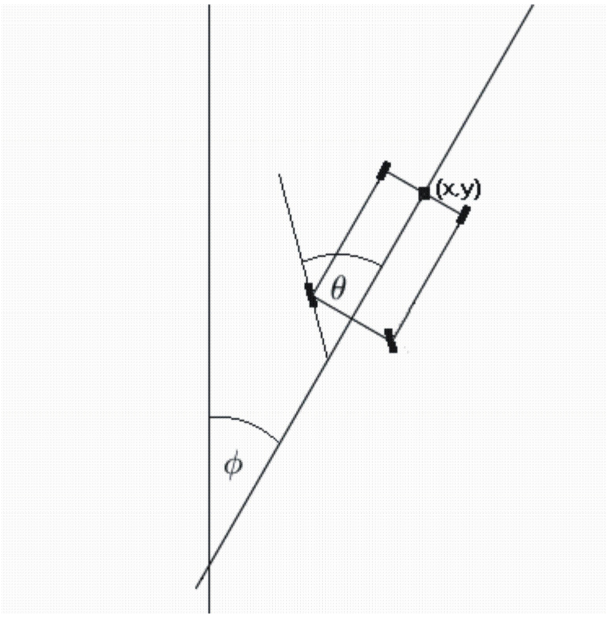
1 Truck parking problem

Let us consider the area $D \subset \mathbb{R}^2$. It is rectangle defined as $D = \{(x, y) \in \mathbb{R}^2 : x \in [-300, 300], y \in [0, 200]\}$. D is the area of the car park in which the vehicle is in motion. Let us embed this area in the coordinate system. It means that every point of this area has coordinates (x, y) defined in the coordinate system. Let the line going through the points $(0, 0)$ and (0.200) be the axis of the parking.

Suppose a vehicle is moving in a parking lot. Let's assume that this vehicle is a truck. It is a rectangular object, the size of which is known. Let a be the width of the vehicle and b be its length. Moreover, let us assume that one of the short sides of the rectangle is the rear of the vehicle. We assume that the vehicle is moving, this means that we have to make some assumptions and determine the physical formulas that describe this movement.

The first element to be determined is the position of the vehicle. We know its size and shape and assume that it is entirely in the area D. Let the straight line dividing the truck into two symmetrical parts parallel to the long side be the axis of symmetry that we will use to determine the location of the truck. Assume that the vehicle position is defined by a triple: (x, y, ϕ), where (x, y) is the center of the rear of the vehicle and ϕ is the angle, which is formed by the axis of the vehicle and the axis of the parking (see Figure 1). Assume that $\phi \in [-180^o, 180^o]$.



Rysunek 1: Parking with the vehicle.

Let the ramp be the top edge of the parking, i.e. the set of points $\{(x, y) : x \in [-300, 300], y = 0\}$. The assumption that $\phi \in [-180^o, 180^o]$ means that a truck positioned in a position where the truck axis coincides with the parking axis and its rear is closer to the ramp is a position where $\phi = 0$. Negative values of this angle correspond to deviations "to the left" from the axis of the parking lot, and positive values - deviations "to the right".

1.1 How do we park the truck?

Our goal is to park the truck. Let us describe it using parameters determining the position of the vehicle. We want to park the truck, that is, generate a finite sequence of steps, the last of which will place the truck at $(x, y, \phi) = (0, 0, 0)$. This is the position where the axis of the truck coincides with the axis of the parking lot and the rear of the truck lies on the top edge of the parking lot such that $(x, y) = (0, 0)$.

We assume a finite sequence of steps. Let $t = 0, 1, \ldots, T$, where $T \in \mathbb{N}$, is an index that determines the successive positions of the truck. The last step of the whole sequence of movements is the position of the truck where we either managed to park or "drove" outside the parking area.

Due to the physical properties and specificity of the vehicle, we assume $\theta \in [-45^o, 45^o]$. $\theta = 0$ this means straight driving. Negative values of the steering angle are the turning of the wheels to the left, and positive values are the turning of the wheels to the right.

1.2 Mathematical model

The parking problem discussed above can be considered from a different point of view. It can be understood as a task in which we are to choose the right angle of turning the wheels. Thus, we are preparing mappings that for the triple $(x, y, \phi) \in [-300, 300] \times [0, 200] \times [-180, 180]$ calculates the value $\theta \in [-45, 45]$.

The following equation describes the moving of the car

$$\begin{cases} x(t+1) &= x(t) + \sin(\theta(t) + \phi(t)) - \sin(\theta(t))\cos(\phi(t)), \\ y(t+1) &= y(t) + \cos(\theta(t) + \phi(t)) - \sin(\theta(t))\sin(\phi(t)), \\ \phi(t+1) &= \phi(t) - \arcsin\left(\frac{2\sin(\theta(t))}{b}\right). \end{cases} \tag{1}$$

or

$$\begin{cases} \frac{dx(t)}{dt} &= \sin(\theta(t) + \phi(t)) - \sin(\theta(t))\cos(\phi(t)), \\ \frac{dy(t)}{dt} &= \cos(\theta(t) + \phi(t)) - \sin(\theta(t))\sin(\phi(t)), \\ \frac{d\phi(t)}{dt} &= -\arcsin\left(\frac{2\sin(\theta(t))}{b}\right). \end{cases} \tag{2}$$

for $t \in [0, T]$, $x(0)-$ the initial position of the truck, $x(T) = (0, 0, 0)$, $T \in \mathbb{R}$.

The equation (2) describes the travel trajectories from the initial position (in time $t = 0$) to the final position (in time $t = T$). The value T is some fixed number depending on the number of steps. It may be different for each initial position.

2 Fuzzy controller

A fuzzy control system is a control system based on fuzzy logica mathematical system that analyzes analog input values in terms of logical variables that take on continuous values between 0 and 1, in contrast to classical or digital logic, which operates on discrete values of either 1 or 0 (true or false, respectively).

Fuzzy logic is widely used in machine control. The term "fuzzy" refers to the fact that the logic involved can deal with concepts that cannot be expressed as the "true" or "false" but rather as "partially true". Although alternative approaches such as genetic algorithms and neural networks can perform just as well as fuzzy logic in many cases, fuzzy logic has the advantage that the solution to the problem can be cast in terms that human operators can understand, so that their experience can be used in the design of the controller. This makes it easier to mechanize tasks that are already successfully performed by humans.

Fuzzy controllers are very simple conceptually. They consist of an input stage, a processing stage, and an output stage. The input stage maps sensor or other inputs, such as switches, thumbwheels, and so on, to the appropriate membership functions and truth values. The processing stage invokes each appropriate rule and generates a result for each, then combines the results of the rules. Finally, the output stage converts the combined result back into a specific control output value.

The most common shape of membership functions is triangular, although trapezoidal and bell curves are also used, but the shape is generally less important than the number of curves and their placement. From three to seven curves are generally appropriate to cover the required range of an input value, or the "universe of discourse" in fuzzy jargon.

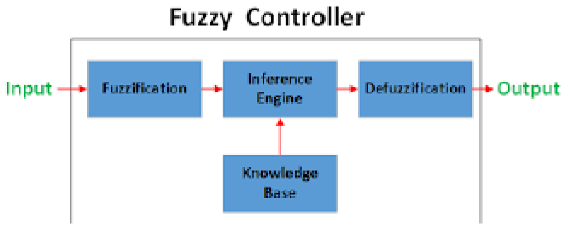
As discussed earlier, the processing stage is based on a collection of logic rules in the form of IF-THEN statements, where the IF part is called the "antecedent" and the THEN part is called the "consequent". Typical fuzzy control systems have dozens of rules.

Consider a rule for a thermostat:

IF (temperature is "cold") THEN turn (heater is "high")

3 Fuzzy controller - model

The model of the fuzzy controller is presented on the diagram of the fuzzy controller.



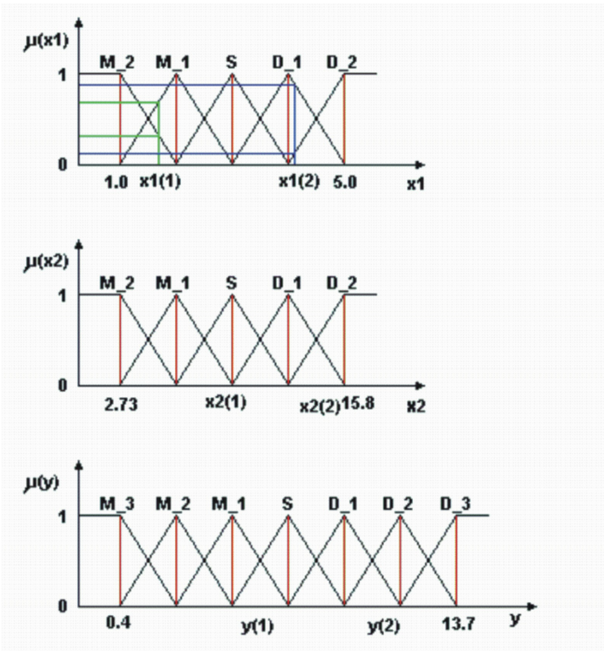
Rysunek 2: The diagram of the fuzzy controller.

4 Algorythm

In order to create a fuzzy controller, several steps of the algorithm must be carried out.

4.1 Step 1 - Fuzzification

Fuzzification is the process of converting a crisp input value to a fuzzy value that is performed by the use of the information in the knowledge base. Although various types of curves can be seen in literature, Gaussian, triangular, and trapezoidal MFs are the most commonly used in the fuzzification process.



Rysunek 3: The diagram of fuzzification.

4.2 Step 2 - Creating fuzzy rules based on training data

We have a data set. We create one fuzzy rule for each vector

$$\textbf{IF } (x_1 \textbf{ is } A_1) \textbf{ AND } (x_2 \textbf{ is } A_2) \textbf{ THEN } (y \textbf{ is } B)$$

4.3 Step 3 - Assignment of degrees of truthfulness to each of the rules

For the rule

$$\textbf{IF } (x_1 \textbf{ is } A_1) \textbf{ AND } (x_2 \textbf{ is } A_2) \textbf{ THEN } (y \textbf{ is } B)$$

the truthfulness is defined as

$$tr = \mu_{A_1}(x_1)\mu_{A_2}(x_2)\mu_B(y).$$

4.4 Step 4 - Creation the database of the fuzzy rules

The fuzzy rules database is a table containing all previously defined rules.

4.5 Step 5 - Defuzzification

To determine the numerical value of the control we have to define some method of defuzzification. For the point (x_1, x_2) we calculate the value $\bar{y} = \bar{y}(x_1, x_2)$ defined as:

$$\bar{y} = \bar{y}(x_1, x_2) = \frac{\sum_{i=1}^{N} \tau^{(k)} y^{(k)}}{\sum_{i=1}^{N} \tau^{(k)}},$$

where

- $N-$ number of rules,
- $\tau^{(k)}-$ the degree of activity of the $k-th$ rule

$$\tau^{(k)} = \mu_{A_1^{(k)}}(x_1)\mu_{A_2^{(k)}}(x_2)$$

- $y^{(k)}-$ the element of the fuzzy set $B^{(k)}$, where the value of membership function is the biggest.