Root locus

1 $\dfrac{s^2 - 3s + 2}{s^2 + 7s + 12} = \dfrac{(s-2)(s-1)}{(s+3)(s+4)}$ Zeroes at $\sigma = 1, 2$
Poles at $\sigma = -3, -4$

$P - Z = 0.$

Break away angles $= \dfrac{180}{Z}$
$= 90°$

Break in angles $= \dfrac{180}{P}$
$= 90°$

Break away/inpoints – Algebraic method

$$\sum_{i=1}^{Z} \frac{1}{\sigma_b - Z_i} = \sum_{j=1}^{P} \frac{1}{\sigma_b - P_j}$$

$$\frac{1}{\sigma_b + 3} + \frac{1}{\sigma_b + 4} = \frac{1}{\sigma_b - 2} + \frac{1}{\sigma_b - 1}$$

$$(2\sigma_b + 7)(\sigma_b - 2)(\sigma_b - 1) = (2\sigma_b - 3)(\sigma_b + 3)(\sigma_b + 4)$$
$$(2\sigma_b^2 - 4\sigma_b + 7\sigma_b - 14)(\sigma_b - 1) = (2\sigma_b^2 + 6\sigma_b + 3\sigma_b - 9)(\sigma_b + 4)$$
$$\cancel{2\sigma_b^3} + 3\sigma_b^2 - 14\sigma_b - 2\sigma_b^2 - 3\sigma_b + 14 = \cancel{2\sigma_b^3} + 3\sigma_b^2 - 9\sigma_b + 8\sigma_b^2 + 12\sigma_b - 36$$
$$\sigma_b^2 - 17\sigma_b + 14 = 11\sigma_b^2 + 3\sigma_b - 36 - 50$$
$$0 = 10\sigma_b^2 + 20\sigma_b - 50$$
$$= \sigma_b^2 + 2\sigma_b - 5$$
$$\sigma_b = -1 \pm \sqrt{6}$$
$$= -3.45 \quad \text{or} \quad 1.45$$

Break away/in points Derivative way

$1 + k\,G(s) = 0$

$$k = \frac{-s^2 - 7s - 12}{s^2 - 3s + 2}$$

$$\frac{dk}{ds} = \frac{(-2s-7)(s^2-3s+2) \mp (4s^2+7s+12)(2s-3)}{(s^2-3s+2)^2}$$

We only care about the numerator here and it's the same Quadratic in the algebraic method so I'm going to skip to the answer

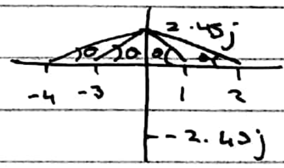
$\sigma = -3.45$ or 1.45

Depending on the function, the derivative method can be more complicated. I personally find it easier as it's got the 'products' semi expanded.

Imaginary crossing Points.

Pick point $\frac{1.45 + 3.45}{2}$ ~~arbitrary~~ average of break in/away val

$\to 2.45j$



$\arctan(-1/2.45) + \arctan(-2/2.45)$ ← Zeros

$+ \arctan(4/2.45) - \arctan(3/2.45)$ Poles.

$= -170.76$

go lower to 2.2. y int = ~~48~~ 181.65

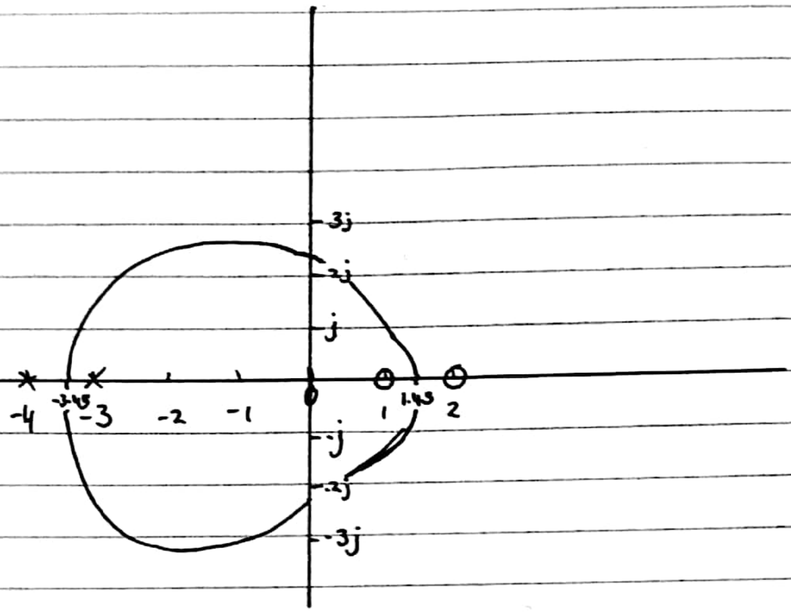
sub above 2.45 for 2.2.

go up to 2.25 y int = ~~-179~~ -179.368

go down to 2.225 y int = -180.5.

so y intercept is ~ 2.225.

(1)



Root locus (2)

$$Sys = \frac{s+3}{0.9s^3 + 3s^2 + 5s + 2}$$

$$= \frac{s+3}{(s+0.55309)(s+1.39+1.44j)(s+1.39-1.44j)}$$

$p = 3$
$z = 1$
$p - z = 2$.

$$\theta_a = \frac{(2k+1)\pi}{2}$$

$$\theta_{a_1} = \frac{\pi}{2} \quad k=0$$

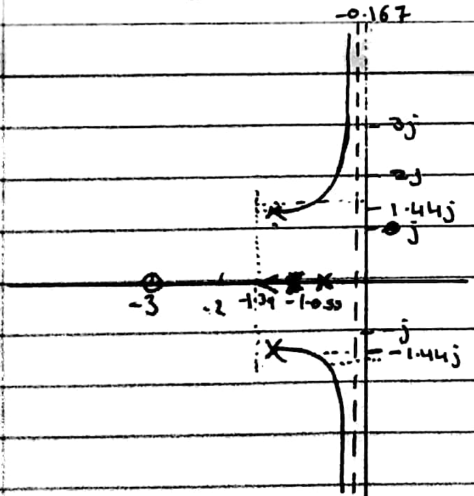
$$\theta_{a_2} = \frac{-\pi}{2} \quad k=-1$$

$$\sigma_a = \cancel{0.8} \ \frac{-0.55309 - 1.39 \times 2 + 3}{2}$$

$$= -0.166545.$$

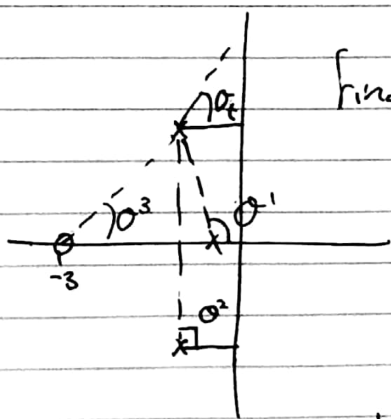
no imaginary intercept.

Final Plot (skipped a step.)



complex
Departure angles

Find θ_t



$$-180 = -\theta_1 - \theta_2 - \theta_t + \theta_3$$

$$\theta_t = 180 - \theta_1 - \theta_2 + \theta_3$$

$$= 180 - 120.167 - 90 + 41.8097$$

$$= 11.64247^\circ$$

$$= 11.6^\circ \quad 1dp.$$

$$\theta_1 = 180 - \arctan\left(\frac{1.44}{1.39 - 0.553}\right)$$

$$= 120.167$$

$$\theta_2 = 90^\circ$$

$$\theta_3 = \arctan\left(\frac{1.44}{3 - 1.39}\right)$$

$$= 41.8097$$

Bode Plots 1

~~at $\omega = 0$~~

eg 3 $\frac{s+30}{s^2+2s} = \frac{30}{2} \frac{\left(\frac{s}{30}+1\right)}{s\left(\frac{s}{2}+1\right)}$

$(s+30)$ gain $= 20\log 30$

$= 29.54$ dB

s^{-1} gain $= 0$

$(s+2)^{-1}$ gain $= -20\log 2$

$= -6.02$ dB.

Stability

total additional gain $= 20\log 15$ ~~directly~~.

$= 23.5$ dB

~~Phase ϕ_m ... $\to \infty$ never crosses~~ $-180°$ phase

$\phi_m \approx 60°$, $\therefore$ is stable.

reading off plot

eq 4

$$\frac{106}{s^3+16s^2+64s} = \frac{106}{s(s^2+16s+64)}$$

$$= \frac{106}{s(s+8)^2}$$

$$= \frac{106}{64}\,\frac{1}{s(\frac{s}{8}+1)^2}$$

106 gain = 20 log 106 = 40.5 dB — 0dB rolloff

s^{-1} gain = 0 dB — 20dB rolloff

$(s+8)^{-2}$ gain = $-40 \log 8$ = -36.12 dB — 40dB rolloff

total additional gain = $20 \log \frac{106}{64}$ = 4.38 dB.

Stability

$\phi_m \simeq 70°$ ∴ is stable

reading off plot

2 $L(f) \approx \cancel{25 \cdot 10^{2}} \dfrac{A}{(s+100)^2}$

$\cancel{\approx 17.75 \times \frac{1}{s+100}}$

ϕ_m ~~as~~ measured at $\omega = 400$ rad/s

$\phi_m = 30°$

~~We want a lag controller as an integrator PI removes SSE~~

To remove steady state error need an I or lag component

Using PI $C(s) = \cancel{C_0 \frac{s+\omega_b}{\omega_b}}$

$= \cancel{C_0} \times \omega_b \left(\frac{s}{\omega_b}+1\right) \times$ $C_0\omega_b$ want zero close to the integrator so low ω_b

$\qquad \frac{s}{\omega_b}$

~~we want $C_0 \times \omega_b = \frac{4}{\omega_b}$ $\therefore C_0 = \frac{1}{\omega_b}$~~

~~we so try $\omega_b = 0.1$ $C_0 = 10$~~

We want $C_0 \times \omega_b = \cancel{\frac{1}{\omega_b}}\, \omega_b$ $\therefore$ $C_0 = 1$

So our compensator will be $C(s) = \dfrac{s+0.1}{s}$

$= 0.1 \dfrac{\left(\frac{s}{0.1}+1\right)}{s}$

you can use the lead controller to make a hump in the phase that ranges between 0-90° which can be placed around your phase margin to raise the phase at that point and increase the phase margin. It also raises the response after the selected frequency ~~increasing~~ decreasing drop off for a bit ~~and~~, & raising the gain at some frequencies, this can also move the phase margin ~~freq~~ location by changing the unity crossing point.

we will add a Zero at $\omega = 400$ and a pole at $\omega = 4000$ so our lead compensator looks like $\frac{s+100}{s+1000} = C(s)$ we want this with unity gain overall so it ~~does~~ doesn't effect existing response / move the phase margin $C(s) = k \times \frac{\frac{s}{400}+1}{\frac{s}{4000}+1} \times \frac{400}{4000}$

So $k = 10$ $\therefore$ $C(s) = \frac{\frac{s}{400}+1}{\frac{s}{4000}+1} = 10\,\frac{s+400}{s+4000}$ ϕ_m now $\sim 90°$