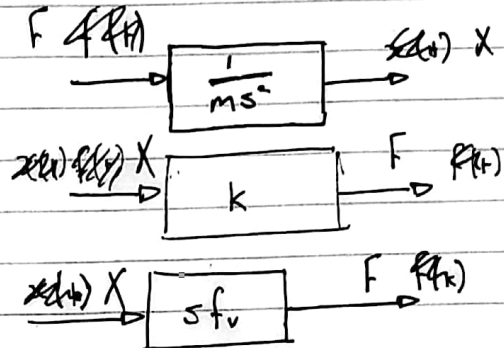
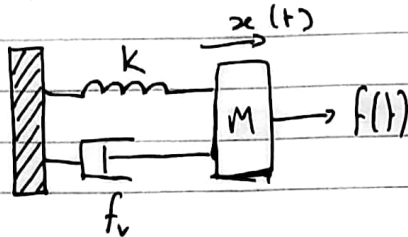


input $f(t)$
output $x(t)$

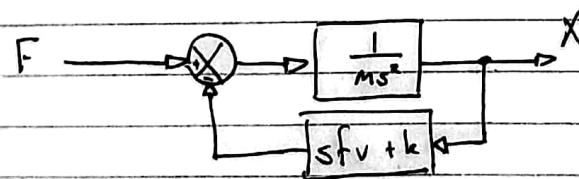
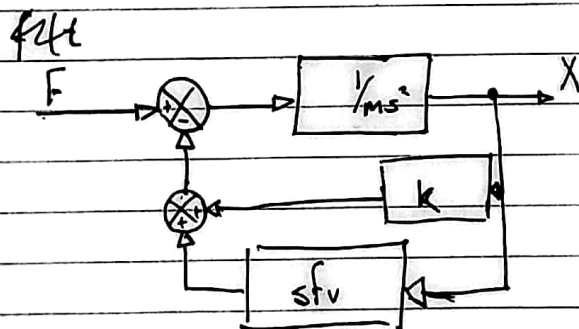


in opposite
Direction to
applied.
in opposite
Direction to
applied.

spring $F = kx$
damper $F = f_v v$
mass $F = ma$

~~Free~~

combine blocks

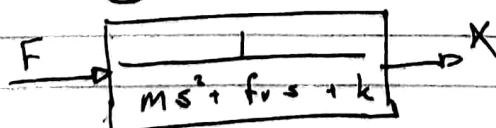


$$\frac{X}{F} = \frac{\text{forward}}{1 - \text{loop}}$$

$$= \frac{1}{ms^2}$$

$$1 - \frac{sf_v + k}{ms^2}$$

$$= \frac{1}{ms^2 + sf_v + k}$$

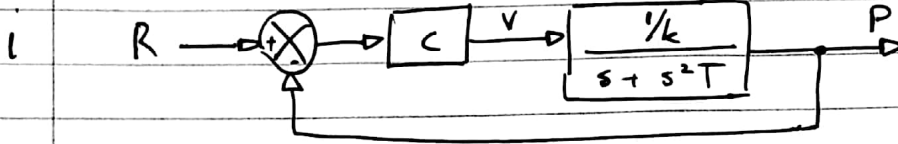


2

$$\frac{P}{V} = \frac{\frac{1}{k}}{s + s^2 T}$$

$$k = 0.2 \frac{Nm}{A}$$

$$T = 0.8s$$



$$\frac{P}{R} = \frac{\text{Forward}}{1 - \text{loop}}$$

$$= \frac{C \times \frac{P}{V}}{1 + C \times \frac{P}{V}}$$

$$= \frac{C \times P}{V + C \times P}$$

$$= \frac{C/k}{s^2 T + s + C/k} = \frac{\frac{C}{0.2 \times 0.8}}{s^2 + \frac{s}{0.8} + \frac{C}{0.2 \times 0.8}}$$

$$= \frac{6.25C}{s^2 + 1.25s + 6.25C}$$

iii

~~$R(s) = \frac{1}{s}$~~

~~$\frac{P}{R} = \frac{6.25C}{s^2 + 1.25s + 6.25C}$~~

~~$\zeta = 1.25$~~

~~discriminant $\Delta = 1.25^2 - 4 \times 1 \times 6.25C$
 $= 1.5625 - 25C$~~

Underdamped when Δ

2ii

A

discriminant $\Delta = b^2 - 4ac$ \leftarrow coef
 $= 1.25^2 - 4 \times 6.25 c$ \leftarrow gain controller c
 $= 1.5625 - 25c$

underdamped when $\Delta < 0$

critically damped when $\Delta = 0$

$$0 = 1.5625 - 25c$$

$$25c = 1.5625$$

$$c = \frac{1.5625}{25}$$

$$= 0.0625$$

under damped when $\Delta < 0$

ie $4ac < b^2$ $0 > b^2 - 4ac$

$25c$

$4ac > b^2$

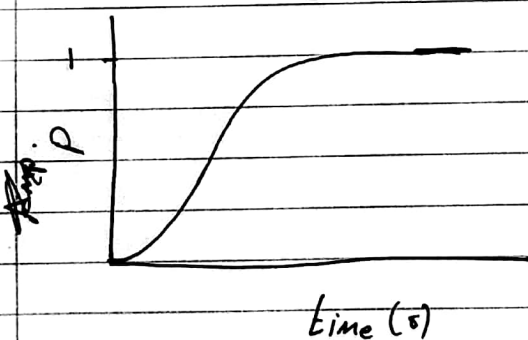
$$c > 0.0625$$

overdamped when $\Delta > 0$

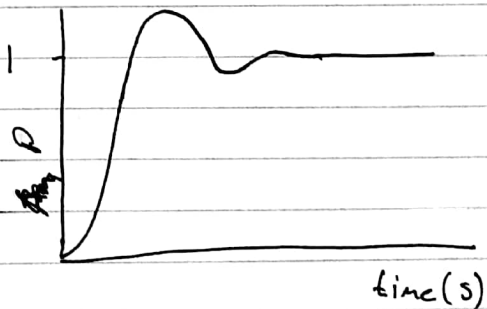
$$c < 0.0625$$

steady state gain for all systems is 1

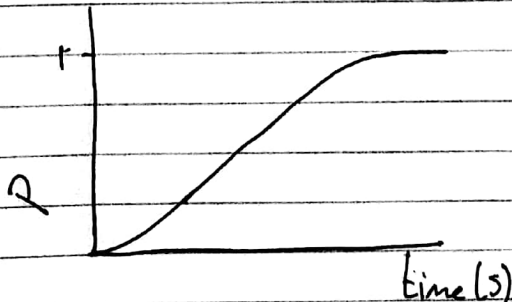
$$c = 0.0625$$



$$c > 0.0625$$



$$c < 0.0625$$



Q3

$$G(s) = \frac{1}{(s+20)(s^2 + 5s + 20)}$$

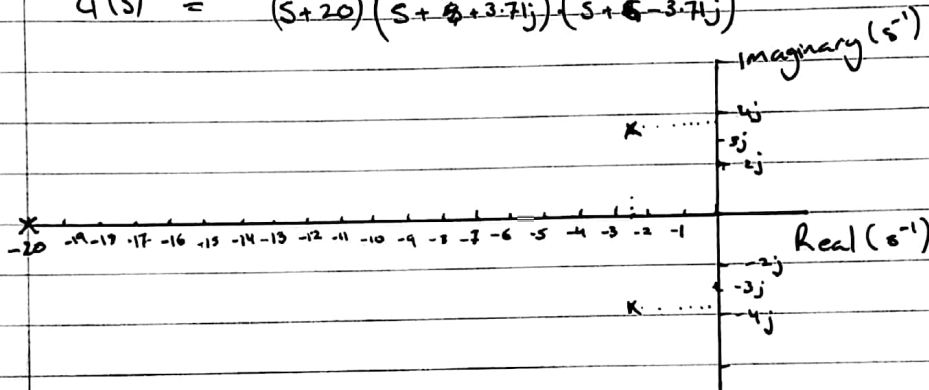
i roots of ~~eq~~ 2nd order component

$$s = \frac{-5 \pm \sqrt{25 - 80}}{2}$$

$$= -2.5 \pm \frac{\sqrt{-55}}{2}$$

$$= -2.5 \pm 3.71j$$

$$G(s) = \frac{1}{(s+20)(s^{2.5} + 3.71j)(s^{2.5} - 3.71j)}$$

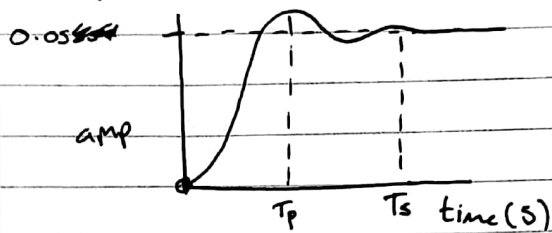


ii we can simplify this system to a second order system by ignoring the response caused by the pole at $s = -20$. We can do this because that pole the poles that dominate the system response are the slower poles which are defined by being closer to the real component origin. As $s = -20$ is much more negative than the real components of the other poles it provides a much quicker response that isn't as dominant in the system.

Q3 iii Plot step response to $G(s) = \frac{1}{s^2 + 5s + 20}$

expect it to look like.

$$\text{input} = u(t) \Rightarrow \frac{1}{s}$$



$$\text{Steady state gain} = \frac{1}{20} = 0.05$$

this is 20 times greater than it would have been had we not ignored the fast pole.

$$G(s) = k \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \frac{4}{2} \sqrt{20} = 2\sqrt{5} \quad k = 1/20$$

$$2\zeta\omega_n = 5$$

$$\zeta\omega_n = \frac{5}{2}$$

$$\zeta = \frac{5}{2} \times \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{4}$$

$$\% OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$$

$$= e^{-\left(\frac{\frac{\sqrt{5}}{4}\pi}{\sqrt{1-\left(\frac{\sqrt{5}}{4}\right)^2}}\right)} \times 100$$

$$= 12.03\%$$

$$T_s = \frac{4}{\zeta\omega_n}$$

$$= \frac{16}{2\sqrt{5} \times \frac{5}{2}}$$

$$= \frac{16 \times 4}{2 \times 5}$$

$$= \frac{16}{5}$$

$$= 3.2 \text{ s}$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$= \frac{\pi}{2\sqrt{5} \sqrt{1-\left(\frac{\sqrt{5}}{4}\right)^2}}$$

$$= 0.847 \text{ s}$$

Q 4 i Negative Feedback when the output (or part of) is fed back into the system and subtracted from the input so that with the goal of reducing fluctuations and increasing stability.

ii advantages

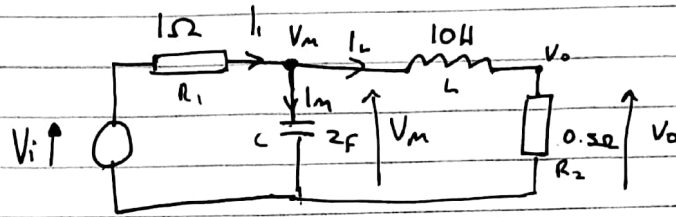
- Reduces effect of external disturbances
- Reduces effect of parameter changes
- improves stability

iii ~~= Schmitt triggers, use positive feedback to create a region where nothing switches so that noise has less effect.~~

- Schmitt trigger, use positive feedback to create a hysteresis region that ~~delays~~ causes the trigger to not respond to small changes and reduce the effect of noise in a signal.

- Radio signals, greatly amplify the signal so that it can be transmitted further.

Q 5



$$\frac{V_o}{V_i} = \frac{1}{40s^2 + 22s + 3}$$

$$V_i - V_m = I_1 R_1$$

$$I_1 = I_m + I_L$$

for

$$V_m = \frac{1}{C} \frac{dI_m}{dt}$$

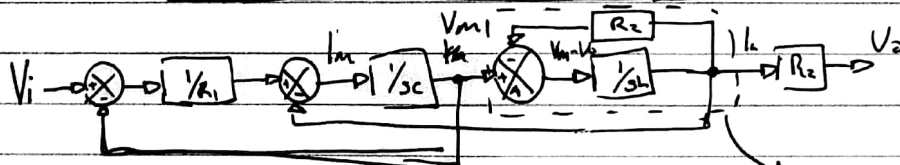
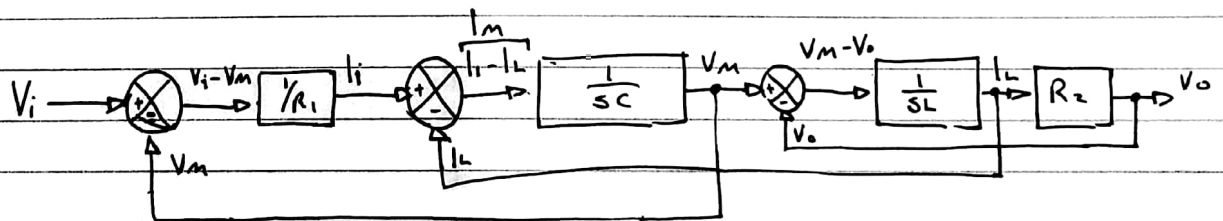
$$V_m = L \int I_L dt + I_L R_2 + V_o \quad V_o = I_L R_2$$

$$I_1 \rightarrow [R_1] \rightarrow V_i - V_m$$

$$I_m \rightarrow \left[\frac{1}{sC} \right] \rightarrow V_m$$

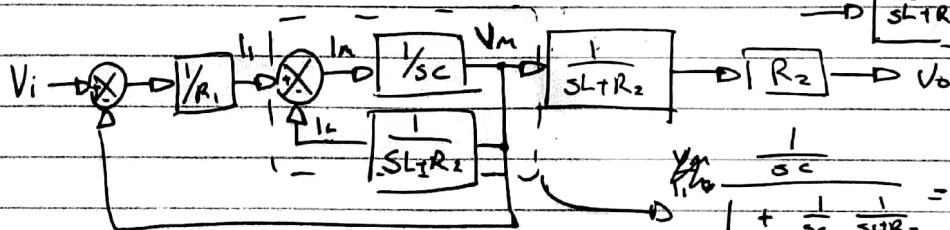
$$I_L \rightarrow [sL] \rightarrow V_m - V_o$$

$$I_L \rightarrow [R_2] \rightarrow V_o$$



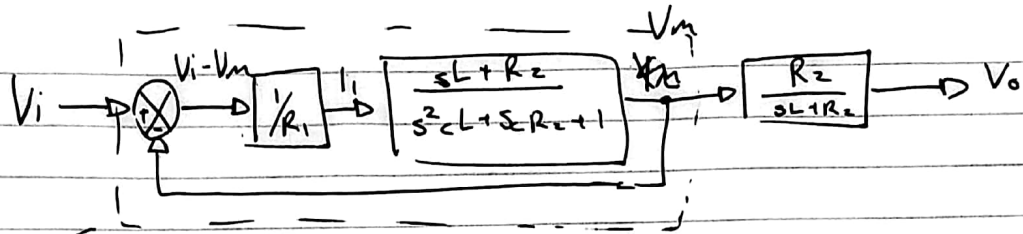
$$\frac{\frac{1}{sC}}{1 + \frac{R_2}{sL}} = \frac{I_L}{V_m}$$

$$\frac{1}{sL + R_2} \rightarrow I_L$$



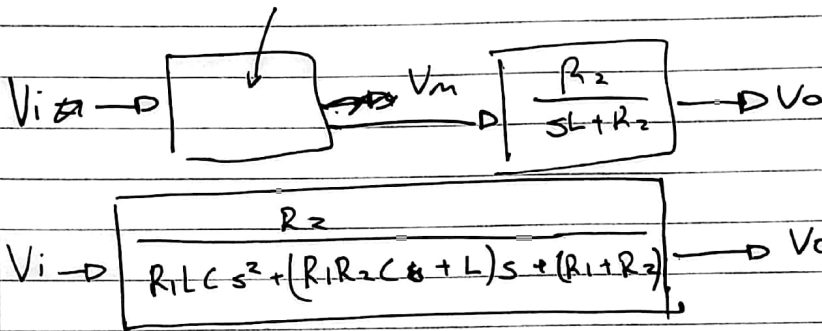
$$\frac{1}{1 + \frac{1}{sC} \frac{1}{sL + R_2}} = \frac{sL + R_2}{s^2 L + sCR_2 + 1} = \frac{V_m}{I_1}$$

Q5 cont



$$\frac{V_m}{V_i} = \frac{\frac{1}{R_1} \frac{sL + R_2}{Ls^2 + R_2s + 1}}{1 + \frac{1}{R_1} \frac{sL + R_2}{Ls^2 + R_2s + 1} \frac{sL + R_2}{sL + R_2}}$$

$$= \frac{R_1 L C s^2 + R_1 R_2 C s + R_1 + sL + R_2}{R_1 L C s^2 + R_1 R_2 C s + R_1 + sL + R_2}$$



To change make it underdamped with out changing steady state gain, change the value of C or L

$$\Delta \Delta = 2^2 - 4 \times 40 \times 3$$

$$= 484 - 480$$

we want $b^2 < 4ac$.

one way to do this is lower L to ^{something like} 9 H

$$b^2 = (1 \times 0.5 \times 2 \times 9)^2 \quad 4ac = 4 \times 9 \times 2 \times 1 \times (1 + 0.5)$$

$$= 10^2 \quad = 108$$

$$= 100$$

so $b^2 < 4ac$.

The other way is to increase C to something like 3F

$$b^2 = (1 \times 0.5 \times 3 \times 9)^2 \quad 4ac = 4 \times 10 \times 3 \times 1 \times (1 + 0.5)$$

$$= 11.5^2 \quad = 180$$

$$= 132.25$$

so $b^2 < 4ac$.