ECEN321 - Lab 3
Estimating Parameters

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1 Introduction

This report investigates estimation of distribution parameters. The purpose of doing this is to understand how to estimate parameters and statistics and how they can help us model complex signals like speech. From this we should also be able to see the accuracy of such estimates.

2 Method

To understand the behaviour of random variables, we will be investigating a many gaussian realisations of a random variable. To do this, two random variables with 10 and 10000 samples taken will be realised 1000 times. The means and variances will then be plotted and the corresponding distribution plotted and analysed.

The distribution of speech will then be modelled. This will be done using a speech sample with an assumed mean of 0 and measured variance $\sigma^2 = 2.4353e7$. The samples in the signal will be considered independent, as this allows us to treat each sample as a random variable and combine it into a joint distribution for the next step. We can make the assumption of independence based on the measurements not being dependent on the previous one based on the fact each sample is independently taken. It is making the assumption that a high frequency to a low frequency happens fast enough to not matter and effect the distribution of the From there, the generalised normal distribution will be used to model the distribution of the speech signal. However, the parameters α and β are unknown, so will have to be approximated. This is done by finding the maximum of β then α, and repeating until they converge. From here it is possible to figure out the type of distribution that matches that of the speech signal.

3 Results

3.1 Gaussian Random Variables

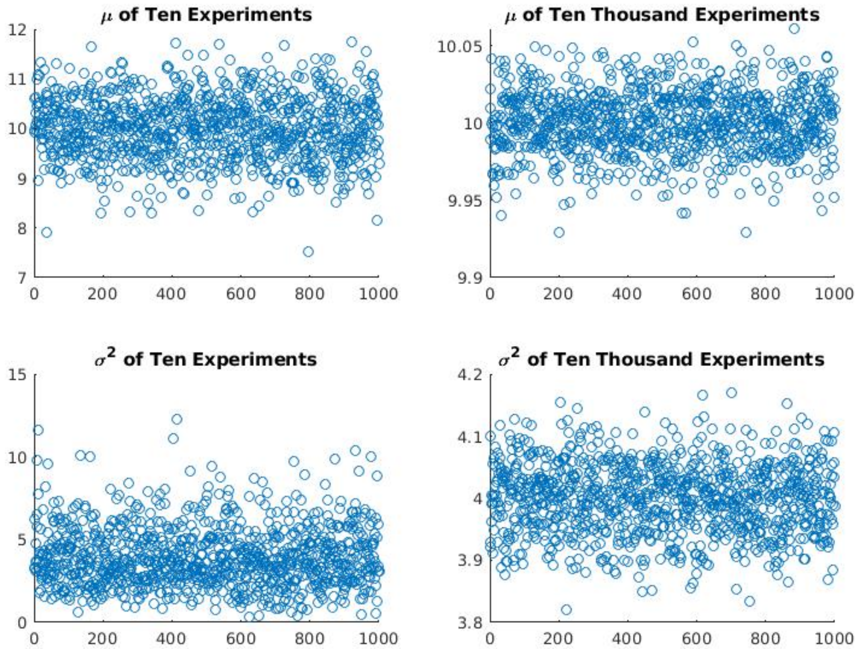

Figure 1: Scatter plot of means and variances of all realisations

From the plots in figs. 1 and 2 it can be seen that the realisations of 10 samples has a wider distribution spanning whole integers, and the realisations of 10000 samples, which spans a range 100 times smaller. All of these, aside from the distribution of the variance for 10 samples appear to have a normal distribution. The outlier has a right skew to the distribtion, which is visible in the scatter plot as there is a dense cluster and a thinner set of points above it, opposed to the other plots which have a dense cluster in the middle and thin tails either side. This indicates that the increase in samples per random variable makes the variance more normally distributed.

One of the differences between the two sets of realisations is the range that they are over. The set with 10 samples is over whole integers and the set with 10000 is over a range 100 times smaller. We know this to be true as the increase in samples reduces the standard deviation by a factor of $\sqrt{N}$ which can be seen in he 10000 set is $\sqrt{10000} = 100$, this also has an effect on the variance. According to [1], the sampled variance of a normally distributed

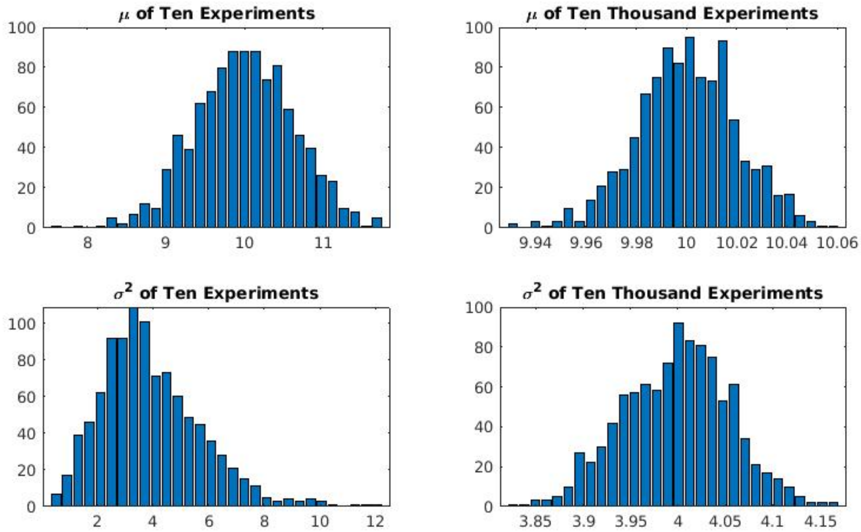

Figure 2: Distributions of means and variances of all realisations

random variable is χ^2 distributed with $N-1$ degrees of freedom. Which can be seen from the fact that we know that the lower the degrees of freedom the more right skew the distribution has, and at 9999 degrees of freedom the χ^2 becomes more centered, like that of the variances distribution for 10000 samples (section A). This can also be explained by the central limit theorem, by summing the χ^2 distributions together, it results in a normal distribution that approximates it.

3.2 Speech

With the initial approximation that $\alpha = \sqrt{2\sigma^2} = 6978.9$, we can find the value that maximises β by comparing the outputs of various β values, resulting in a first $\beta = 1.62$. Using this value of β, we can then find a value of α that maximises the new function, doing this results in $\alpha = 5868.6$. Continuing with this maximisations for each value one at a time, until it converges results in a final parameter pair $\theta = [\alpha = 1727.1, \beta = 0.707]$.

The PDF for the converged values of α and β is the blue line plotted in fig. 3. Alongside this is the PMF of speech signal is plotted in orange. We can see that these two line up pretty well, indicating that the distribution from the found values is close to or exactly the distribution of this speech signal. This distribution is in the Laplace family of distributions.

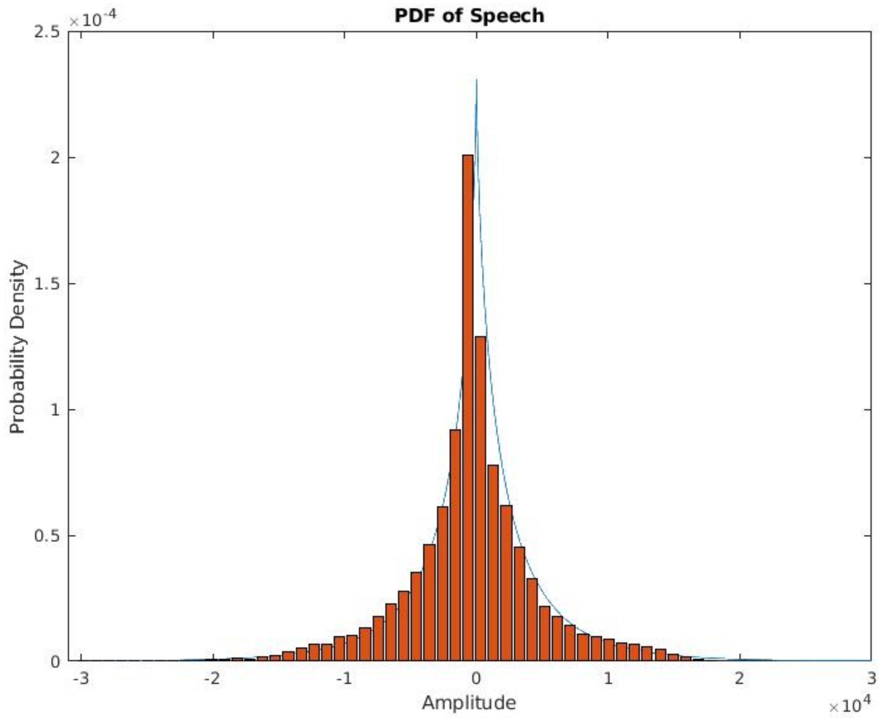

Figure 3: Distribution of the speech signal

4 Conclusion

From esimating statistics for a series of gaussian random variables, we can see that as we increase the number of samples in a realisation the more accurate our estimate of the mean and variance are. It is also seen that the distribution becomes more normally distributed with more samples.

Additionally, we have seen an outline for a method of approximating the distribution of a speech signal. We do this by computing the maximum likelihood estimation by maximising both α and β one after the other until they both converge. From here we can see that the distribution estimated is a laplacian distribution that comes very close to the distribution of the signal and is a good model for this signal.

References

[1] 2020. [Online]. Available: https://www.csus.edu/indiv/j/jgehrman/courses/stat50/samplingdists/7samplingdist.htm#top

Appendices

A χ^2 Distributions

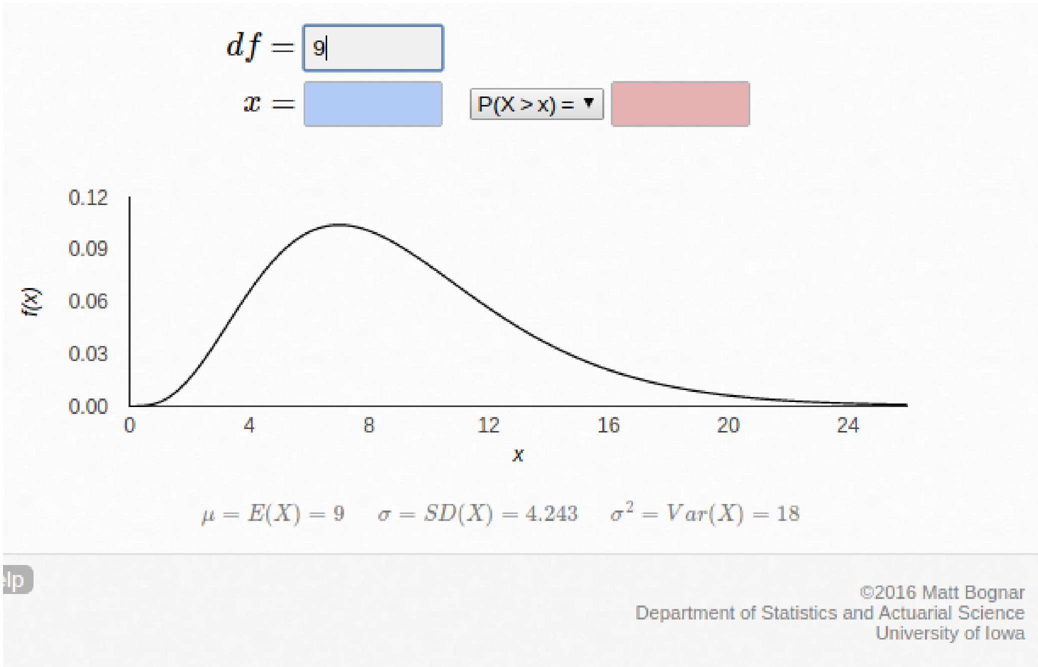

Figure 4: χ^2 distribution with $\nu = 9$

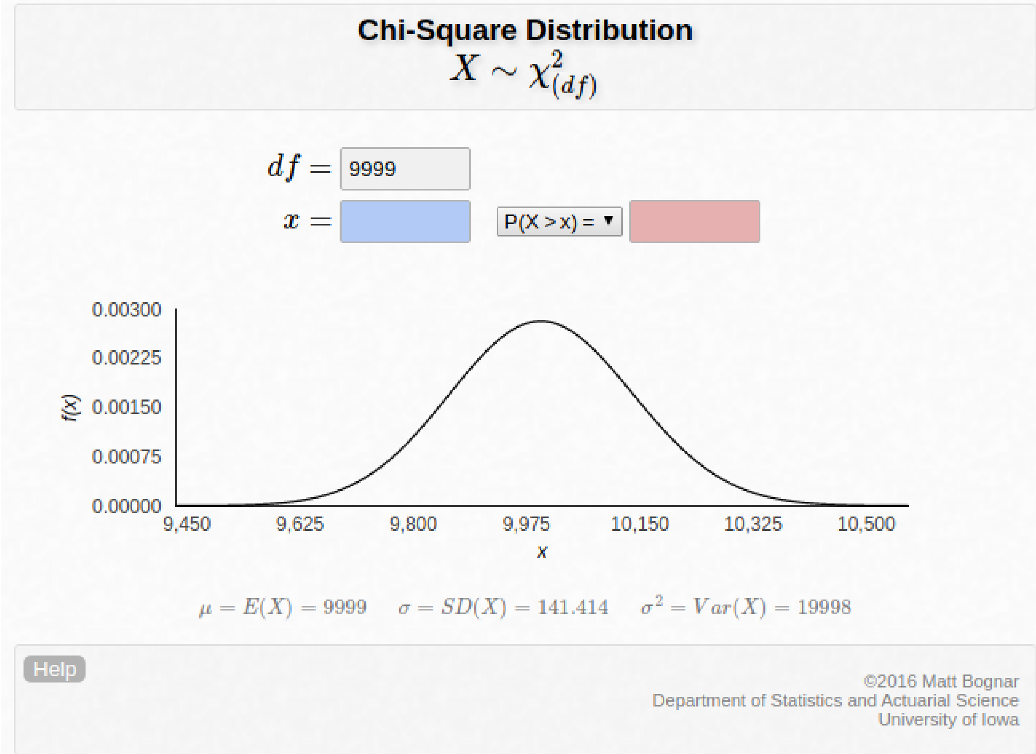

Figure 5: χ^2 distribution with $\nu = 9999$

B MATLAB code for Gaussian Random Variables

```
%Gaussian Distribution
close all
clear
mu = 10;
variance = 4;
sigma = sqrt(variance);

ten = sigma * randn(1000, 10) + mu;
tenThou = sigma * randn(1000, 10000) + mu;

%1
tenMu = mean(ten, 2);
tenThouMu = mean(tenThou, 2);
%2
```

```
tenVar = var(ten, 0, 2);
tenThouVar = var(tenThou, 0, 2);
%3
index = 1:1:1000;

figure(1)

subplot(2,2,1)
scatter(index, tenMu)
title("\mu of Ten Experiments")

subplot(2,2,3)
scatter(index, tenVar)
title("\sigma^2 of Ten Experiments")

subplot(2,2,2)
scatter(index, tenThouMu)
title("\mu of Ten Thousand Experiments")

subplot(2,2,4)
scatter(index, tenThouVar)
title("\sigma^2 of Ten Thousand Experiments")

%4
figure(2)

subplot(2,2,1)
[N, X] = hist(tenMu, 30);
binWidth = X(2) - X(1);
bar(X, N - binWidth/2) %plot bars shifted back from the
    center half a bin width
title("\mu of Ten Experiments")

subplot(2,2,3)
[N, X] = hist(tenVar, 30);
binWidth = X(2) - X(1);
bar(X, N - binWidth/2)
title("\sigma^2 of Ten Experiments")

subplot(2,2,2)
[N, X] = hist(tenThouMu, 30);
```

```
binWidth = X(2) - X(1);
bar(X, N - binWidth/2)
title("\mu of Ten Thousand Experiments")

subplot(2,2,4)
[N, X] = hist(tenThouVar, 30);
binWidth = X(2) - X(1);
bar(X, N - binWidth/2)
title("\sigma^2 of Ten Thousand Experiments")
```

C MATLAB code for Speech Estimation

```
% Speech Distribution
close all
clear

%1
fid = fopen("Speech.pcm");
[signal N] = fread(fid, 'int16');
figure(1)
plot(signal)
%2
variance = var(signal);
%4
mu = 0;
%5

%parameters
alphaEsti = sqrt(2 * variance); % Initial Guess
betaEsti = 2; % Initial Guess
absx = abs(signal);

maxIters = 100;

%betaConditions
betaEnd = 5;
betaStep = 0.001;
betaStart = 0;
lastBeta = 0;

%alphaConditions
```

```
alphaStart = 0;
alphaEnd = 7e3;
alphaStep = 0.1;
lastAlpha = 0;

%array of past values
alphaVals = [alphaEsti];
betaVals = [];


figure(2)
for iters = 0:maxIters
    subplot(1,2,1)
    hold on
    f = zeros(betaEnd/betaStep+1, 1);
    i = 1;
    for beta = betaStart:betaStep:betaEnd
        if length(betaVals) > 1 && betaVals(end-1) ==
            betaVals(end)
            break;
        end
        f(i) = N * (log(beta) - log(2*alphaEsti) -
            gammaln(1/beta)) - sum((absx/alphaEsti).^
            beta);
        if i > 1 && f(i) < f(i-1)
            betaEsti = lastBeta;
            betaVals(end+1) = betaEsti;
            break
        end
        lastBeta = beta;
        i=i+1;
    end
    plot(betaStart:betaStep:betaEnd, f);
    f = zeros(alphaEnd/alphaStep+1, 1);
    i = 1;
    subplot(1,2,2);
    hold on
    for alpha = alphaStart:alphaStep:alphaEnd
        if length(alphaVals) > 1 && alphaVals(end-1) ==
            alphaVals(end)
            break;
```

```
        end
        f(i) = N * (log(betaEsti) - log(2*alpha) -
          gammaln(1/betaEsti)) - sum((absx/alpha).^
          betaEsti);
        if i > 1 && f(i) < f(i-1)
            alphaEsti = lastAlpha;
            alphaVals(end+1) = alphaEsti;
            break
        end
        lastAlpha = alpha;
        i=i+1;
    end
    plot(alphaStart:alphaStep:alphaEnd, f);
end
subplot(1,2,1)
title("\beta")
subplot(1,2,2)
title("\alpha")

figure(3)
[H, barCenters] = hist(signal, 50);
dx = barCenters(2)-barCenters(1);
histVals = H/(N*dx);
barEdges = barCenters - dx;
x = -30000:10:30000;
y = (betaEsti / (2 * alphaEsti * gamma(1/betaEsti)) *
   exp(-power(abs(x)./alphaEsti, betaEsti)));
plot(x,y);
hold on
bar(barEdges, histVals);
title("PDF of Speech")
ylabel("Probability Density")
xlabel("Amplitude")
```