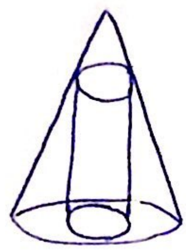
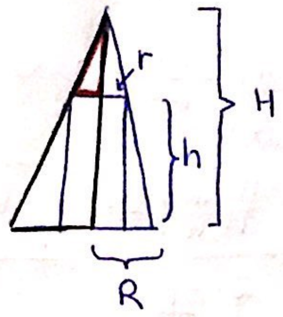
* Determinar las dimensiones del cilindro circular recto de volumen máximo que pueda inscribirse en un cono circular recto de radio R y altura H.





El triángulo rectángulo de catetos r y H−h es semejante al triángulo rectángulo de catetos R y H.

$\downarrow$

$\Downarrow$

$$\frac{R}{r} = \frac{H}{H-h} \rightarrow \quad r = \frac{R\cdot(H-h)}{H} = \frac{RH}{H} - \frac{Rh}{H} = R - \frac{Rh}{H}$$

De esta forma, el volumen del cilindro sería:

$$V = \pi r^2 h = \pi \cdot \left(R - \frac{Rh}{H}\right)^2 \cdot h$$

$$V = \pi \cdot \left(R^2 - 2\cdot R \cdot \frac{Rh}{H} + \left(\frac{Rh}{H}\right)^2\right) \cdot h$$

$$V = \pi \cdot \left(R^2 - 2\frac{R^2h}{H} + \frac{R^2h^2}{H^2}\right) h$$

$$V' = \pi \cdot \left(R^2 - \frac{2R^2h}{H} + \frac{R^2h^2}{H^2}\right) + \pi h \cdot \left(\frac{2R^2}{H^2}h - \frac{2R^2}{H}\right)$$

$$V' = \pi\left(R^2 - \frac{2R^2h}{H} + \frac{R^2h^2}{H^2} + \frac{2R^2h^2}{H^2} - \frac{2R^2h}{H}\right)$$

$$V' = \pi\left(R^2 - 4\frac{R^2h}{H} + 3\frac{R^2h^2}{H^2}\right)$$

$$\pi\left(R^2 - \frac{4R^2h}{H} + \frac{3R^2h^2}{H^2}\right) = 0$$

$$R^2 - \frac{4R^2h}{H} + \frac{3R^2h^2}{H^2} = 0$$

$$h = \frac{\frac{4R^2}{H} \pm \sqrt{\left(\frac{4R^2}{H}\right)^2 - 4 \cdot \frac{3R^2}{H^2} \cdot R^2}}{2 \cdot \frac{3R^2}{H^2}}$$

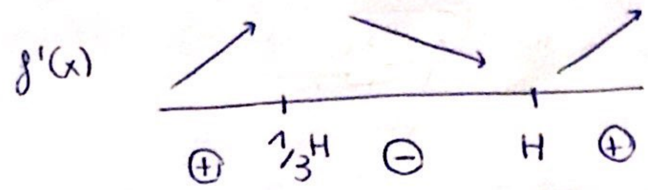
$$h = \frac{\frac{4R^2}{H} \pm \sqrt{\frac{16R^4}{H^2} - \frac{12R^4}{H^2}}}{\frac{6R^2}{H^2}} = \frac{\frac{4R^2}{H} \pm \sqrt{\frac{4R^4}{H^2}}}{\frac{6R^2}{H^2}}$$

$$h = \frac{\frac{4R^2}{H} \pm \frac{2R^2}{H}}{\frac{6R^2}{H^2}}$$

$$h_1 = \frac{\frac{6R^2}{H}}{\frac{6R^2}{H^2}} = \frac{\cancel{6R^2}H^{\cancel{2}}}{\cancel{6R^2}\cdot\cancel{H}} = H$$

$$h_2 = \frac{\frac{2R^2}{H}}{\frac{6R^2}{H^2}} = \frac{2\cancel{R^2}H^{\cancel{2}}}{6\cancel{R^2}\cancel{H}} = \frac{1}{3}H$$

Comprobamos si en estos puntos hay un máximo relativo.



$$f'(0) = \pi \cdot \left(R^2 - \frac{4 \cdot R^2 \cdot 0}{H} + \frac{3R^2 \cdot 0}{H^2}\right) = \pi R^2 \rightarrow +$$

$$f'(1/2\,H) = \pi \cdot \left(R^2 - \frac{4R^2 \cdot \frac{1}{2}\cancel{H}}{\cancel{H}} + \frac{3R^2\left(\frac{1}{2}H\right)^2}{H^2}\right) = \pi\left(R^2 - \frac{4R^2}{2} + \frac{3R^2}{4}\right) = \pi\left(-\frac{1}{4}R^2\right) \rightarrow -$$

$$f'(2H) = \pi\left(R^2 - \frac{4R^2 2\cancel{H}}{\cancel{H}} + \frac{3R^2 \cdot 4\cancel{H^2}}{\cancel{H^2}}\right) = \pi(17R^2) \rightarrow +$$

Por tanto, el cilindro de volumen máximo tendrá $h = \frac{1}{3}H$ y $r = 2/3\,R$.