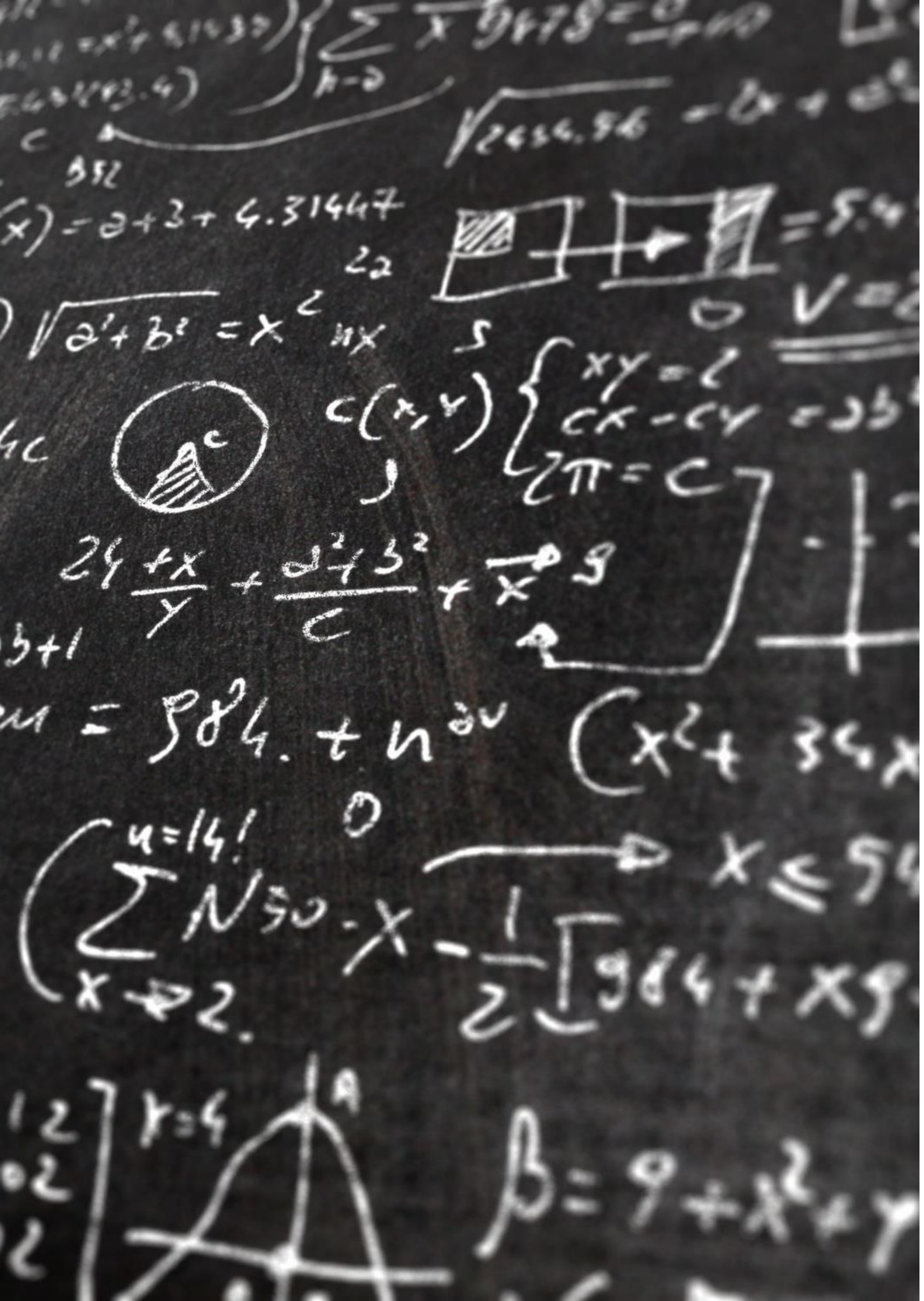
Fuzzy logic

Introduction

What is fuzzy logic?

Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" (1 or 0) Boolean logic on which the modern computer is based.



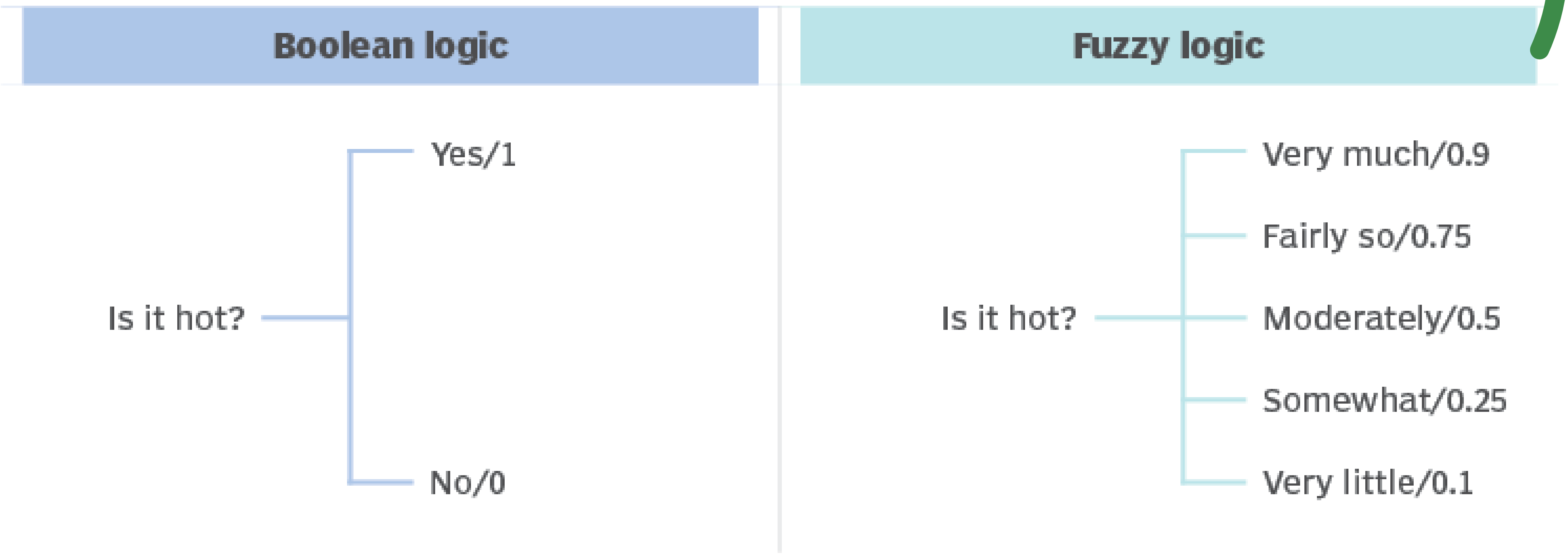
What is fuzzy logic?

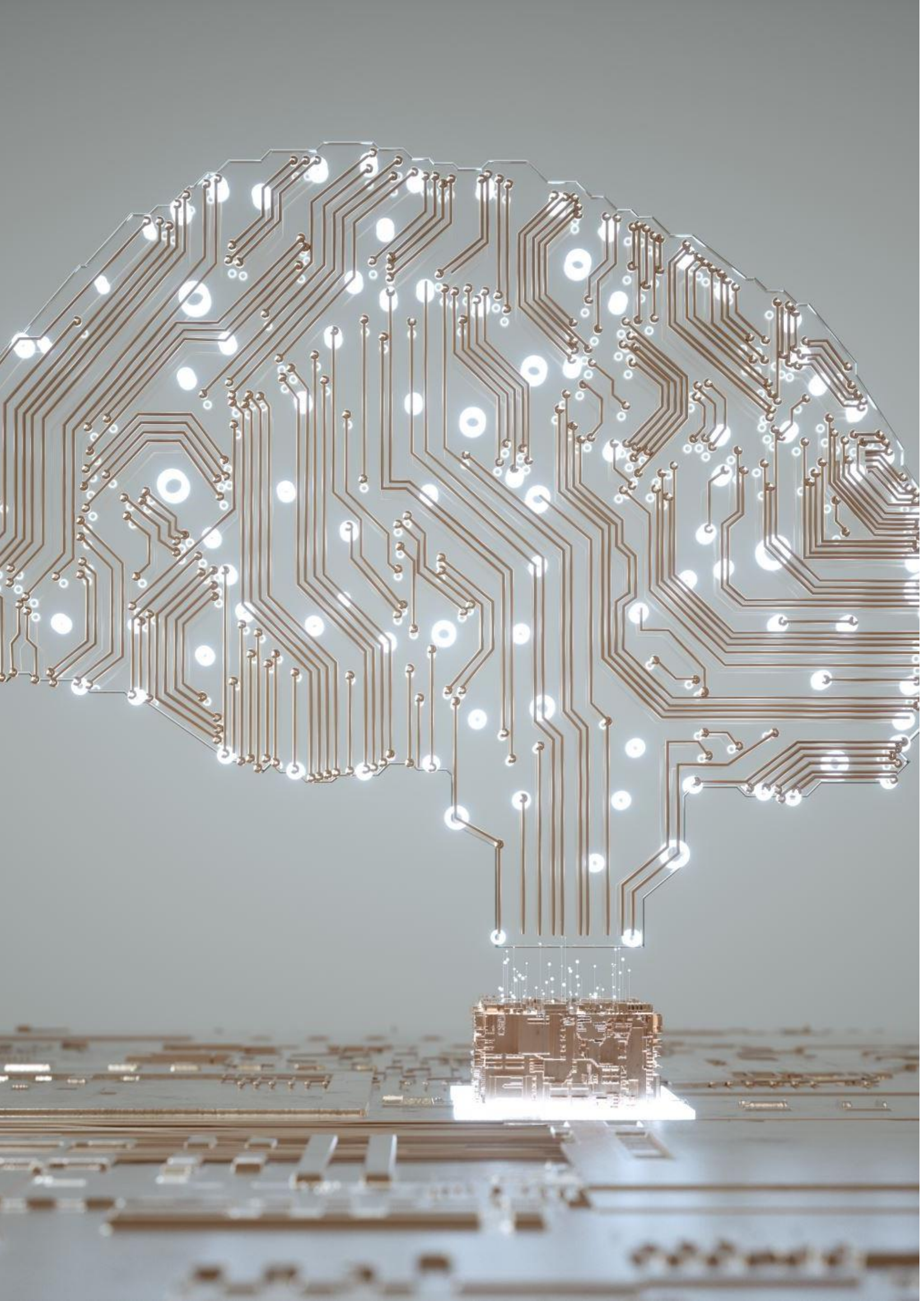
The idea of fuzzy logic was first advanced by Lotfi Zadeh of the University of California at Berkeley in the 1960s. Zadeh was working on the problem of computer understanding of natural language. Natural language -- like most other activities in life and indeed the universe -- is not easily translated into the absolute terms of 0 and 1.

What is fuzzy logic?

Whether everything is ultimately describable in binary terms is a philosophical question worth pursuing, but in practice, much data we might want to feed a computer is in some state in between and so, frequently, are the results of computing. It may help to see fuzzy logic as the way reasoning really works and binary, or Boolean, logic is simply a special case of it.

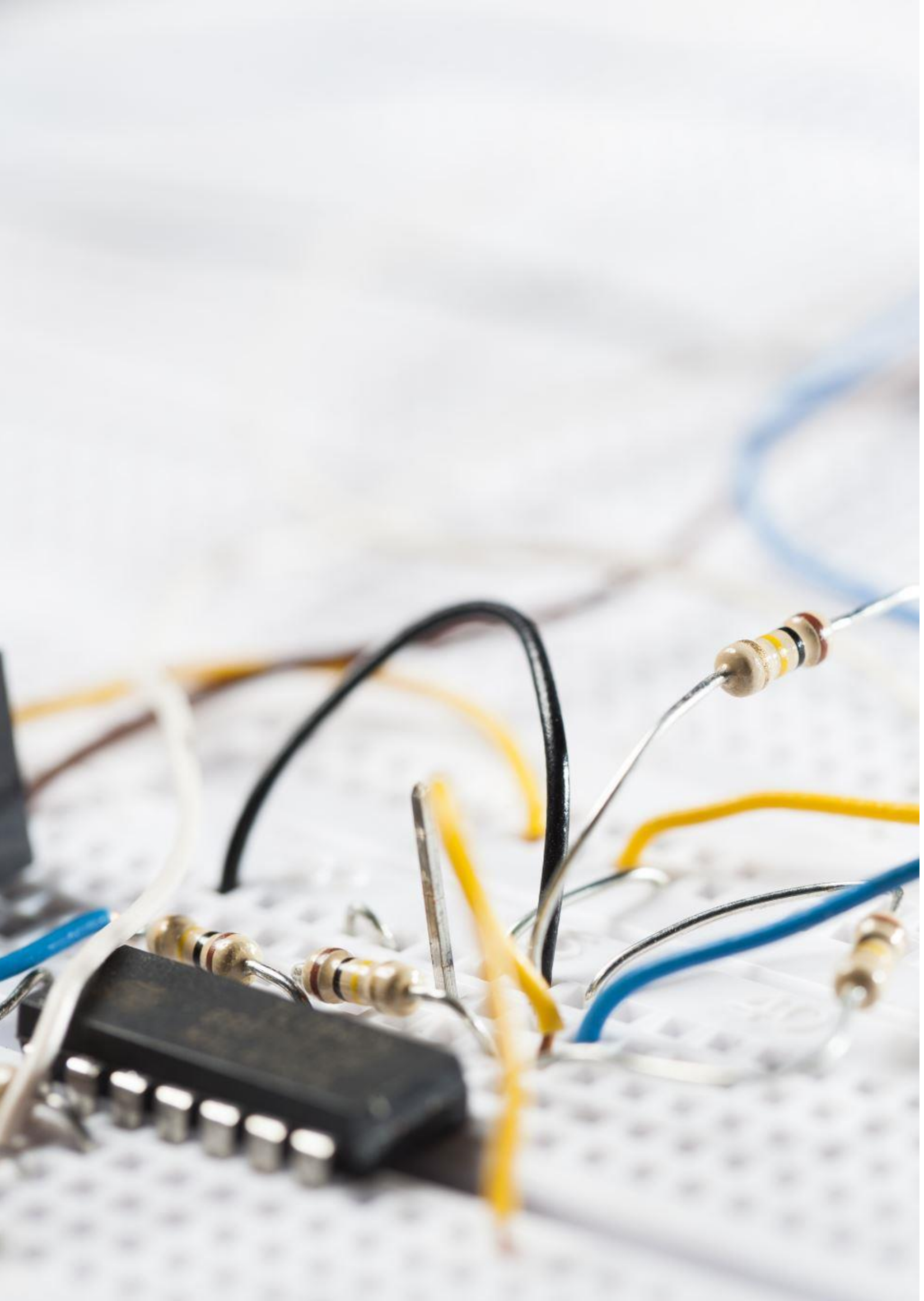
Boolean logic vs. fuzzy logic





Fuzzy logic in AI

In artificial intelligence (AI) systems, fuzzy logic is used to imitate human reasoning and cognition. Rather than strictly binary cases of truth, fuzzy logic includes 0 and 1 as extreme cases of truth but with various intermediate degrees of truth.



Fuzzy logic in AI

As a result, fuzzy logic is well-suited for the following:

- engineering for decisions without clear certainties and uncertainties, or with imprecise data - such as with natural language processing technologies; and
- regulating and controlling machine outputs, according to multiple inputs/input variables - such as with temperature control systems.

What are the uses of fuzzy logic?

Automobile systems

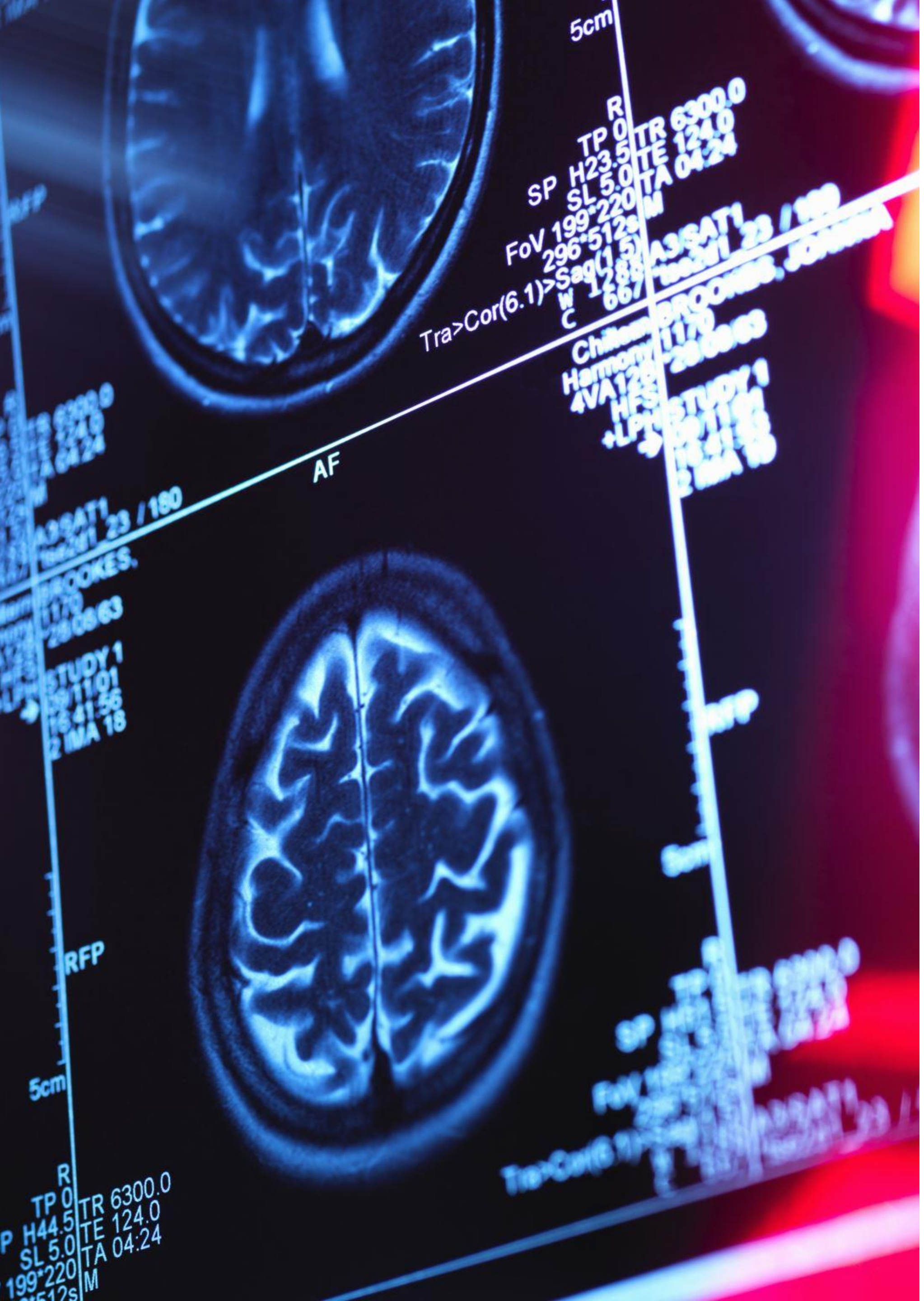
Dishwasher

Copy machine

Aerospace

Medicine

Chemicals



Fuzzy logic applications

Various types of AI systems and technologies use fuzzy logic. This includes vehicle intelligence, consumer electronics, medicine, software, chemicals, aerospace and … .

Fuzzy logic applications

In automobiles, fuzzy logic can be used for gear selection and is based on factors such as engine load, road conditions and style of driving.

In dishwashers, fuzzy logic is used to determine the washing strategy and power needed, which is based on factors such as the number of dishes and the level of food residue on the dishes.

In copy machines, fuzzy logic is used to adjust drum voltage based on factors such as humidity, picture density and temperature.

In aerospace, fuzzy logic is used to manage altitude control for satellites and spacecrafts based on environmental factors.

In medicine, fuzzy logic is used for computer-aided diagnoses, based on factors such as symptoms and medical history.

Fuzzy logic applications

In chemical distillation, fuzzy logic is used to control pH and temperature variables.

In natural language processing, fuzzy logic is used to determine semantic relations between concepts represented by words and other linguistic variables.

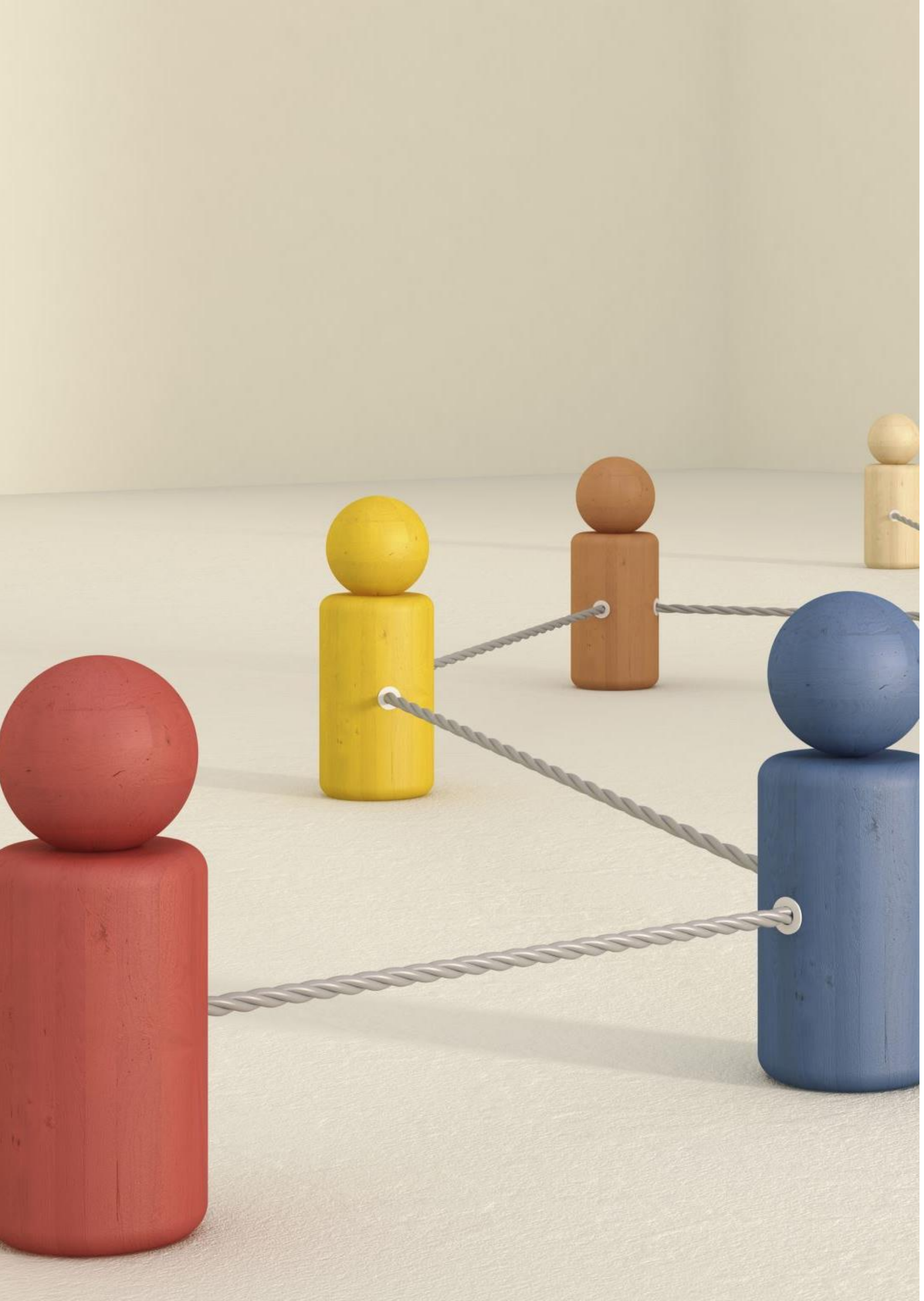
In environmental control systems, such as air conditioners and heaters, fuzzy logic determines output based on factors such as current temperature and target temperature.

In a business rules engine, fuzzy logic may be used to streamline decision-making according to predetermined criteria.

Fuzzy logic

is a form of many-valued logic in which the truth value of variables may be any real number between 0 and 1. It is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false.

By contrast, in Boolean logic, the truth values of variables may only be the integer values 0 or 1.

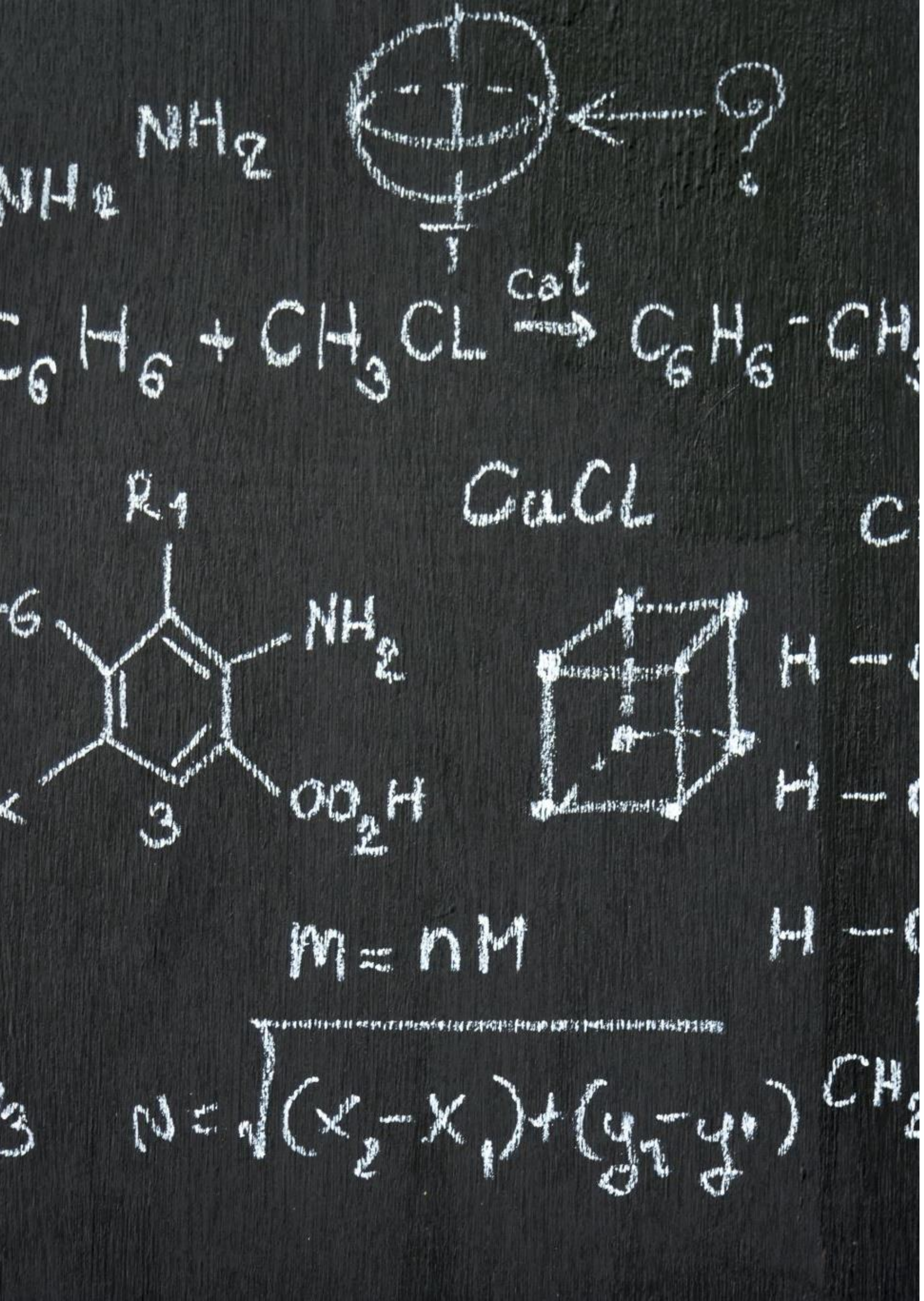


Fuzzy logic

is based on the observation that people make decisions based on imprecise and non-numerical information. Fuzzy models or sets are mathematical means of representing vagueness and imprecise information (hence the term fuzzy). These models have the capability of recognising, representing, manipulating, interpreting, and using data and information that are vague and lack certainty.

Fuzzy logic

has been applied to many fields, for example control theory.

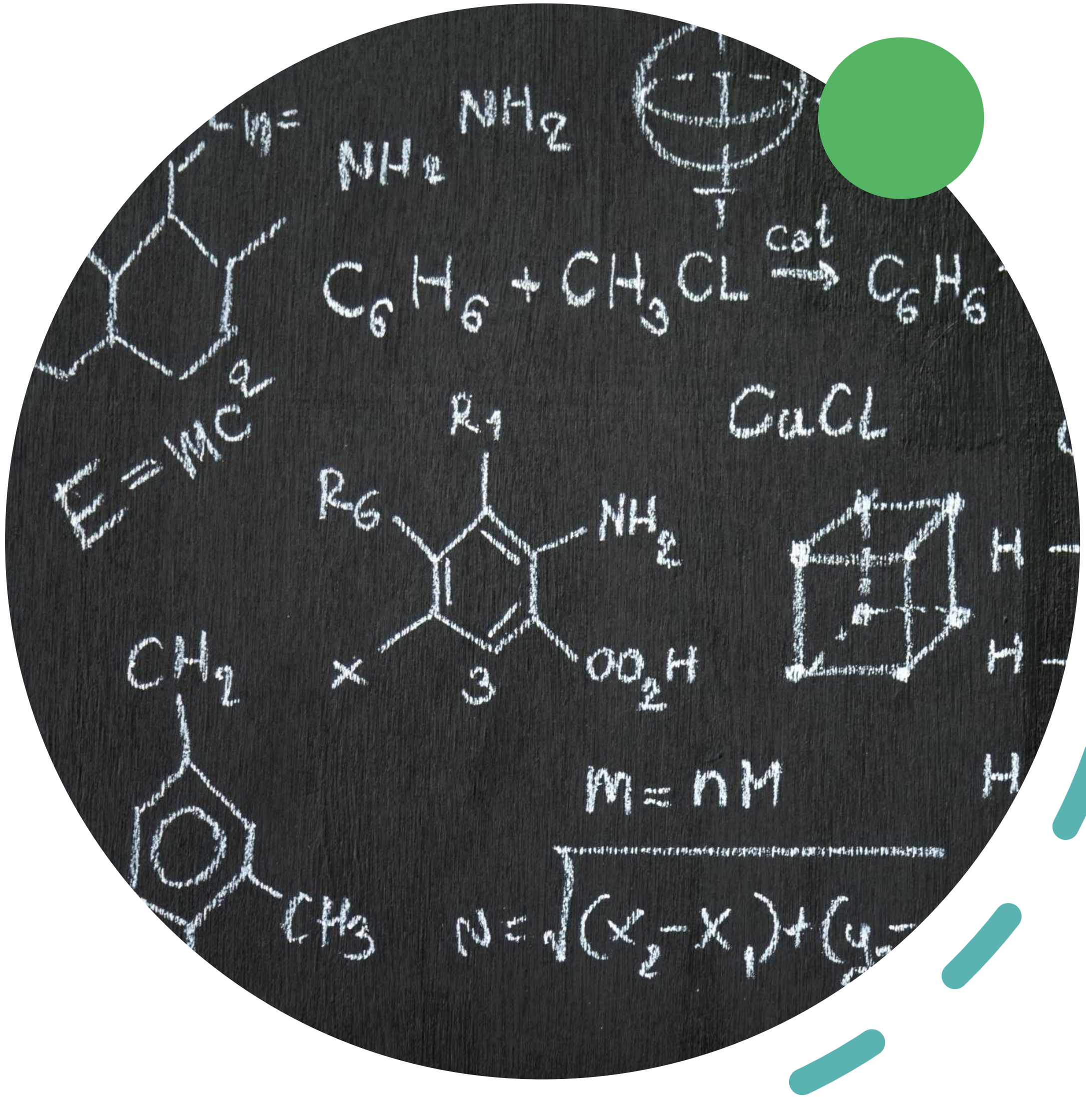


Fuzzy set

In mathematics, fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets were introduced independently by Lotfi A. Zadeh in 1965 as an extension of the classical notion of set. At the same time, Salii (1965) defined a more general kind of structure called an L-relation, which he studied in an abstract algebraic context. Fuzzy relations, which are now used throughout fuzzy mathematics are special cases of L-relations when L is the unit interval [0, 1].

Definition

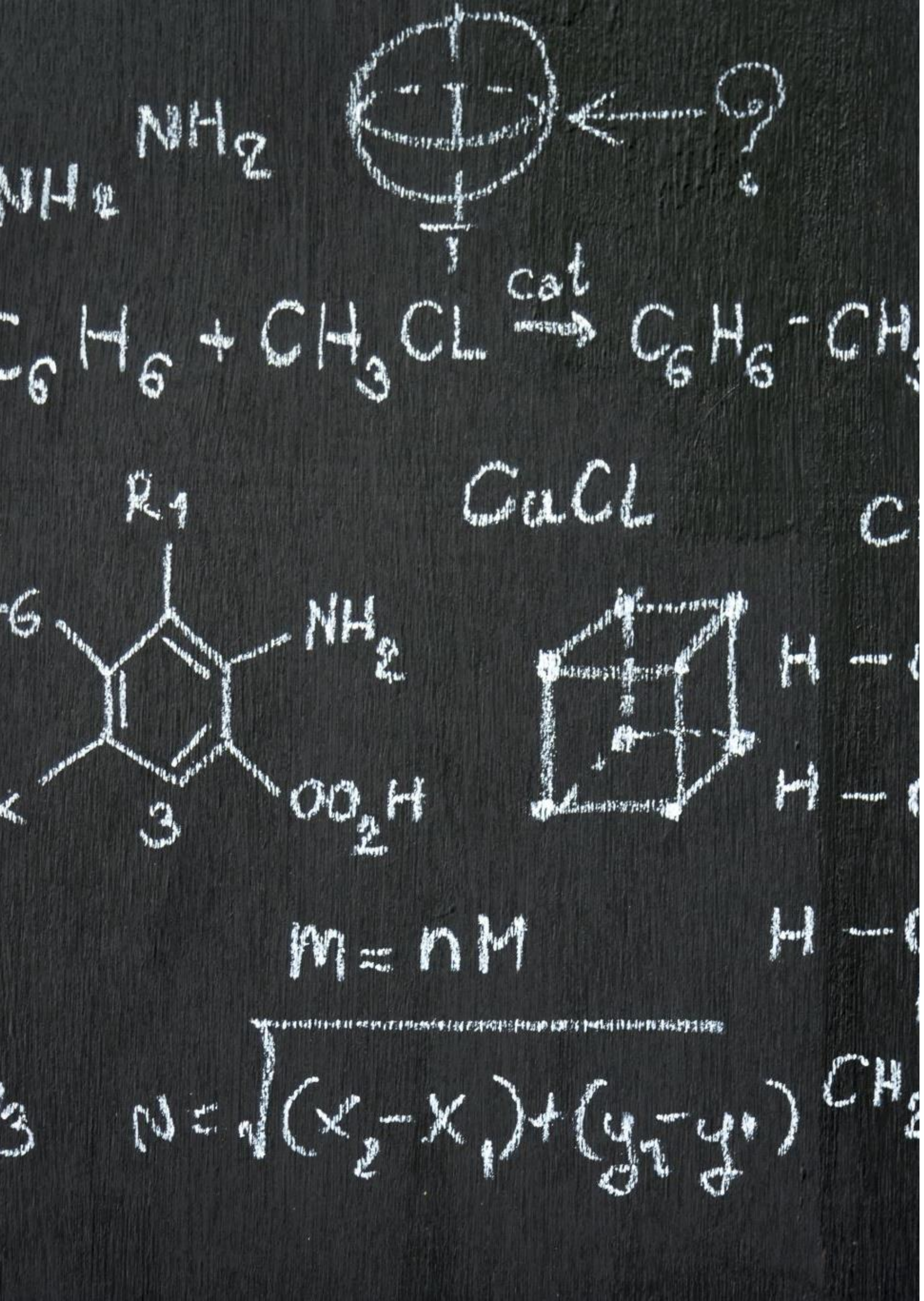
A fuzzy set is a pair (U,m) where U is a set (often required to be non-empty) and $m : U \rightarrow [\,0\,,\,1\,]$ a membership function. The reference set U (sometimes denoted by Ω or X) is called universe of discourse, and for each $x \in U$, the value $m(x)$ is called the grade of membership of x in (U,m). The function $m = \mu_A$ is called the membership function of the fuzzy set A=(U,m).



Fuzzy set

Let $x \in U$. Then x is called

- **not included** in the fuzzy set (U, m) if $m(x) = 0$ (no member),
- **fully included** if $m(x) = 1$ (full member),
- **partially included** if $0 < m(x) < 1$ (fuzzy member).



Fuzzy set

If X is the domain and x is a particular element of X, then a fuzzy set A defined on X and can be written as a collection of ordered pairs

$$A = \{(x, \mu_A(x)), x \in X \}$$

where $\mu_A : X \rightarrow [0, 1]$ is a membership function.

Fuzzy set

discrete and finite

$$\tilde{A} = \sum_{i=1}^{n} \frac{\mu_{\tilde{A}}(x_i)}{x_i} = \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \ldots + \frac{\mu_{\tilde{A}}(x_n)}{x_n}$$

Fuzzy set

Example

- Let X = {g1, g2, g3, g4, g5} be the reference set of students.

- Let Ã be the fuzzy set of “smart” students, where “smart” is
fuzzy term.

Ã = {(g1,0.4)(g2,0.5)(g3,1)(g4,0.9)(g5,0.8)}

Here Ã indicates that the smartness of g1 is 0.4 and so on

Fuzzy set

continuous and Infinite

$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x)}{x}$$

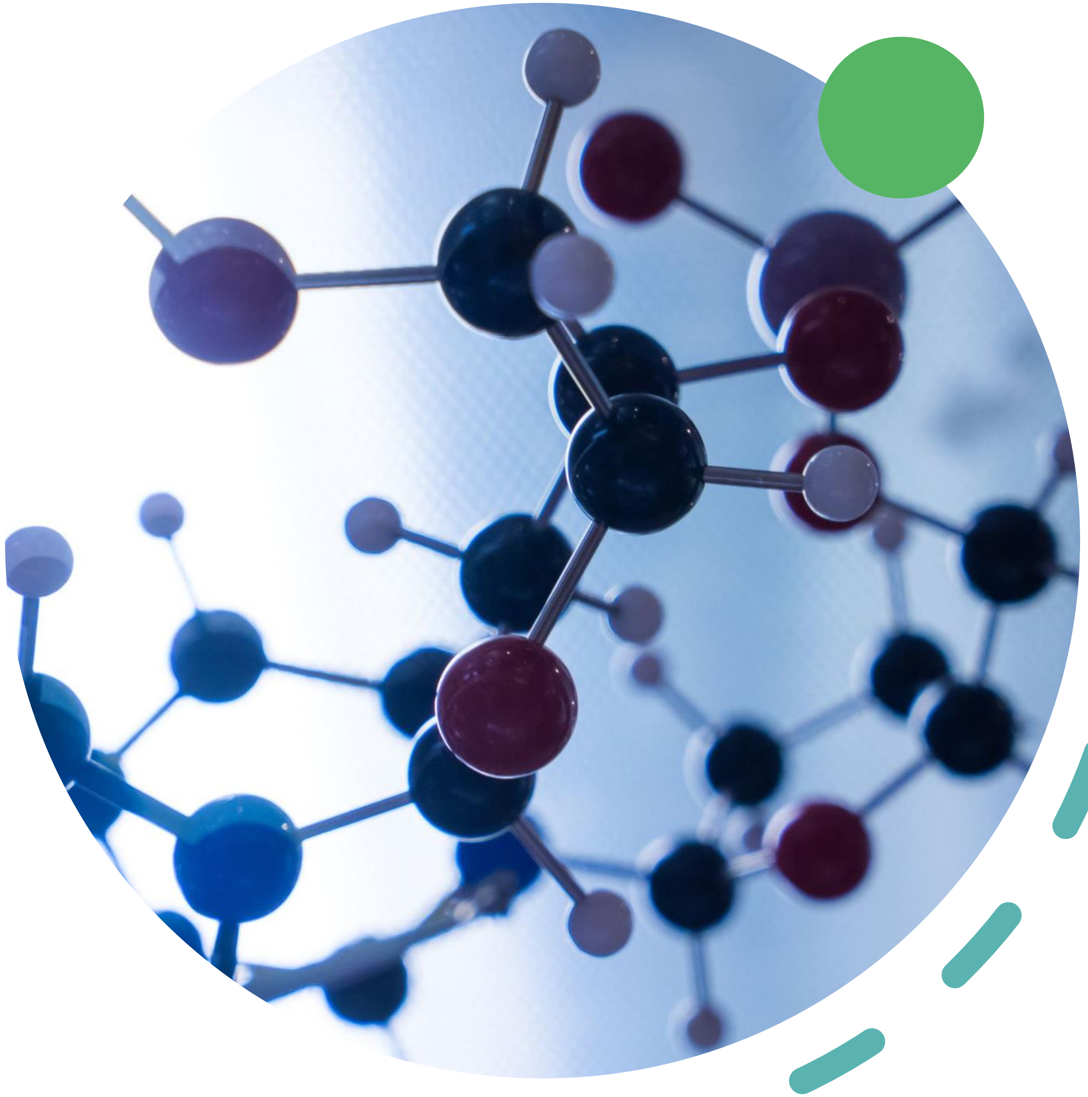
Membership Function

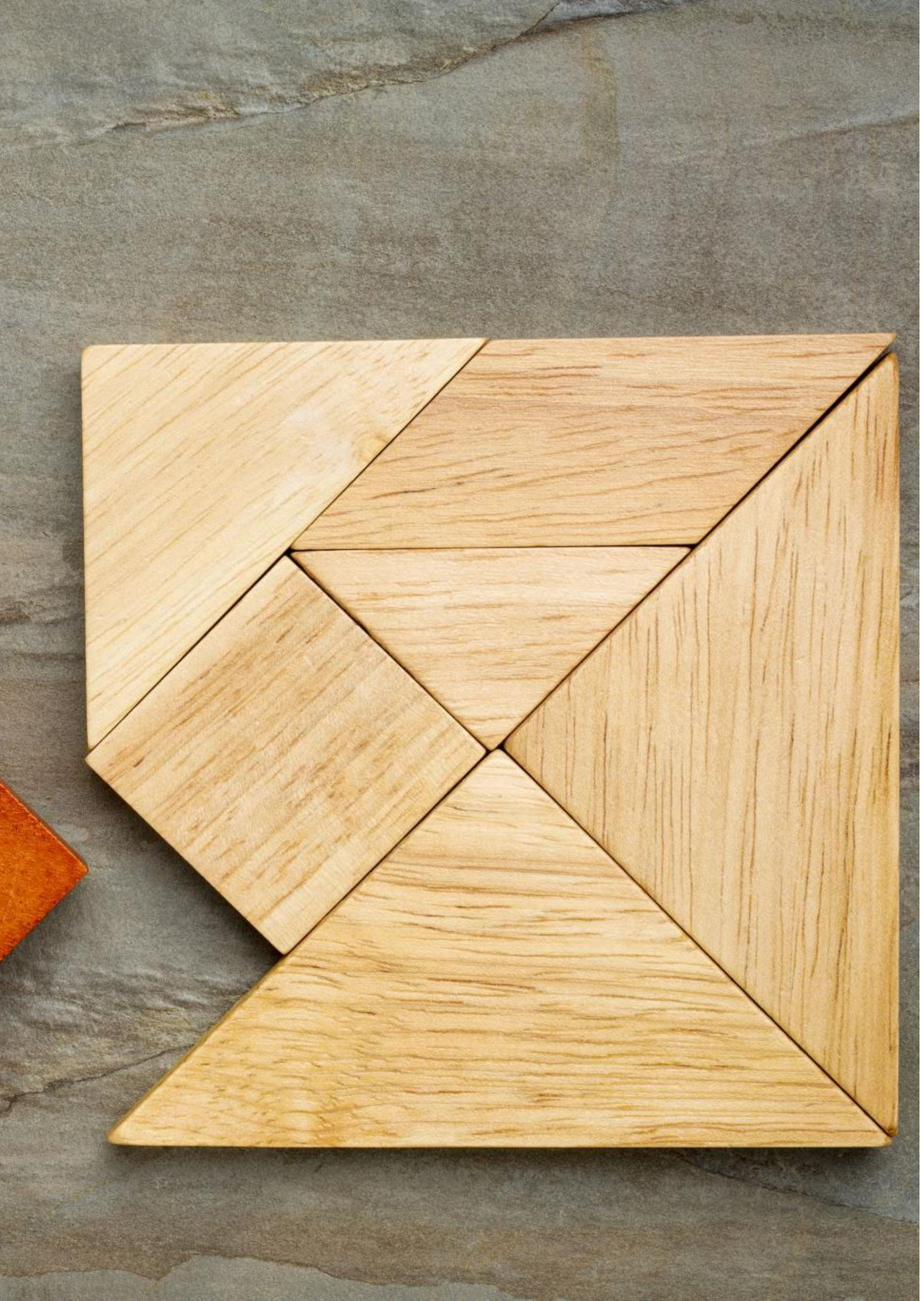
- The membership function fully defines the fuzzy set
- A membership function provides a measure of the degree of similarity of an element to a fuzzy set

Membership functions

can

- either be chosen by the user arbitrarily, based on the user's experience (chosen by two users could be different depending upon their experiences, perspectives, etc.)
- or be designed using machine learning methods (e.g., artificial neural networks, genetic algorithms, etc.)



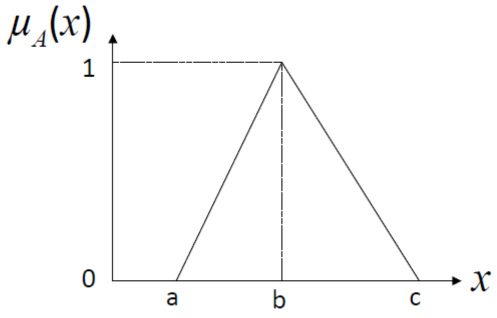


Membership functions

There are different shapes of membership functions:

- Triangular,
- Trapezoidal,
- Gaussian, etc

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x \ge c \end{cases}$$



Fuzzy Set

Triangular membership function
A triangular membership function is specified by three parameters {a, b, c}
a, b and c represent the x coordinates of the three vertices of μA(x) in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b: the centre

Fuzzy Set

- **Trapezoid membership function** is specified by four parameters {a, b, c, d} as follows:

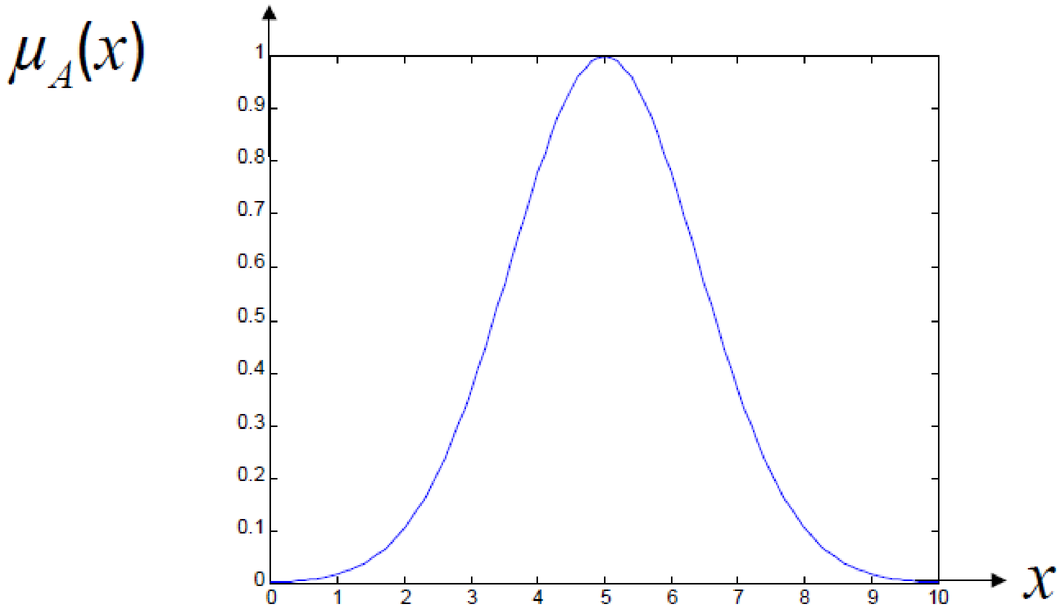
$$\mu_A(x) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{b-a} & if\ a \le x \le b \\ 1 & if\ b \le x \le c \\ \dfrac{d-x}{d-c} & if\ c \le x \le d \\ 0 & if\ d \le x \end{cases}$$

Fuzzy set

Gaussian membership function

$$\mu_A(x,c,s,m) = \exp\left[-\frac{1}{2}\left|\frac{x-c}{s}\right|^m\right]$$

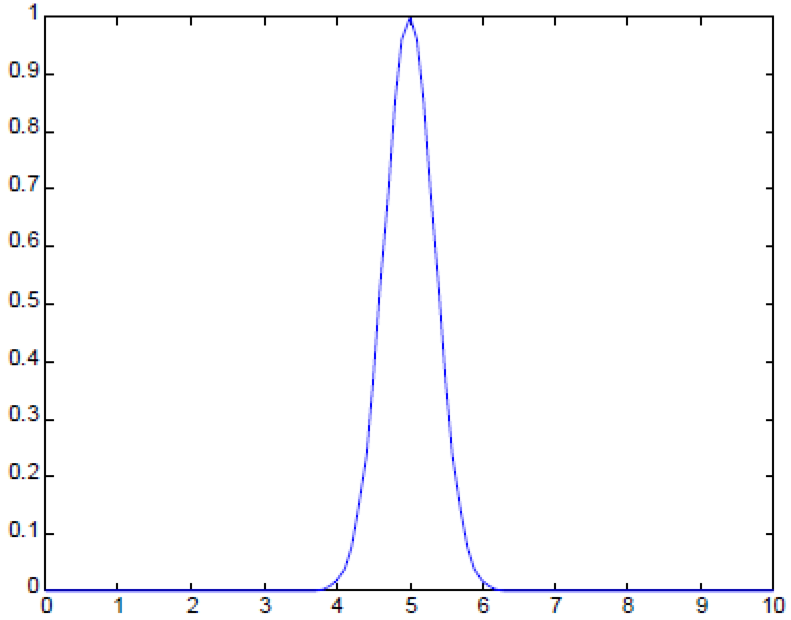
- c: centre
- s: width
- m: fuzzification factor (e.g., m=2)



$c=5$

$s=2$

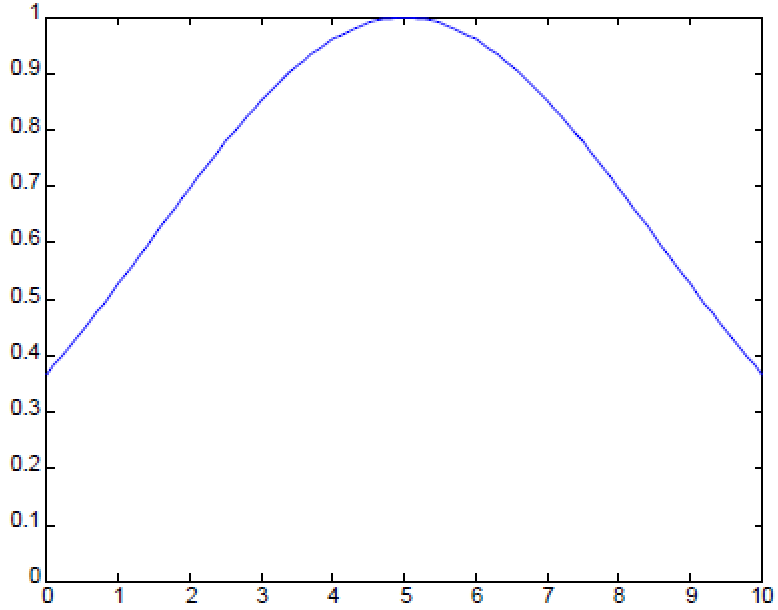
$m=2$



$c=5$

$s=0.5$

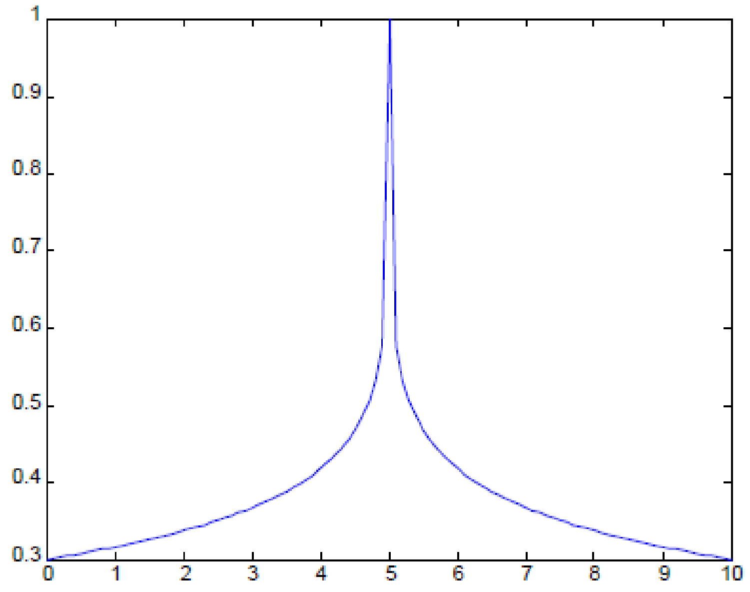
$m=2$



$c=5$

$s=5$

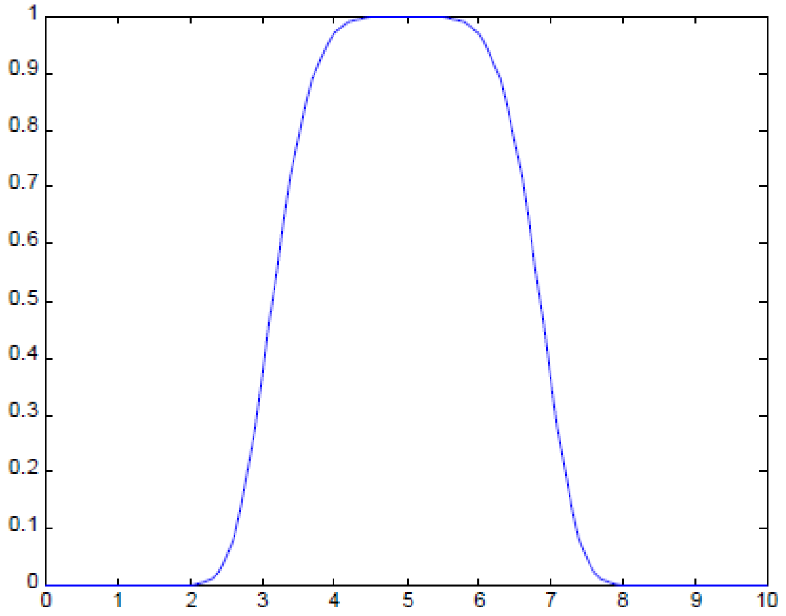
$m=2$



$c=5$

$s=2$

$m=0.2$

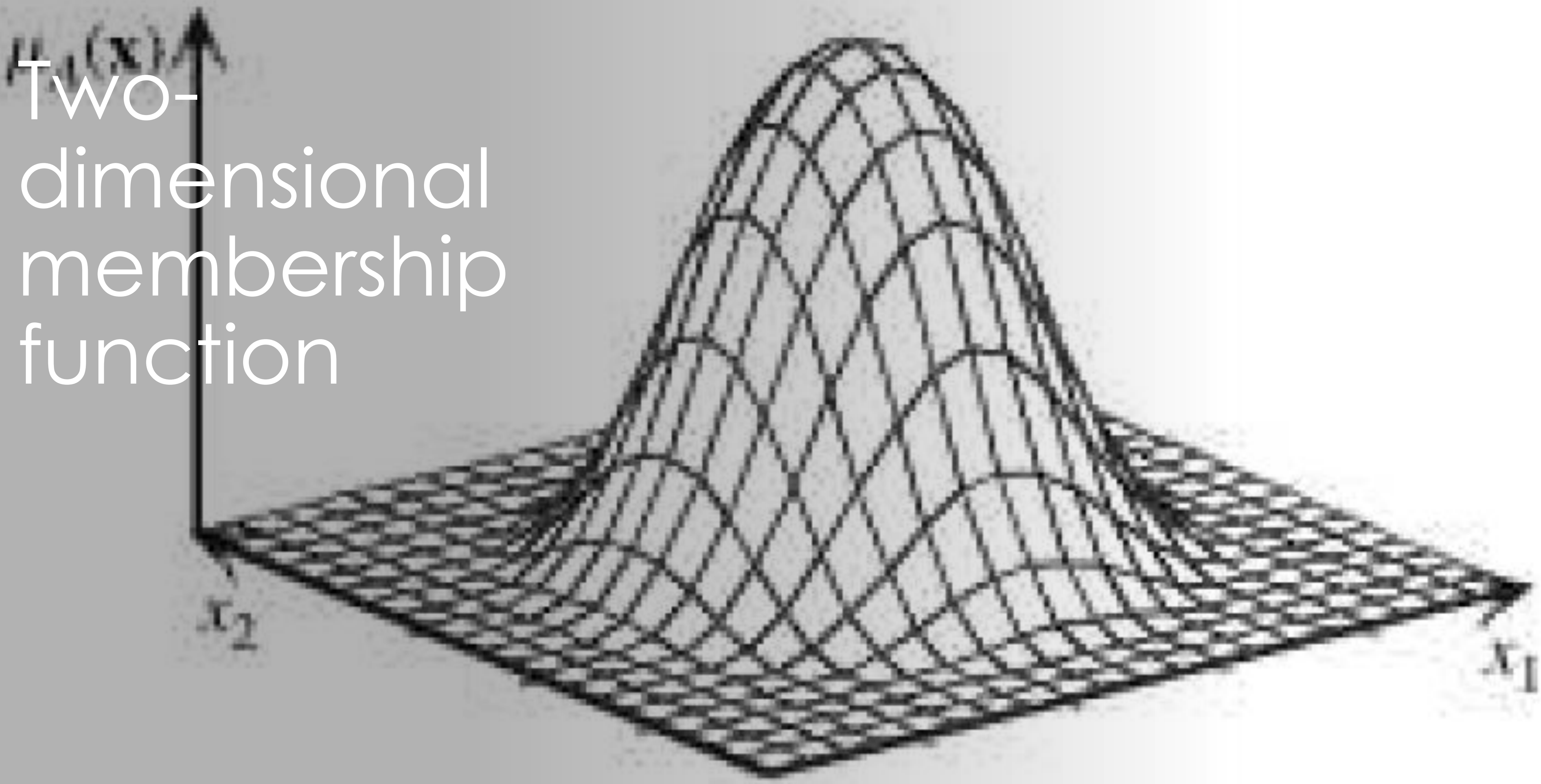


$c=5$

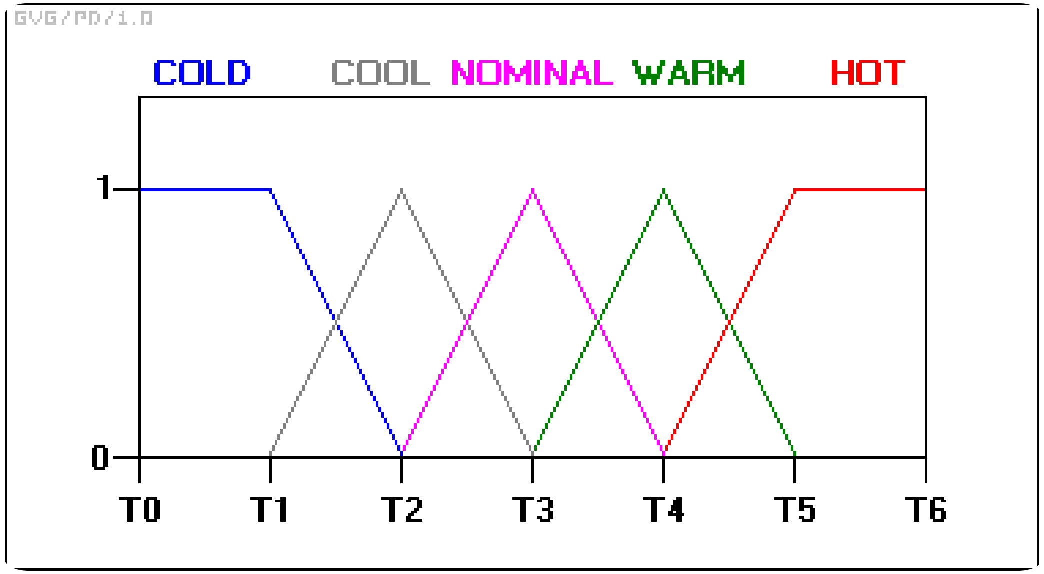
$s=5$

$m=5$

Two-dimensional membership function



Membership function

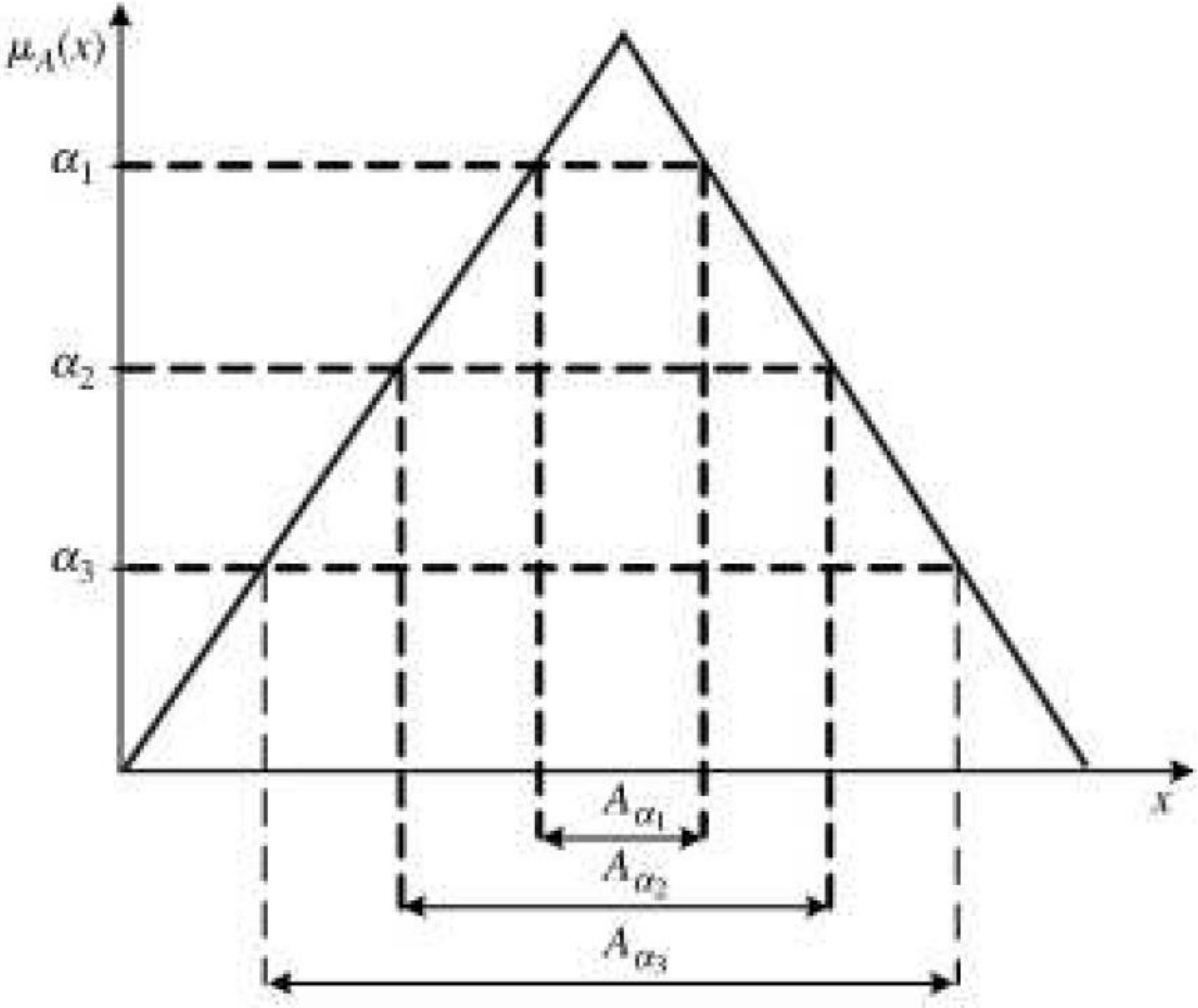


Crisp sets related to a fuzzy set

For any fuzzy set $A = (U, m)$ and $\alpha \in [0, 1]$ the following crisp sets are defined:

- $A^{\geq\alpha} = A_\alpha = \{x \in U \mid m(x) \geq \alpha\}$ is called its **α-cut** (aka **α-level set**)
- $A^{>\alpha} = A'_\alpha = \{x \in U \mid m(x) > \alpha\}$ is called its **strong α-cut** (aka **strong α-level set**)
- $S(A) = \mathrm{Supp}(A) = A^{>0} = \{x \in U \mid m(x) > 0\}$ is called its **support**
- $C(A) = \mathrm{Core}(A) = A^{=1} = \{x \in U \mid m(x) = 1\}$ is called its **core**

α-cut



Other definitions

- A fuzzy set $A = (U, m)$ is **empty** ($A = \varnothing$) (if and only if)

  $$\forall x \in U : \mu_A(x) = m(x) = 0$$

- Two fuzzy sets A and B are **equal** ($A = B$) iff

  $$\forall x \in U : \mu_A(x) = \mu_B(x)$$

- A fuzzy set A is **included** in a fuzzy set B ($A \subseteq B$) iff

  $$\forall x \in U : \mu_A(x) \leq \mu_B(x)$$

- For any fuzzy set A, any element $x \in U$ that satisfies

  $$\mu_A(x) = 0.5$$

  is called a **crossover point**.

Fuzzy Set Operation

Given X to be the universe of discourse and A and B to be fuzzy sets with $\mu_A(x)$ and $\mu_B(x)$ are their respective membership function, the fuzzy set operations are as follows:

Union:

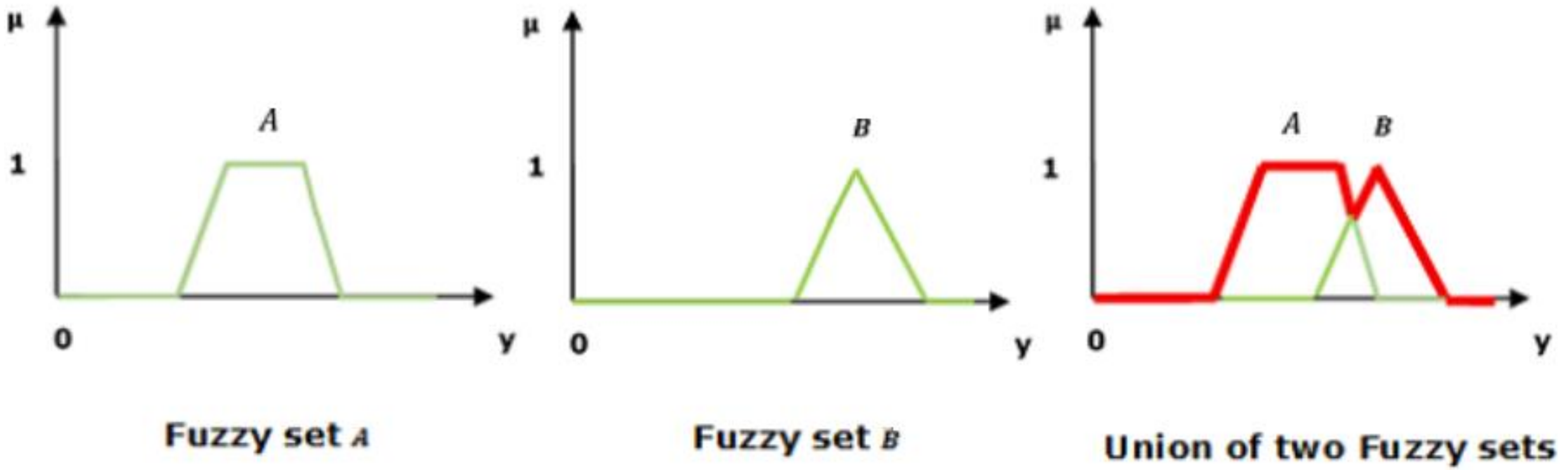
$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

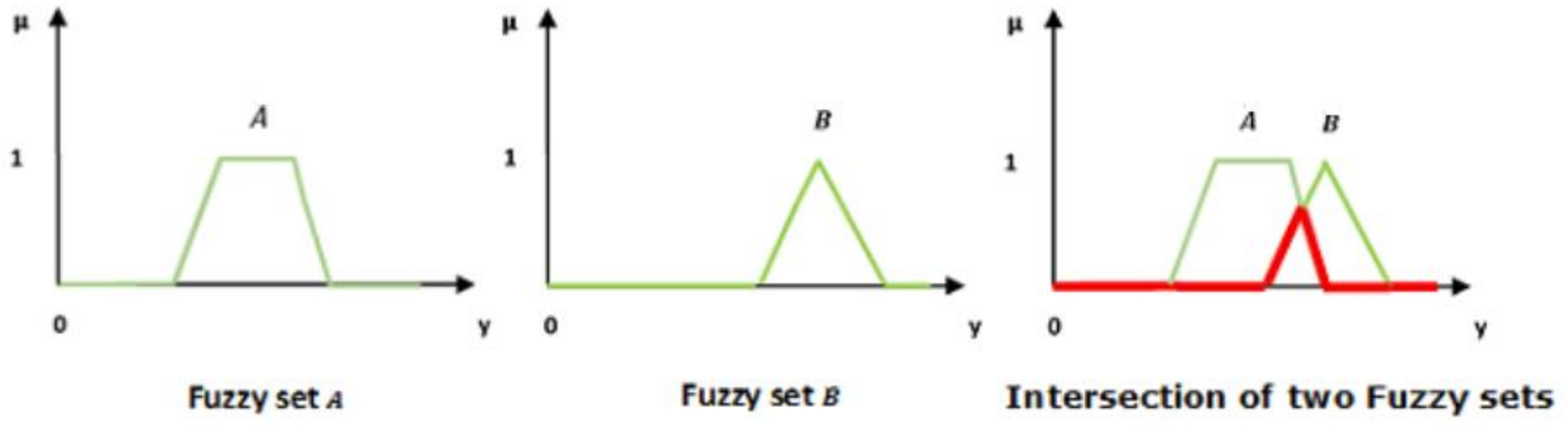
Intersection:

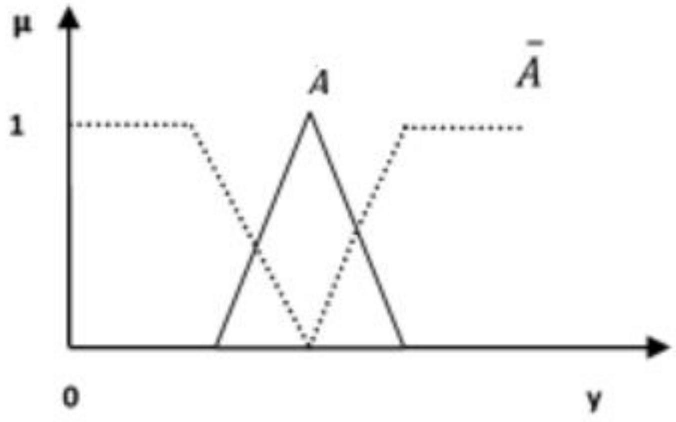
$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Complement:

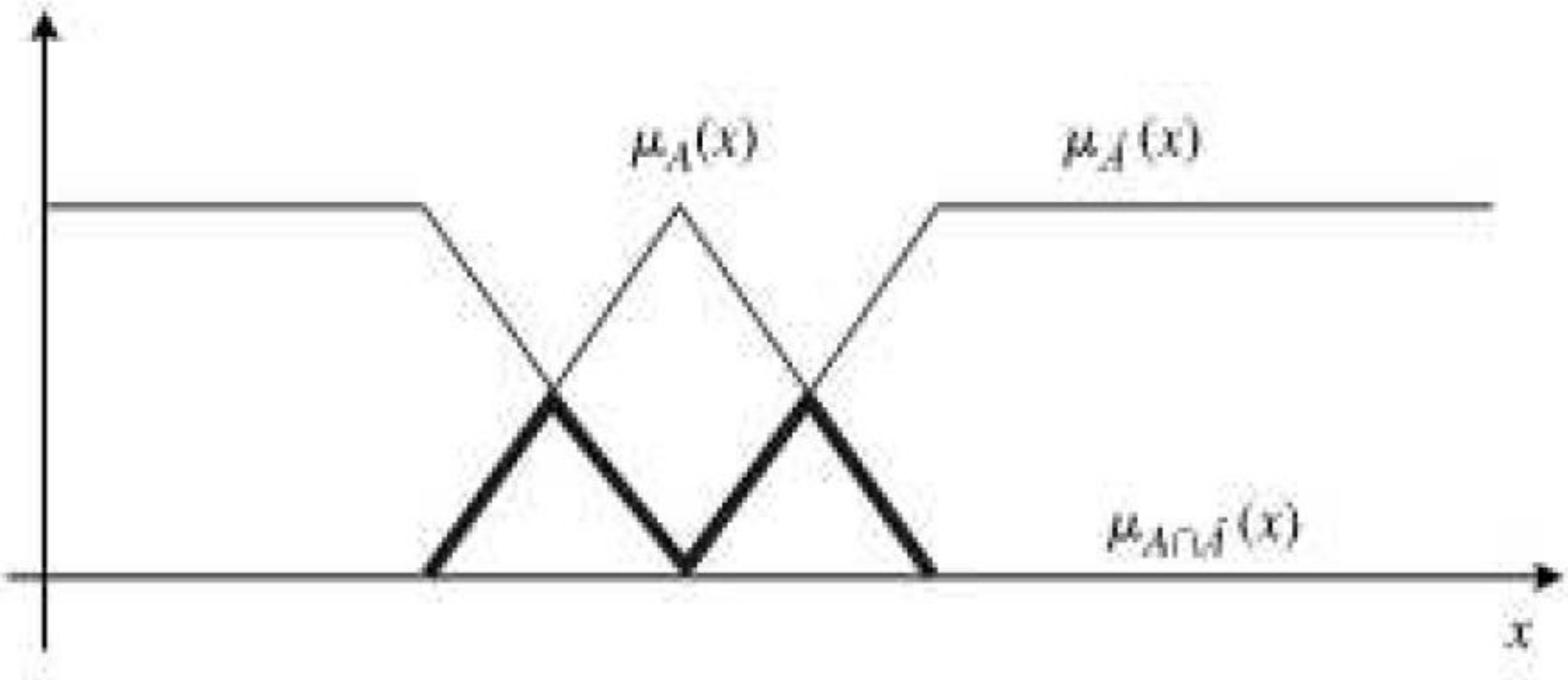
$$\mu_A(x) = 1 - \mu_A(x)$$

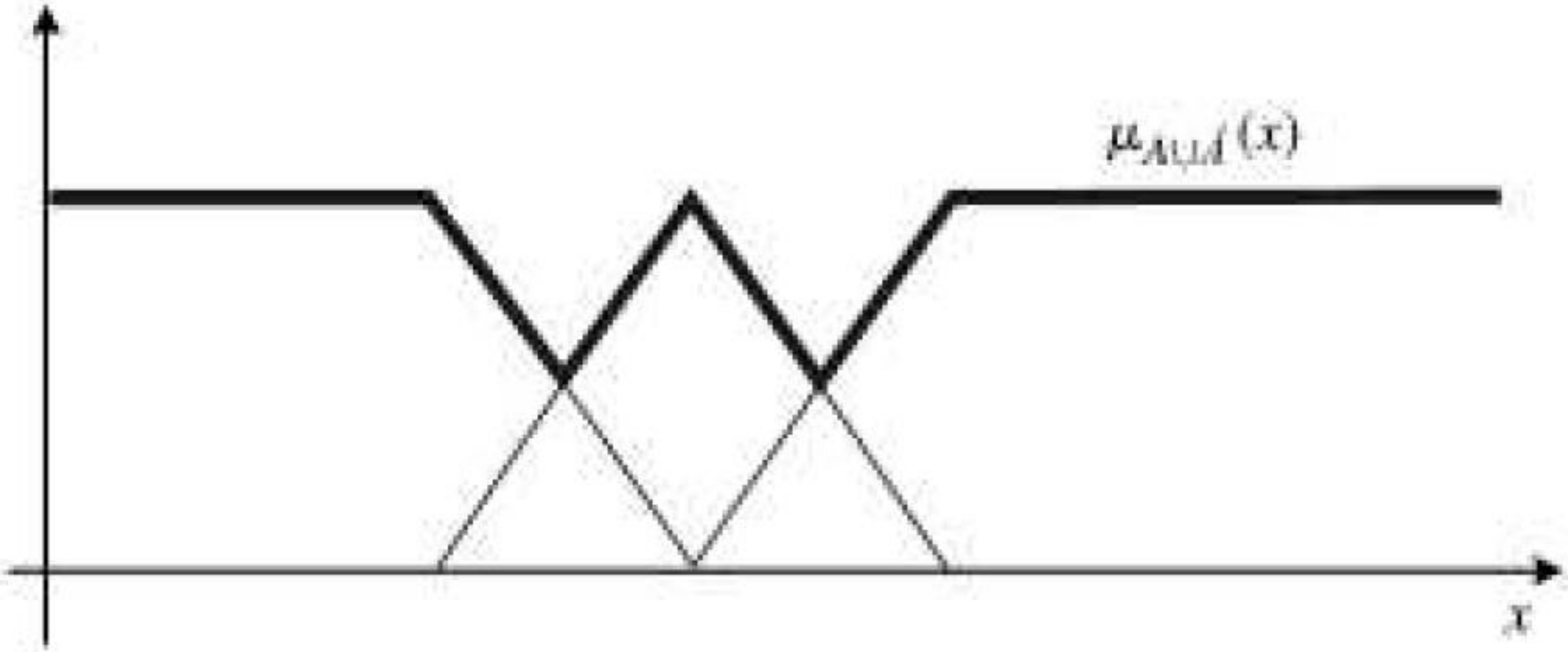

μ
1
A
0
y
Fuzzy set A
μ
1
B
0
y
Fuzzy set B
μ
1
A
B
0
y
Union of two Fuzzy sets


μ
1
0
y
A
Fuzzy set A
μ
1
0
y
B
Fuzzy set B
μ
1
0
y
A
B
Intersection of two Fuzzy sets



Complement of a fuzzy set


$\mu_A(x)$
$\mu_{\bar{A}}(x)$
$\mu_{A\cap\bar{A}}(x)$
x



$\mu_{A \cup A'}(x)$
x

Fuzzy Set Operation - Example

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$ $B = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$

Union:

$$A \cup B = \{(x_1,0.8),(x_2,0.7),(x_3,1)\}$$

Because

$$\mu_{A \cup B}(x_1) = \max(\mu_A(x_1), \mu_B(x_1))$$

$$= \max(0.5,0.8)$$

$$= 0.8$$

$$\mu_{A \cup B}(x_2) = 0.7 \text{ and } \mu_{A \cup B}(x_3) = 1$$

Fuzzy Set Operation - Example

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$ $B = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$

<u>Intersection:</u>

$$A \cap B = \{(x_1,0.5),(x_2,0.2),(x_3,0)\}$$

Because

$$\mu_{A\cap B}(x_1) = \min\,(\mu_A(x_1),\, \mu_B(x_1))$$

$$= \max(0.5,0.8)$$

$$= 0.5$$

$$\mu_{A\cap B}(x_2) = 0.2 \quad \text{and} \quad \mu_{A\cap B}(x_3) = 0$$

Fuzzy Set Operation - Example

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$

<u>Complement:</u>

$$A^c = \{(x_1,0.5),(x_2,0.3),(x_3,1)\}$$

Because

$$\mu_A(x_1) = 1 - \mu_A(x_1)$$

$$= 1 - 0.5$$

$$= 0.5$$

$$\mu_A(x_2) = 0.3 \quad \text{and} \quad \mu_A(x_3) = 1$$

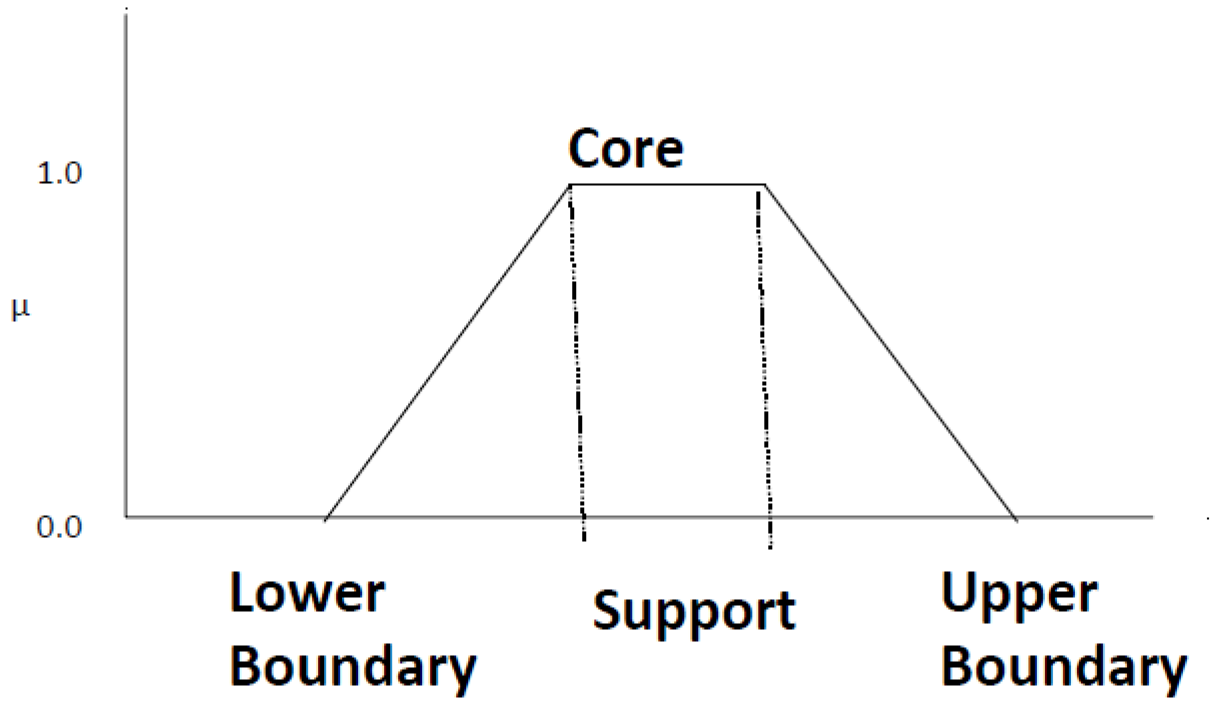
- **Support(A)** is set of all points x in X such that

$$\{(x|\ \mu_A(x) > 0\ \}$$

- **core(A)** is set of all points x in X such that

$$\{(x|\ \mu_A(x) = 1\ \}$$

- Fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called fuzzy singleton


Core
1.0
μ
0.0
Lower
Boundary
Support
Upper
Boundary

Linguistic variable, linguistic term

- **Linguistic variable: A** *linguistic variable* is a variable whose values are sentences in a natural or artificial language.
- **For example**, the values of the fuzzy variable *height* could be *tall, very tall, very very tall, somewhat tall, not very tall, tall but not very tall, quite tall, more or less tall.*
- *Tall is a linguistic* value or primary term

Linguistic variable, linguistic term

If ***age*** is a linguistic variable then its term set is

T(age) = { young, not young, very young, not very young,…… middle aged, not middle aged, … old, not old, very old, more or less old, not very old,…not very young and not very old,…}.