



VICTORIA UNIVERSITY OF  
**WELLINGTON**  
TE HERENGA WAKA

**CLASS TEST 1 – 2020**

**TRIMESTER 1**

**ECEN 321**

**ENGINEERING STATISTICS**

**Time Allowed:** THIRTY FOUR HOURS

**OPEN BOOK**

**Permitted materials:** Calculators permitted.

**Instructions:** Attempt ALL Questions.  
There are 40 marks in total.

## 1. Sample statistics (8 marks)

Measurements of signal strength  $y$  versus distance  $x$  for a given wireless channel were made, giving the results below:

$x$ (km)	8	5	9	8	4	8
$y$ (dB)	18	22	16	9	25	12

(a) Calculate the sample means  $\bar{x}$  and  $\bar{y}$ . (2 marks)

(b) Calculate the sample standard deviations  $s_x$  and  $s_y$  (4 marks)

(c) Calculate the correlation coefficient between  $x$  and  $y$ . (2 marks)

## 2. Conditional Probability

**(10 marks)**

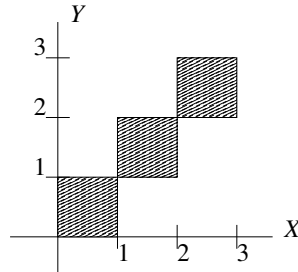
Items are inspected for flaws by two quality inspectors. If a flaw is present, it will be detected by the first inspector with probability 0.9, and by the second inspector with probability 0.7. Assume the inspectors function independently.

- (a) If an item has a flaw, what is the probability that it is found by both inspectors? **(1 mark)**
  
- (b) If an item has a flaw, what is the probability that it is found by at least one inspector? **(1 mark)**
  
- (c) If the second inspector were to only examine those items that have been passed by the first inspector, what is the probability, for an item that has a flaw, that the second inspector will find it? **(1 mark)**
  
- (d) If the probability that an item has a flaw is 0.1, and an item is passed by the first inspector, what is the probability that it actually has a flaw? **(3 marks)**
  
- (e) If the probability that an item has a flaw is 0.1, and an item is passed by the both inspectors, what is the probability that it actually has a flaw? **(4 marks)**

## 3. Joint Probability Density

(12 marks)

Consider the density  $f_{X,Y}(x,y)$  shown below. The shaded area has a constant density,  $c$ . Elsewhere the density is zero. **As always explain all answers: only providing answers results in no marks.**



- (a) Compute the value  $c$  of the density, where the density is not zero. (1 mark)
- (b) Determine the density  $f_X(x)$  (a careful plot is ok). (1 mark)
- (c) Determine the function  $f_{X,Y}(\frac{1}{2}, y)$  (a careful plot is ok). (1 mark)
- (d) Determine the density  $f_{Y|X}(y|\frac{1}{2})$  (a careful plot is ok). (2 marks)
- (e) Determine  $E\{X\}$  (2 marks)
- (f) Determine the expectation  $E\{XY\}$ . (3 marks)
- (g) Compute the covariance between  $X$  and  $Y$ . (2 marks)

## 4. Joint Probability Density

(10 marks)

There are two fuses in an electrical device. Let  $X$  denote the lifetime of the first use, and let  $Y$  denote the lifetime of the second fuse (both in years). Assume that the joint probability density function of  $X$  and  $Y$  is

$$f(x, y) = \begin{cases} \frac{1}{6}e^{-x/2-y/3} & x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find  $P(X \leq 2 \text{ and } Y \leq 3)$ .

(2 marks)

(b) Find the probability that both fuses last at least 3 years.

(2 marks)

(c) Find the marginal probability density function of  $X$

(2 marks)

(d) Find the marginal probability density function of  $Y$

(2 marks)

(e) Are  $X$  and  $Y$  independent? Explain.

(2 marks)

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