1. $\pi: \ x+3y+2z+7=0$

$A(0,1,2)$

Halla el punto simétrico de A respecto a π

2. Halla la distancia entre l_1 y l_2:

$l_1 = \{(1,2,3) + t\cdot[1,7,2] : t\in\mathbb{R}\}$

$l_2 = \{(2,0,7) + s\cdot[2,0,9] : s\in\mathbb{R}\}$

3. Halla la longitud de la curva $\varphi(t) = \left(t, \dfrac{e^t + e^{-t}}{2}\right)$ en $[-1,1]$.

4. Calcula la curvatura de $\gamma(t) = (t, t^2, \cos t)$ en el punto $(0,0,1)$.

5. Sea el paraboloide $z = x^2 + y^2$ y la parábola $y^2 = x$. Considera la curva que resulta de la intersección de estos dos y calcula $t(s), n(s), b(s), k(s)$ y $\tau(s)$.

1. Primero calculamos la proyección de A en π:

Sea $l \perp \pi$ y $A \in l \Rightarrow l: \begin{cases} x = 0 + \alpha\cdot 1 \\ y = 1 + \alpha\cdot 3 \\ z = 2 + \alpha\cdot 2 \end{cases}$

$$\alpha + 3(1+3\alpha) + 2(2+2\alpha) + 7 = 0$$

$$\alpha + 9\alpha + 4\alpha + 3 + 4 + 7 = 0$$

$$14\alpha = -14 \Rightarrow \alpha = -1$$

$$A' = (-1,-2,0)$$

$$\left(\frac{0+x''}{2}, \frac{1+y''}{2}, \frac{1+z''}{2}\right) = (-1,-2,0)$$

$x'' = -2 \quad y'' = -5 \quad z'' = -1 \Rightarrow A'' = (-2,-5,-1)$

2. Consideramos 2 planos paralelos tales que $\ell_1 \subset \pi_1$ y $\ell_2 \subset \pi_2$:

$d(\ell_1, \ell_2) = d(\pi_1, \pi_2) = d(p_1, \pi_2)$ con $p_1 = (1,2,3)$

Los vectores directores de π_2 son $[1,7,2]$ y $[2,0,9]$.

$\pi_2 : Ax + By + Cz + D = 0$

$$[A,B,C] = [1,7,2] \times [2,0,9] = \left[\det\begin{pmatrix} 7 & 2 \\ 0 & 9 \end{pmatrix}, -\det\begin{pmatrix} 1 & 2 \\ 2 & 9 \end{pmatrix}, \det\begin{pmatrix} 1 & 7 \\ 2 & 0 \end{pmatrix}\right] =$$

$$= [63, -5, -14]$$

$(2,0,7) \in \pi_2 \Longrightarrow 63 \cdot 2 - 5 \cdot 0 - 14 \cdot 7 + D = 0$

$D = -28$

$$d(p_1, \pi_2) = \frac{|63 \cdot 1 - 5 \cdot 2 - 14 \cdot 3 - 28|}{\sqrt{63^2 + 5^2 + 14^2}} = \frac{17}{64.73} \approx 0.26263 u$$

3. $$\varphi'(t) = \left(1, \frac{\overbrace{-e^{-t} + e^{t}}^{\text{senh}(t)}}{2}\right)$$

$$\|\varphi'(t)\| = \sqrt{1 + \text{senh}^2 t} = \cosh(t)$$

$$\mathcal{L}(\varphi) = \int_{-1}^{1} \cosh(t)\,dt = \text{senh}(t)\Big]_{-1}^{1} = \frac{e - \frac{1}{e}}{2} - \frac{\frac{1}{e} - e}{2} = \frac{2e - \frac{2}{e}}{2} =$$

$$= e - \frac{1}{e} \approx 2.3504 u$$

4. $\gamma'(t) = (1, 2t, -\text{sen}t)$

$\gamma''(t) = (0, 2, -\cos t)$

$k(t) = \|\gamma''(t)\| = \sqrt{\cos^2 t + 4}$

En el punto $(0,0,1)$, $t = 0 \Longrightarrow k(0) = \sqrt{5}$

5. La parametrización de la curva es $\varphi(s) = (s^2, s, s^4 + s^2)$

Vector tangente: $t(s) = \varphi'(s) = (2s, 1, 4s^3 + 2s)$

$$\varphi''(s) = (2, 0, 12s^2 + 2)$$

$$\|\varphi''(s)\| = \sqrt{4 + 144s^4 + 4 + 48s^2} = 2\sqrt{36s^4 + 12s^2 + 2} = k(s)$$

Vector normal: $n(s) = \dfrac{\varphi''(s)}{\|\varphi''(s)\|} = \left(\underbrace{\dfrac{1}{\sqrt{36s^4 + 12s^2 + 2}}}_{\alpha}, 0, \underbrace{\dfrac{6s^2 + 1}{\sqrt{36s^4 + 12s^2 + 2}}}_{\beta}\right)$

$$b(s) = t(s) \times n(s) = \left[\det\begin{pmatrix} 1 & 4s^3 + 2s \\ 0 & \beta \end{pmatrix}, -\det\begin{pmatrix} 2s & 4s^3 + 2s \\ \alpha & \beta \end{pmatrix}, \det\begin{pmatrix} 2s & 1 \\ \alpha & 0 \end{pmatrix}\right] =$$

$$= [\beta, -2s\beta + \alpha\cdot(4s^3 + 2s), -\alpha] =$$

$$= \left[\frac{6s^2 + 1}{\sqrt{36s^4 + 12s^2 + 2}}, -\frac{12s^3 + 2s}{\sqrt{36s^4 + 12s^2 + 2}} + \frac{4s^3 + 2s}{\sqrt{36s^4 + 12s^2 + 2}}, -\frac{1}{\sqrt{36s^4 + 12s^2 + 2}}\right] =$$

$$= \frac{1}{\sqrt{36s^4 + 12s^2 + 2}} \cdot [-6s^2 - 1, -8s^3, -1]$$