**DEF:** Let $R \subset S$ be a region in $S$ and $S$ a regular surface given by parametrization $x(u,v)$. Then, the number

$$\text{Area}(R) = \iint_Q \|x_u \times x_v\| \, du\, dv$$ is called the area of the region $R = x(Q)$

$$\|x_u \times x_v\| = \sqrt{\|x_u\|^2 \cdot \|x_v\|^2 - \langle x_u, x_v\rangle^2} = \sqrt{E\cdot G - F^2}$$

$$N(p) \overset{def}{:=} \frac{x_u \times x_v}{\|x_u \times x_v\|}(p)$$ Normal vector

$$e = \langle N, x_{uu}\rangle \; ; \; f = \langle N, x_{uv}\rangle \; ; \; g = \langle N, x_{vv}\rangle$$

$$E = \langle x_u, x_u\rangle \; ; \; F = \langle x_u, x_v\rangle \; ; \; G = \langle x_v, x_v\rangle$$

Weingarten formulas

$$N_u = a_{11} x_u + a_{21} x_v$$
$$N_v = a_{12} x_u + a_{22} x_v$$

$$a_{11} = -\frac{e\cdot G - f\cdot F}{EG - F^2} \qquad a_{12} = -\frac{f\cdot G - g\cdot F}{EG - F^2}$$

$$a_{21} = -\frac{-e\cdot F + f\cdot E}{EG - F^2} \qquad a_{22} = -\frac{-f\cdot F + g\cdot E}{EG - F^2}$$

$$K = \det(dN) = \frac{e\cdot g - f^2}{EG - F^2}$$ Gauss curvature

$$H = \tfrac{1}{2}(k_1 + k_2) = -\tfrac{1}{2}(a_{11} + a_{22}) = \tfrac{1}{2}\,\frac{eG - 2fF + gE}{EG - F^2}$$ Mean curvature

$$\begin{cases} k_1 = H + \sqrt{H^2 - K} \\ k_2 = H - \sqrt{H^2 - K} \end{cases}$$ Principal curvatures $\Longrightarrow$ Fact: $K = k_1 \cdot k_2$

$$\begin{cases} \Gamma_{11}^1 E + \Gamma_{11}^2 F = \langle x_{uu}, x_u\rangle = \tfrac{1}{2} E_u \\ \Gamma_{11}^1 F + \Gamma_{11}^2 G = \langle x_{uu}, x_v\rangle = F_u - \tfrac{1}{2} E_v \end{cases}$$

$$\begin{cases} \Gamma_{12}^1 E + \Gamma_{12}^2 F = \langle x_{uv}, x_u\rangle = \tfrac{1}{2} E_v \\ \Gamma_{12}^1 F + \Gamma_{12}^2 G = \langle x_{uv}, x_v\rangle = \tfrac{1}{2} G_u \end{cases}$$

$$\Gamma_{12}^1 = \Gamma_{21}^1 \qquad \Gamma_{12}^2 = \Gamma_{21}^2$$

$$\begin{cases} \Gamma_{22}^1 E + \Gamma_{22}^2 F = \langle x_{vv}, x_u\rangle = F_v - \tfrac{1}{2} E_u \\ \Gamma_{22}^1 F + \Gamma_{22}^2 G = \langle x_{vv}, x_v\rangle = \tfrac{1}{2} G_v \end{cases}$$

$$\chi_T(R) \overset{def}{=} \underbrace{F}_{\text{number of triangles}} - \underbrace{E}_{\text{number of edges}} + \underbrace{V}_{\text{number of vertices}}$$ Euler characteristic of the region $R$