle X\langle 0|2|0\rangle + \langle -.5\sin(\Psi)|0|A_z\rangle X\langle 0|4|0\rangle + \langle 0|0|{-F_4}\rangle X\langle 0|4|0\rangle$$

$$\sum M_c = \langle 0|0|0\rangle = \langle -2F_3\cos(\Psi)\,|0|{-2F_3}\sin(\Psi)\rangle + \langle -4F_4|0|0\rangle + \langle 4A_z|0|2F_3\sin(\Psi)\rangle$$

Resulting in only one equation:

$$0 = -2F_3\cos(\Psi) - 4F_4 + 4A_z$$

By combining this equation with the last, we can solve for F_4 and A_z through the use of matrices:

$$\begin{vmatrix} 1 & -2 \\ -4 & 4 \end{vmatrix} * \begin{vmatrix} F_4 \\ A_z \end{vmatrix} = \begin{vmatrix} -F_3\cos(\Psi) \\ 2F_3\cos(\Psi) \end{vmatrix}$$

$$\begin{vmatrix} F_4 \\ A_z \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ -4 & 4 \end{vmatrix}^{-1} * \begin{vmatrix} -F_3\cos(\Psi) \\ 2F_3\cos(\Psi) \end{vmatrix}$$

MATLAB BROKEN DOWN:

To better help the individual understand our Matlab code, we have included a broken up version of our code after our theoretical analysis, to fortify understanding of the matrix operations used in our theoretical analysis, as both are very similar in operation.

FORCES AND MASS

To find the unknown Forces for many different angles, we have to switch from singular variables to vectors, which will let us store our calculations for graphing later.

`F1= [];` The force vector for F1 storing the force of F1 for various angles

`F3= [];` Force vector for F3 storing the force of F3 for various angles

`F4= [];` Force vector for F4 storing the force of F4 for various angles

`AZ= [];` Force vector for Az storing the force of Az for various angles

DISTANCE VECTORS AND WEIGHT

As Seen in Our F.B.D on page 6, we can see that we have only two known distance vectors, those can be seen as the Forearm and the Humerus

`W = 2;` Weight of the mass being hung, in Kilograms (at the hand)

`L1 = 15;` Length of the Forearm in inches (From the elbow to the hand)

`L2 = 13;` Length of the Humerus in inches (From the elbow to the shoulder)

FOR LOOP : VALUES OF THETA

Theta is set to be a vector from 1 to 179 degrees, that lets us find all force values between and at the two angles, then we can compute for the actual unknowns afterwards. The distance from 1 to 179 degrees represents theta = 1 , or the angle of the arm fully bent upwards, with a completed bicep curl, and incredibly flexed bicep muscle. At theta = 179, the arm is nearly fully unflexed, as though the arm is getting ready to start lifting the weight, this is where the muscle is least compressed, and at its longest.

RESULTS

The results of our matlab data are collected and stored in multiple vectors, and reapplied for all 180 values of theta . The resulting code below is then read and computed and then reapplied until all of the calculations are complete.

```
a = sqrt(L1^2+L2^2-(2*L1*L2*cosd(Theta)));
psi = asind((15*sind(Theta))/a);
```

A is the length of our Muscle

```
M1 = [-2*cosd(90-Theta),-sind(psi);2*sind(90-Theta),cosd(psi)];
M2 = [0;-W];

M3 = inv(M1)*M2;

F1(Theta) = M3(1);
F3(Theta) = M3(2);

M1 = [1,-2;-4,4];

M2 = [F3*cosd(psi);-2*F3*sind(psi)]; M3 = inv(M1)*M2;

F4(Theta) = M3(1);
AZ(Theta) = M3(2);
AX(Theta) = -.5*sind(psi);
```

DECLARATION OF GRAPH VARIABLES, FIGURE TITLE ECT. FOR THETA VS. F4 & AX

Our first graph represents the comparision of theta to F4 and Ax, F4 being the force of our Humerus, and Ax being the x component of our shoulders resultant force.

```
Theta=1:1:179;

plot(Theta,F4)
hold on

plot(Theta,AX,'red')

xlabel('Theta');
ylabel('F4 and AX');
legend('F4','AX');
title('Plot of Theta vs F4 and AX')

hold off
figure
```

DECLARATION OF GRAPH VARIABLES, FIGURE TITLE ECT. FOR THETA VS. F1 & F3

Our second graph represents the comparison of theta to F4 and Ax, F1 being the force of our forearm, and F3 being the tension force of our bicep as it lifts the weight.

```
plot(Theta,F1,'green')
hold on

plot(Theta,F3,'black')

legend('F1','F3');
xlabel('Theta');
ylabel('F1 and F3');

title('Plot of Theta vs F1 and F3')
```

MATLAB RESULTS: 500G MASS AND THETA VS. F1 & F3

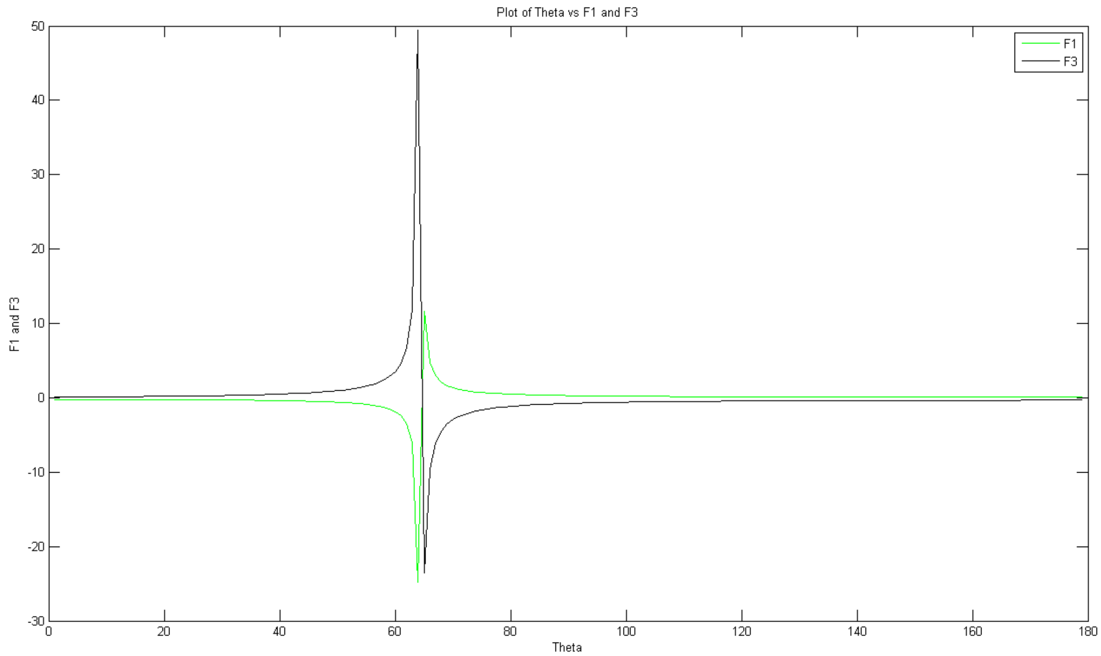


Figure 2: 500 gram mass and Theta vs. F1 & F3 (Matlab)

This figure was produced in Matlab to show the forces of the forearm and the bicep muscle, as you can see, the two forces peak when they switch from compression to tension, and vice versa, since they get close to infinity, this is still applicable for our results however, since the two forces would cancle their extremely high values out. As seen in the next graph.

To differentiate the two weights on the full Matlab display better for the reader, this figure displays only the resultant formed by the 500 gram mass (or monkey). The 1000 gram mass's resultant can be found on page 11.

MATLAB RESULT: 500 GRAM MASS AND THETA VS. F4 & AX.

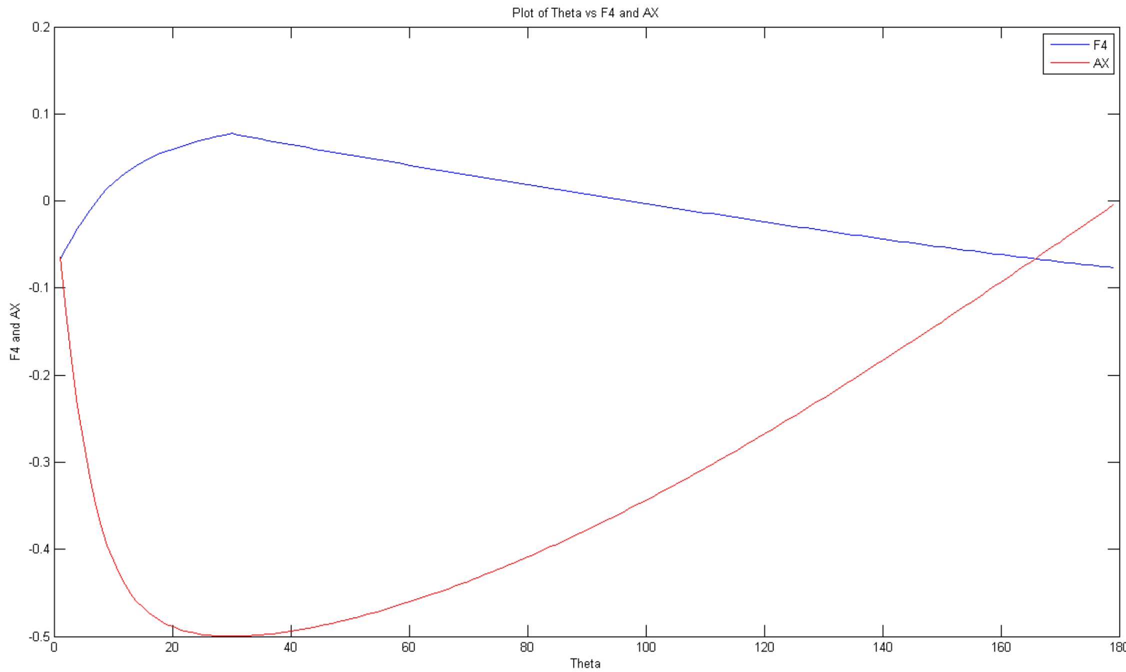


Figure 3 : 500 gram mass and Theta vs. F4 & Ax (Matlab)

This figure was produced in Matlab to show the forces of the Shoulders x resultant and the Humerus, as you can see, the two forces have much lower values than the Forearm or Bicep, The Humerus and Shoulder have much less weight to hold than the bicep or forearm, and as the angle increases, they peak quickly, and then become incredibly small as the forearm is raised. This is still applicable for our results however, since the two forces conflict each other as well, and that should be well fitted to our results on page 21.

MATLAB RESULT: 1000 GRAM MASS AND THETA VS. F1 & F3

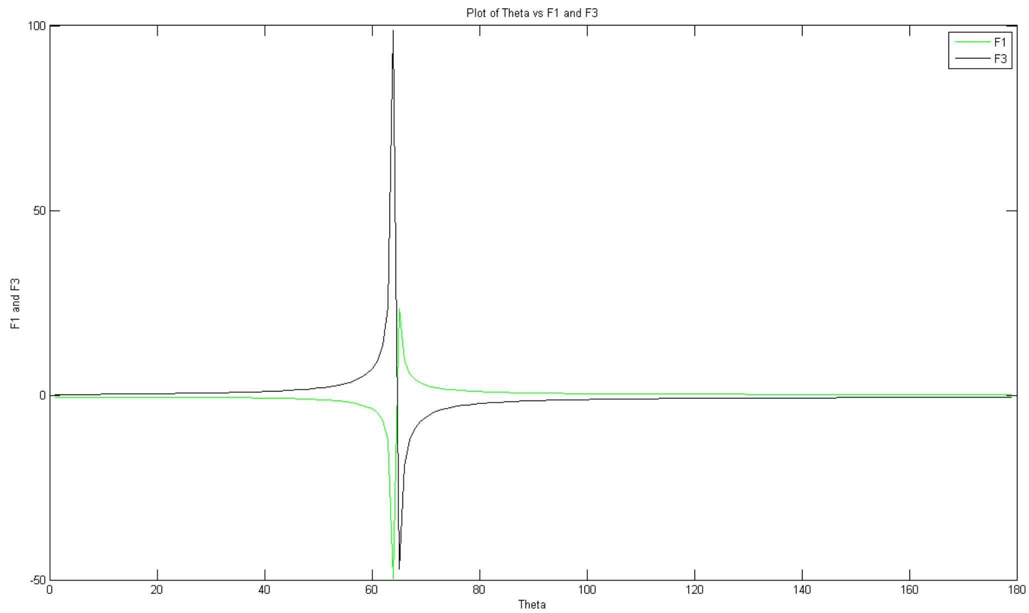


Figure 4: 1000 gram mass Theta vs. F3 and F3

This figure was produced in Matlab with twice the weight being held, to show that the forces all scale linearly in the program, as they should. Since they do it allows us to better ensure that the code is accurate to our results. Since the forces spike near infinity at 60 degrees until 70 degrees, we know also to not try to take values in this degree range, since this would result in extremely inaccurate data in our results portion.

MATLAB RESULT: 1000 GRAM MASS AND THETA VS. F4 & AX.

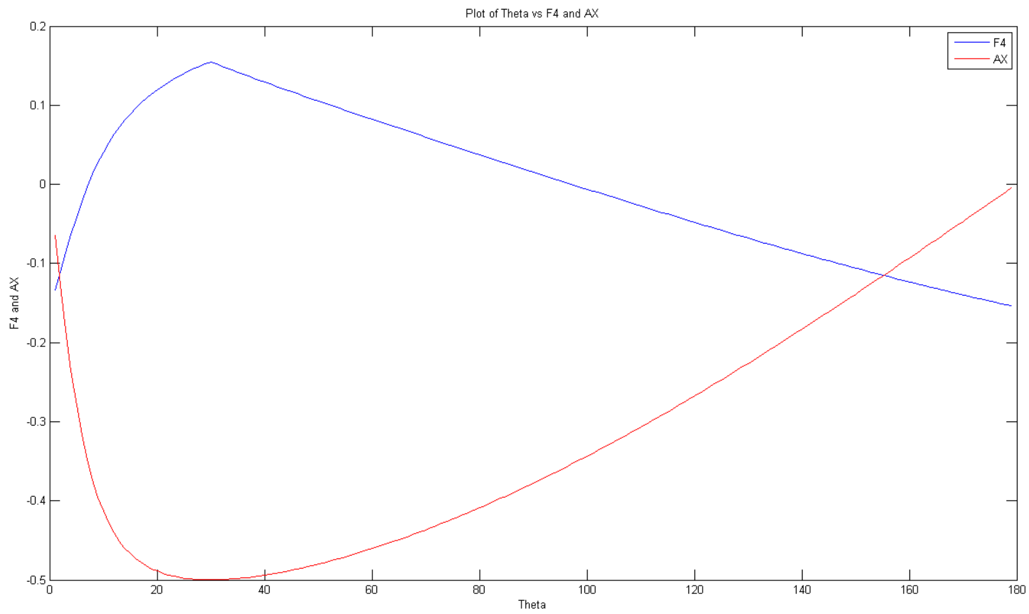


Figure 5: 1000 gram Theta vs. F4 and Ax

The scaling of our second graph is also present, which allows us to state the same for the calculations done to this as the first graphs equations. We can also better view our peaks in the two forces now, with peaks at 30 degrees for both forces, this is due to the weight in the arm driving into the forearm more, and as such push the Humerus backwards into the shoulder.

METHODS AND MATERIALS

To complete this project we made use of Pascos Model structure kits, provided to us for use through MCC's Engineering Science Department. This kit allowed us to properly simulate our initial assignment, with flexibility on what masses or lengths we set our weight at. The model we produced was original and new to us, since it needed to be properly fixed before testing. The errors we found in our model are based on the weight of our arm, and the flexability The design we produced for our model wasdifficult to produce, and streched the abilities of our pasco Kit, we use multiple axis pieces to better secure the model, while using less weight, allowing us to better increase our physical results. This model has weaknesses though, with the model being much smaller than other models, and still having a large weight placed upon it. The model would initially flex substaintally.Also, Since our project is capable of moving when not being tested, its movement makes it prone to losing screws, and loose pieces.

Our model was produced with an array of pieces , with exactly 135 different pieces in our model, and 37 inches of string used to simulate cables. This was what allowed us to have such a sturdy Model. It also allowed us to get usable data to compare our theoritcal results with. It also contributed to our heavy mass, and inability to easily move the model across both the campus and lab enviroments we tested in.

Of this total part count we can break our model down into many individual parts.

- 3 Coad Cells
- 8 5 Inch I-Beams
- 4 7 Inch I-Beams
- 4 2 inch I- Beams
- 8 3 ½ inch I-Beams
- 10 5 Inch Flexable I-Beams
- 37 inches of string
- 18 Half Circle Connectors
- 4 Full circle Connectors
- 1 5Inch Axle
- 1 9 Inch Axle
- 1 10 Inch Axle
- 72½ Inch Mounting Screws for I-Beams
- 1 I-beam Connector

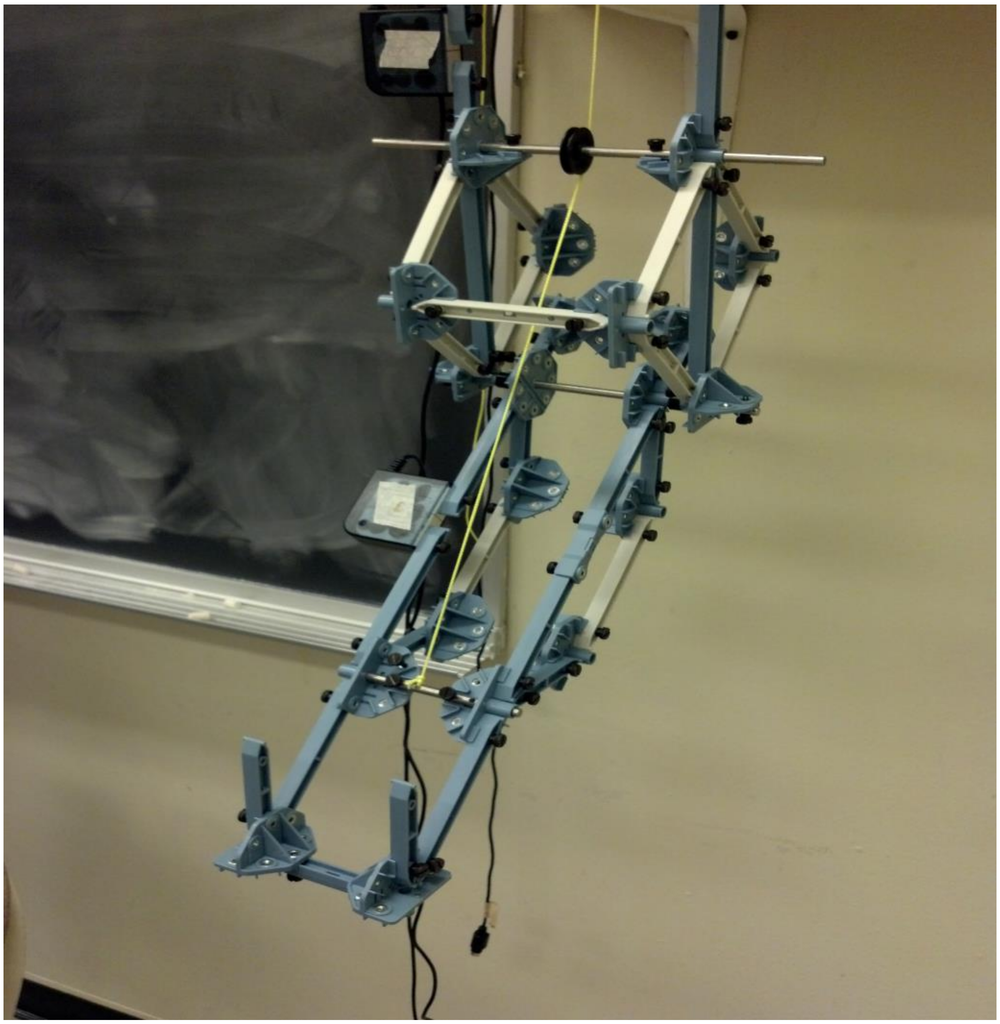


Figure 6: Model during data collection stage

We tested our model inside the crystal palace, or Mcc's Engineering design lab, as well as 7-111 making use of pasco's Capstone software and 6 port amplifiers to properly collect and input our data into the computer. This position allowed the entire model to to move around with easy, while letting the forearm hang supported by the bicep muscle.It was with this orientation that we reached the low errors we collected and have presented during our results section. This allowed room for the cables not being in the way.

RESULTS

To better understand what was happening with our model of the arm we used three force probes to determine what is going on in the structure. We place the first force probe where the forearm bone is located in the arm. The second one is located in bicep and the third force probe is located in the shoulder. This would allow use to see what is going on in the arm as it curls upward. With these force probes placed in the proper spots it allowed us to determine the force that would act like just like in a arm.

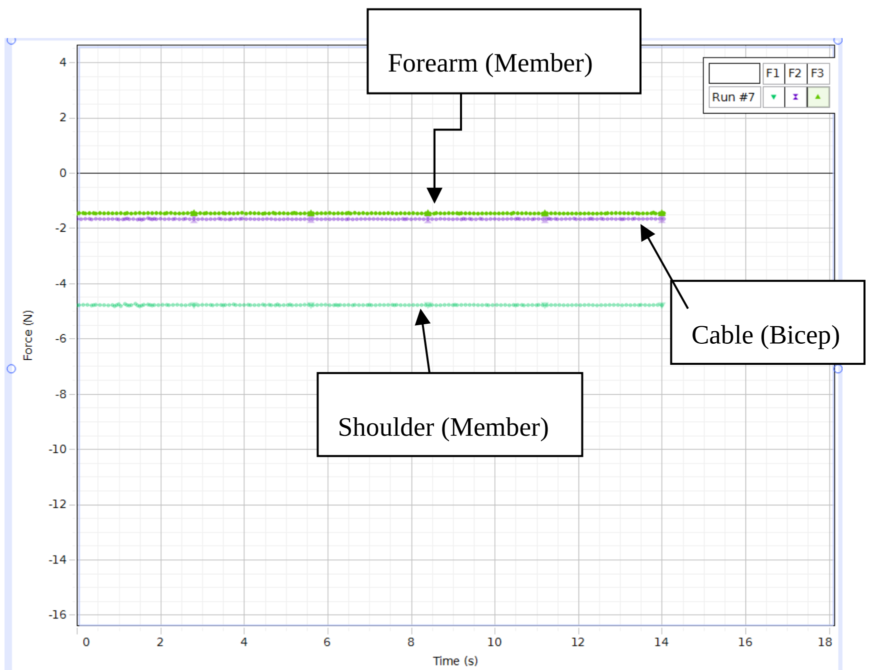
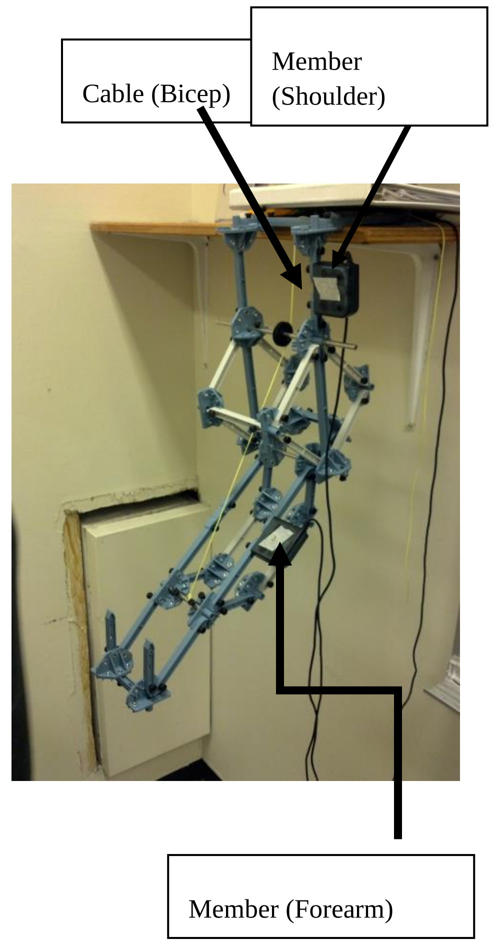


Figure 7 Test Control



The above diagram is showing the result of the arm as it hangs in it neutral position or other wise down. As you can see the bicep and forearm are under the same amount of tension. This ment that our arm was reading the right forces. We knew that if the bicep and the shoulder werent under the same tension our sturcture was built wronge. As for the Shoulder it was holding the entire weight of our arm. So due to the fact that our structure was around .5kg we knew that our force sensor should be reading around 5N.

Min.	-4.83	-1.70	-1.48
Max	-4.74	-1.65	-1.44
Mean	-4.79	-1.68	-1.47
Std. Dev.	0.01	0.01	0.01

The above data conforms was the finding form the arm in our natural position with no weight on it.

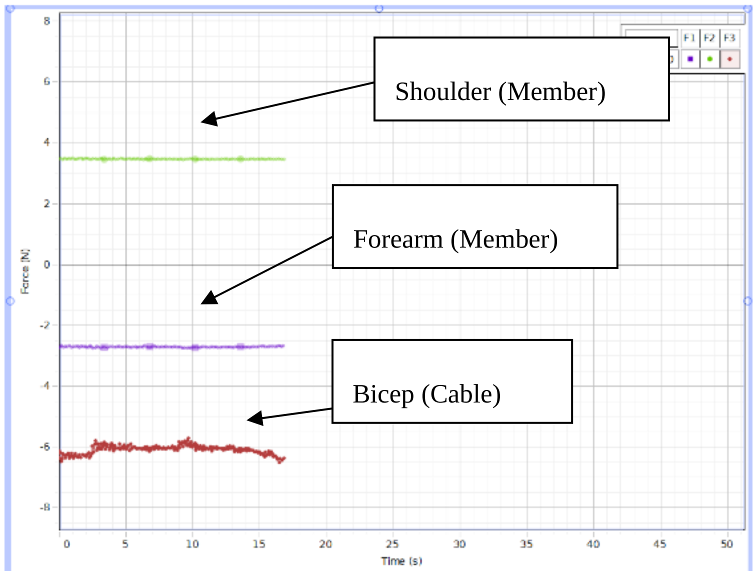


Figure 8 three forces curl no weight

The graph to the left is shown the arm (structure) fully curled. With no weight the shoulder data is matching the non curl data. The Bicep is taken the majority of the forces when curled which is what is should be doing. Then the forearm is taken the stress of the rest of the weight of the structure.

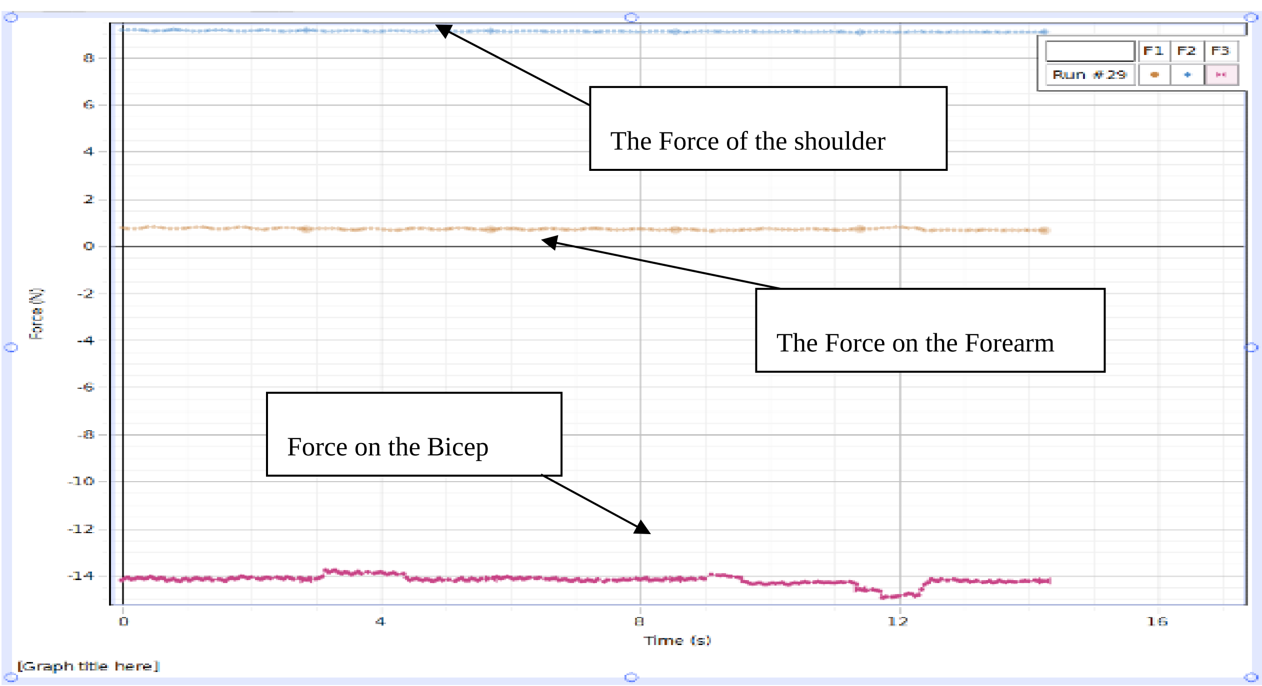


Figure 9 Three forces curl 500 grams

The graph above is shown the different forces on the arm with a .5 kg weight when the arm is at a full 90 degree curl. When looking at the shoulder forces does it make sense? The shoulder force has doubled and is now in compression. This is a good thing since we add .5kg and the structure had a weight of .5kg it should be around 9.8N which it was. The Bicep has now jump greatly to 14N. This is showing the Bicep is taken all the weight of the .5kg (Dumbbell) which matched up with our Matlab. Then the Forearm is taken almost no weight. This is because the bicep is taken the load from the forearm. This as well matched up with our math calculations.

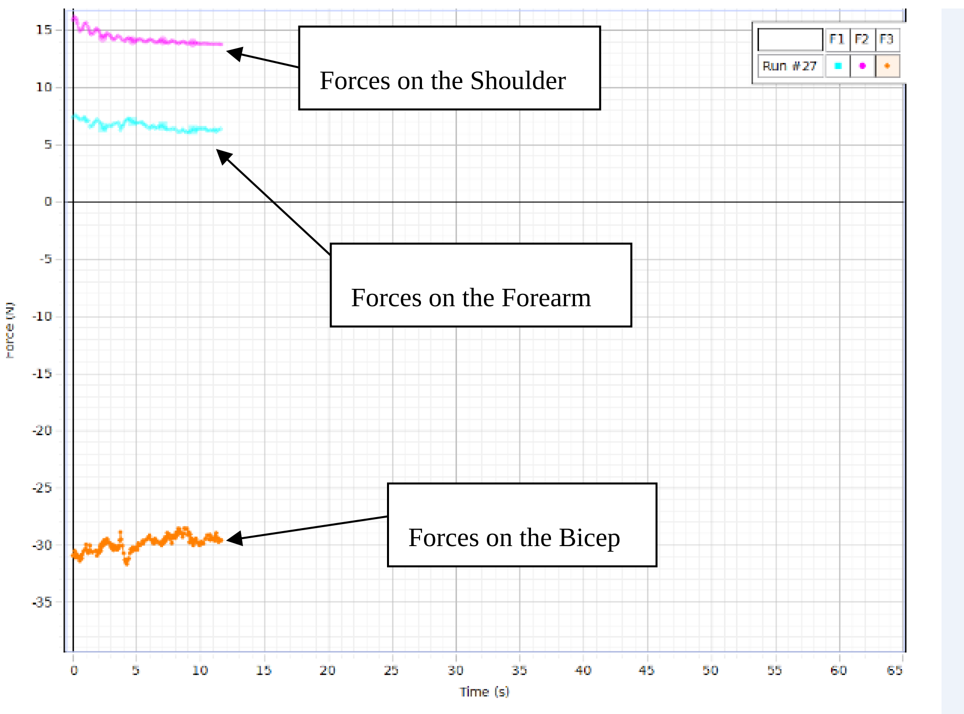


Figure 10 three forces curl 1000 grams

The Forces on the shoulder probe came out to be around 14N after it began to steady out. With a told weight of around 1.5kg this meant the forces on the shoulder where correct. The forces on the forearm began to talk more of the load as the weight increase it took around 7N. As for the bicep it once again took force from the forearm and the extra added weight (1kg). The probe read an average of 30N of force on the bicep.

DISCUSSION

The experimental findings we recorded where very close to our experimental values along with our Matlab values. We found that original we were of by 25% until we added the calculated weight of the arm. That number then greatly decrease down to 11%. We where able to even bring that percent error down more when we took data with the arm wasn't moving. This gave us a better average only putting off by 7%. This is extremely close when calculating the force on a certain body part.

All of our data was very linearly which means its easy to predict what the next result come be around. If you look at the graph with no weight when we are in the curling motion and compare it to .5kg and 1kg. You can see how the force on the bicep doubles each time. As well as the forearm does the same thing. As for the shoulder it takes the total weight on structure which shows that the force probes are all reading accurate numbers.

The major obstacle that our team faced was being able to get the correct readings from the load cells. We found that the zero would sometimes change calibration over a period of time when testing multiple forces on the structure to compare results. The zero calibration would change once the force was taken off of the structure, so the forces wouldn't go back to true zero. The second obstacle that our team faced was making the structural model sound. If any other member on the structure bent or moved in any way. We found that it resulted in a much greater force on our cables and support member.

In conclusion the structural model was successful in determining the forces on the cables and on the support member. The theoretical and experimental data matched up perfectly proving that the structure is in static equilibrium. We have also proven that the when changing the weight on structure it will be linear to any other weight added to the structure.

REFERENCES

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MATLAB CODE

- Matlab was used to check our theoretical values, and was also used to supply us with the graphs seen earlier in this report.
- We used Matlab 2014 Student Edition to produce our .mat file, which was provided to us as students through the Monroe Community College Engineering Science Department.
- Matlab pre-generated GUI library was used, and our 3d graphs were constructed from this through modifications made by Josh Wilkins for this project.

```
%%%%%%%%%%%%%%%%%%%%%% Project 3 -- Upper Body Model %%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%% By Josh Wilkins, Tom Remier, and Ryan Spies %%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear
clc

%%%%%%%%%%%%%% Input Points and Weight %%%%%%%%%%%%%

W = 2;

L1 = 15;

L2 = 13;


F1=[];

F3=[];

F4=[];

AZ=[];



for Theta=1:1:179


a = sqrt(L1^2+L2^2-(2*L1*L2*cosd(Theta)));

psi = asind((15*sind(Theta))/a);

M1 = [-2*cosd(90-Theta),-sind(psi);2*sind(90-Theta),cosd(psi)];

M2 = [0;-W];

M3 = inv(M1)*M2;


F1(Theta) = M3(1);

F3(Theta) = M3(2);


M1 = [1,-2;-4,4];

M2 = [F3*cosd(psi);-2*F3*sind(psi)];

M3 = inv(M1)*M2;


F4(Theta) = M3(1);
```

```
AZ(Theta) = M3(2);
AX(Theta) = -.5*sind(psi);

End



Theta=1:1:179;
plot(Theta,F4)
hold on

plot(Theta,AX,'red')
xlabel('Theta');
ylabel('F4 and AX');
legend('F4','AX');

title('Plot of Theta vs F4 and AX')
hold off

figure
plot(Theta,F1,'green')
hold on

plot(Theta,F3,'black')
legend('F1','F3');
xlabel('Theta');
ylabel('F1 and F3');

title('Plot of Theta vs F1 and F3')
```