Root locus

1. $$\frac{s^2-3s+2}{s^2+7s+12} = \frac{(s-2)(s-1)}{(s+3)(s+4)}$$

Zeroes at $\sigma = 1, 2$
Poles at $\sigma = -3, -4$

$P - Z = 0$

Break away angles $= \frac{180}{Z}$

$= 90°$

Break in angles $= \frac{180}{P}$

$= 90°$

Break away/in points — Algebraic method

$$\sum_{i=1}^{Z} \frac{1}{\sigma_b - Z_i} = \sum_{j=1}^{P} \frac{1}{\sigma_b - P_j}$$

$$\frac{1}{\sigma_b+3} + \frac{1}{\sigma_b+4} = \frac{1}{\sigma_b-2} + \frac{1}{\sigma_b-1}$$

$$(\sigma_b + 2\sigma_b + 7)(\sigma_b-2)(\sigma_b-1) = (\sigma_b - 2\sigma_b - 3)(\sigma_b+3)(\sigma_b+4)$$

$$(2\sigma_b^2 \cancel{- 4\sigma_b} + 7\sigma_b - 14)(\sigma_b-1) = (2\sigma_b^2 \cancel{+ 6\sigma_b} + 3\sigma_b - 9)(\sigma_b+4)$$

$$\cancel{2\sigma_b^3} + 3\sigma_b^2 - 14\sigma_b \cancel{- 20\sigma_b^2} \cancel{- 3\sigma_b} + 14 = \cancel{2\sigma_b^3} + 3\sigma_b^2 \cancel{- 9\sigma_b} + 8\sigma_b^2 + \cancel{12\sigma_b} - 36$$

$$\sigma_b^2 - 17\sigma_b + 14 = 11\sigma_b^2 + 3\sigma_b \cancel{- 36} - 50$$

$$0 = 10\sigma_b^2 + 20\sigma_b - 50$$

$$= \sigma_b^2 + 2\sigma_b - 5$$

$$\sigma_b = -1 \pm \sqrt{6}$$

$$= -3.45 \quad \text{or} \quad 1.45$$

Break away/in points   Derivative way

$1 + kG(s) = 0$

$$k = \frac{-s^2 - 7s - 12}{s^2 - 3s + 2}$$

$$\frac{dk}{ds} = \frac{(-2s-7)(s^2-3s+2) \mp (4s^2+7s+12)(2s-3)}{(s^2-3s+2)^2}$$

We only care about the numerator here and it's the same Quadratic in the algebraic method so I'm going to skip to the answer

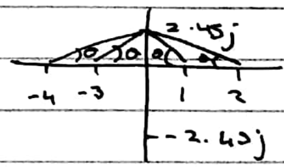
$\sigma = -3.45$ or 1.45

Depending on the function, the derivative method can be more complicated. I personally find it easier as it's got the 'products' semi expanded.

Imaginary crossing Points.

Pick point $\frac{1.45 + 3.45}{2}$ ~~arbitrary~~ average of break in/away vals

→ $2.45j$



$\arctan(-1/2.45) + \arctan(-3/2.45)$ ← Zeros

$-\arctan(4/2.45) - \arctan(3/2.45)$ Poles.

$= -170.76$

go lower to 2.2. y int $=$ ~~4.84~~ 181.65

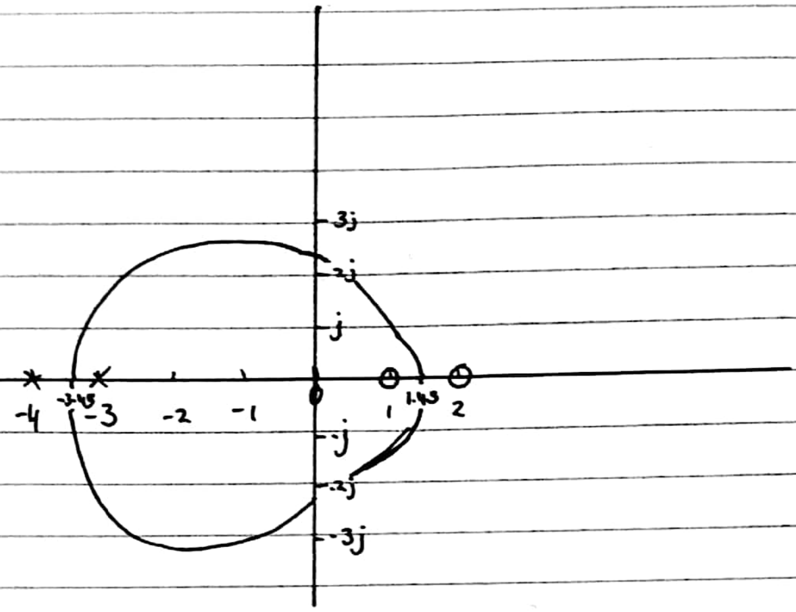
sub above 2.45 for 2.2.

go up to 2.25 y int $=$ ~~-179~~ -179.368

go down to 2.225 y int $= -180.5$.

so y intercept is ~ 2.225.

(1)



Root locus (2)

$$Sys = \frac{s+3}{0.9s^3+3s^2+5s+2}$$

$$= \frac{s+3}{(s+0.55309)(s+1.39+1.44j)(s+1.39-1.44j)}$$

$p=3$
$z=1$
$p-z=2.$

$$\theta_a = \frac{(2k+1)\pi}{2}$$

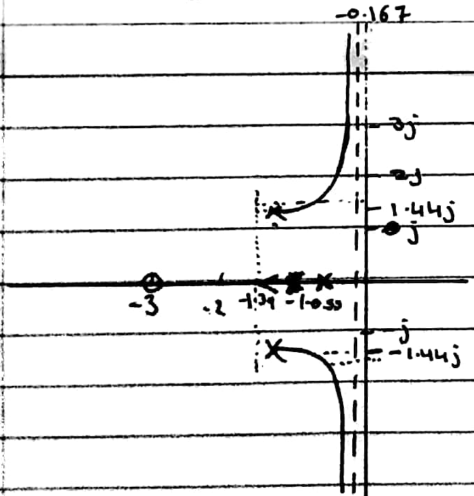
$\theta_{a_1} = \frac{\pi}{2} \quad k=0$

$\theta_{a_2} = \frac{-\pi}{2} \quad k=-1$

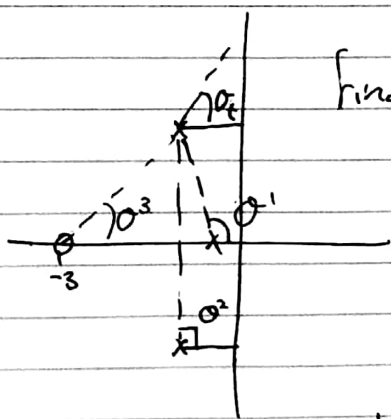
$$\sigma_a = \cancel{0.5}\ \frac{-0.55309-1.39\times2+3}{2}$$

$= -0.16654s.$

no imaginary intercept.



Final Plot (skipped a step)



Find θ_t

complex
Departure angles

$-180 = -\theta_1 - \theta_2 - \theta_t + \theta_3$

$\theta_t = 180 - \theta_1 - \theta_2 + \theta_3$

$= 180 - 120.167 - 90 + 41.8097$

$= 11.64247^\circ$

$= 11.6^\circ$ 1dp.

$\theta_1 = 180 - \arctan\left(\frac{1.44}{1.39-0.553}\right)$

$= \cancel{44} \; 120.167$

$\theta_2 = 90^\circ$

$\theta_3 = \arctan\left(\frac{1.44}{3-1.39}\right)$

$= 41.8097$

Bode Plots 1

~~at $\omega = 0$~~

eg 3 $\frac{s+30}{s^2+2s} = \frac{30}{2} \frac{\left(\frac{s}{30}+1\right)}{s\left(\frac{s}{2}+1\right)}$

$(s+30)$ gain $= 20 \log 30$

$= 29.54$ dB

s^{-1} gain $= 0$

$(s+2)^{-1}$ gain $= -20 \log 2$

$= -6.02$ dB.

Stability

total additional gain $= 20 \log 15$

$= 23.5$ dB

~~Phase ϕ_m $\to \infty$ = never crosses -180° phase~~

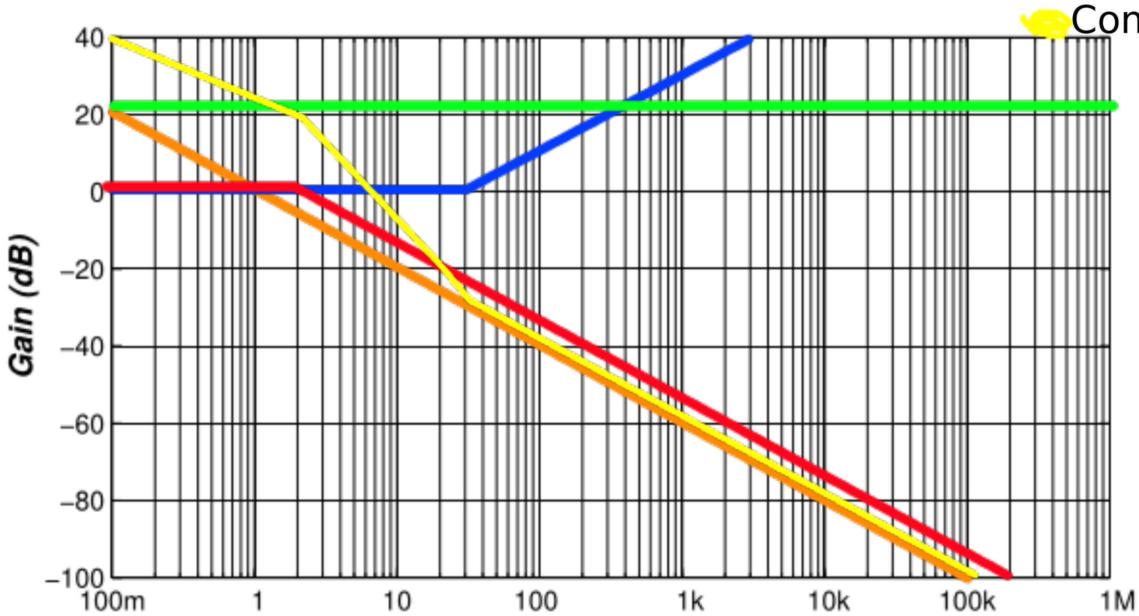
$\phi_m \approx 60°$, $\therefore$ is stable.

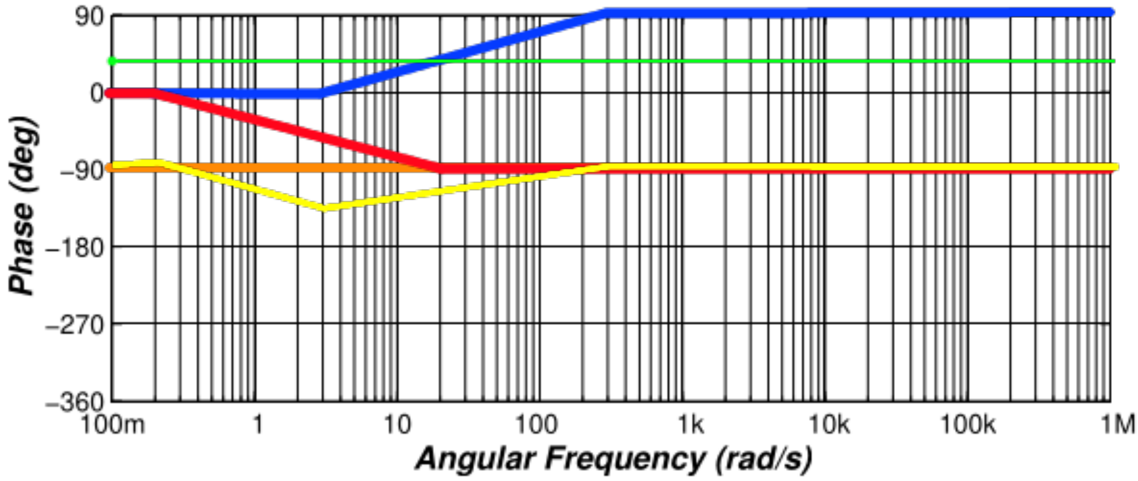
reading off plot

No Gain Margin

Gain = 23.5dB
1/s
1/(s+2)
(s+30)
Combined

Name: Student Number:





eq 4

$$\frac{106}{s^3+16s^2+64s} = \frac{106}{s(s^2+16s+64)}$$

$$= \frac{106}{s(s+8)^2}$$

$$= \frac{106}{64}\,\frac{1}{s(\frac{s}{8}+1)^2}$$

106 gain $= 20\log 106$

$= 40.5$ dB 0dB rolloff

s^{-1} gain $= 0$ dB 20dB rolloff

$(s+8)^{-2}$ gain $= -40\log 8$ 40dB rolloff

$= -36.12$ dB

total additional gain $= 20\log\frac{106}{64}$

$= 4.38$ dB.

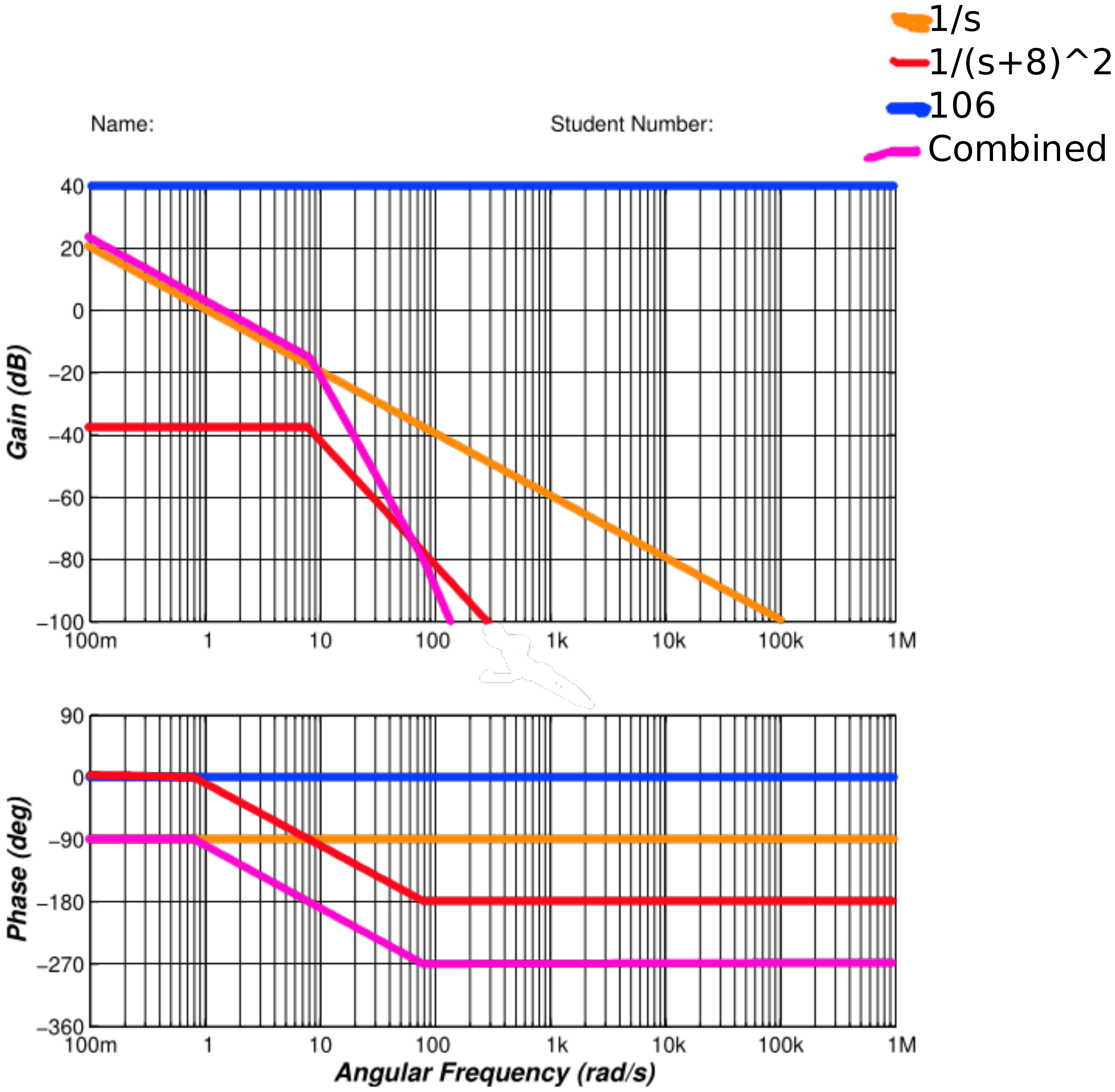
Stability

$\phi_m \simeq 70°$ $\therefore$ is stable

reading off plot

Gain Margin of -13dB

Name: Student Number:



2 $\angle F \approx \cancel{25} \frac{A}{(s+100)^2}$

~~$\approx 17.75 \times \frac{1}{s+100}$~~

ϕ_m as measured at $\omega = 400$ rad/s

$\phi_m = 30°$

~~We want a lag controller as an integrator removes~~ SSE

PI

To remove steady state error need an I or lag component

Using PI $C(s) = \cancel{C_0 \frac{s+\omega_b}{\omega_b}}$

$= C_0 \times \omega_b \left(\frac{s}{\omega_b} + 1\right) \times \frac{1}{s}$ Cowb want zero close to the integrator so low wb

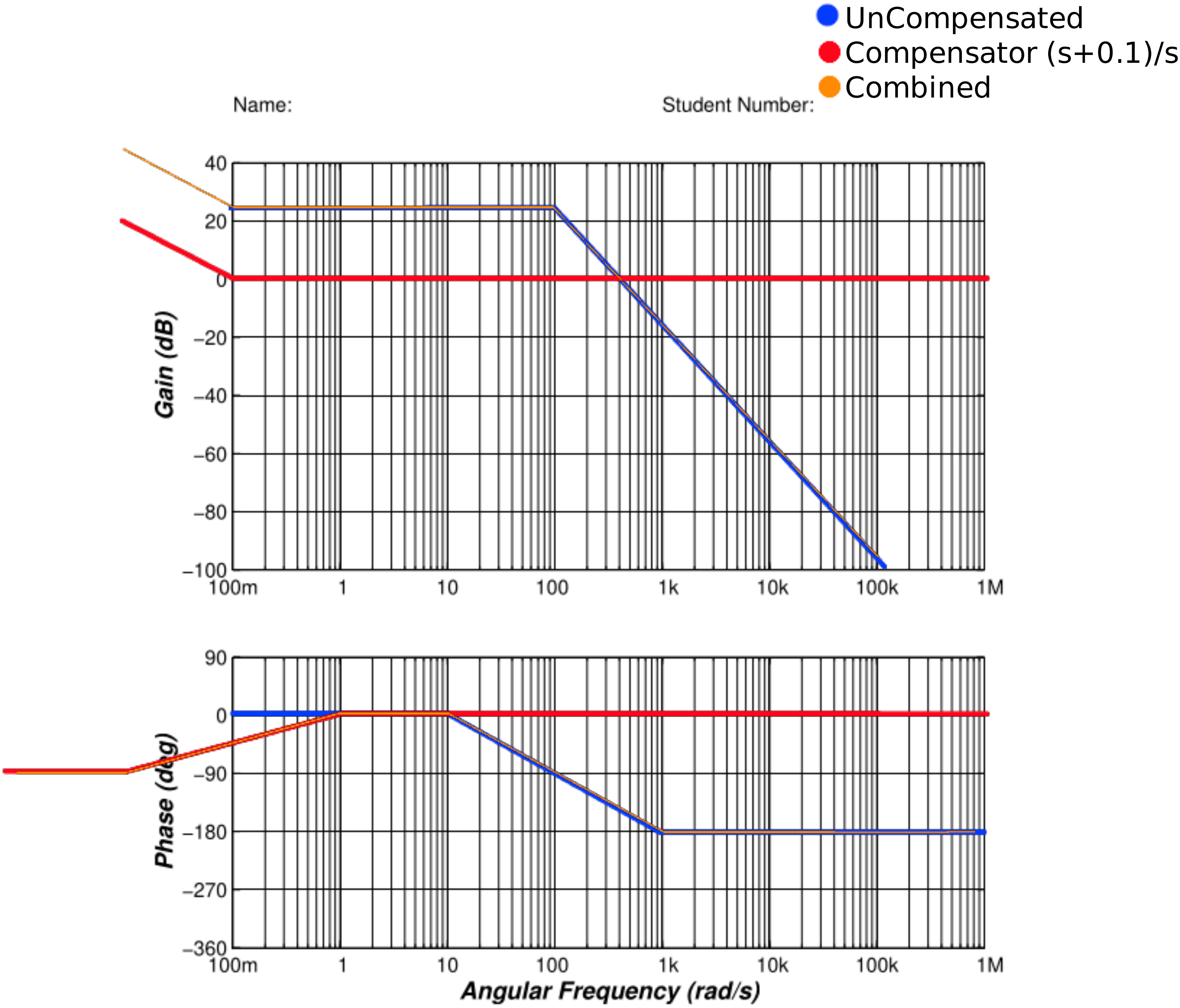
~~we want $C_0 \times \omega_b = \frac{4}{\omega_b}$ $\therefore C_0 = \frac{1}{\omega_b}$~~

~~we so try $\omega_b = 0.1$ $C_0 = 10$~~

we want $C_0 \times \omega_b = \omega_b$ $\therefore C_0 = 1$

So our compensator will be $C(s) = \frac{s+0.1}{s}$

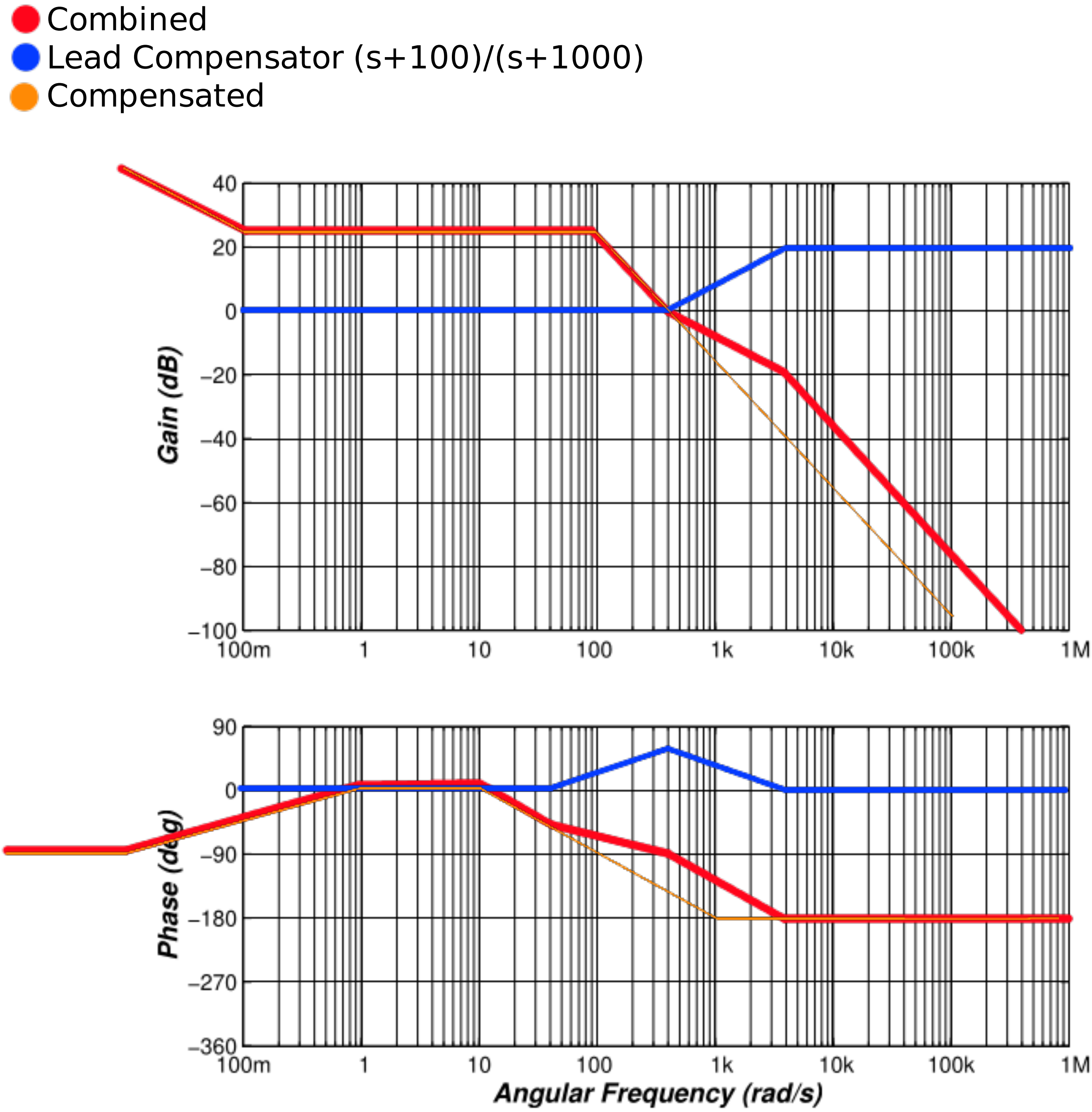
$= 0.1 \frac{\left(\frac{s}{0.1} + 1\right)}{s}$


UnCompensated
Compensator (s+0.1)/s
Combined
Name:
Student Number:
Gain (dB)
40
20
0
−20
−40
−60
−80
−100
100m
1
10
100
1k
10k
100k
1M
Phase (deg)
90
0
−90
−180
−270
−360
Angular Frequency (rad/s)

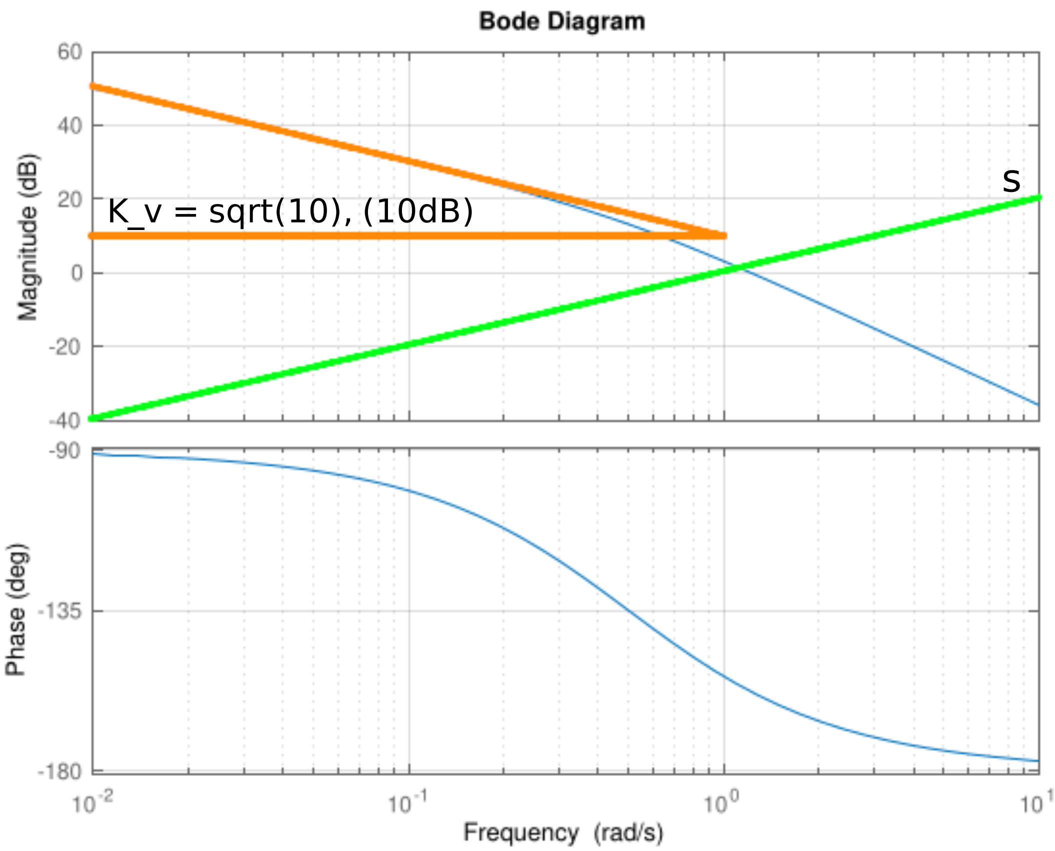
you can use the lead controller to make a hump in the phase that ranges between 0-90° which can be placed around your phase margin to raise the phase at that point and increase the phase margin. It also raises the response after the selected frequency ~~increasing~~ decreasing drop off for a bit ~~and~~, & raising the gain at some frequencies, this can also move the phase margin ~~freq~~ location by changing the unity crossing point.

we will add a zero at $\omega = 400$ and a pole at $\omega = 4000$ so our lead compensator looks like $\frac{s+100}{s+1000} = C(s)$
we want this with unity gain overall so it ~~does~~ doesn't effect existing response / move the phase margin $C(s) = k \times \frac{\frac{s}{400}+1}{\frac{s}{4000}+1} \times \frac{400}{4000}$

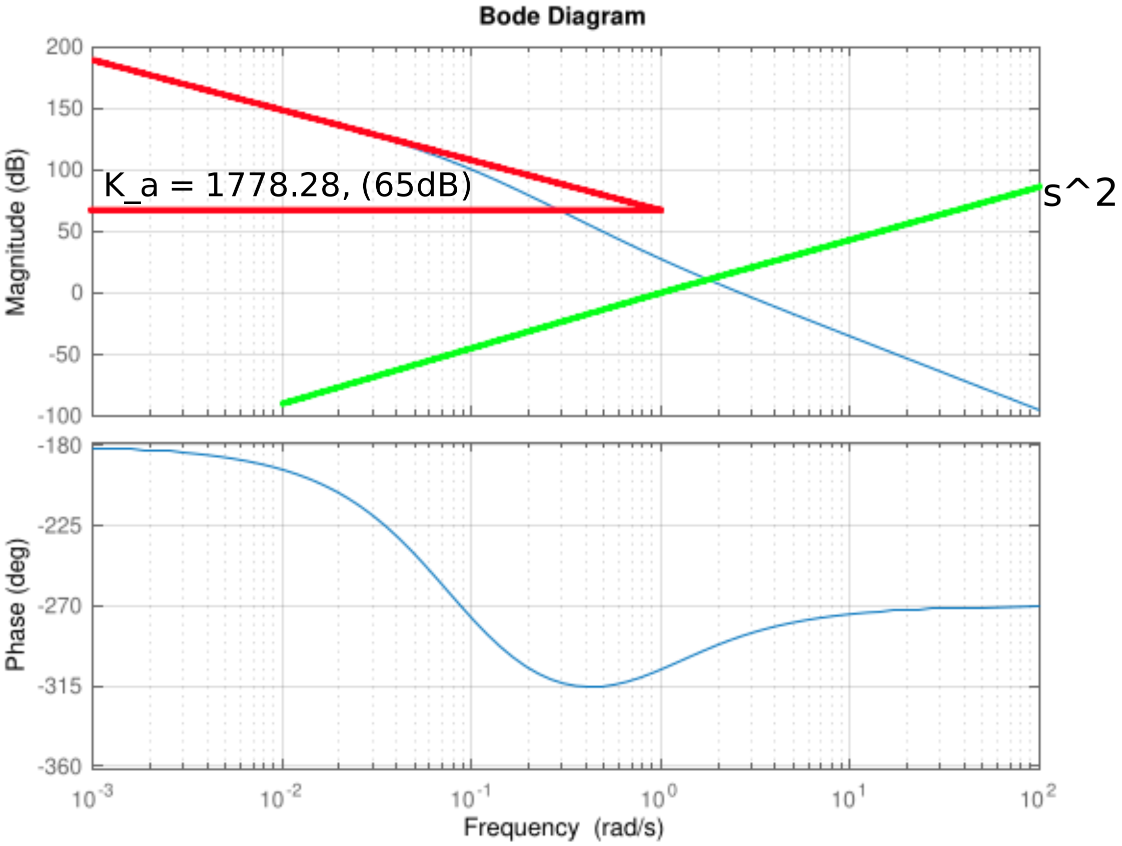
So $k = 10$ $\therefore$ $C(s) = \frac{\frac{s}{400}+1}{\frac{s}{4000}+1} = 10\,\frac{s+400}{s+4000}$ ϕ_m now $\sim 90°$


Combined
Lead Compensator (s+100)/(s+1000)
Compensated
Gain (dB)
40
20
0
−20
−40
−60
−80
−100
100m
1
10
100
1k
10k
100k
1M
Phase (deg)
90
0
−90
−180
−270
−360
Angular Frequency (rad/s)

Type: 1
Steady State Error
Step Input: 0 Given it's a type 1 system.
Ramp Input: 1/K_v = 1/sqrt(10) = 0.316
Parabolic Input: Infinite



Type: 2
Steady State Error
Step Input: 0
Ramp Input: 0
Parabolic Input: 1/K_a = 1/1778.28 = 5.6 * 10 ^ -4



Type: 0
Steady State Error
Step Input: 1/(1 + K_p) = 1/(1+5.012) = 0.16634
Ramp Input: Infinite
Parabolic Input: Infinite