Repaso Geometría

$\boxed{1.}$ (Ejercicio 27 Relación)

$$U_k = \mathcal{L}\{(0,-1,k,3),(0,k,-2-k,3),(k-2,-1,-2,3)\}$$

a) ¿ $\dim_{\mathbb{R}}(U_k)$ en función de k? Base y ecuaciones de U_k para cada k.

$$\begin{vmatrix} 0 & 0 & k-2 \\ -1 & k & -1 \\ k & -2-k & -2 \\ 3 & 3 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & k-2 \\ -1 & k & -1 \\ k & -2-k & -2 \end{vmatrix} = (2-k)(-2-k) - k^3 + 2k^2 = -4 - 2k + 2k + k^2 - k^3 + 2k^2 = -k^3 + 3k^2 - 4$$

$$k^3 - 3k^2 + 4 = 0$$

$$\begin{array}{c|cccc} & 1 & -3 & 0 & 4 \\ 2 & \downarrow & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

$$k^2 - k - 2 = 0$$

$$k = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$$\begin{vmatrix} 0 & 0 & k-2 \\ -1 & k & -1 \\ 3 & 3 & 3 \end{vmatrix} = -3k + 6 - 3k^2 + 6k = -3k^2 + 3k + 6$$

$$k^2 - k - 2 = 0 \Rightarrow k = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$$\begin{vmatrix} 0 & -3 \\ -1 & -1 \end{vmatrix} = 3 \neq 0 \Rightarrow k = -1,\ rg(A) = 2$$

$$\begin{vmatrix} -4 & -2 \\ 3 & 3 \end{vmatrix} = -6 \neq 0$$

$k = 2$

$rg(A) = 2$

$k=-1 \Rightarrow \dim_{\mathbb{R}}(U_{-1})=2 \Rightarrow \text{Base}_{-1} = \{(0,-1,-1,3),(-3,-1,-2,3)\}$

$\cancel{k=2 \Rightarrow \dim_{\mathbb{R}}(U_2)=2 \Rightarrow \text{Base } 2 = \{}$

$k \neq -1 \Rightarrow \dim_{\mathbb{R}}(U_k)=3 \Rightarrow \text{Base} = \{(0,-1,k,3),(0,k,-2-k,3),$
$(k-2,-1,-2,3)\}$

$$\operatorname{rg}\begin{pmatrix} 0 & -3 & x \\ -1 & -1 & y \\ -1 & -2 & z \\ 3 & 3 & t \end{pmatrix} = 2 \Rightarrow \begin{vmatrix} 0 & -3 & x \\ -1 & -1 & y \\ -1 & -2 & z \end{vmatrix} = 0 \Rightarrow 3y+2x+x-3z=0$$

$$3x+3y-3z=0$$

$$\boxed{x+y-z=0}$$

$$\begin{vmatrix} -1 & -1 & y \\ -1 & -2 & z \\ 3 & 3 & t \end{vmatrix} = 0 \Rightarrow 2t-3z-3y+6y-t+3z=0$$

$$\boxed{3y+t=0}$$

Para $k \neq -1$ solo se necesita una ecuación:

$$\begin{vmatrix} 0 & 0 & k-2 & x \\ -1 & k & -1 & y \\ k & -2-k & -2 & z \\ 3 & 3 & 3 & t \end{vmatrix} = 0 \Rightarrow (k-2)\begin{vmatrix} -1 & k & y \\ k & -2-k & z \\ 3 & 3 & t \end{vmatrix} + x \begin{vmatrix} -1 & k & -1 \\ k & -2-k & -2 \\ 3 & 3 & 3 \end{vmatrix} =$$

$$(k-2)\big((2+k)t+(3k)z+(3k)y+(6+3k)y-k^2t+3z\big)$$
$$+x\big((2+k)3-6k-3k-6-3k-3k^2-6\big)=$$
$$(k-2)\big((-k^2+k+2)t+(6k+6)y+(3k+3)z\big)+x(-3k^2-6k-6)=0$$

$$\boxed{(-3k^2-6k-6)x+(k-2)(6k+6)y+(k-2)(3k+3)z+(k-2)(-k^2+k+2)t=0}$$

b) Encontrar para k con $\dim_{\mathbb{R}}(U_k)=2$, encontrar un subespacio de $\mathbb{R}^4$ tal que $\mathbb{R}^4 = U_k \oplus W$. Determina unas ec. para W.

<u>Fórmula dimensiones</u>

$$\underbrace{\dim_{\mathbb{R}}(U_k)}_{=2} + \underbrace{\dim_{\mathbb{R}}(W)}_{=2} = \overbrace{\dim_{\mathbb{R}}(U_k+W)}^{4 \text{ para que sea } \mathbb{R}^4 \; =} + \overbrace{\dim_{\mathbb{R}}(U_k \cap W)}^{0 \; =}$$

W tendrá dimensión 2, y para que $U_k \oplus W$ sea $\mathbb{R}^4$, necesitamos que la base de W sea L.I. con la de U_k.

$B_W = \{(0,0,1,0),(0,0,0,1)\}$ pues:

$$\left(\begin{array}{ccc|c} 0 & -3 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -1 & -2 & 1 & 0 \\ \hline 3 & 3 & 0 & 1 \end{array}\right) \qquad \begin{vmatrix} 0 & -3 & 0 \\ -1 & -1 & 0 \\ -1 & -2 & 1 \end{vmatrix} = -3 \neq 0$$

y por tanto el determinante de 4×4 también, por lo que $W \oplus U_k = \mathbb{R}^4$

Unas ec. cartesianas de W serán:

$$rg\begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 1 & 0 & z \\ 0 & 1 & t \end{pmatrix} = 2 \Rightarrow \begin{vmatrix} 0 & 0 & x \\ 1 & 0 & z \\ 0 & 1 & t \end{vmatrix} = 0 \Rightarrow \boxed{x=0}$$

$$\begin{vmatrix} 0 & 0 & y \\ 1 & 0 & z \\ 0 & 1 & t \end{vmatrix} = 0 \Rightarrow \boxed{y=0}$$

$\boxed{2.}$ (Ejercicio 23 Relación)

En $M_2(\mathbb{C})$:

$$A = \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix} \quad B = \begin{pmatrix} -1 & -i \\ 1 & 2i \end{pmatrix} \quad C = \begin{pmatrix} \alpha & 0 \\ 1 & -i \end{pmatrix}$$

¿Valores de $\alpha \in \mathbb{C}$ para los que $U = \mathcal{L}(A,B,C)$ tiene dimensión 2? Calcula una base de U y las coordenadas de la matriz $v = \begin{pmatrix} 2i-1 & -i \\ 3 & 0 \end{pmatrix}$

$$B_u = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}$$

$A = (i,0,1,-i) \quad B = (-1,-i,1,2i) \quad C = (\alpha,0,1,-i)$

$$\begin{vmatrix} i & -1 & \alpha \\ 0 & -i & 0 \\ 1 & 1 & 1 \\ -i & 2i & -i \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} i & -1 & \alpha \\ 0 & -i & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1 + \alpha i = 0 \Rightarrow \boxed{\alpha = i}$$

$$\Downarrow$$

$\dim_{\mathbb{C}} M_2 = 2$

$B = \{(i,0,1,-i),(-1,-i,1,2i)\}$

$a(i,0,1,-i) + b(-1,-i,1,2i) = (2i-1,-i,3,0)$

$$\left.\begin{array}{l} ai - b = 2i - 1 \Rightarrow a = 2 \; b = 1 \\ -bi = -i \Rightarrow b = 1 \\ a + b = 3 \Rightarrow a = 2 \; b = 1 \\ -ai + b2i = 0 \Rightarrow a = 2 \; b = 1 \end{array}\right\} (2,1)$$

$\boxed{3.}$ (Ejercicio 24 Relación)

$U_1 = \mathcal{L}\{(3,6,1,0),(1,0,-1,2),(2,3,0,1)\}$

$U_2 = \mathcal{L}\{(2,0,-1,3),(3,3,-2,4)\}$

$U_4 = \{(x,y,z,t) \in \mathbb{R}^4 \mid x-2y+t=0,\ 3x+y+6z=0\}$

a) Calcular bases y dimensión.

$\boxed{U_1}$ $\begin{pmatrix} 3 & 1 & 2 \\ 6 & 0 & 3 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \Rightarrow \begin{vmatrix} 3 & 1 & 2 \\ 6 & 0 & 3 \\ 1 & -1 & 0 \end{vmatrix} = 3-12+9=0$

$\begin{vmatrix} 3 & 1 & 2 \\ 6 & 0 & 3 \\ 0 & 2 & 1 \end{vmatrix} = 24-6-18=0$

$\dim_{\mathbb{R}}(U_1) = 2 \quad B = \{(3,6,1,0),(1,0,-1,2)\}$

$\boxed{U_2}$ $\begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix} = 6 \neq 0 \Rightarrow \dim_{\mathbb{R}}(U_2) = 2$

$B = \{(2,0,-1,3),(3,3,-2,4)\}$

$\boxed{U_4}$ $\dim_{\mathbb{R}}(U_4) = 2$

Una base de U_4 vendrá dada por dos vectores L.I. que cumplan sus ec. cartesianas:

$B = \{(-1,3,0,-5),(1,0,-\frac{1}{2},-1)\}$

$\hookrightarrow$ [illegible]

$B = \{(-1,-3,1,-5),(2,0,-1,-2)\}$

b) Base y dimensión de $U_1 \cap U_2$ y $U_2 \cap U_4$

Ec. cartesianas de U_1

$rg\begin{pmatrix} 3 & 1 & x \\ 6 & 0 & y \\ 1 & -1 & z \\ 0 & 2 & t \end{pmatrix} = 2 \Rightarrow \begin{vmatrix} 3 & 1 & x \\ 6 & 0 & y \\ 1 & -1 & z \end{vmatrix} = 0 \Rightarrow y-6x-6z+3y=0 \Rightarrow -6x+4y-6z=0$ $\boxed{3x-2y+3z=0}$

$\begin{vmatrix} 3 & 1 & x \\ 6 & 0 & y \\ 0 & 2 & t \end{vmatrix} = 0 \Rightarrow 12x-6t-6y=0 \Rightarrow \boxed{2x-y-t=0}$

Ec. cartesianas de U_2

$$rg\begin{pmatrix} 2 & 3 & x \\ 0 & 3 & y \\ -1 & -2 & z \\ 3 & 4 & t \end{pmatrix} = 2 \Rightarrow \begin{vmatrix} 2 & 3 & x \\ 0 & 3 & y \\ -1 & -2 & z \end{vmatrix} = 0 \Rightarrow 6z - 3y + 3x + 4y = 0$$

$$\boxed{3x + y + 6z = 0}$$

$$\begin{vmatrix} 2 & 3 & x \\ 0 & 3 & y \\ 3 & 4 & t \end{vmatrix} = 0 \Rightarrow 6t + 9y - 9x - 8y = 0$$

$$-9x + y + 6t = 0$$

$$\boxed{9x - y - 6t = 0}$$

$\boxed{U_1 \cap U_2}$

$$\begin{pmatrix} 3 & -2 & 3 & 0 \\ 2 & -1 & 0 & -1 \\ 3 & 1 & 6 & 0 \\ 9 & -1 & 0 & -6 \end{pmatrix} \qquad \begin{vmatrix} 3 & -2 & 3 \\ 2 & -1 & 0 \\ 3 & 1 & 6 \end{vmatrix} = -18 + 6 + 9 + 24 \neq 0$$

$$\Rightarrow \begin{pmatrix} 3 & -2 & 3 & 0 \\ 2 & -1 & 0 & -1 \\ 3 & 1 & 6 & 0 \\ 0 & 5 & & \end{pmatrix}$$

$$-1\begin{vmatrix} 3 & -2 & 3 \\ 3 & 1 & 6 \\ 9 & -1 & 0 \end{vmatrix} - 6\begin{vmatrix} 3 & -2 & 3 \\ 2 & -1 & 0 \\ 3 & 1 & 6 \end{vmatrix} = -1(-108 - 9 - 27 + 18) - 6(21) = 0$$

$$\begin{cases} 3x - 2y + 3z = 0 \\ 2x - y - t = 0 \\ 3x + y + 6z = 0 \end{cases}$$

Necesitamos un vector $\Rightarrow$ $\begin{cases} 3x - 2y = -3 \\ 2x - y = 0 \\ 3x + y = -6 \end{cases}$

$$\begin{cases} 3x - 2y = -3 \\ -4x + 2y = 0 \end{cases}$$
$$-x \quad = -3 \Rightarrow x = 3$$
$$y = 6$$

$$\begin{cases} 3x - 2y + 3z = 0 \\ 2x - y = 1 \Rightarrow y = 2x - 1 \\ 3x + y + 6z = 0 \end{cases}$$

$$\begin{cases} 3x - 4x + 3z = 0 \\ 3x + 2x + 6z = 0 \end{cases} \Rightarrow \begin{cases} -x + 3z = 0 \\ 5x + 6z = 0 \end{cases}$$

$$2x - 6z = 0$$
$$5x + 6z = 0$$
$$7x \quad = 0 \Rightarrow$$

$$(3x - 2(2x - 1) + 3z) = 0$$

$$3x + 2x - 1 + 6z = 0$$

$$\Rightarrow \begin{cases} -x + 3z = -2 \\ 5x + 6z = 1 \end{cases} \Rightarrow \begin{matrix} -2x - 6z = 4 \\ 5x + 6z = 1 \\ \hline 3x \quad = 5 \end{matrix} \Rightarrow x = \frac{5}{3} \quad y = \frac{7}{3} \quad z = -\frac{1}{9}$$

$$B = \{(5/3, 7/3, -1/9, 1)\}$$

$\boxed{U_1+U_2}$

$$\overset{2}{\overset{\shortparallel}{\dim_{\mathbb{R}}(U_1)}}+\overset{2}{\overset{\shortparallel}{\dim_{\mathbb{R}}(U_2)}}=\overset{3}{\overset{\shortparallel}{\dim_{\mathbb{R}}(U_1+U_2)}}+\overset{1}{\overset{\shortparallel}{\dim(U_1\cap U_2)}}$$

$$\begin{pmatrix} 3 & 1 & 2 & 3 \\ 6 & 0 & 0 & 3 \\ 1 & -1 & -1 & -2 \\ 0 & 2 & 3 & 4 \end{pmatrix} \qquad \begin{vmatrix} 3 & 1 & 2 \\ 6 & 0 & 0 \\ 1 & -1 & 1 \end{vmatrix} = -12-6=-18\neq 0$$

$$B=\{(3,6,1,0),(1,0,-1,2),(2,0,-1,3)\}$$

$\boxed{\mathbb{R}^4/U_1}$ Buscamos un complementario de U_1:

$$\begin{pmatrix} 3 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix}$$

Ambos determinantes son no nulos ya que $\begin{vmatrix} 3 & 1 \\ 6 & 0 \end{vmatrix}\neq 0$, por lo tanto $W=\mathcal{L}\{(0,0,1,0),(0,0,0,1)\}$ y $U_1\oplus W=\mathbb{R}^4$.

$$\mathbb{R}^4/U_1=\mathcal{L}\{((0,0,1,0)+U_1,(0,0,0,1)+U_1)\}$$

$\boxed{\mathbb{R}^4/U_2}$

$$\begin{pmatrix} 2 & 3 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ -1 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{pmatrix}$$

Ambos determinantes son no nulos ya que $\begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix}\neq 0$, por lo tanto $W=\mathcal{L}\{(0,0,1,0),(0,0,0,1)\}$ y $U_2\oplus W=\mathbb{R}^4$

$$\mathbb{R}^4/U_2=\mathcal{L}\{\{(0,0,1,0)+U_2,(0,0,0,1)+U_2\}\}$$

$\boxed{U_1\cap U_2\cap U_4}$

$(U_1\cap U_2)\cap U_4$

$$\begin{cases} 3x-2y+3z=0 \\ 2x-y-t=0 \\ 3x+y+6z=0 \\ x-2y+t=0 \\ 3x+y+6z=0 \end{cases} \qquad \begin{pmatrix} 3 & -2 & 3 & 0 \\ 2 & -1 & 0 & -1 \\ 3 & 1 & 6 & 0 \\ 1 & -2 & 0 & 1 \\ 3 & 1 & 6 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 3 & -2 & 3 & 0 \\ 2 & -1 & 0 & -1 \\ 3 & 1 & 6 & 0 \\ 1 & -2 & 0 & 1 \end{vmatrix} = -1\begin{vmatrix} 3 & -2 & 3 \\ 3 & 1 & 6 \\ -1 & 2 & 0 \end{vmatrix}+1\begin{vmatrix} 3 & -2 & 3 \\ 2 & -1 & 0 \\ 3 & 1 & 6 \end{vmatrix}=$$

$$=-1(12+18+3-36)+1(-18+6+9+24)=24$$

4 ecuaciones L.I. $\Rightarrow U_1\cap U_2\cap U_4=\{0\}$

$\boxed{4.}$ (Ejercicio 18d Relación)

$$U = \left\{ p(x) \in \mathbb{R}_2[x] \mid \int_0^1 p(x)\,dx = 0 \right\}, \ V = \mathbb{R}_2[x]$$

$$U = \{ p(x) \in \mathbb{R}_2[x] \mid 2a_2 + 3a_1 + 6a_0 = 0 \}$$

$$a_0 + a_1x + a_2x^2 \Rightarrow \int (a_0 + a_1x + a_2x^2)\,dx = \frac{a_2x^3}{3} + \frac{a_1x^2}{2} + a_0x$$

$$\int_0^1 p(x)_{dx} = 0 \quad \left[\frac{a_2x^3}{3} + \frac{a_1x^2}{2} + a_0x \right]_0^1 = \underbrace{\frac{a_2}{3} + \frac{a_1}{2} + a_0 = 0}_{2a_2 + 3a_1 + 6a_0 = 0}$$

Como solo tenemos una ecuación y $\mathbb{R}_2[x]$ tiene dimensión 3, $\dim(U) = 2$.

$$B = \{2x-1,\ 3x^2-1\} \qquad \begin{vmatrix} -1 & 2 & 0 \\ -1 & 0 & 3 \\ 0 & 1 & 0 \end{vmatrix} = 3 \neq 0 \Rightarrow L.I.$$

$$\downarrow \qquad \downarrow$$

$$(-1,2,0) \quad (-1,0,3)$$

$(0,1,0) \Rightarrow x$ Subespacio complementario $= \mathcal{L}\{\{x\}\}$

$\boxed{5.}$ (Ejercicio 8c Relación)

$$V = \mathbb{R}_n[x] \quad U_1 = \{p(x) \in \mathbb{R}_n[x] \mid p(1) + p'(1) = 0\}$$
$$U_2 = \{p(x) \in \mathbb{R}_n[x] \mid p(0) + p''(0) = 0\}$$

$\boxed{U_1}$ $a_0 + a_1x + \dots + a_nx^n$

$$p(1) = a_0 + a_1 + \dots + a_n$$
$$p'(x) = a_1 + 2a_2x + \dots + (n)a_nx^{n-1}$$
$$p'(1) = a_1 + 2a_2 + \dots + na_n$$
$$p(1) + p'(1) = a_0 + 2a_1 + 3a_2 + \dots + (n+1)a_n = 0$$

$p(x) \in U_1 \quad q(x) \in U_1 \quad p(x) + q(x) = r(x)$

$$r(1) + r'(1) = 0 \Rightarrow p(1) + q(1) + p'(1) + q'(1) = 0$$
$$\Rightarrow \underbrace{p(1) + p'(1)}_{0} + \underbrace{q(1) + q'(1)}_{0} = 0 \Rightarrow U_1 \text{ es subespacio}$$

$\boxed{U_2}$ $a_0+a_1x+a_2x^2+\dots+a_nx^n$

$p(0)=a_0$

$p'(x)=a_1+2a_2x+\dots+na_nx^{n-1}$

$p''(x)=2a_2+\dots+n(n-1)a_nx^{n-2}$

$p''(0)=2a_2$

$$p(0)+p''(0)=\underbrace{a_0+2a_2=0}_{\text{Una ecuación}}$$

$$U_2=\{p(x)\in\mathbb{R}_n[x]\,|\,a_0+2a_2=0 \text{ ó } p(0)+p''(0)=0\}$$

¿$ap(x)+bq(x)\in U_2$?

$\hookrightarrow$ ¿$\underbrace{ap(0)+bq(0)+ap''(0)+bq''(0)=0}$?

$$ap(0)+ap''(0)+bq(0)+bq''(0)$$

$$a\underbrace{(p(0)+p''(0))}_{0}+b\underbrace{(q(0)+q''(0))}_{0}=0$$

¿$U_1\oplus U_2=V$?

<u>Fórmula de las dimensiones</u>

$$\underset{n}{\underset{\parallel}{\dim(U_1)}}+\underset{n}{\underset{\parallel}{\dim(U_2)}}=\dim(U_1+U_2)+\dim(U_1\cap U_2)$$

$$2n=\underset{n+1}{\underset{\wedge|}{\dim(U_1+U_2)}}+\dim(U_1\cap U_2)$$

$$\dim(U_1\cap U_2)\geq 2n-n-1=n-1$$

Para $n=1$, $\dim(U_1\cap U_2)\geq 0$, entonces $U_1\oplus U_2=\mathbb{R}_n[x]$ se puede dar que

Si $n\geq 2$, no se puede dar.

$\boxed{6}$ (Ejercicio Examen)

En $\mathbb{R}_2[x]$: $u_1=1-x+x^2$, $u_2=1+x^2$, $u_3=1+x$

Probar que $B=\{u_1,u_2,u_3\}$ es base

Hallar $M_{B\leftarrow B_u}$.

$B_u = \{1, x, x^2\}$

$u_1 = (1,-1,1) \quad u_2 = (-1,0,1) \quad u_3 = (1,1,0)$

$$\begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -1-1-1 = -3 \neq 0 \Rightarrow \text{Son L.I., por lo tanto son base}$$

$\boxed{M_{B_a - B_u}}$ $\quad 1 = a(1-x-x^2) + b(-1+x^2) + c(1+x)$

$1 = a - ax - ax^2 + b + bx^2 + c + cx$

$1 = (-a+b)x^2 + (c-a)x + a+b+c$

$-a+b=0 \Rightarrow a=b$

$c-a=0$ $\quad \begin{cases} c-b=0 \\ 2b+c=1 \end{cases}$

$a+b+c=1$

$$\begin{array}{r} c-b=\cancel{0}\ -1 \\ -2b-c=\cancel{0} \\ \hline -3b = \cancel{0}\ -1 \end{array} \Rightarrow b = \cancel{0}\ \tfrac{1}{3}$$

$a=\frac{1}{3}$

$c=\cancel{0}$

$c=\frac{1}{3}$

$x = (-a+b)x^2 + (c-a)x + a+b+c$

$-a+b=0 \Rightarrow a=b$

$c-a=1$ $\quad \begin{cases} c-b=1 \\ 2b+c=0 \end{cases} \Rightarrow$

$a+b+c=0$

$$\begin{array}{r} c-b=1 \\ -2b-c=0 \\ \hline -3b = 1 \end{array} \Rightarrow b=-\tfrac{1}{3}$$

$a=-\frac{1}{3}$

$c=\frac{2}{3}$

$x^2 = (-a+b)x^2 + (c-a)x + a+b+c$

$-a+b=1$

$c-a=0 \Rightarrow a=c$ $\quad \begin{cases} b-c=1 \\ b+2c=0 \end{cases}$

$a+b+c=0$

$$\begin{array}{r} b-c=1 \\ -b-2c=0 \\ \hline -3c=1 \end{array} \Rightarrow c=-\tfrac{1}{3}$$

$a=-\frac{1}{3}$

$b=\frac{2}{3}$

$$M_{B_a - B_u} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$$