

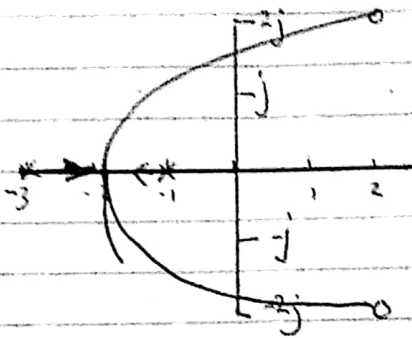
Ever 315 test 2

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1a $G(s) = \frac{s^2 - 4s + 8}{s^2 + 4s + 3}$

$$= \frac{(s - 2 + 2i)(s - 2 - 2i)}{(s + 3)(s + 1)}$$

$P - Z = 0$ no asymptotes.



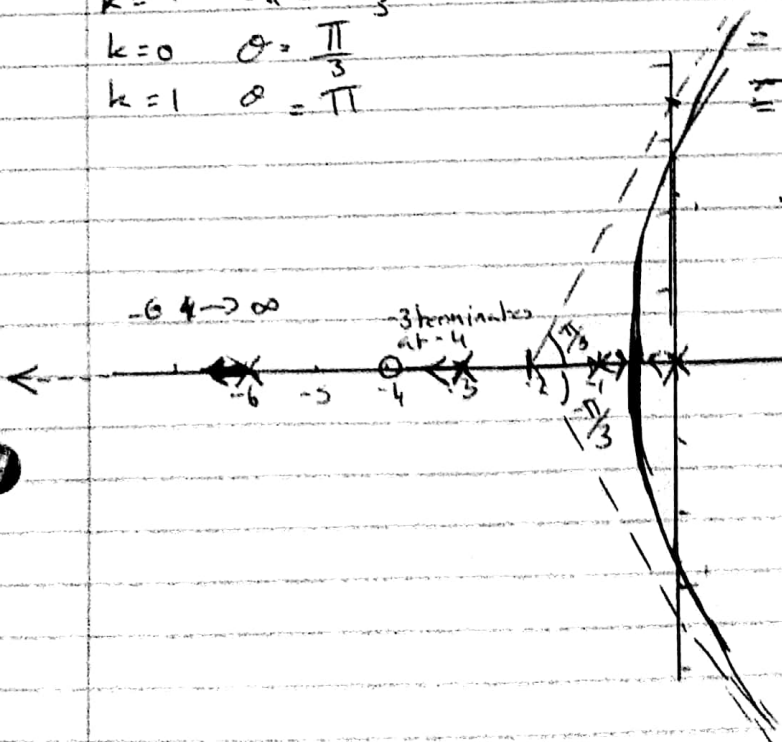
$G(s) = \frac{s + 4}{s(s + 6)(s + 3)(s + 1)}$

$P - Z = 4 - 1 = 3$

$\theta_a = \frac{(2k+1)\pi}{3}$
 $k = -1 \quad \theta_a = -\frac{\pi}{3}$
 $k = 0 \quad \theta = \frac{\pi}{3}$
 $k = 1 \quad \theta = \pi$

$\theta_a = \frac{\overbrace{0 - 6 - 3 - 1}^{\text{Poles}} - \underbrace{-4}_{\text{Zeros}}}{3}$

$= \frac{-6}{3} = -2$



$$1b) \quad G(s) = \frac{s^2 + 10s + 24}{s^2 + 3s + 2} = \frac{(s+6)(s+4)}{(s+2)(s+1)}$$

$$k = \frac{-1}{G(s)}$$

$$= \frac{-1}{G(s)}$$

$$\frac{dk}{ds} = \frac{d}{ds} \left(\frac{-1}{s^2 + 10s + 24} \right)$$

$$= \frac{(-2s - 3)(s^2 + 10s + 24) + (s^2 + 3s + 2)(2s + 10)}{(s^2 + 10s + 24)^2}$$

ignore denominator

$$= -2s^3 - 20s^2 - 48s - 72 + 2s^3 + 6s^2 + 4s + 10s^2 + 30s + 20$$

$$= (-20 + 6 + 10)s^2 + (-48 + 4 + 30)s + (-72 + 20)$$

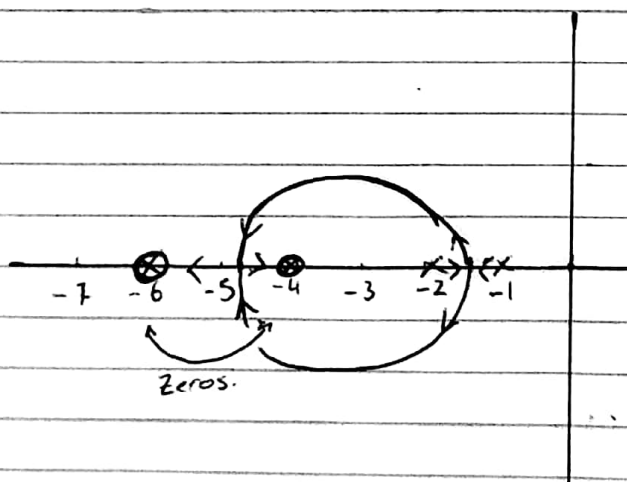
$$= -7s^2 - 44s - 52$$

$$= 0$$

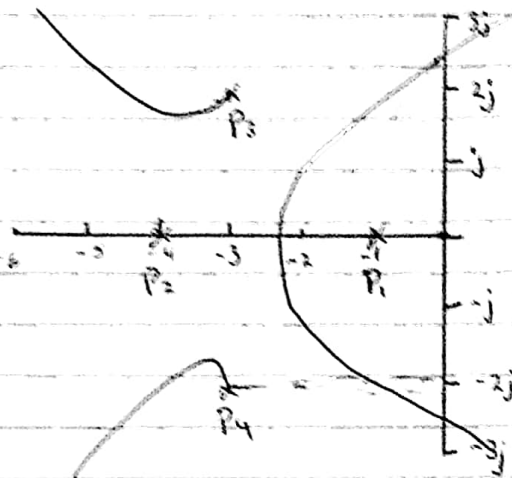
$$7s^2 + 44s + 52 = 0$$

$$(4s + 1.577)(s + 4.707) = 0$$

$$\left(s + \frac{22 + 2\sqrt{33}}{7}\right) \left(s + \frac{22 - 2\sqrt{33}}{7}\right) = 0$$



$$1c. \quad G(s) = \frac{1}{(s+4)(s+1)(s+3+2j)(s+3-2j)}$$



try $j\omega = 2j \cdot 5j$

~~arg(P1)~~

$$\theta_{p1} = 4 \arctan \frac{2 \cdot 5}{1}$$

$$= 68.199^\circ$$

$$\theta_{p2} = \arctan \left(\frac{2 \cdot 5}{4} \right)$$

$$= 32^\circ$$

$$\theta_{p3} = \arctan \left(\frac{0.5}{3} \right)$$

$$= 9.46$$

$$\theta_{p4} = \arctan \left(\frac{4 \cdot 5}{3} \right)$$

$$= 56.31$$

$$-4\theta_{p1} - \theta_{p2} - \theta_{p3} - \theta_{p4} = -68.199 - 32 - 9.46 - 56.31$$

$$= -165.969$$

too low so try $j\omega = 2.8j$

$$\theta_{p1} = \arctan \frac{2.8}{1} \quad \theta_{p2} = \arctan \left(\frac{0.5}{3} \right)$$

$$= 70.346 \quad = 14.934$$

$$\theta_{p3} = \arctan \left(\frac{2.8}{4} \right) \quad \theta_{p4} = \arctan \left(\frac{4 \cdot 2.8}{3} \right)$$

$$= 34.99 \quad = 57.995$$

$$-\theta_{p1} - \theta_{p2} - \theta_{p3} - \theta_{p4} = -178.26$$

Bit too low try 2.85

$$\theta_{p1} = \arctan \left(\frac{2.85}{1} \right) \quad \theta_{p2} = \arctan \left(\frac{0.5}{3} \right)$$

$$= 70.665 \quad = 15.8192$$

$$\theta_{p3} = \arctan \left(\frac{2.85}{4} \right) \quad \theta_{p4} = \arctan \left(\frac{4 \cdot 2.85}{3} \right)$$

$$= 40.665 \quad = 58.26086$$

$$-\theta_{p1} - \theta_{p2} - \theta_{p3} - \theta_{p4} = -180.235$$

Imag intercept
at $\sim j\omega = 2.85j$
and $j\omega = -2.85j$

1c cont

imag intercept $= \pm 2.85j$

Gain this occurs at.

$$|k| = \frac{1}{|G(s)|}$$

$$= |(s+4)| \times |s+1| \times |s+3+2j| \times |s+3-2j|$$

$$= \sqrt{2.85^2 + 4^2} \times \sqrt{2.85^2 + 1^2} \times \sqrt{4.85^2 + 3^2} \times \sqrt{0.85^2 + 3^2}$$

~~$s = 2.85j$~~
 $s = 2.85j$

$$= 263.78$$

$$20 \log(k) = 48.42 \text{ dB.}$$

$$2a \quad G(s) = \frac{s+300}{s^2 + 8s + 100}$$

$$= \frac{s+300}{s(s+10)}$$

2b Roots $(s+3)$ & $(s+8)$, $(s+100)$
all poles

$$G(s) = k \frac{1}{(s+3)(s+8)(s+100)}$$

$$= \frac{k}{3 \times 8 \times 100} \left(\frac{1}{\left(\frac{s}{3}+1\right)\left(\frac{s}{8}+1\right)\left(\frac{s}{100}+1\right)} \right)$$

$$k = 24000$$

$$20 \log \frac{k}{2400} = 20 \text{ dB}$$

$$\frac{k}{10} = 10$$

$$G(s) = \frac{24000}{(s+3)(s+8)(s+100)}$$

2. System is type 1 as phase starts at -90°

SSE

Step input : 0

$$316.23 \therefore SSE = \frac{1}{316.23} = 3.16 \times 10^{-3}$$

Ramp input : $1/k_v$ where $k_v = 281.74$ from graph : $SSE = \frac{1}{281.74} =$

parabolic input : ∞

See attached graph

$$2e \quad G(s) = \frac{20000}{s(s+100)(s+10)}$$

$$= \frac{20000}{s(s+100)(s+10)}$$

$$40000 \left(\frac{s}{100} + 1 \right) \left(\frac{s}{10} + 1 \right)$$

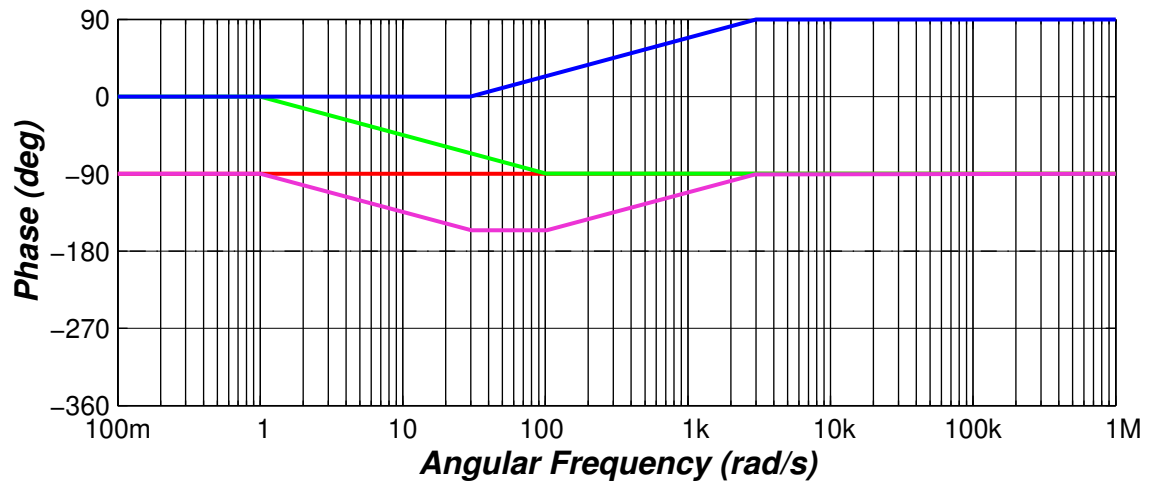
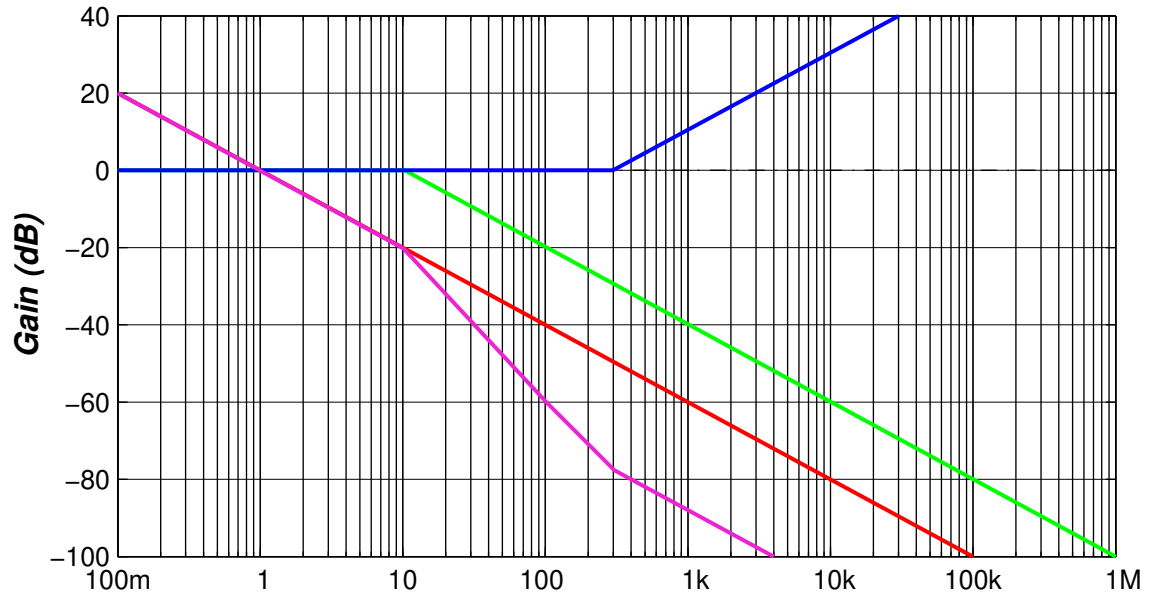
ϕ_m happens at $\sim 80 \text{ rad/s}$ ^{15 rad/s}

$$\text{lead compensator} = \frac{s+15}{s+100}$$

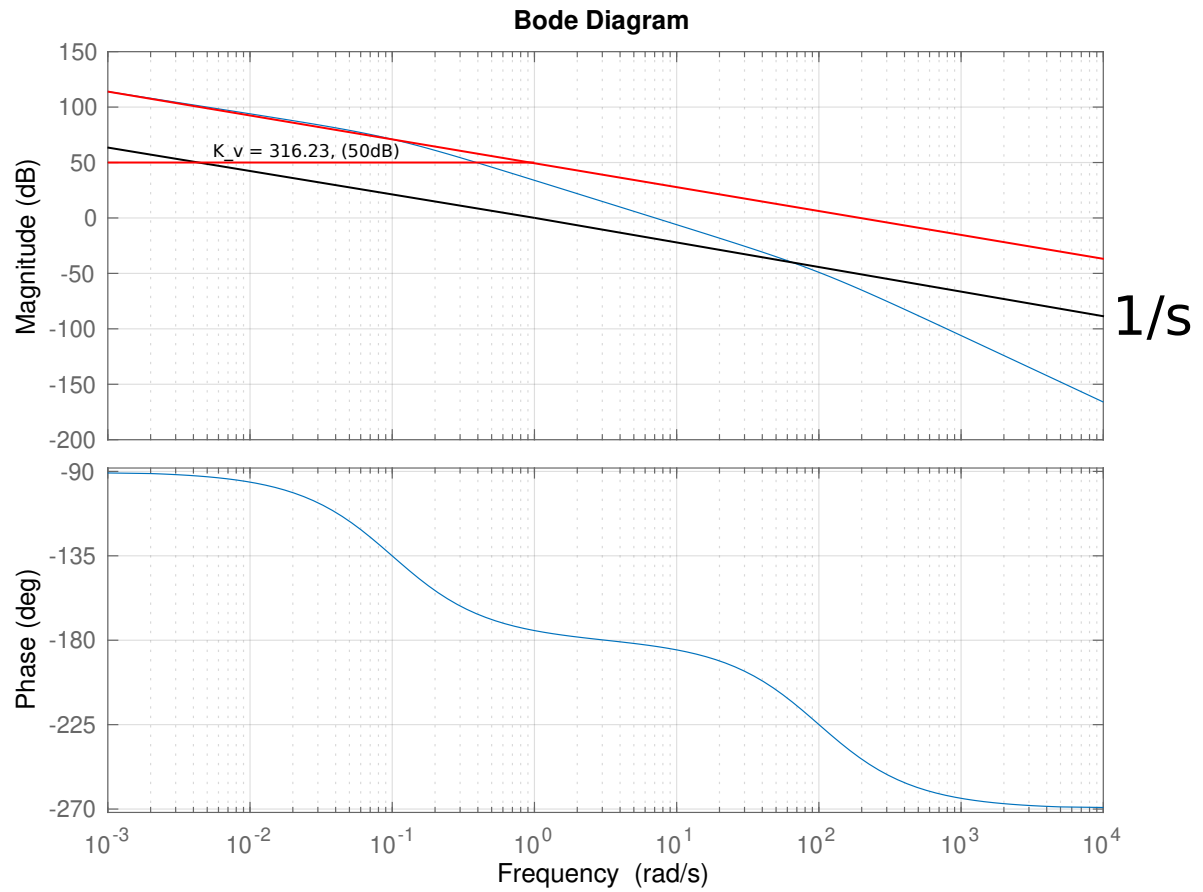
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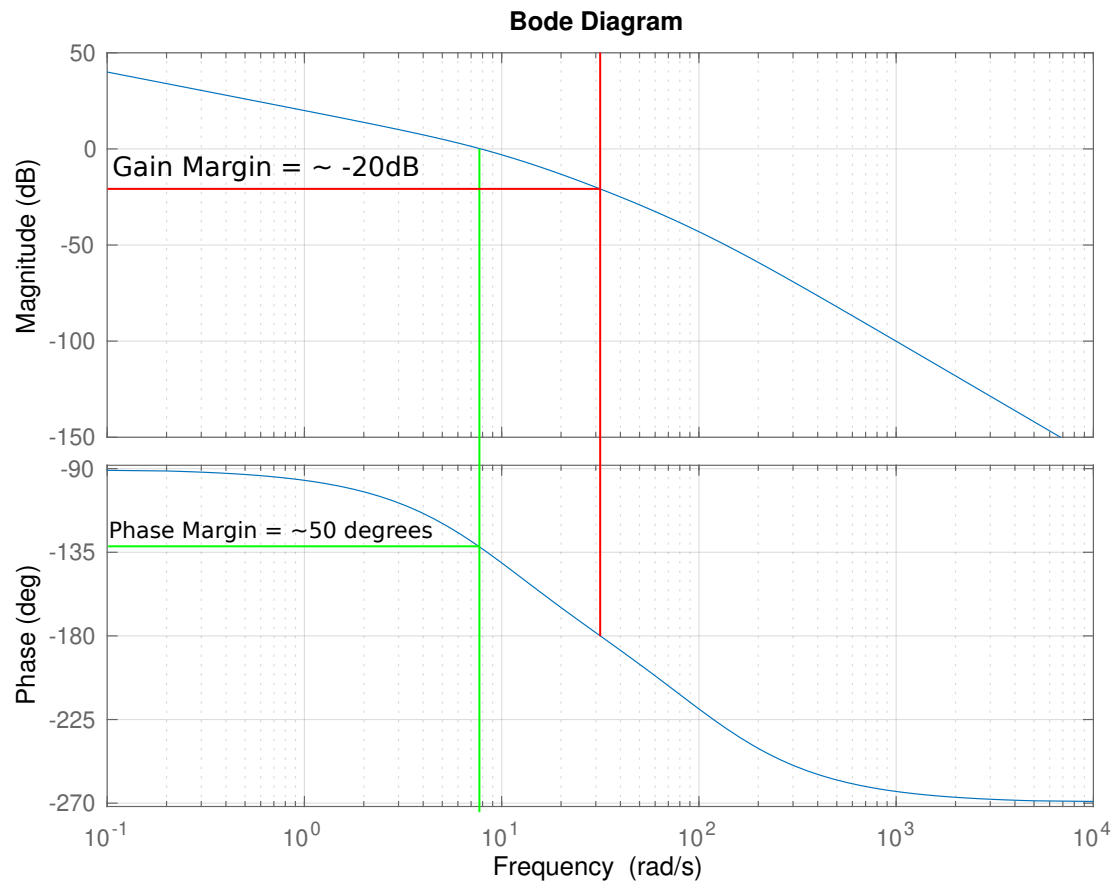
- 1/s
- 1/(s+10)
- (s+300)
- Combined



- (c) (5 points) For the Bode plot displayed below, give the type of the system and the steady state error for a step, ramp and parabolic input. Show or explain how you got your result.



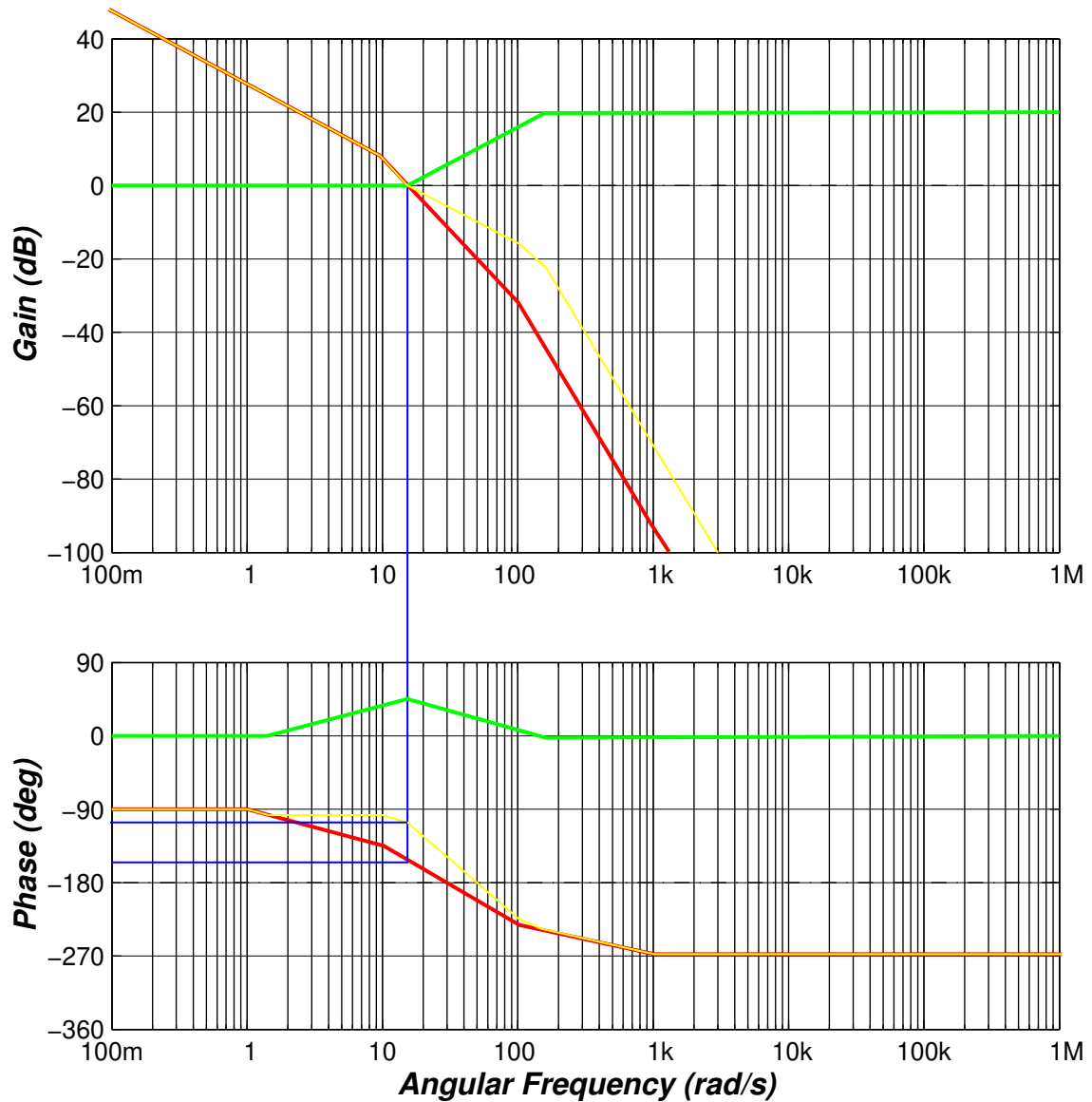
- (d) (5 points) For the Bode plot displayed below, give the approximate gain and phase margin, as well as the approximate damping ratio.



$$\begin{aligned}\text{Damping Ratio} &= \text{Phase Margin}/100 \\ &= 50 / 100 \\ &= 0.5\end{aligned}$$

Name:

Student Number:



Initial Phase Margin of 30

Final Phase Margin of 75 degrees

- Plant
- Compensator = $(s+15)/(s+150)$
- Phase Margin Indicator
- Combined