1.5 c) $f:]0,1[\to \mathbb{R}$ $\quad f(x) = \dfrac{2x-1}{x(x+1)}$, $\forall x \in]0,1[$

$x \neq 0$, $x \neq -1$, $\text{Dom } f =]0,1[$

$$f'(x) = \frac{(2x-1)'(x^2+x) - (2x-1)(x^2+x)'}{(x^2+x)^2} = \frac{2x^2+2x-(2x-1)(2x+1)}{(x^2+x)^2} =$$

$$= \frac{2x^2+2x-4x^2+1}{(x^2+x)^2} = -\frac{2x^2-2x-1}{(x^2+x)^2} = 0 \qquad 2x^2-2x-1=0 \quad D = 4+8 = 12$$

$$x_{1,2} = \frac{1 \pm \sqrt{3}}{2} \qquad \begin{cases} x = \frac{1+\sqrt{3}}{2} \approx 1,32 \\ x = \frac{1-\sqrt{3}}{2} \approx -0,37 \end{cases}$$

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$$\lim_{x \to 0^+} \frac{2x-1}{x^2+x} = \frac{-1}{0^+} = -\infty \qquad f(1) = \frac{1}{2}$$

$$\text{Im}(f) = (-\infty, \tfrac{1}{2})$$

d) $f: [-1,1] \to \mathbb{R}$, $f(x) = \dfrac{x^2}{1+x^2}$, $\forall x \in [-1,1]$

$\text{Dom}(f) = [-1,1]$ $\qquad f'(x) = \dfrac{(x^2)'(1+x^2) - x^2(1+x^2)'}{(1+x^2)^2} = \dfrac{2x(1+x^2) - 2x \cdot x^2}{(1+x^2)^2} =$

$$= \frac{2x}{(1+x^2)^2} = 0 \qquad x = 0$$

$$f(-1) = f(1) = \frac{1}{2} \qquad \text{Im}(f) = [0, \tfrac{1}{2}]$$