Rochester Institute of Technology

EE414 – Classical Control Technical Memorandum

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Subject: Laboratory #5: The Effects of an Additional Pole and/or Zero Inside a Loop

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Abstract

The ability to analyze complex system responses is vital to a core understanding of system behavior in nearly every field of engineering. Modeling the transient response of the LJE motor module provides a great introduction into this form of analysis. The focus of this report is on the effect of the addition of a pole or zero into the system inside a loop. Specifically, what are the conditions under which the effects of this addition to the transfer function be reasonably neglected. Additionally, this report compares the effect of a pole or zero when within the feedback loop to the effect of those added outside the feedback loop. The results tended to agree with simulated analysis and calculations of the system with minor inconsistencies caused by the simplification of parameter acquisition. The results show that the larger the pole or zero added, the less it has of an effect. This makes it more reasonable to approximate the system without adding the pole or zero.

Theory

To model and acquire a simulated transient response of the motor unit, the closed loop transfer function below was implemented as shown below.

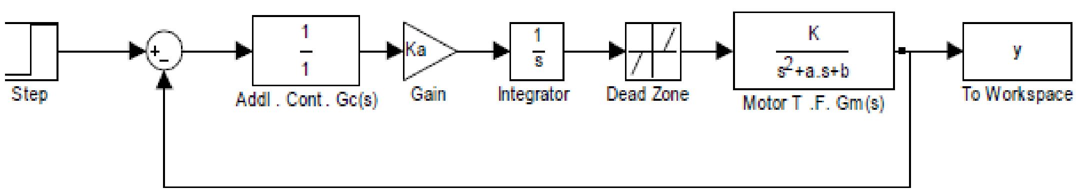

Figure 1: Closed Loop Transfer Function Model

In order to use root-locus techniques to predict the effect of a zero into the loop of the system, the closed loop characteristic equation needs to be rewritten into a form that appears like root locus analysis. This means the closed loop denominator needs to be set equal to zero as follows:

$$s^3 + as^2 + bs + 2.5K + 2.5Ks/z = 0 \tag{1}$$

Note the value of *a* and *b* were determined in a previous lab and implemented in analysis. It follows that:

$$\left\{1+\frac{(2.5K/z)\ s}{s^3+as^2+bs+2.5K}\right\}=0 \quad \Rightarrow \quad \hat{G} = \frac{\frac{1}{z}(2.5K)\ s}{s^3+as^2+bs+2.5K} \tag{2}$$

From a root locus analysis, it can be seen that as the zero (z) decreases, rise and peak times increase while settling time and overshoot decrease.

Similarly, if a pole is added within the loop of the system:

$$(s^3+as^2+bs)(s/p+1)+2.5K=0 \tag{3}$$

Which can again be rewritten in the form suitable for root locus:

$$1+\frac{(1/p)\ s\ (s^3+as^2+bs)}{s^3+as^2+bs+2.5K}=0 \quad \Rightarrow \quad \hat{G} = \frac{(1/p)\ s\ (s^3+as^2+bs)}{s^3+as^2+bs+2.5K} \tag{4}$$

From a root locus analysis, it can be seen that as the pole (p) decreases, overshoot, peak time, and settling time increase, but rise time will remain somewhat constant.

Additionally, several parameters were calculated including rise time, percent overshoot, peak time, and settling time to facilitate a more accurate comparison between models. The formulas for a second order system are shown below:

$$\%OS = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \tag{5}$$

$$t_P = \frac{\pi}{|Im(pole)|} \tag{6}$$

$$t_s = 4\tau = \frac{-4}{Re(pole)} \tag{7}$$

$$t_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)/\omega_n \tag{8}$$

It is important to note that the results of this experiment will not be analogous to all other systems, since there are unique non-linearities in the motor such as deadzone and voltage limits.

Results

The original closed loop transfer function, with an additional zero added to the system within the feedback loop, was modeled and simulated varying the zero value as z = [100, 10, 5, 2, 1, 0.5].

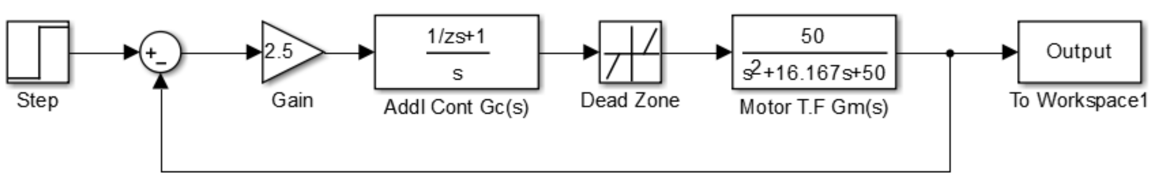

Figure 2: Original transfer function model with the addition of a zero inside the feedback loop

Simulating the above transfer function model with the additional zero implemented. Note that the original transfer function, without the additional zero, is included for comparison:

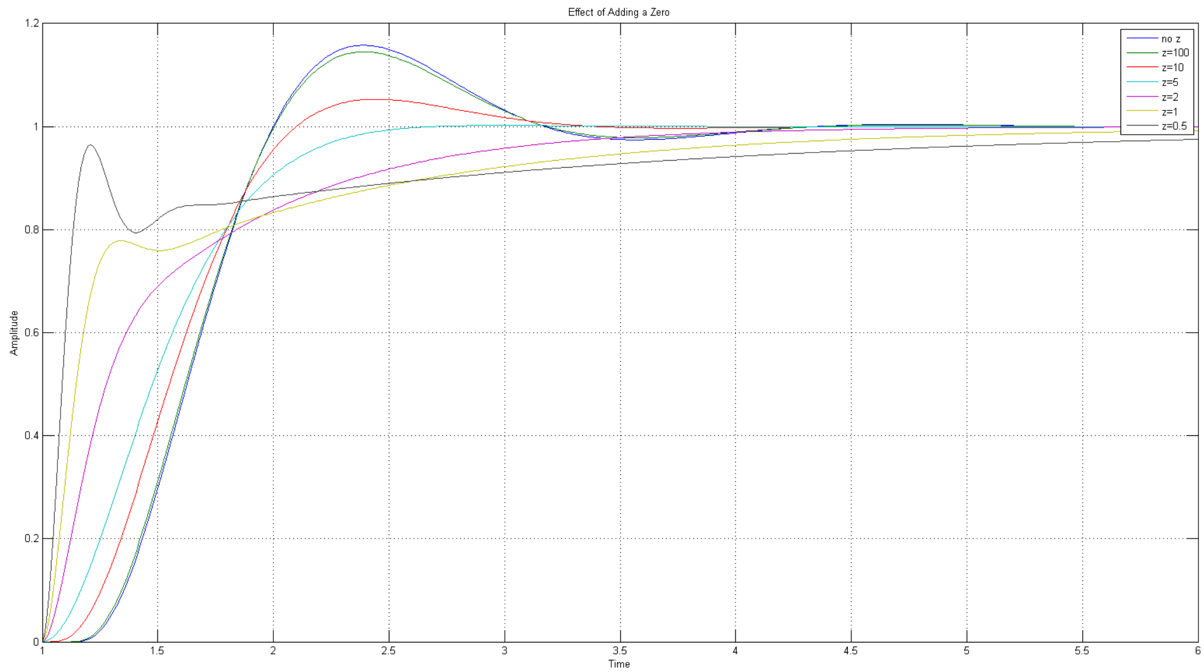

Figure 3: Original closed loop transfer function with the addition of a zero

Similarly, the apparent open loop function in (2) was simulated for verification of analysis:

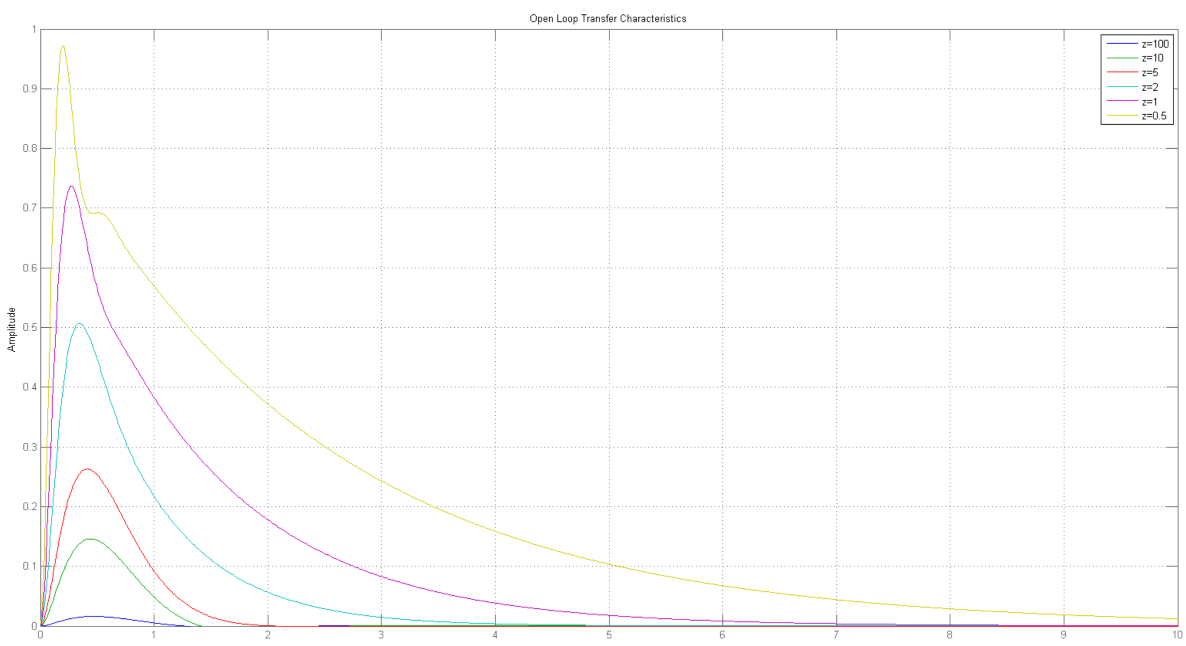

Figure 4: Apparent open loop transfer function of an additional zero

From the closed-loop transfer function simulation in figure 3, the characteristics of the transfer curves were found, tabulated as shown below:

Zero	Rise Time	Peak Time	Overshoot	Settling Time
no z	0.547	2.390	15.698	2.223
100	0.556	2.390	14.368	2.167
10	0.650	2.440	5.851	1.488
5	0.808	2.980	0.505	0.831
2	1.247		0.000	2.111
1	1.567		0.000	3.327
0.5	0.125		0.000	4.526

Table 1: Closed loop transfer function parameters with the addition of a zero

The original closed loop transfer function, with an additional pole added to the system within the feedback loop, was then modeled and simulated varying the pole value as p = [100, 10, 5, 2, 1, 0.5].

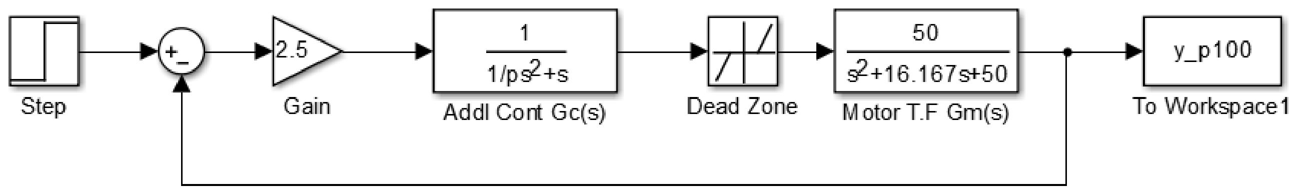

Figure 5: Original transfer function model with the addition of a pole inside the feedback loop

Simulating the above transfer function model with the additional zero implemented. Note that the original transfer function, without the additional zero, is included for comparison:

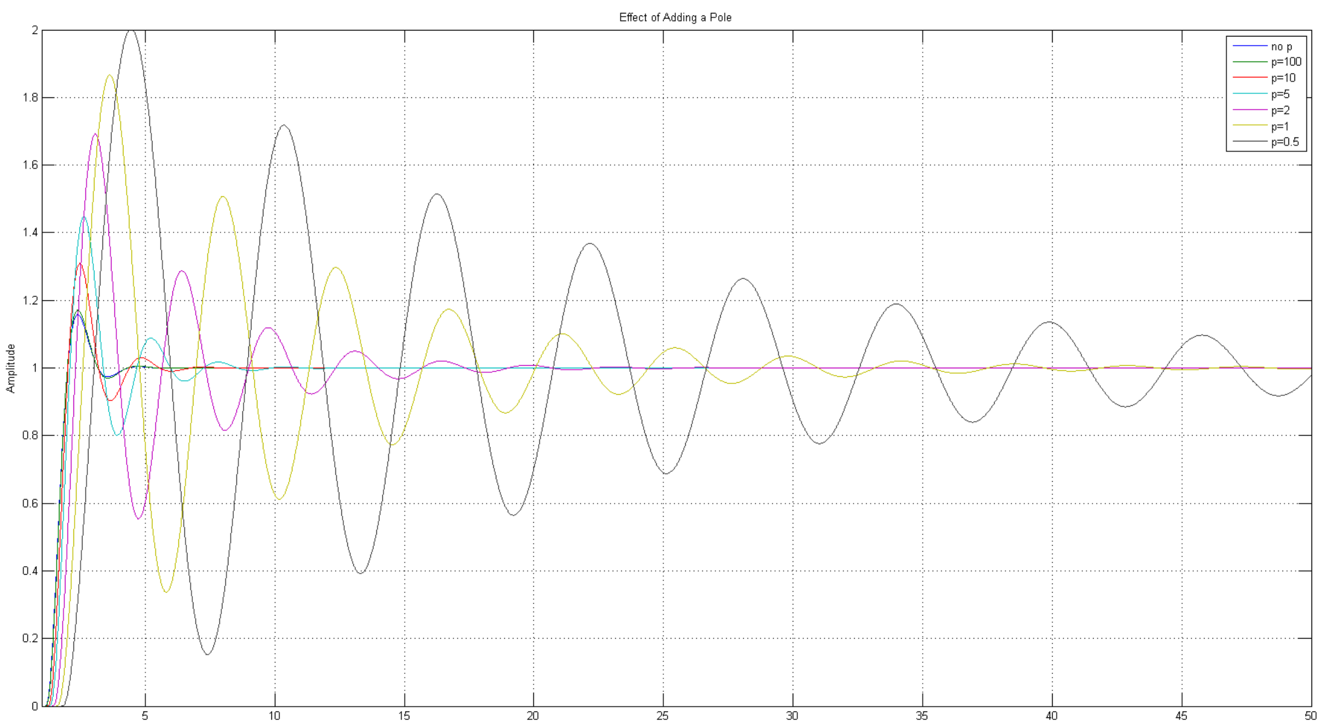

Figure 6: Original closed loop transfer function with the addition of a pole

The figure below helps to illustrate the effect of a higher pole values' effect on the system by magnifying the scope of the figure:

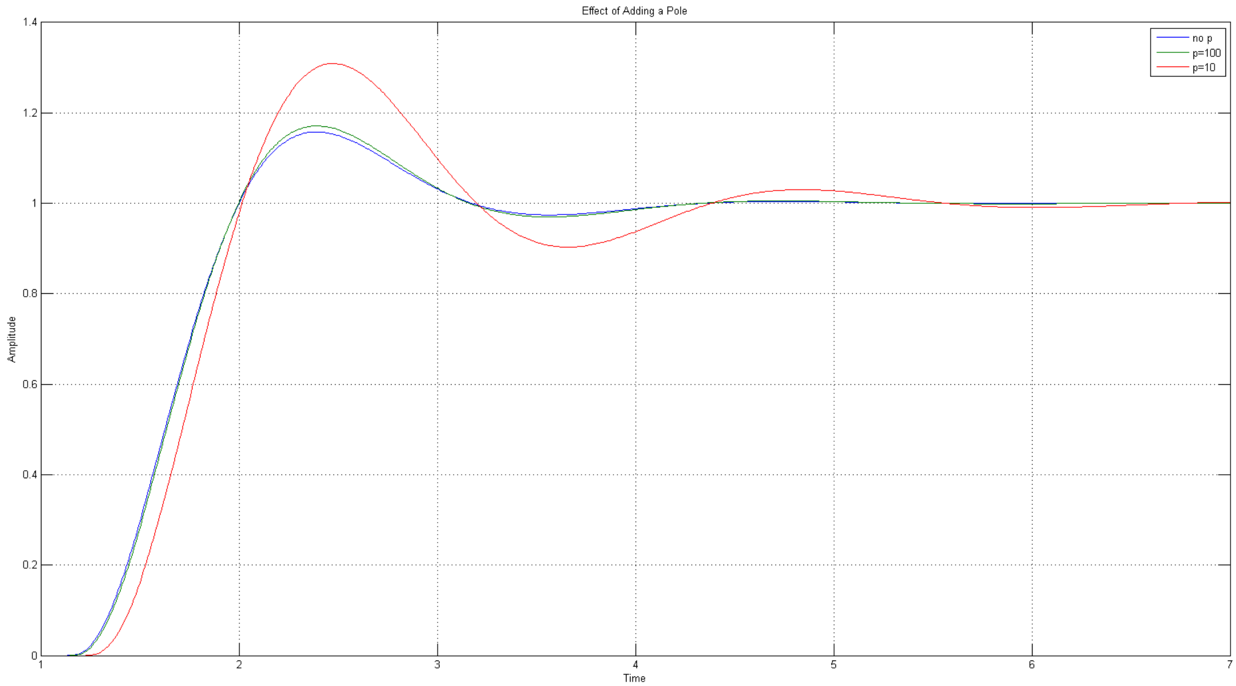

Figure 7: Original closed loop transfer function with the addition of high-valued poles

Similarly, the apparent open loop function in (4) was simulated for verification of analysis:

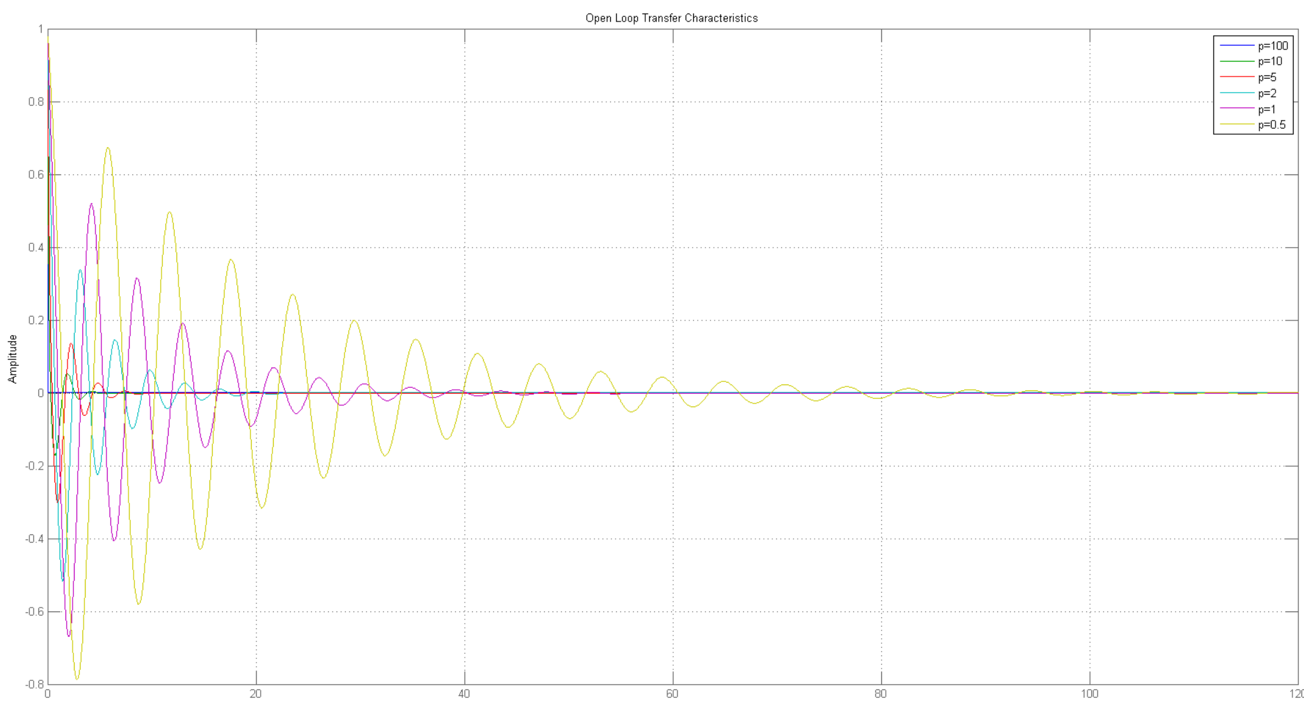

Figure 8: Apparent open loop transfer function of an additional pole

Again, to help illustrate the effect of a higher pole values' effect on the system, the scope of the figure has been magnified:

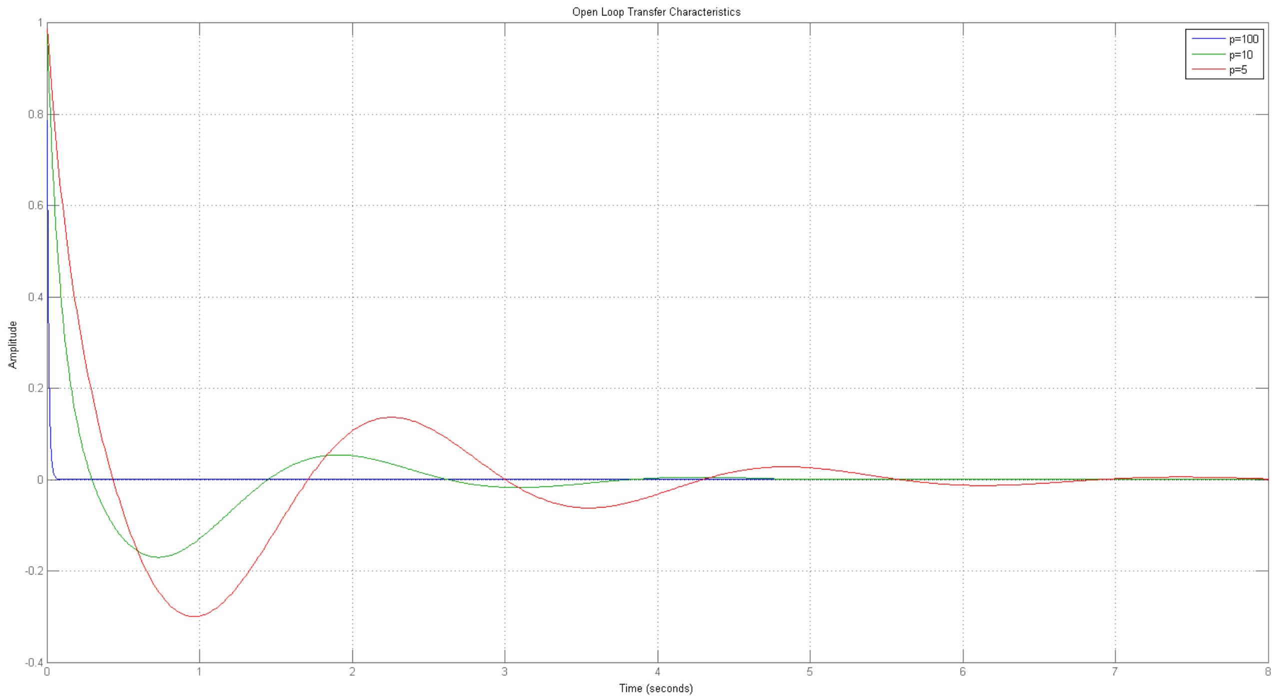

Figure 9: Apparent open loop transfer function with the addition of high-valued poles

From the closed-loop transfer function simulation in figure 6, the characteristics of the transfer curves were found, tabulated as shown below:

Pole	Rise Time	Peak Time	Overshoot	Settling Time
no p	0.547	2.390	15.698	2.223
100	0.538	2.390	17.059	2.269
10	0.496	2.470	30.921	3.437
5	0.499	2.630	44.203	5.261
2	0.554	3.070	68.644	16.211
1	0.644	3.630	87.736	32.261
0.5	0.859	4.344	108.084	

Table 2: Closed loop transfer function parameters with the addition of a pole

Similarly, a hardware analysis was performed to verify calculated and simulated analysis. The Simulink test script for the original transfer function with the addition of a zero into the loop with an integral controller is shown in the figure below:

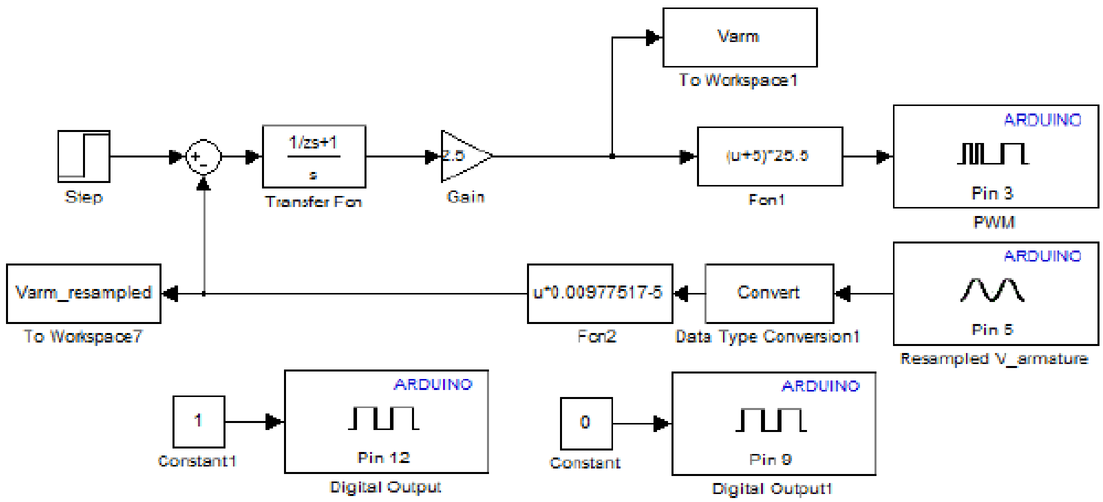

Figure 10: Hardware approach for the original transfer function with an integral controller and added zero

After collecting all the data for the same zeroes as the simulation, the hardware transfer curve with this additional zero are shown below. Note that the data was passed first through a high pass filter to remove some of the unnecessary noise.

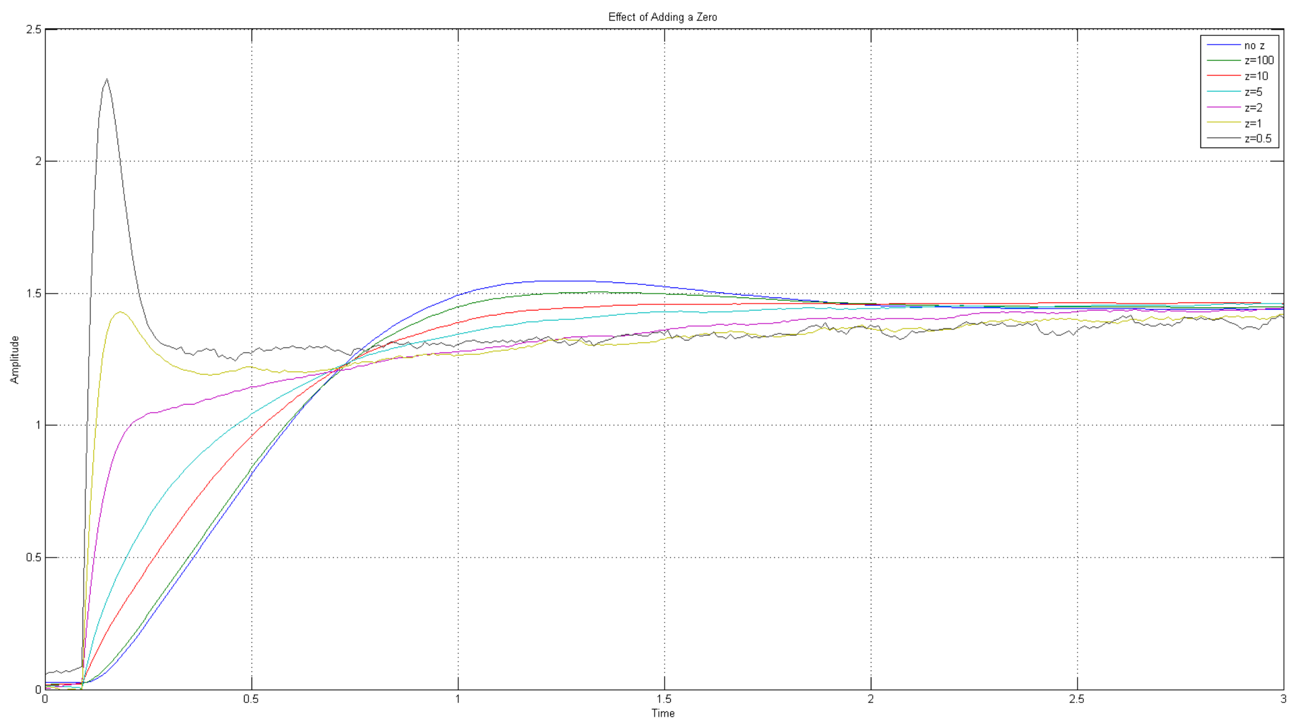

Figure 11: Hardware analysis with the addition of a zero

From the collected data, the characteristics of the transfer function with the additional zero were found and tabulated below:

Zero	Rise Time	Peak Time	Overshoot	Settling Time
no z	0.5473	1.200	8.523	1.491
100	0.5814	1.300	4.945	1.466
10	0.6865			0.794
5	0.7430			1.432
2	0.9734			2.488
1	0.0047	0.100	73.636	3.485
0.5	0.0165	0.150	240	4.955

Table 3: Hardware closed loop transfer function parameters with the addition of a zero

Similarly, a hardware analysis was performed for the original transfer function with the addition of a pole into the loop with an integral controller as modeled and shown in the figure below:

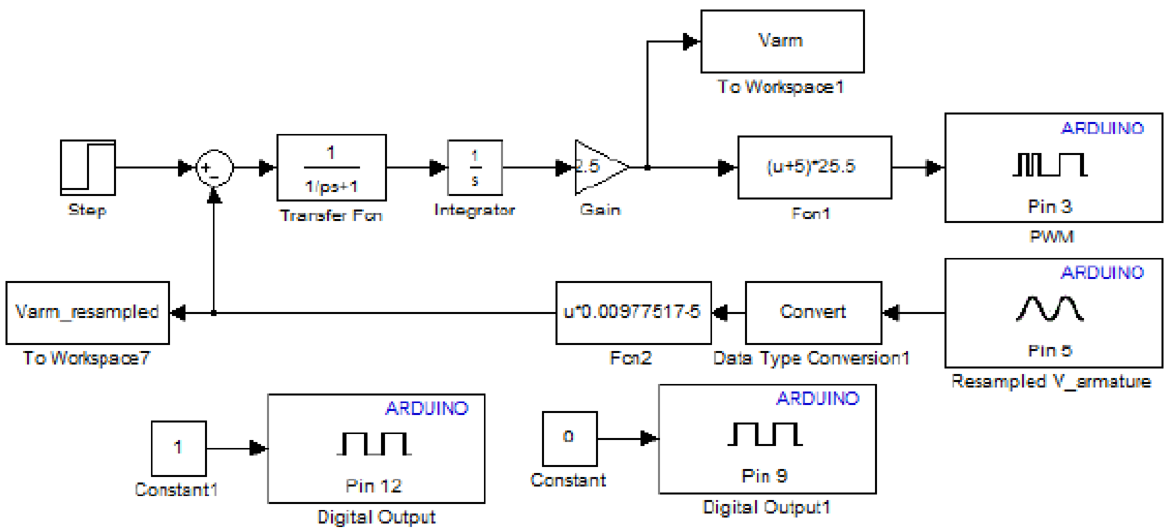

Figure 12: Hardware approach for the original transfer function with an integral controller and added pole

After collecting all the data for the same poles as the simulation, the hardware transfer curve with this additional pole are shown in figure 13 below. From the collected data, the characteristics of the transfer function with the additional pole were found and tabulated below:

Hardware				
Pole	Rise Time	Peak Time	Overshoot	Settling Time
no p	0.547	1.200	8.523	1.491
100	0.588	1.230	5.978	1.215
10	0.534	1.280	15.823	2.222
5	0.572	1.390	26.000	3.547
2	0.682	1.900	44.167	8.086
1	0.940	2.360	60.484	14.977
0.5	1.242	3.200	64.286	

Table 4: Hardware closed loop transfer function parameters with the addition of a pole

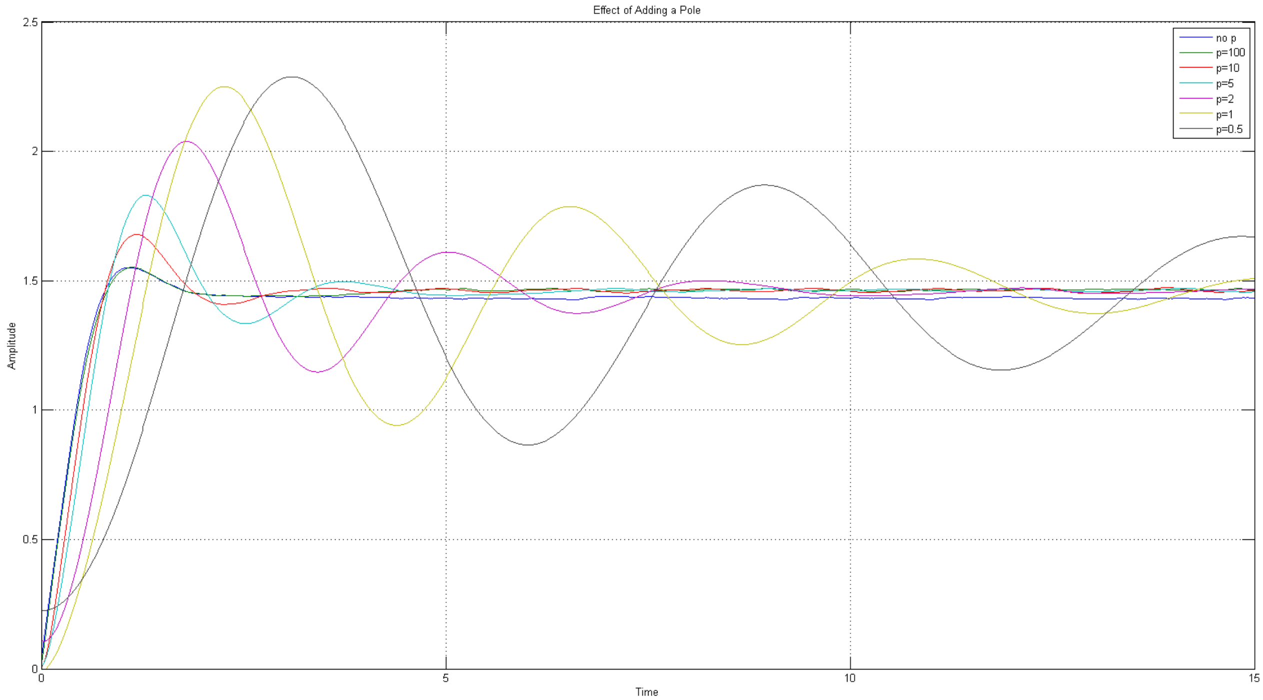

Figure 13: Hardware analysis with the addition of a pole

Discussion

From each of the above graphs and corresponding data, it is clear that as the value of the pole or zero that is added increases, the effect of that pole/zero decreases. This indicates that the larger the value of the pole or zero, the more appropriate it would be to ignore it for the sake of a reduction in complexity. Previously, when the pole was added outside the feedback loop, this would drastically increase the rise time and settling time, but decrease the overshoot of the system. The addition of a small zero however, would have the opposite affect for each characteristic.

With the pole or zero inside the feedback loop however, the characteristics behave differently according to the placement of the pole or zero. When the zero was added inside the feedback loop, it is hard to quantify its effect due to various noise issues. However, it would appear as if rise time and settling time should increase, while overshoot decreases as the magnitude of the zero added is decreased. The addition of a pole into the loop however appears to increase every characteristic.

If any benefit to the system is needed, the addition of a pole inside the feedback loop would not help. If the pole is added outside the loop however, the systems overshoot would be reduced at the cost of rise and settling times. The addition of a zero inside the feedback loop appears to have this same effect. If the opposite effects are needed, the addition of a zero outside the loop should be implemented. Depending on the type of response required, it is clear that the user can adjust the gain factor in addition to the zeroes or poles a system has to fit a wide range of applications.

Conclusion

The transient response of the LJE motor module was tested and compared against a simulated version as well as against calculated values. The simulation was performed in matlab and the hardware was used to verify the general correlation of data. Overall, the results the lab were very similar and tended to reiterate the initial conceptions about dc-motor responses. Furthermore, it was shown that the addition of the pole or zero to the integral feedback controller changed the parameters of the system. It is clear that the user has some control over the response of the system by the addition of a pole or zero. Any minor inconsistencies within the results can be accounted for in the inherent nature of the simplification methods to obtain parameters. This would be the largest source contributing to any errors found within the system. If this experiment were to be completed again, a method to find the actual parameters of the system should be implemented to remove the largest source of error.