EXAMEN AÑO PASADO

Determinar si los siguientes lenguajes son regulares.

a) $\mathcal{L} = \{ a^m b^n c^p d^q \,/\, m+n \geq p+q \}$

b) $\mathcal{L} = \{ 0^i 1^j 0^k,\ i,j,k \geq 0 \ (i > j \Rightarrow j = k) \}$

a) $\forall n \in \mathbb{N} \ \exists z \in \mathcal{L} \,/\, |z| \geq n,\ z = uvw$ con $|uv| \leq n$ $|v| \geq 1$ $\exists i \geq 0 /\ uv^i w \notin \mathcal{L}$

$z = a^n b c^n d$ $\quad u = a^k$ $\quad v = a^\ell$ $\quad \ell \geq 1$ $\quad k+\ell \leq n$ $\quad w = a^{n-k-\ell} b c^n d$

Para $i = 0$, $uv^0 w = uw = a^k a^{n-k-\ell} b c^n d = a^{n-\ell} b c^n d \notin \mathcal{L}$ porque $n-\ell+1 < n+1$ pues $\ell \geq 1$

b) $i > j \Rightarrow j = k$, es lo mismo que $\underbrace{j \neq k \Rightarrow i \leq j}_{z = 0^n 1^n 0^{n+1}}$

$u = 0^k$ $\quad v = 0^\ell$ $\quad \ell \geq 1$ $\quad k + \ell \leq n$ $\quad w = 0^{n-k-\ell} 1^n 0^{n+1}$

$\exists i \geq 0 / uv^i w \notin \mathcal{L} \Rightarrow i = 2$, $uv^2 w = 0^k 0^{2\ell} 0^{n-k-\ell} 1^n 0^{n+1} = 0^{n+\ell} 1^n 0^{n+1} \notin \mathcal{L}$