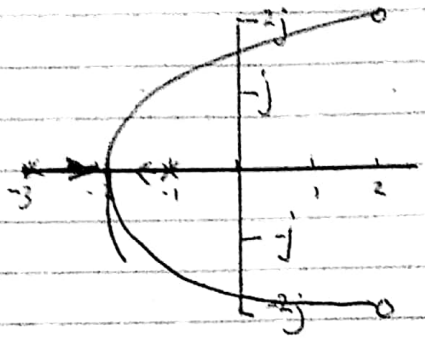
Exam 315 test 2 Joshua Benfell 30043322

1a $G(s) = \frac{s^2-4s+8}{s^2+4s+3}$

$= \frac{(s-2+2i)(s-2-2i)}{(s+3)(s+1)}$

$P-Z=0$ no asymtotes.



$G(s) = \frac{s+4}{s(s+6)(s+3)(s+1)}$

$P-Z = 4-1$

$= 3$

$\theta_a = \frac{(2k+1)\pi}{3}$

$k=-1 \quad \theta_a = \frac{-\pi}{3}$

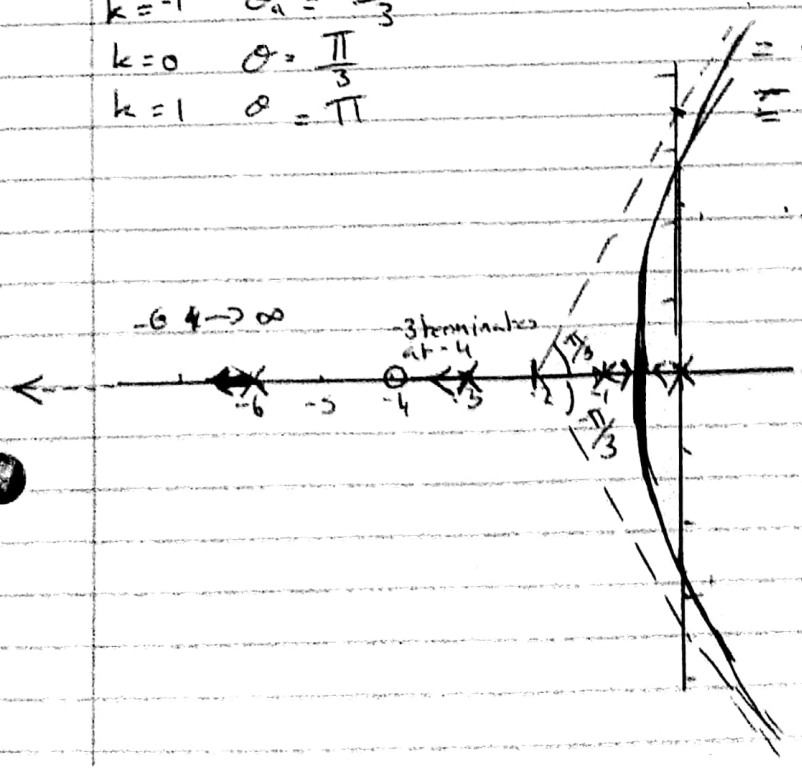
$k=0 \quad \theta = \frac{\pi}{3}$

$k=1 \quad \theta = \pi$

$\sigma_a = \frac{\overbrace{0-6-3-1}^{\text{Poles}} - \overbrace{-4}^{\text{Zeros}}}{3}$

$= \frac{-6}{3}$

$= -2.$



1bb $G(s) = \dfrac{s^2+10s+24}{s^2+3s+2} = \dfrac{(s+6)(s+4)}{(s+2)(s+1)}$

$k = \dfrac{-1}{G(s)}$

$= \dfrac{-1}{G(\sigma)}$

$\dfrac{dk}{d\sigma} = \dfrac{d}{d\sigma}\left(\dfrac{-\sigma^2-3\sigma-2}{\sigma^2+10\sigma+24}\right)$

$= \dfrac{(-2\sigma-3)(\sigma^2+10\sigma+24)+(\sigma^2+3\sigma+2)(2\sigma+10)}{(\sigma^2+10\sigma+24)^2}$

ignore denominator

$= -2\sigma^3-20\sigma^2-48\sigma-3\sigma^2-30\sigma-72+2\sigma^3+6\sigma^2+4\sigma+10\sigma^2+30\sigma+20$

$= (-2+6-3+10)\sigma^2+(-48-30+4+30)\sigma+(-72+20)$

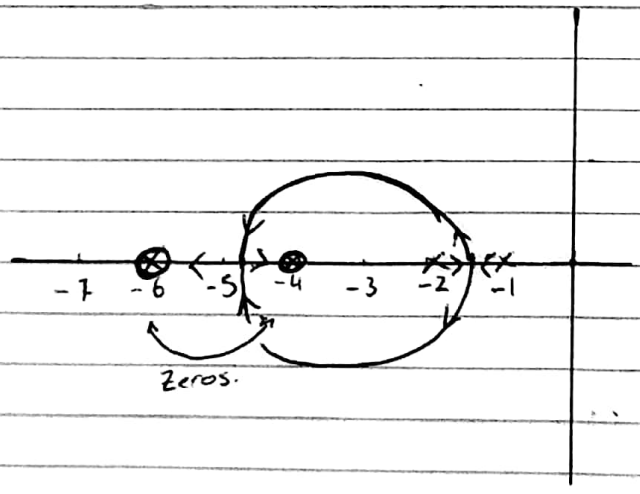
$= -7\sigma^2-44\sigma-52$

$= 0$

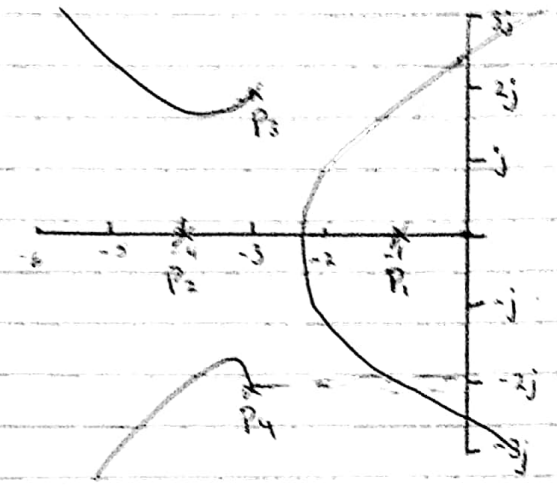
$7\sigma^2+44\sigma+52=0$

$(\sigma+1.577)(\sigma+4.707)=0$

$(\sigma+\frac{22+2\sqrt{30}}{7})(\sigma+\frac{22-2\sqrt{30}}{7})=0$



1c. $G(s) = \frac{1}{(s+4)(s+1)(s+3+2j)(s+3-2j)}$



try $j\omega = 2j$ 2.5j

~~arg(P4)~~

$\theta_{p1} = 4\arctan\frac{2.5}{1}$
$= 68.199°$

$\theta_{p2} = \arctan\left(\frac{2.5}{4}\right)$
$= 32°$

$\theta_{p3} = \arctan\left(\frac{0.5}{3}\right)$
$= 9.46$

$\theta_{p4} = \arctan\left(\frac{4.5}{3}\right)$
$= 56.31$

~~-4~~ $-\theta_{p1} - \theta_{p2} - \theta_{p3} - \theta_{p4} = -68.199 - 32 - 9.46 - 56.31$
$= -165.969$

too low so try $j\omega = 2.8j$

$\theta_{p1} = \arctan\frac{2.8}{1}$ $\theta_{p3} = \arctan\left(\frac{0.2}{3}\right)$
$= 70.346$ $= 14.934$

$\theta_{p2} = \arctan\left(\frac{2.8}{4}\right)$ $\theta_{p4} = \arctan\left(\frac{4.8}{3}\right)$
$= 34.99$ $= 57.995$

$-\theta_{p1} - \theta_{p2} - \theta_{p3} - \theta_{p4} = -178.26$ Bit too low try 2.85

$\theta_{p1} = \arctan\left(\frac{2.85}{1}\right)$ $\theta_{p3} = \arctan\left(\frac{0.85}{3}\right)$
$= 70.665$ $= 15.8192$

$\theta_{p2} = \arctan\left(\frac{2.85}{4}\right)$ $\theta_{p3} = \arctan\left(\frac{4.85}{3}\right)$
$= 70.665 - 34.354899$ $= 58.26086$

$-\theta_{p1} - \theta_{p2} - \theta_{p3} - \theta_{p4} = -180.235$

D imag intercept at ~ $j\omega = 2.85j$ and $j\omega = -2.85j$

1c cont

imag intercept $= \pm 2.85j$

Gain this occurs at.

$$|k| = \frac{1}{|G(s)|}$$

$$= |(s+4)| \times |s+1| \times |s+3+2j| \times |s+3-2j| \quad \Big| \; s = 2.85j$$

$$= \sqrt{2.85^2+4^2} \times \sqrt{2.85^2+1^2} \times \sqrt{4.85^2+3^2} \times \sqrt{0.15^2+3^2}$$

$$= 263.78$$

$20\log(k) = 48.42$ dB.

2a $G(s) = \frac{s+300}{s^2+s(a+10)s}$

$= \frac{s+300}{s(s+10)}$

2b Roots $(s+3)$ (1), $(s+8)$, $(s+100)$
all poles

$G(s) = k \frac{1}{(s+3)(s+8)(s+100)}$

$= \frac{k}{3\times8\times100}\left(\frac{1}{\left(\frac{s}{3}+1\right)\left(\frac{s}{8}+1\right)\left(\frac{s}{100}+1\right)}\right)$

$k = 24000$

$20\log\frac{k}{2400} = 20\,dB$

$\frac{k}{10} = 10$

$G(s) = \frac{24000}{(s+3)(s+8)(s+100)}$

2c System is type 1 as phase starts at -90°

SSE

Step input: 0

Ramp input: $1/k_v$ where $k_v = 316.23$ ∴ $SSE = \frac{1}{316.23} = 3.16 \times 10^{-3}$

~~$k_v = 251.84$ from graph ∴ $SSE = \frac{1}{251.84} =$~~

parabolic input: ∞

↳ See attached graph

2e $G(s) = \frac{20000}{s(s+100)(s+10)}$

ϕ_m happens at ~ 80 rads/s

$= \frac{20000}{1000 s\left(\frac{s}{100}+1\right)\left(\frac{s}{10}+1\right)}$

lead compensator $= \frac{s+15}{s+150}$