Ecen 321 Assignment 4

1a No as both sit 0.2 away from the expected result

b They second thermometer is more precise as it has a lower uncertainty

2a yes it's the standard deviation

$\bar{x} = \frac{25.6 + 26.8 + 26.2 + 26.8 + 25.4}{5}$

$= 26.16$

$s = \sqrt{\frac{(25.6-26.16)^2 + (26.8-26.16)^2 + (26.2-26.16)^2 + (26.8-26.16)^2 + (25.4-26.16)^2}{4}}$

$= 0.6542$

b yes bias is μ - true value.
seeing as our mean is 26.16 and our true value is 0 in this case as our measure values are done with reference to 1 kg

so Bias $= 26.16 - 0$

$= 26.16$.

3 9 as we want to do $\frac{1.5}{3}$ and more measurements decrease the uncertainty by a factor of $\sqrt{N}$

Scanned with CamScanner

4 Max clearance $= \frac{20+0.01-19.9-0.02}{2}$

$= 0.065$

$\frac{\partial C}{\partial h} = 1$

$\frac{\partial C}{\partial p} = 1$

Min Clearance $= \frac{20-0.01-19.9+0.02}{2}$

$= 0.035$

~~uncertainty~~ average clearance $= \frac{0.065+0.035}{2}$

$= 0.05$

~~uncertainty = (0.065 - 0.035)/2 = 0.015~~

uncertainty $= \frac{\sqrt{1^2 \times 0.01^2 + 0.02^2 \times 1^2}}{2}$

$= \frac{0.022}{2}$

$= 0.011$

clearance = ~~0.05 ± 0.015~~

$= 0.05 \pm 0.011$

5 $T = 300 \pm 0.4k$

$\frac{0.4}{300} \times 100 = 0.13\dot{3}\%$

$T = 300k \pm 0.13\%$ convert to a percent.

$V = 20.04\sqrt{300k \pm 0.13\%}$

$= 20.04 \times 10\sqrt{3} \pm 0.0667\%$

$= 347.1029818 \pm 0.066\%$

$= 347.10 \pm 0.23$

$0.066\% \times 347.1029818 = 0.23140198799$

$= 0.23$

6 1

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6a $V = \frac{\pi r^2 h}{3}$ $\quad r = 5 \pm 0.02$ cm

$h = 6 \pm 0.01$ cm

$$V = \frac{\pi\, r \times r \times h}{3} \pm \sigma_u$$

$$= \frac{\pi \times 5 \times 5 \times 6}{3} \pm \sigma_u$$

$$= 50\pi \pm \sigma u$$

$$\sigma u = \sqrt{\left(\frac{\partial V}{\partial r}\right)^2 \times \sigma_r^2 + \left(\frac{\partial V}{\partial h}\right)^2 \times \sigma_h^2}$$

$$= \sqrt{\left(\frac{2\pi r h}{3}\right)^2 \times 0.02^2 + \left(\frac{\pi r^2}{3}\right)^2 \times (0.01)^2}$$

$$= \sqrt{\left(\frac{2 \times \pi \times 5 \times 6}{3}\right)^2 \times 0.02^2 + \left(\frac{\pi \times 5^2}{3}\right)^2 \times 0.01^2}$$

$$= \cancel{1.796\ cm^3}\ 1.28\ cm^3$$

~~V = 50π ± 1.796 cm³~~

$V = 50\pi \pm 1.28\ cm^3$

b When $r = 5 \pm 0.01$ cm

$V = 50\pi \pm \sigma_u$

$$\sigma_u = \sqrt{\left(\frac{2\pi r h}{3}\right)^2 \times 0.01^2 + \left(\frac{\pi r^2}{3}\right)^2 \times (0.01)^2}$$

$$= \cancel{0.926\ cm^3}\ 0.68\ cm^3$$

~~V = 50π ± 0.926 cm³~~ $\quad V = 50\pi \pm \cancel{0.68}\ 0.68\ cm^3$

When $h = 6 \pm 0.005$

$$\sigma_u = \sqrt{\left(\frac{2\pi r h}{3}\right)^2 \times 0.02^2 + \left(\frac{\pi r^2}{3}\right)^2 \times 0.005^2}$$

$$= \cancel{1.7820}\ 1.26\ cm^3$$

~~V = 50π ± 1.7820 cm³~~ $\quad V = 50\pi \pm 1.26\ cm^3$

~~V = 1.260~~ r

Reducing the uncertainty of r has greater effect and reduces the uncertainty of V more

7a $R = k\frac{L}{d^2} = kLd^{-2}$ $\quad \frac{\partial R}{\partial L} = kd^{-2}$

$\frac{\partial R}{\partial d} = -2kLd^{-3}$

$L = 14 \pm 0.1\,cm \qquad d = 4.4 \pm 0.1\,cm$

$R = k \times \frac{14}{4.4^2} \pm \sigma_R$

$= 0.72314 \pm \sigma_R$

$\sigma_R = \sqrt{\left(\frac{\partial R}{\partial L}\right)^2 \sigma_L^2 + \left(\frac{\partial R}{\partial d}\right)^2 \sigma_d^2} = \sqrt{(kd^{-2})^2(0.1)^2 + (\cancel{kd}\,-2kLd^{-3})^2(0.1)^2}$

$= \sqrt{\frac{k^2}{19.36} \times 0.01 + \left(\frac{-28k}{85.184}\right)^2 \times 0.01}$

$= \sqrt{5.16\cancel{8}\times10^{-4}k^2 + 1.08\times10^{-3}k^2}$

$= \sqrt{1.597\times10^{-3}k^2}$

$= 0.03996k\ \Omega$

$R = 0.72314k \pm 0.03996k\,\Omega$

b $L = 0.005$

$\sigma_R = \sqrt{\frac{k^2}{19.36} \times (0.005)^2 + 1.08\times10^{-3}k^2}$

$= \cancel{2.7\times10^{-4}k^2}$

$= \sqrt{1.29\times10^{-4}k^2 + 1.08\times10^{-3}k^2}$

$= \sqrt{1.2096\times10^{-3}k^2}$

$= 0.0348k\ \Omega$

$R = 0.72314k \pm 0.0348k\ \Omega$

~~Changing~~

Reducing the uncertainty of d had the greatest reduction in the uncertainty of R.

$d = 0.05$

$\sigma_R = \sqrt{\frac{k^2}{19.36} \times 0.01 + \left(\frac{-28k}{85.184}\right)^2 \times 0.05^2}$

$= \sqrt{5.165\times10^{-4}k^2 + 2.7\times10^{-4}k^2}$

$= \sqrt{7.866\times10^{-4}k^2}$

$= \cancel{0.0249}\ 0.028k$

$R = 0.72314k \pm 0.028k\ \Omega$