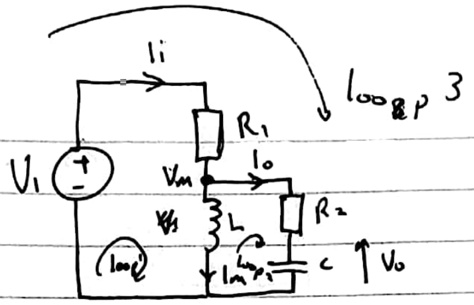
Q1

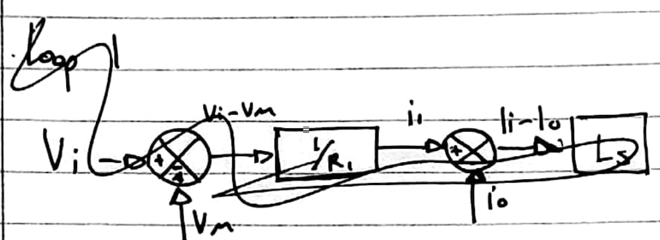


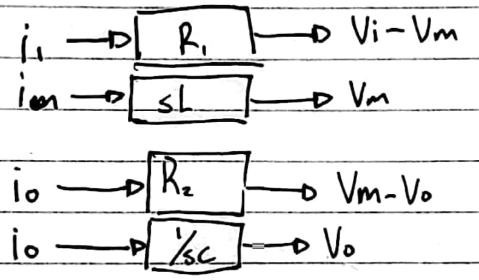
$R_1 = 0.6\Omega$

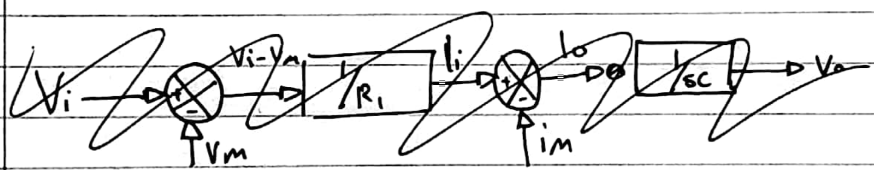
$R_2 = 0.1\Omega$

$L = 4H$

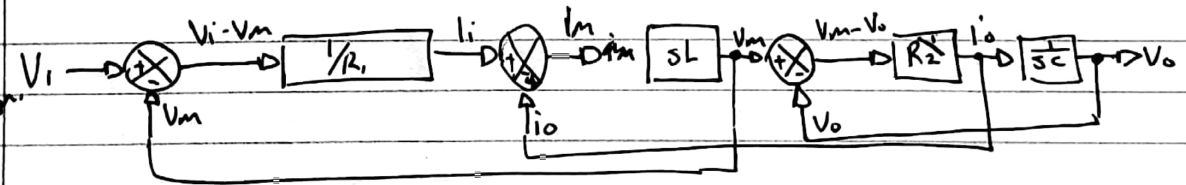
$C = 1F$

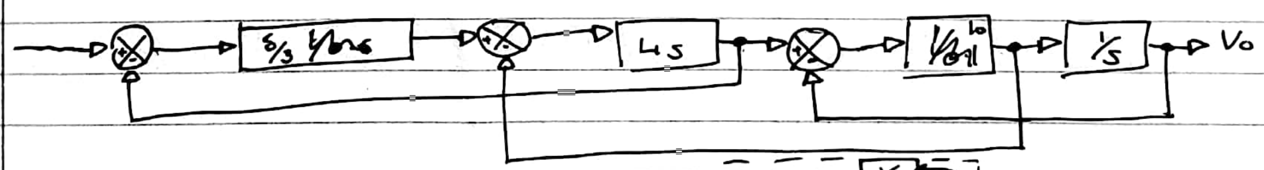


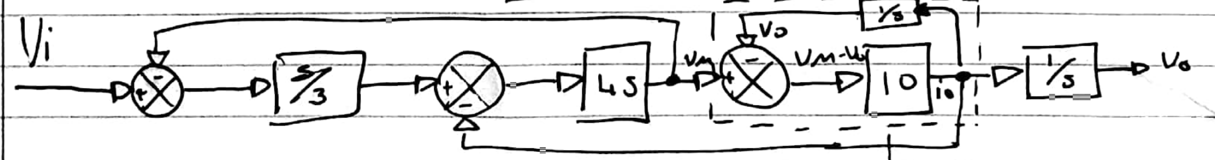




Starting Block Diagram

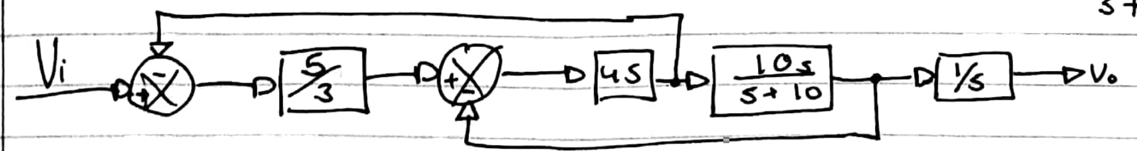


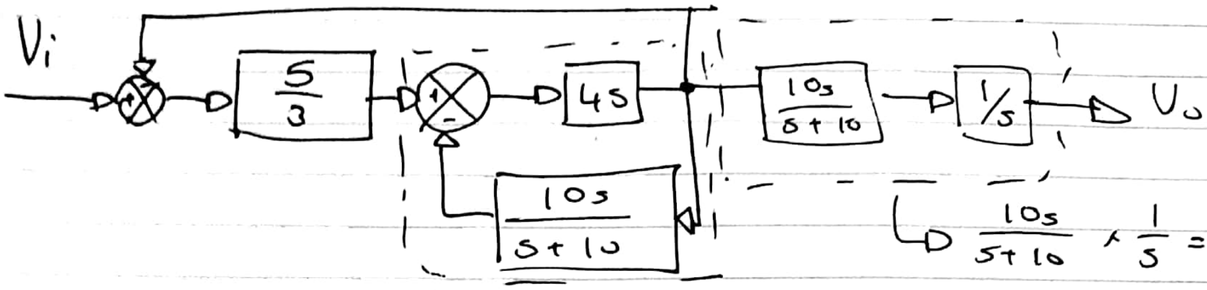




$$\frac{\text{forward}}{1 - \text{Loop}} = \frac{10}{1 + \frac{10}{s}}$$

$$= \frac{10s}{s+10}$$

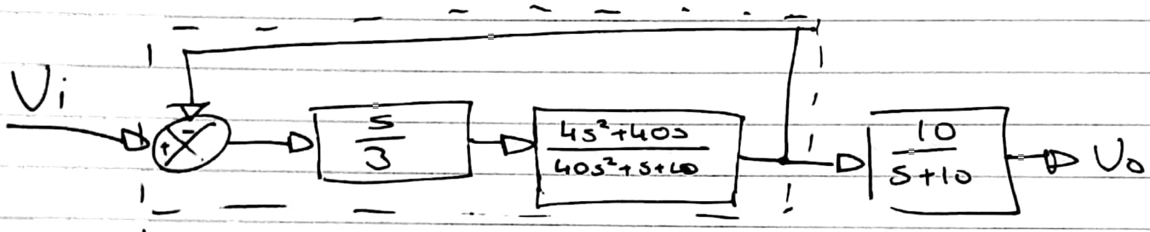




$$\rightarrow \frac{10s}{s+10} \times \frac{1}{s} = \frac{10}{s+10}$$

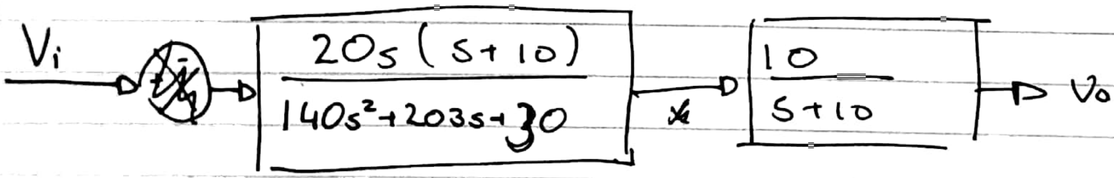
$$\rightarrow \frac{4s}{1+\frac{4s\times 10s}{s+10}} = \frac{4s\times(s+10)}{s+10+40s^2}$$

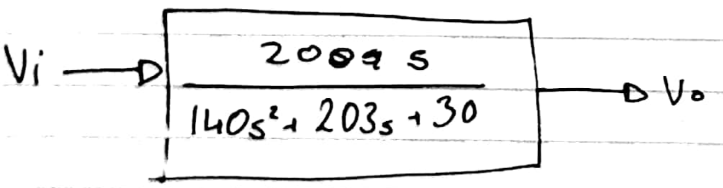
$$= \frac{4s^2+40s}{40s^2+s+10}$$



$$\rightarrow \frac{\frac{5}{3}\times\frac{4s^2+40s}{40s^2+s+10}}{1+\frac{5}{3}\times\frac{4s^2+40s}{40s^2+s+10}} = \frac{5\times 4s(s+10)}{120s^2+3s+30+\cancel{20s(s+10)}}$$

$$20s^2+200s$$





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$$\frac{V_o}{V_i} = \frac{200s}{140s^2 + 203s + 30}$$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1, & t \geq 0 \end{cases}$$

$$\mathcal{L}\{u\} = \frac{1}{s}$$

let $V_i(s) = \mathcal{L}\{u\}$
$= 1/s$

$$V_o = \frac{200s\, V_i}{140s^2 + 203s + 30}$$

$$= \frac{200}{140s^2 + 203s + 30}$$

$$= \frac{\frac{200}{140}}{s^2 + \frac{203}{140}s + \frac{30}{140}}$$

$$= \frac{10}{7} \frac{1}{s^2 + \frac{58}{40}s + \frac{3}{14}}$$

$$= \frac{10}{7} \frac{1}{s^2 + \frac{58}{40}s + \left(\frac{29}{40}\right)^2 - \left(\frac{29}{40}\right)^2 + \frac{3}{14}}$$

$$= \frac{10}{7} \frac{1}{\underbrace{\left(s + \frac{29}{40}\right)^2 - \frac{3487}{11200}}} \qquad \frac{1}{\omega} \frac{\omega}{(s+a)^2 + \omega^2}$$

$$\mathcal{L}^{-1}\{V_o\} = \frac{10}{7}\sqrt{\frac{11200}{3487}}\, j\, \mathcal{L}\left\{ \frac{j\sqrt{\frac{3487}{11200}}}{\left(s + \frac{29}{40}\right)^2 + j\sqrt{\frac{3487}{11200}}} \right\}$$

↳ from tables

$$= \frac{10}{7}\sqrt{\frac{11200}{3487}}\, j\, e^{-\frac{29}{40}t} \sin\left(j\sqrt{\frac{3487}{11200}}\, t\right) \qquad e^{-at}\sin(\omega t)$$

$$j\sin(jt) = \sinh(t)$$

$$= \frac{10}{7}\sqrt{\frac{11200}{3487}}\, e^{-\frac{29}{40}t} \sinh\left(\sqrt{\frac{3487}{11200}}\, t\right) \qquad \frac{\sqrt{3487}}{\sqrt{11200}} = \frac{\sqrt{3487}}{40\sqrt{7}}$$

$$= \frac{4000}{\sqrt{24409}}\, e^{-\frac{29}{40}t} \sinh\left(\sqrt{\frac{3487}{11200}}\, t\right)$$

$$sQ - Q_0 = hA(T_i - T)$$

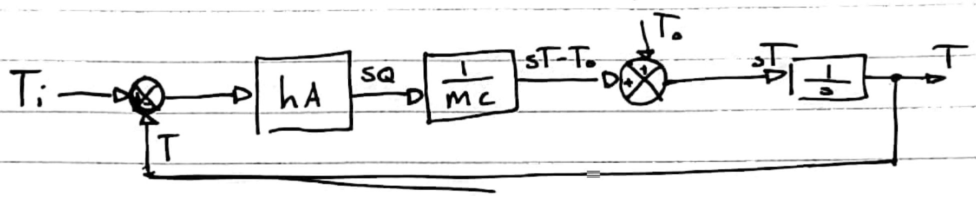
$$\dot{q} = \frac{dQ}{dt} = hA(T_i - T)$$

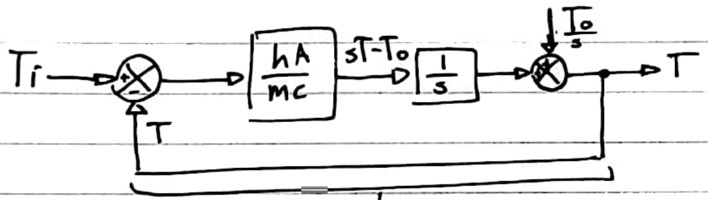
in: T_i
out: T

$$\frac{dQ}{dt} = mc\frac{dT}{dt} \Rightarrow \cancel{sQ - Q_0 = mc\,sT}$$

$$sQ - Q_0 = mc(sT - T_0)$$

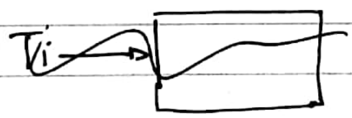
initial condition as we don't know what ~~the~~ temp ~~the~~ dish starts at.

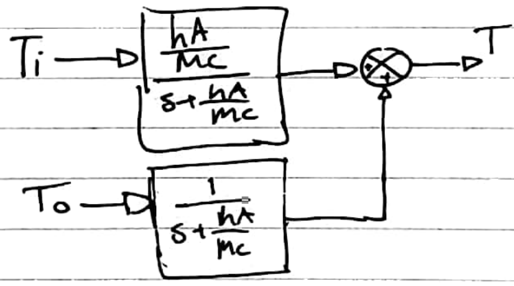




~~Ti~~

$$\frac{\frac{hA}{mcs}}{1 + \frac{hA}{mcs}} + \frac{\frac{T_0}{s}}{1 + \frac{hA}{mcs}} = \frac{\frac{hA}{mc}}{s + \frac{hA}{mc}} + \frac{T_0}{s + \frac{hA}{mc}}.$$





~~Ti~~

$$T = T_i\frac{\frac{hA}{mc}}{s + \frac{hA}{mc}} + \frac{T_0}{s + \frac{hA}{mc}}$$

When $T_i = \frac{200}{s}$ a step ~~response~~ input the step response is

$$T = \frac{\frac{200hA}{mc}}{s\left(s + \frac{hA}{mc}\right)} + \frac{T_0}{s + \frac{hA}{mc}}$$

$$T = \frac{A}{s} + \frac{B}{s + \frac{hA}{mc}} + \frac{T_0}{s + \frac{hA}{mc}}$$

$$A = \left.\frac{\frac{200hA}{mc}}{s+\frac{hA}{mc}}\right|_{s=0} \qquad B = \left.\frac{\frac{200hA}{mc}}{s}\right|_{s=-\frac{hA}{mc}}$$

$$= 200 \qquad\qquad = -200$$

$$T = \frac{200}{s} - \frac{200}{s+\frac{hA}{mc}} + \frac{T_0}{s+\frac{hA}{mc}}$$

inverse laplace.

$$T(t) = 200 + (T_0 - 200)\,e^{-\frac{hA}{mc}t}$$

We have:

$h = 100 \frac{W}{m^2 K}$

$C = 0.45 \frac{J}{gK}$

We don't have

M

A

T_0

so the exact time it takes for the dish to reach 80°C varies on the values for those variables.

we shall assume $M = 1\,kg$ or $1000\,g$, $A = 1\,m^2$ and T_0 is a room temp of 20°C

∴

$$80 = 200 + (20-200)\,e^{\frac{-100\times 1}{1000\times 0.45}t}$$

$$-120 = -180\times e^{\frac{-1}{4.5}t}$$

$$e^{-\frac{2}{9}t} = \frac{2}{3}$$

$$-\frac{2}{9}t = \ln\left(\frac{2}{3}\right)$$

$$= -0.405465$$

$t = 1.824593$ seconds.

so for the above conditions it takes 1.83 seconds to go from 20°C to 80°C.

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