29. Calcular $\lim\limits_{x\to 0} \dfrac{x\int_0^x \sin(t^2)dt}{\sin(x^4)}$.

Sea $F(x) = \displaystyle\int_0^x \sin(t^2)dt$. Por el Primer Teorema Fundamental del Cálculo sabemos que $F'(x) = \sin(x^2)$.

$$\begin{aligned}
\lim_{x\to 0} \frac{x\int_0^x \sin(t^2)dt}{\sin(x^4)} = \left[\frac{0}{0}\right] &\overset{L'H}{=} \lim_{x\to 0} \frac{\int_0^x \sin(t^2)dt + x\sin(x^2)}{4x^3\cos(x^4)} = \left[\frac{0}{0}\right] \\
&\overset{L'H}{=} \lim_{x\to 0} \frac{2\sin(x^2) + 2x^2\cos(x^2)}{12x^2\cos(x^4) - 16x^6\sin(x^4)} = \left[\frac{0}{0}\right] \\
&\overset{L'H}{=} \lim_{x\to 0} \frac{8x\cos(x^2) - 4x^3\sin(x^2)}{24x\cos(x^4) - 144x^5\sin(x^4) - 64x^9\cos(x^4)} = \left[\frac{0}{0}\right] \\
&\overset{L'H}{=} \lim_{x\to 0} \frac{8\cos(x^2) - 16x^2\sin(x^2) - 12x^2\sin(x^2) - 8x^4\cos(x^2)}{(24 - 1152x^8)\cos(x^4) - (96x^3 + 720x^4 - 256x^{12})\sin(x^4)} \\
&= \frac{8}{24} = \frac{1}{3}
\end{aligned}$$