Ecen 321
Assignment 5 Joshua Benfell

1 a red 20% $X=1$ } $Z=1$
white 45% $Y=1$
Blue 35% ~~Z=0~~

a $p_X(X=1) = 0.2$

b $p_Y(Y=1) = 0.45$

c $p_Z(Z=1) = 0.45 + 0.2$
$= 0.65$

d ~~yes~~ ~~yes if a single transaction is what is considered.~~ ~~(Buy 2 sets)~~
~~no if you are tracking individual purchases of the set.~~ ~~[Buy 1 set then Buy another]~~
no as each purchase of a Dinner set is separate so they can't buy two at a time.

e yes

f ~~[illegible]~~

yes because if someone buys a red set X is 1, Y is 0, white set Y is 1, X is 0. Both cases $X+Y=1$ which is also Z for these conditions if a Blue set is bought $X=0$, $Y=0$ $X+Y=0=Z$ as Z also is 0.

2a $X=1$ if doubles (1,1) (2,2) (3,3) (4,4) (5,5) (6,6)
$X=0$ otherwise
$Y=1$ sum is 6 (1,5) (5,1) (2,4) (4,2) (3,3)
$Y=0$ otherwise
$Z=1$ Both X and Y happen. (3,3)
$Z=0$ otherwise

a ~~$\frac{6}{36}$~~ $= 1/6$ $P_X(X=1) = \frac{1}{6}$ $\frac{6}{36}$ -> 36 combinations of the two dice

b $P_Y(Y=1) = \frac{5}{36}$

c $P_Z(Z=1) = 1/36$

d

$P(X \wedge Y) = 1/36$
$P(X)P(Y) = 1/6 \times 5/36$
$= 5/216 \neq 1/36$ ∴ not independent

e ~~as shown~~
$P(Z) = 1/36$ $P(X)P(Y) = 5/216$
$\neq P(Z)$

no

f $Z = XY$ yes.
Because X and Y will only both be successes (1) on the event (3,3). Any other event has one as a failure (0) which is also when Z fails.

$$\frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

3 75% on time $P = 0.75$

Sample of 10 flights. $N = 10$

$x = 10$

a $P(X = 10) = \frac{10!}{10!(10-10)!} 0.75^{10} (1-0.75)^{10-10}$

$= \frac{1}{0!} 0.75^{10} (0.25)^{0}$

$= 1 \times 1 \times 0.75^{10}$

$= 0.05631$

$x = 8$

b $P(X = 8) = \frac{10!}{8!(10-8)!} \times 0.75^{8} (0.25)^{10-8}$

$= \frac{10 \times 9}{2!} \times 0.75^{8} \times 0.25^{2}$

$= 0.281567$

c $P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$

$P(X = 9) = \frac{10!}{9!(10-9)!} \times 0.75^{9} \times 0.25^{1}$

$= \frac{10}{1} \times 0.75^{9} \times 0.25$

$= 0.1877$

$P(X \geq 8) = 0.281567 + 0.1877 + 0.05631$

$= 0.525577$

4 $N = 100$ samples 92 passed

~~$\hat{p} = 0.92$~~

a $\hat{p} = \frac{92}{100}$

$= 0.92.$

$\cancel{\sigma_x = \sqrt{np(1-p)}}$ $\sigma_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{N}}$

$\cancel{= \sqrt{100 \times 0.92 \times 0.08}}$

$\cancel{=}$

$\sigma_p = \sqrt{\frac{0.92(1-0.92)}{100}}$

$= 0.0271$

0.92 ± 0.0271

b assuming $\hat{p} = 0.92$ still

$\sigma_p = 0.01$

$0.01 = \sqrt{\frac{0.92 \times 0.08}{N}}$

$\frac{0.01^2}{0.92 \times 0.08} = \frac{1}{N}$

$N = \frac{0.92 \times 0.08}{0.01^2}$

$= 736$

Scanned with CamScanner

5 6 tracks/cm^2

X = number counted / cm^2

a $P(X=7) = e^{-6} \times \frac{6^7}{7!}$ $\lambda = 6$

$= 0.13768$

b $P(X \geq 3) = 1 - (P(X=0) + P(X=1) + P(X=2))$

$P(X=0) = e^{-6} \times \frac{6^0}{0!}$

$= e^{-6}$

$P(X=1) = e^{-6} \times \frac{6^1}{1!}$

$= 6e^{-6}$

$P(X=2) = e^{-6} \times \frac{6^2}{2!}$

$= 18e^{-6}$

$P(X \geq 3) = 1 - 25e^{-6}$

$= 0.938$

c $P(2 < X < 7) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$P(X=3) = e^{-6} \frac{6^3}{3!}$ $P(X=4) = e^{-6} \frac{6^4}{4!}$ $P(X=5) = e^{-6} \frac{6^5}{5!}$

$= e^{-6} \frac{6^3}{6}$ $= 54e^{-6}$ $= 64.8e^{-6}$

$= 36e^{-6}$

$P(X=6) = e^{-6} \frac{6^6}{6!}$

$= 64.8e^{-6}$

$P(2 < X < 7) = 219.6e^{-6}$

$= 0.54433$

Scanned with CamScanner

6 48 particles per 3 mls

16 particles per ml.

$\hat{r} = 16$ particles per ml

$$\sigma_r = \sqrt{\frac{16}{3}}$$
$$= \frac{4}{\sqrt{3}}$$
$$= 2.309$$

concentration $\approx 16 \pm 2.309$ particles per ml