ECEN 321 Assignment 2 Joshua Bartell

1 ~~Permutations~~ Combinations : Dividing into sets

$N = 15$

$k_1 = 6$

$k_2 = 5$

$k_3 = 4$

let A = num of ways assignments can be made

$$A = \frac{15!}{6!\,5!\,4!}$$

$$= 630630 \text{ combinations}$$

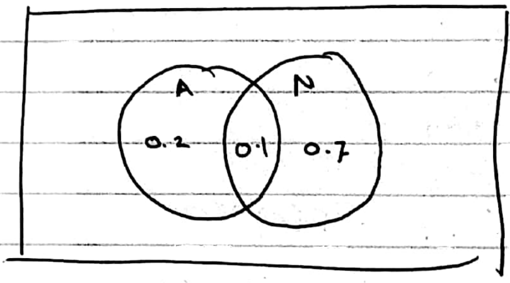
2a ~~File~~ allocation = A

nonessential = N

~~$P(A) = 0.2$~~ ~~$P(A$~~

~~$P(N) = 0.7$~~

~~$P(A \cap N) = 0.1$~~



a $P(A) = 0.2 + 0.1$

$= 0.3$

b $P(N) = 0.7 + 0.1$

$= 0.8$

c $$P(N \mid A) = \frac{P(N \cap A)}{P(A)}$$

$$= \frac{0.1}{0.3}$$

$$= 0.\overline{3}$$

d $$P(A \mid N) = \frac{P(A \cap N)}{P(N)}$$

$$= \frac{0.1}{0.8}$$

$$= 1/8$$

e $P(\cancel{A} N^c \mid A) = \dfrac{P(A \cap N^c)}{P(A)}$

$= \dfrac{0.2}{0.3}$

$= 2/3.$

f $P(A^c \mid N) = \dfrac{P(A^c \cap N)}{P(N)}$

$= \dfrac{0.7}{0.8}$

$= 7/8.$

3. $\cancel{P(G) = 0.7}$
$\cancel{P(M) = 0.2}$
$\cancel{P(P) = 0.1}$

$P(G) = 0.7$ $\cancel{P(F \mid G) = 0.0}$ $P(F \mid G) = 0.005$

$P(M) = 0.2$ $P(F \mid M) = 0.01$

$P(P) = 0.1$ $P(F \mid P) = 0.025$

a $P(G \cap F) = P(G)P(F \mid G)$

$= 0.005 \times 0.7$

$= 3.5 \times 10^{-3}$

b $P(F) = P(F \mid G)P(G) + P(F \mid M)P(M) + P(F \mid P)P(P)$

$= 0.005 \times 0.7 + 0.01 \times 0.2 + 0.025 \times 0.1$

$= 8 \times 10^{-3}$

c $P(G \mid F) = \dfrac{P(G)P(F \mid G)}{P(F)}$

$= \dfrac{0.005 \times 0.7}{8 \times 10^{-3}}$

$= 0.4375.$

4. ~~PC~~

let flaw = E for error

~~let P = Pass~~

let F = fail

a $P(E) = 0.0002$

$P(F \mid E) = 0.995$

$P(\bar{F} \mid \bar{E}) = 0.99$

$$P(E \mid F) = \frac{P(F \mid E)P(E)}{P(F)}$$

$P(F) = P(F \mid E)P(E) + P(F \mid \bar{E})P(\bar{E})$

$P(F \mid \bar{E}) = 1 - P(\bar{F} \mid \bar{E})$

$$\therefore \quad P(E \mid F) = \frac{P(F \mid E)P(E)}{P(F \mid E)P(E) + [1 - P(\bar{F} \mid \bar{E})]\, P(\bar{E})}$$

$$= \frac{0.995 \times 0.0002}{0.995 \times 0.0002 + [1 - 0.99] \times (1 - 0.0002)}$$

$= \cancel{0.9904497512}\ 0.01952$

~~0.99~~.

b i

c $$P(\bar{E} \mid \bar{F}) = \frac{P(\bar{F} \mid \bar{E})\, P(\bar{E})}{P(\bar{F})}$$

$$= \frac{P(\bar{F} \mid \bar{E})\, P(\bar{E})}{[1 - (P(F \mid E)P(E) + [1 - P(\bar{F} \mid \bar{E})]P(\bar{E}))]}$$

$$= \frac{0.99 \times (1 - 0.0002)}{1 - \left(0.995 \times 0.0002 + (1 - 0.99)(1 - 0.0002)\right)}$$

$= 0.999999.$

d ii

4 e $P(F) = 0.995 \times 0.0002 + (1-0.99)(1-0.0002)$

$= 0.010197$

The probability in a) indicates that there are a lot of false negatives which means some good bottles will be thrown away in the small likelihood that a fail ~~is~~ ~~de~~ happens. ~~It~~ This isn't an issue because ~~in~~ of the passing bottles we want as little false positives as possible to ensure a few defective products ~~to~~ pass. So long as we have very good ~~[illegible]~~ ~~[illegible]~~ no flaw pass rates whether we chuck away bottles regardless of flaws doesn't matter as we care less about that.

5

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.1	~~0.15~~ 0.05

a $p(X \leq 2) = 0.4 + 0.3 + 0.15$

$= 0.85$

b $P(X > 1) = 0.15 + 0.1 + 0.05$

$= 0.3$

c $\mu_x = 0 \times 0.4 + 1 \times 0.3 + 2 \times 0.15 + 3 \times 0.1 + 4 \times 0.05$

$= 1.1$

d $\sigma_X^2 = \sum_i x_i^2 f_X(x_i) - \mu_x^2$

$= 0^2 \times (0.4) + 1^2(0.3) + 2^2(0.15) + 3^2(0.1) + 4^2(0.05) - 1.1^2$

$= 1.39$

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6 $f(x) = \begin{cases} \frac{x}{250}, & 20 < x < 30 \\ 0, & \text{otherwise.} \end{cases}$

a $p(X > 25\%) = \int_{25}^{30} \frac{x}{250}\, dx$

$= \frac{x^2}{\cancel{250}\ 500}\Big|_{25}^{30}$

$= \frac{30^2}{\cancel{250}\ 500} - \frac{25^2}{\cancel{250}\ 500}$

$= 0.55$

b $\mu_x = \int_{20}^{30} x f(x)\, dx$

$= \int_{20}^{30} \frac{x^2}{250}\, dx$

$= \frac{x^3}{750}\Big|_{20}^{30}$

$= \frac{30^3 - 20^3}{750} = \frac{76}{3}$

$= 25.33\%$

c $\sigma_x^2 = \int_{20}^{30} x^2 f_x(x)\, dx - \mu_x^2$

$= \int_{20}^{30} \frac{x^3}{250}\, dx - \cancel{25.33}\ \frac{76}{3}^2$

$= \frac{x^4}{1000}\Big|_{20}^{30} - \frac{76}{3}^2$

$= \frac{30^4 - 20^4}{1000} - \frac{76}{3}^2$

$= \frac{\cancel{1074}}{3}\ \frac{74}{9}$

$= 8.22\%$

6 d $\sigma = \sqrt{\sigma^2}$

$= \sqrt{\frac{74}{9}}$

$= \frac{\sqrt{74}}{3}$

$= 2.867\ \%$

e $F_X(x < 20) = \int_{-\infty}^{20} 0\, dt$

$= 0$

$F_X(20 \leq x \leq 30) = F_X(x < 0) + \int_{20}^{x} \frac{t}{250}\, dt$

$= \frac{t^2}{500} \Big|_{20}^{x}$

$= \frac{x^2}{500} - \frac{400}{500}$

$= \frac{x^2}{500} - 4/5$

$F_X(x > 30) = \cancel{F_X(0) +} F_X($

$= 0 + \int_{20}^{30} \frac{t}{500}\, dt + \int_{0}^{\infty} 0\, dt$

$= 0 + \frac{30^2 - 20^2}{500} + 0.$

$= \frac{900 - 400}{500}$

$= 1$

f $F_X(x > 28) = \int_{28}^{30} \frac{x}{250}\, dx$

$= \frac{30^2 - 28^2}{500}$

$= \frac{29}{125}$

$= 0.232.$

7 $\sigma = 7 \times 10^{-15}$ s

↳ of each measurement.

From slides

$\mu_{\bar{X}} = \mu$

$\sigma^2_{\bar{X}} = \frac{\sigma^2}{N}$

$\therefore$

$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$

$N = 2.$

$\therefore \sigma_{\bar{X}} = \frac{7 \times 10^{-15}}{\sqrt{2}}$

$= 4.95 \times 10^{-15}.$