Assignment 3 ECEN321 Joshua Benfell

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		y			
		0	1	2	3
x	0	0.15	0.12	0.11	0.1
	1	0.09	0.07	0.05	0.04
	2	0.06	0.05	0.04	0.02
	3	0.04	0.03	0.02	0.01

a $p_X(x) = P(X = x) = \sum_y p(x,y)$

	$p_X(x)$	
0	0.12 + 0.15 + 0.11 + 0.1	= 0.48
1	0.09 + 0.07 + 0.05 + 0.04	= 0.25
2	0.06 + 0.05 + 0.04 + 0.02	= 0.17
3	0.04 + 0.03 + 0.02 + 0.01	= 0.10

b $p_Y(y) = P(Y=y) = \sum_x p(x,y)$

	$P_Y(y)$	
0	0.15 + 0.09 + 0.06 + 0.04	= 0.34
1	0.12 + 0.07 + 0.05 + 0.03	= 0.27
2	0.11 + 0.05 + 0.04 + 0.02	= 0.22
3	0.1 + 0.04 + 0.02 + 0.01	= 0.17

c No

$P_{X,Y}(\overset{0}{X}, \overset{0}{Y}) = 0.15 \neq P_X(X=0)\,P_Y(Y=0) = 0.48 \times 0.34$

$= 0.1632.$

d $\mu_X = E\{X\} = 0 \times p_X(0) + 1 \times p_X(1) + 2 \times p_X(2) + 3 \times p_X(3)$

$= 0 + 0.25 + 2 \times 0.17 + 0.1 \times 3$

$= 0.89$

$\mu_Y = E\{Y\} = 0 \times p_Y(0) + 1 \times p_Y(1) + 2 \times p_Y(2) + 3 \times p_Y(3)$

$= 0 + 1 \times 0.27 + 2 \times 0.22 + 3 \times 0.17$

$= 1.22.$

1e $\sigma_x = \sqrt{\sigma_x^2}$

$$\sigma_x^2 = \sum_x x^2 P_x(X=x) - \mu_x^2$$

$$\sigma_x = \sqrt{(0^2 \times 0.48 + 1^2 \times 0.25 + 2^2 \times 0.17 + 3^2 \times 0.10) - 0.89^2}$$

$= 1.01877$

$$\sigma_y = \sqrt{(0^2 \times 0.34 + 1^2 \times 0.27 + 2^2 \times 0.22 + 3^2 \times 0.17) - 1.22^2}$$

$= 1.0916$

f $\text{Cov}(X,Y) = E\{(X-\mu_X)(Y-\mu_Y)\}$

$$= \sum_x \sum_y (x-\mu_x)(y-\mu_y)\, p_{X,Y}(x,y)$$

$$= \mu_{XY} - \mu_X \mu_Y$$

$$= (0\times0\times0.13 + 0\times1\times0.12 + 0\times2\times0.11 + 0\times3\times0.01 + 1\times0\times0.09 + 1\times1\times0.07 + 1\times2\times0.05 + 1\times3\times0.04 + 2\times0\times0.06 + 2\times1\times0.05 + 2\times2\times0.04 + 2\times3\times0.02 + 3\times0\times0.04 + 3\times1\times0.03 + 3\times2\times0.02 + 3\times3\times0.01) - (0.89)(1.22)$$

$= -0.1158$

g $\rho(X,Y) = \dfrac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$

$$= \frac{-0.1158}{(1.0188)(1.0916)} = -0.1041$$

2.

$$f_{XY}(x,y) = \begin{cases} xe^{-(x+xy)} & x>0 \text{ and } y>0 \\ 0 & \text{otherwise} \end{cases}$$

a $$P(X>1 \text{ and } Y>1) = \int_1^\infty \int_1^\infty xe^{-(x+xy)}\, dy\, dx$$

$$\int_1^\infty xe^{-(x+xy)}\, dy = \int_1^\infty xe^{-x}e^{-xy}\, dy = -e^{-(x+xy)}\Big|_1^\infty$$

$$= \underbrace{-e^{-(x+\infty)}}_{\to 0} + e^{-(x+x)}$$

$$= e^{-(x+x)} = e^{-2x}$$

$$P(X>1 \wedge Y>1) = \int_1^\infty e^{-2x}\, dx$$

$$= -\frac{1}{2}e^{-2x}\Big|_1^\infty$$

$$= \frac{-1}{2}\left(e^{-2\times\infty} - e^{-2}\right)$$

$$= \frac{e^{-2}}{2}$$

$$\int u'v = uv - \int uv'$$

c $$f_{xy}(y) = \int_0^\infty xe^{-(x+xy)}\, dx$$

let $v = x$ $\quad u' = e^{-(x+xy)} = e^{-x(1+y)}$

$\frac{dv}{dx} = 1$ $\quad \int u'dx = \frac{-e^{-x(1+y)}}{y+1}$

$$= -x \cdot \frac{xe^{-(x+xy)}}{y+1} + \int \frac{e^{-(x+xy)}}{y+1}\, dx$$

$$= \frac{-xe^{-(x+xy)}}{y+1} - \frac{e^{-(x+xy)}}{(y+1)^2}\Big|_0^\infty$$

h $P_{Y|X}(y|1) = \frac{P_{YX}(y,1)}{P(X=1)} = \frac{P_{YX}(0,1)}{0.25}, \frac{P_{YX}(1,1)}{0.25}, \frac{P_{YX}(2,1)}{0.25}, \frac{P_{YX}(3,1)}{0.25}$

$= \frac{0.09}{0.25}, \frac{0.07}{0.25}, \frac{0.05}{0.25}, \frac{0.04}{0.25}$

	y			
	0	1	2	3
$P_{Y|X}(y,1)$	0.36	0.28	0.2	0.16

i ~~$P_{Y|X}(y|2)$~~ $P_{X|Y}(x|2)$

~~$P_X(x=2) = 0.17$~~

$P_Y(y=2) = 0.22$

	y			
	0	1	2	3
	$\cancel{\frac{0.09}{0.17}}$	$\cancel{\frac{0.03}{0.17}}$	$\cancel{\frac{0.04}{0.17}}$	$\cancel{\frac{0.02}{0.17}}$

~~0.11~~

x			
0	$\frac{0.11}{0.22}$	=	$\frac{1}{2}$
1	$\frac{0.05}{0.22}$	=	$\frac{5}{22}$
2	$\frac{0.04}{0.22}$	=	$\frac{4}{22}$
3	$\frac{0.02}{0.22}$	=	$\frac{1}{11}$

j $E\{Y|X=1\} = 0 \times 0.36 + 1 \times 0.28 + 2 \times 0.2 + 3 \times 0.16$

$= 1.16$

k $E\{X|Y=2\} = 0 \times \frac{1}{2} + 1 \times \frac{5}{22} + 2 \times \frac{4}{22} + 3 \times \frac{1}{11}$

$= \frac{19}{22}$

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