0.1 Ballistic curve

Let us consider a practical physical model: the ballistic curve. Ballistic curve - the curve that a fired material point would move without propulsion, taking into account the air resistance acting on it.

In practice, the ballistic curve traces the flight path of the projectile's center of mass from the point of exit from the barrel to the point of impact. The shape of this path, as in the case of the material point, depends on the angle of inclination and initial velocity of the projectile and the height from which it was launched.

The ballistic curve roughly corresponds to a parabola, but is actually somewhat asymmetric.

Let us assume the following symbols:

- $g-$ gravitational acceleration (defined as $9.81m/s^2$),
- $\theta-$ the angle at which the projectile was fired,
- $v-$ initial velocity,
- y_0- initial projectile height,
- $d-$ distance (on the Earth's surface) traveled by the projectile.

Disregarding air resistance, the curvature of the Earth, and other forces besides gravity, the projectile moves along a parabola.

Road traveled

Total horizontal distance traveled:

$$d = \frac{v\cos(\theta)}{g}\left(v\sin(\theta) + \sqrt{(v\sin(\theta)^2 + 2gy_0)}\right).$$

When the projectile is launched from the surface of the Earth (the initial height is zero), the distance traveled is equal to:

$$d = \frac{v2\sin(2\theta)}{g}.$$

In a specific case, the distance is given as:

$$d = \frac{v^2}{g},$$

when the angle θ is 45^o and the initial height y_0 is 0.

Flight time

The flight time t is the time it takes for the projectile to complete its trajectory:

$$t = \frac{d}{v\cos(\theta)} = \frac{v\sin(\theta) + \sqrt{(v\sin(\theta))^2 + 2gy_0}}{g}.$$

This formula can be simplified to:

$$t = \frac{\sqrt{2}\cdot v}{g},$$

when θ is 45^o and y_0 is 0.

Shooting angle

Shooting angle is the angle θ, at which a projectile must be launched to travel distance d, at a given initial velocity v.

$$\sin(2\theta) = \frac{gd}{v^2}$$

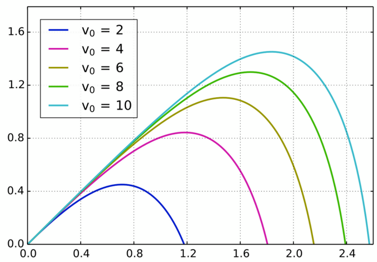
$$\theta = \frac{1}{2}\arcsin\left(\frac{gd}{v^2}\right).$$

0.2 Ballistic curve in practice

As it is known from theory, we can consider the ballistic curve in many ways. We can assume that both the initial velocity and the angle of inclination are the quantities we control. We can also assume that one (chosen) quantity is fixed at some constant value and the other changes. For example, we can assume that the angle of inclination is fixed at 45^o, while the initial velocity of the projectile changes. Similarly, we can assume that the initial velocity of the projectile is fixed, and the parameter that affects the trajectory is the angle of inclination. From a theoretical point of view, both cases are equivalent.

Suppose the angle of inclination is fixed at 45^o and the initial velocity of the projectile changes. The model we are considering can be described as follows. For a given distance, we are looking for the initial velocity of the projectile for which this distance has been reached. Let $D \subset \mathbb{R}$ such that $D = [0, d_{max}]$ and $V \subset \mathbb{R}$ such that $V = [0, v_{max}]$ will be the domain for distance and initial velocity, respectively. Suppose other parameters such as the weight of the projectile are fixed.

Our goal is to find an approximation of the function that will determine the initial velocity of the projectile for a given distance.



Rysunek 1: Trajectories of a projectile with air drag and varying initial velocities.