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$$x_n = \lim_{n \rightarrow \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n} \right)$$

Sea $a=0$ $b=1$ y se considera la partición

$$\frac{b-a}{n} = \frac{1}{n} \Rightarrow P_n = \{ a=0, \frac{1}{n}, \frac{2}{n}, \dots, b=1 \}$$

$$x_n = \sum_{k=1}^n \frac{1}{n+k} = \frac{1}{n} \sum_{k=1}^n \frac{n}{n+k} = \frac{1}{n} \sum_{k=1}^n \frac{1}{1+\frac{k}{n}} = \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right)$$

Es una suma de Riemann de.

$$f(x) = \frac{1}{1+x} \Rightarrow \int_0^1 f(x) dx = \int_0^1 \frac{1}{1+x} dx =$$

$$= \left[\log |1+x| \right]_0^1 = \underline{\log 2}$$

Ahora bien,

$$\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} f\left(\frac{k}{n}\right) = \int_0^1 \frac{1}{1+x} dx = \underline{\underline{\log 2.}}$$