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1a $n=10$

$\sigma^2 = 0.01$

$$\bar{x} = \frac{12.18 + 11.77 + 12.09 + 12.03 + 11.87 + 11.96 + 12.03 + 12.36 + 12.28 + 11.85}{10}$$

$= 12.042$

~~$s = 0.1905$~~

H_0: ~~$\sigma^2 \neq 0.01$~~ $\sigma^2 = 0.01$

H_a: $\sigma^2 \neq 0.01$

$$\chi^2 = \frac{n-1}{\sigma_0^2} s^2$$

$$= \frac{10-1}{0.01} 0.1905^2$$

$= 32.661$

$df = 9$

$P < 2 \times 0.005 = 0.01$

P val less than 0.02 and 0.05 so reject null hypothesis.

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2

	$\bar{X}$	s
A	12.938	1.1168
B	10.890	1.2395
C	11.721	2.3433
D	10.473	1.3996
E	11.524	1.0892

$H_0: \mu \geq 12$

$H_a: \mu < 12$

$\mu = 12$

$n = 10$

a

	t
pA	2.655
pB	-2.832
pC	-0.377
pD	-3.450
pE	-1.382

$t = \frac{x - \mu}{\frac{s}{\sqrt{n}}}$

$\frac{12.938 - 12}{\frac{1.1168}{\sqrt{10}}}$

$\frac{10.890 - 12}{\frac{1.2395}{\sqrt{10}}}$

$\frac{11.721 - 12}{\frac{2.3433}{\sqrt{10}}}$

$\frac{10.473 - 12}{\frac{1.3996}{\sqrt{10}}}$

$\frac{11.524 - 12}{\frac{1.0892}{\sqrt{10}}}$

$P_a > 0.40 = 0.987$

$0.005 < P_B < 0.01 = 0.00983$

$0.25 < P_c < 0.40 = 0.35745$

$0.001 < P_D < 0.005 = 0.00364$

$0.10 < P_e < 0.25 = 0.10015$

b. $P_{ra} = n P_a = 5 \times 0.987$ ~~Raso~~ Bonferroni correction

$= 4.93$ reject

$P_{rb} = n P_b = 5 \times 0.00983$

$= 0.04915$ pass < 0.05

$P_{rc} = n P_c = 5 \times 0.35745$

$= 1.78725$ reject

$P_{rd} = n P_d = 5 \times 0.00364$

$= 0.0182$ pass $<$ ~~reject~~ 0.05

$P_{re} = n P_e = 5 \times 0.10015$

$= 0.50075$ reject

~~at least one provides an improvement~~

iii Some provide improvement but it's inconclusive

3 $n = 27$ $\bar{x} = 0.8404$ $\bar{y} = 0.8267$

$$a = \sum (x_i - \bar{x})^2 = 2.9679$$
$$b = \sum (y_i - \bar{y})^2 = 2.9446$$
$$c = \sum (y_i - \bar{y})(x_i - \bar{x}) = 2.8972$$

a $$\hat{\beta}_1 = \frac{c}{a} = \frac{2.8972}{2.9679}$$

$$= 0.9762$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}\bar{x} = 0.8267 - 0.9762 \times 0.8404$$
$$= 0.0063$$

b $$r = \frac{c}{\sqrt{a}\sqrt{b}}$$

$$= \frac{2.8972}{\sqrt{2.9679}\sqrt{2.9446}}$$

$$= 0.9800$$

$H_0: \beta_0 = 0$
$H_a: \beta_0 \neq 0$

$$s^2 = \frac{\sum (y_i - \hat{y})^2}{n-2}$$

$$= \frac{(1-r^2)\,b}{n-2}$$

$$= \frac{(1 - 0.98^2) \times 2.9446}{25}$$

$$= 0.00466$$

$$t = \frac{\hat{\beta}_0 - \beta_0}{s\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{a}}}$$

$$= \frac{0.0063 - 0}{\sqrt{0.00466}\sqrt{\frac{1}{27} + \frac{0.8404^2}{2.96789}}}$$

$$= 0.176$$

$P > 2 \times 0.40 = 0.80$

$P > 0.05 \Rightarrow$ fail to reject

3 c $H_0: \beta_1 = 1$
$H_1: \beta_1 \neq 1$

$$t = \frac{0.9762 - 1}{\sqrt{0.00466}/\sqrt{2.9679}}$$

$= -0.601$

$df = 25$

$p < 2 \times 0.4 = 0.8$

$p > 0.05$ fail to reject H_0

d Yes. method is accurate when $\beta_0 = 0$ and $\beta_1 = 1$
this is not the case so is inaccurate

e $x_0 = 0.8$ $ci = 0.95$

$\hat{y} = \hat{\beta} + \hat{\beta}_1 x$
$= 0.063 + 0.9762(0.8)$
$= 0.7873$

$t_{\alpha/2} = 2.060$

$$E = t_{\alpha/2}\sqrt{s^2\left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{a}\right)}$$

$$= 2.06\sqrt{0.00466\left(1/27 + \frac{(0.8 - 0.8404)^2}{2.9676}\right)}$$

$= 0.0273$

$(0.769, 0.8146)$

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3f CI doesn't contain 0.75, so it is unlikely and there is sufficient evidence to say this claim is false

4

$n = 23$ $\bar{x} = 28.0783$ $\bar{y} = 29.2217$

$a = \sum (x_i - \bar{x})^2$ $c = \sum (x_i - \bar{x})(y_i - \bar{y})$

$= 287.9991$ $= -347.2191$

$b = \sum (y_i - \bar{y})^2$

$= 1692.1791$

a $\hat{\beta}_1 = \frac{c}{a}$ $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$= \frac{-347.2191}{287.9991}$ $= 29.2217 + (1.2056)(28.0783)$

$= -1.2056$ $= 63.0736$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

$= 63.0736 - 1.2056x$

b $r = \frac{c}{\sqrt{b}\sqrt{a}}$ $s^2 = \frac{\sum (y_i - \bar{y})^2}{n-2}$ $df = 21$

$= \frac{-347.2191}{\sqrt{287.9991}\sqrt{1692.1791}}$ $= \frac{(1 - r^2)(b)}{n-2}$

$= -0.4474$ $= 66.6439$

$t = 2.080$

$E_{\beta_0} = 2.080 \times s \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{a}}$ $E_{\beta_1} = t \times \frac{s}{\sqrt{a}}$

$= 27.0118$ $= 0.9545$

$\beta_0 \Rightarrow (36.0618, 90.0854)$ $\beta_1 \Rightarrow (-2.1601, -0.2511$

c $2\hat{\beta}_1 = 2 \times (-1.2056) = -2.4112$

differs by 2.4112 days.

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d Claim slope is negative

$H_0: \beta_1 \geq 0$

$H_1: \beta_1 < 0$

$$t = \frac{\hat{\beta}_1 - \beta_1}{\frac{s}{\sqrt{a}}} = \frac{-1.2056 - 0}{\frac{\sqrt{60.6439}}{\sqrt{287.9991}}}$$

$= -2.6273$

$0.005 < p < 0.01$

less than 0.05 so reject Ho, slope is negative

e $x_0 = 30$ $CI = 0.95$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_0$

$= 63.0736 - 1.2056(30)$

$= 26.9048$

$H_0: \mu \geq 28$

$H_a: \mu < 28$

$$t = \frac{\hat{y} - \mu_0}{\sqrt{s^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{a}\right)}} = \frac{26.9048 - 28}{\sqrt{60.6439\left(\frac{1}{23} + \frac{(30 - 28.0783)^2}{287.9991}\right)}}$$

$= -0.593$

$0.25 < p < 0.40$

so fail to reject Ho as this isn't less than 0.05

4 f $\quad x_0 = 28.5 \qquad CI = 0.95$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 63.0736 - 1.2036(28.5) = 28.7133$

$t_{\alpha/2} = 2.08$

$E = t_{\alpha/2} \sqrt{s^2 \left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{a}\right)}$

$= 2.08 \sqrt{60.6439 \left(1 + \frac{1}{23} + \frac{(30 - 28.0783)^2}{287.991}\right)}$

$= 16.6476$

$(12.0657, 45.3609)$