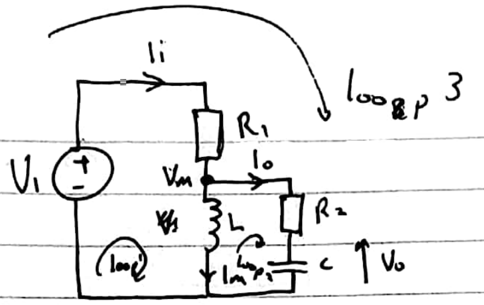
Q1

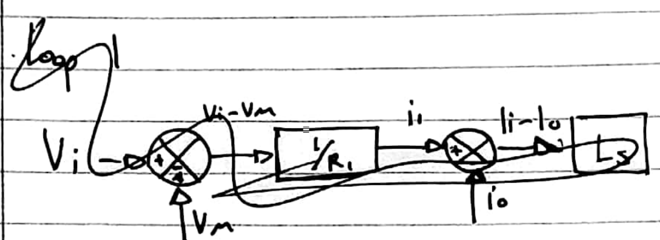


$R_1 = 0.6\Omega$

$R_2 = 0.1\Omega$

$L = 4H$

$C = 1F$

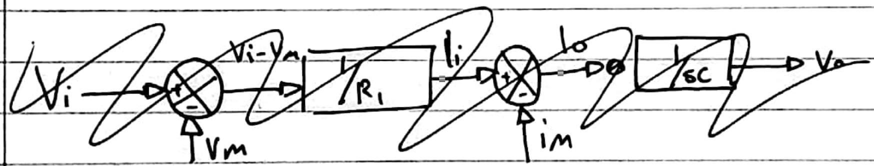


$i_1 \to \boxed{R_1} \to V_i - V_m$

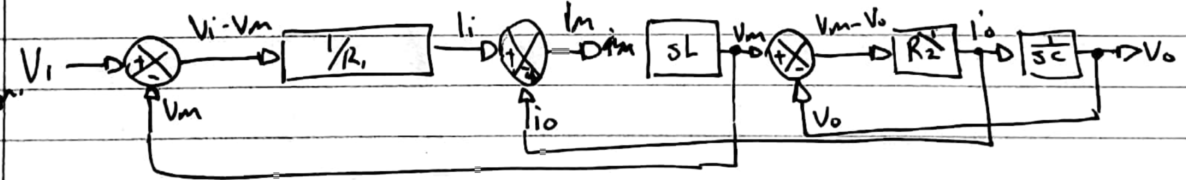
$i_m \to \boxed{sL} \to V_m$

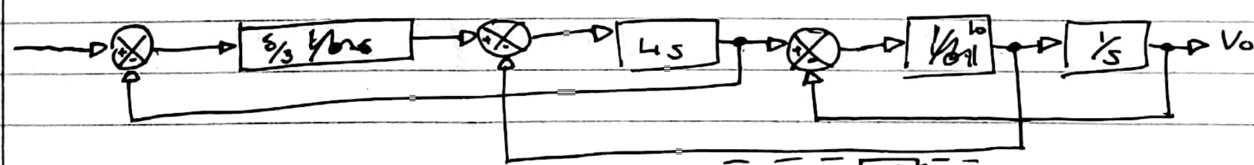
$i_o \to \boxed{R_2} \to V_m - V_o$

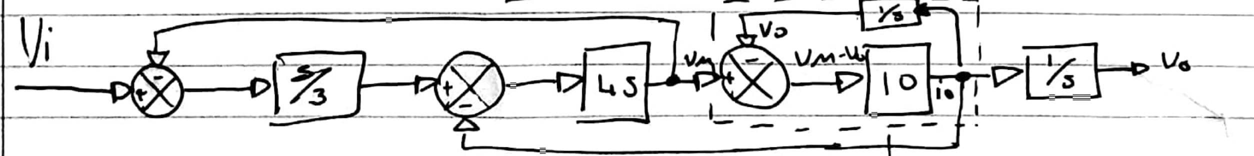
$i_o \to \boxed{1/sC} \to V_o$



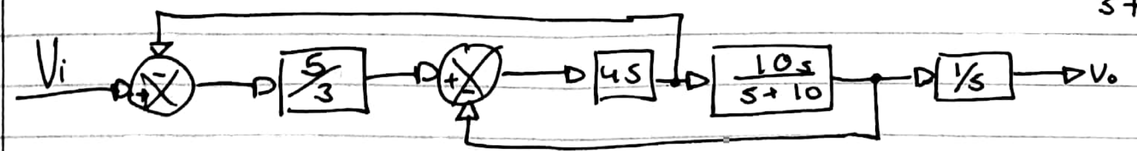
Starting Block Diagram.

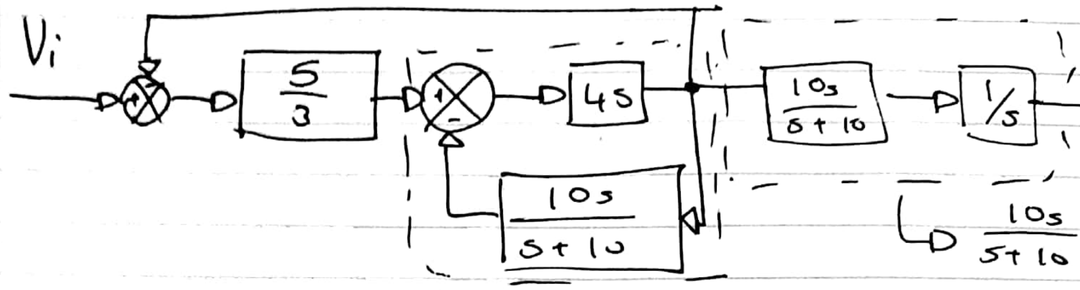






$$\frac{\text{Forward}}{1 - \text{Loop}} = \frac{10}{1 + \frac{10}{s}} = \frac{10s}{s+10}$$

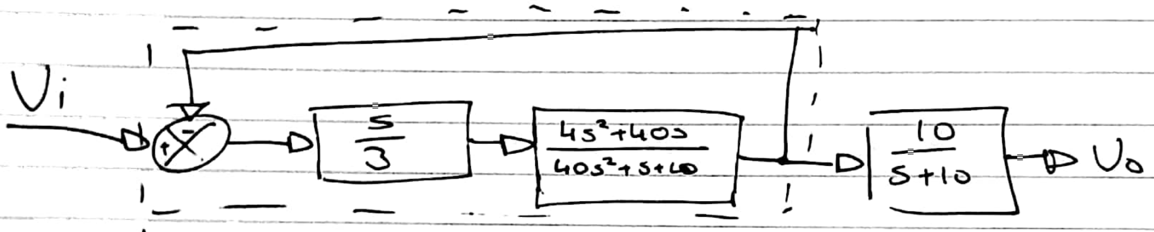




$$\frac{10s}{s+10} \times \frac{1}{s} = \frac{10}{s+10}$$

$$\frac{4s}{1+\frac{4s\times 10s}{s+10}} = \frac{4s\times(s+10)}{s+10+40s^2}$$

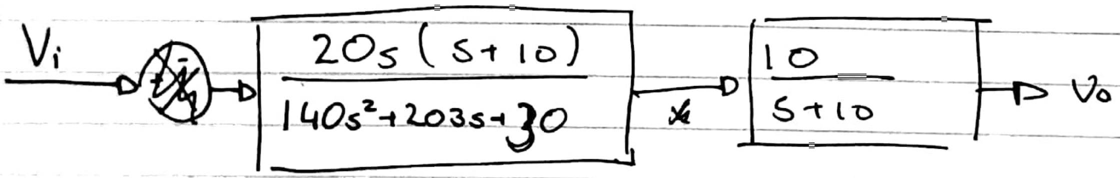
$$= \frac{4s^2+40s}{40s^2+s+10}$$

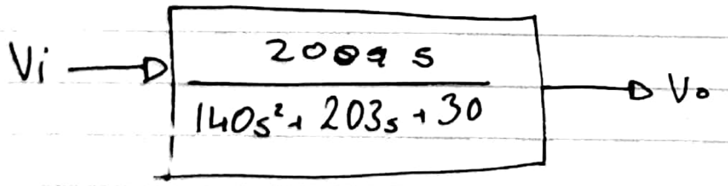


Vi

$$\frac{\frac{5}{3}\times\frac{4s^2+40s}{40s^2+s+10}}{1+\frac{5}{3}\times\frac{4s^2+40s}{40s^2+s+10}} = \frac{5\times 4s\,(s+10)}{120s^2+3s+30+\cancel{20s(s+10)}}$$

$$20s^2+200s$$





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$$\frac{V_o}{V_i} = \frac{200s}{140s^2 + 203s + 30}$$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1, & t \geq 0 \end{cases}$$

$$\mathcal{L}\{u\} = \frac{1}{s}$$

let $V_i(s) = \mathcal{L}\{u\}$
$= 1/s$

$$V_o = \frac{200s\,V_i}{140s^2 + 203s + 30}$$

$$= \frac{200}{140s^2 + 203s + 30}$$

$$= \frac{\frac{200}{140}}{s^2 + \frac{203}{140}s + \frac{30}{140}}$$

$$= \frac{10}{7}\,\frac{1}{s^2 + \frac{58}{40}s + \frac{3}{14}}$$

$$= \frac{10}{7}\,\frac{1}{s^2 + \frac{58}{40}s + \left(\frac{29}{40}\right)^2 - \left(\frac{29}{40}\right)^2 + \frac{3}{14}}$$

$$= \frac{10}{7}\,\frac{1}{\left(s + \frac{29}{40}\right)^2 - \frac{3487}{11200}} \qquad \frac{1}{\omega}\,\frac{\omega}{(s+a)^2 + \omega^2}$$

$$\mathcal{L}^{-1}\{V_o\} = \frac{10}{7}\sqrt{\frac{11200}{3487}}\,j\,\mathcal{L}\left\{\frac{j\sqrt{\frac{3487}{11200}}}{\left(s+\frac{29}{40}\right)^2 + j\sqrt{\frac{3487}{11200}}}\right\}$$

↳ from tables

$$= \frac{10}{7}\sqrt{\frac{11200}{3487}}\,j\,e^{-\frac{29}{40}t}\sin\left(j\sqrt{\frac{3487}{11200}}\,t\right) \qquad e^{-at}\sin(\omega t)$$

$$j\sin(jt) = \sinh(t)$$

$$= \frac{10}{7}\sqrt{\frac{11200}{3487}}\,e^{-\frac{29}{40}t}\sinh\left(\sqrt{\frac{3487}{11200}}\,t\right) \qquad \frac{\sqrt{3487}}{\sqrt{11200}} = \frac{\sqrt{3487}}{40\sqrt{7}}$$

$$= \frac{4000}{\sqrt{24409}}\,e^{-\frac{29}{40}t}\sinh\left(\sqrt{\frac{3487}{11200}}\,t\right)$$

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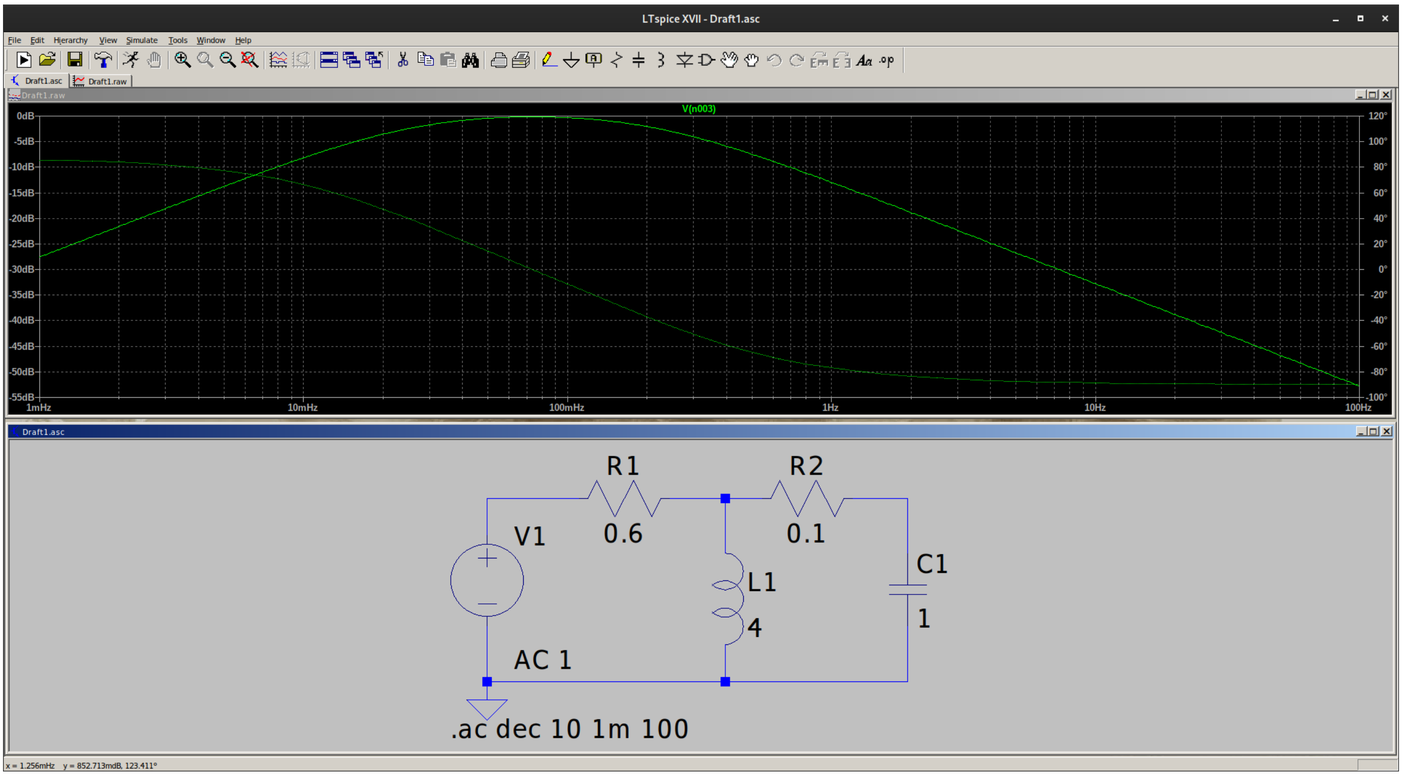


Figure 1: Frequency response of the circuit in LTSpice

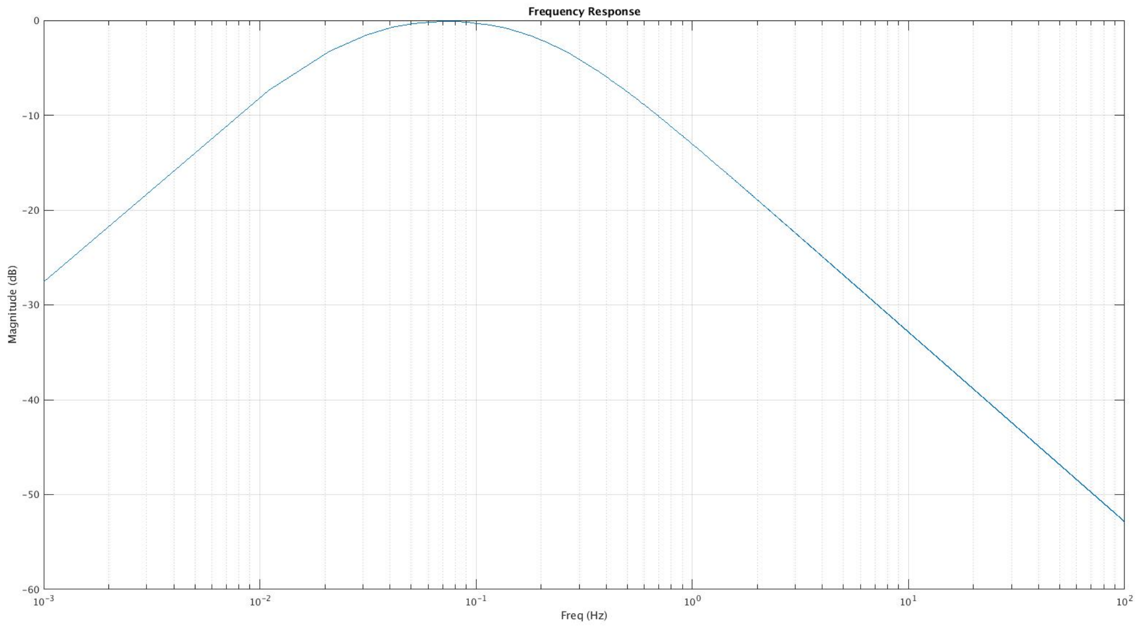


Figure 2: Frequency response of the transfer function in Matlab

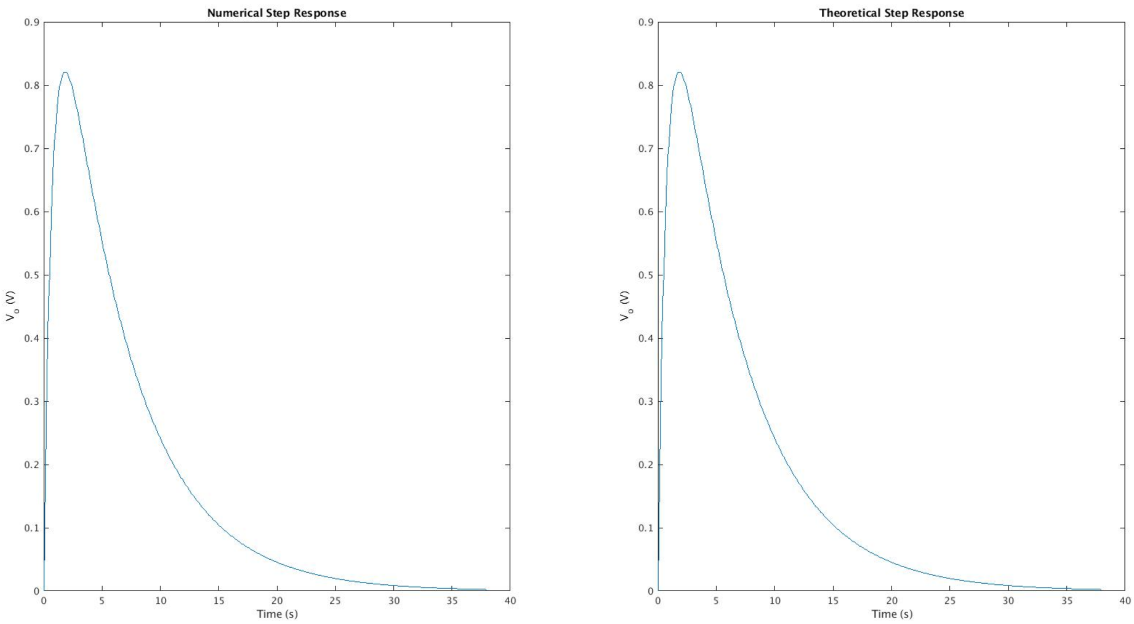


Figure 3: Time response to a step input in Matlab

Figures 1 and 2 show the frequency response of the circuit that was made in spice and the transfer function that was derived above. These plots are the same with gain in dB vs frequency in Hz and have the same intercepts and shape. This indicates a correct derivation.

Figure 3 contains both the numerical (left plot) and theoretical (right plot) time responses to a step input. The numerical one was found using the step function in matlab and the theoretical one was found by inverse laplace transform of the equation with a step response (shown above). As we can see though these are both pretty much identical indicating that the numerical methods are fairly accurate.

Appendices

A Matlab Code

```
clear
W = linspace(0.001*2*pi, 100*2*pi, 10000);
H = freqs([200 0], [140 203 30], W);
figure(1)
semilogx(W./(2*pi), 20*log10(abs(H)))
grid on
xlabel("Freq (Hz)")
ylabel("Magnitude (dB)")
title("Frequency Response")

sys = tf([200 0], [140 203 30]);
[Y,T] = step(sys);
figure(2)
subplot(1,2,1)
plot(T,Y)
title("Numerical Step Response")
xlabel("Time (s)")
ylabel("V_{o} (V)")
```

```

subplot(1,2,2)
y = (400./sqrt(24409)).*exp(T.*(-29/40)).*sinh((sqrt(3487).*T)./(40*sqrt
    (7)));
plot(T,y)
title("Theoretical Step Response")
xlabel("Time (s)")
ylabel("V_{o} (V)")
%stepFunction = (1: T >= 0);
```

$sQ - Q_0 = hA(T_i - T)$

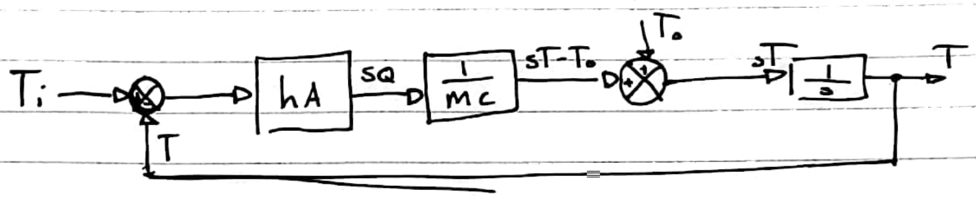
$\dot{q} = \frac{dQ}{dt} = hA(T_i - T)$ in: T_i

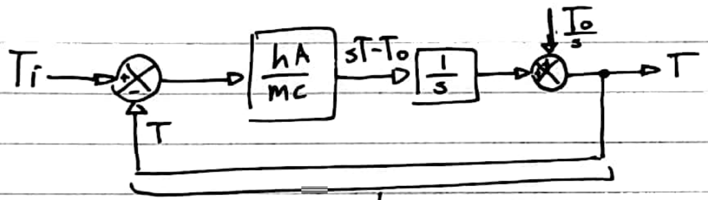
out: T

$\frac{dQ}{dt} = mc\frac{dT}{dt} \Rightarrow \cancel{sQ - Q_0 = mc\,sT}$

$sQ - Q_0 = mc(sT - T_0)$

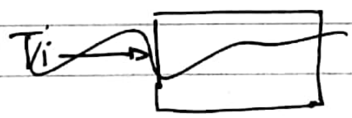
initial condition as we don't know what ~~the~~ temp ~~the~~ dish starts at.

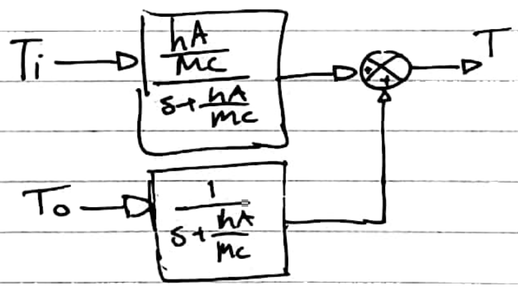




$$\frac{\frac{hA}{mcs}}{1+\frac{hA}{mcs}} + \frac{\frac{T_0}{s}}{1+\frac{hA}{mcs}} = \frac{\frac{hA}{mc}}{s+\frac{hA}{mc}} + \frac{T_0}{s+\frac{hA}{mc}}.$$

~~Ti →~~





$$T = T_i\frac{\frac{hA}{mc}}{s+\frac{hA}{mc}} + \frac{T_0}{s+\frac{hA}{mc}}$$

When $T_i = \frac{200}{s}$ a step ~~response~~ input the step response is

$$T = \frac{\frac{200hA}{mc}}{s\left(s+\frac{hA}{mc}\right)} + \frac{T_0}{s+\frac{hA}{mc}}$$

$$T = \frac{A}{s} + \frac{B}{s+\frac{hA}{mc}} + \frac{T_0}{s+\frac{hA}{mc}}$$

$$A = \left.\frac{\frac{200hA}{mc}}{s+\frac{hA}{mc}}\right|_{s=0} \qquad B = \left.\frac{\frac{200hA}{mc}}{s}\right|_{s=-\frac{hA}{mc}}$$

$$= 200 \qquad\qquad = -200$$

$$T = \frac{200}{s} - \frac{200}{s+\frac{hA}{mc}} + \frac{T_o}{s+\frac{hA}{mc}}$$

inverse laplace.

$$T(t) = 200 + (T_o - 200)e^{-\frac{hA}{mc}t}$$

We have:

$h = 100 \frac{W}{m^2 k}$

$C = 0.45 \frac{J}{gk}$

We don't have

M

A

T_o

so the exact time it takes for the dish to reach 80°C varies on the values for those variables.

we shall assume $M = 1kg$ or $1000g$, $A = 1m^2$ and T_o is a room temp of 20°C

$\therefore$

$$80 = 200 + (20-200)e^{\frac{-100\times 1}{1000\times 0.45}t}$$

$$-120 = -180 \times e^{\frac{-1}{4.5}t}$$

$$e^{-\frac{2}{9}t} = \frac{2}{3}$$

$$-\frac{2}{9}t = \ln\left(\frac{2}{3}\right)$$

$$= -0.405465$$

$$t = 1.825943 \text{ seconds.}$$

so for the above conditions it takes 1.83 seconds to go from 20°C to 80°C.