Econ 321 Ass 7 Joshua Benfell

1a concentration = 30 /mL

a poisson distribution

$\lambda = 2 \times 30$

$= 60$

normal approximation of poisson (λ) when $\lambda > 10$

$X \sim N(\lambda, \lambda)$

$\mu = \lambda = 60$

$\sigma = \sqrt{\lambda}$

$= \sqrt{60}$

$= 2\sqrt{15}$

$x = 50$

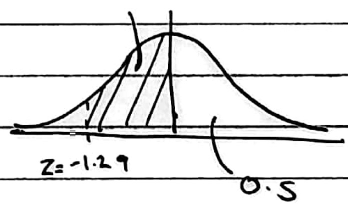
$Z = \frac{x - \mu}{\sigma}$

$= \frac{50-60}{\sqrt{60}}$

$= -\frac{\sqrt{15}}{3}$

$= -1.29099.$

from table = 0.4015



$P(X > 50) = P(Z > -1.29099)$

$= 0.4015 + 0.5$

$= 0.9015.$

b $n = 10$

$p = P(X > 50) = 0.9015$

$P(X = 9) = \frac{10!}{9!(10-9)!} \cdot 0.9015^{9} \cdot (1 - 0.9015)^{10-9}$

$= 0.38737$

$P(X \geq 9) = P(X = 9) + P(X = 10)$ as these are independent events so can add.

$P(X = 10) = \frac{10!}{10!(10-10)!} \times 0.9015^{10} \times 1$

$= 0.9015^{10}$

$= 0.35453$

$P(X \geq 9) = 0.3874 + 0.3545$

$= 0.7419.$

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1c $N = 100$ Find $P(X \geq 90)$ ~~pdf~~

$p = P(X > 50)$
$= 0.9015$

$Np = \cancel{9025}$ $n(1-p) = 100 \times (1 - 0.9015)$
$= 90.15$ $= 9.85.$
> 10

not quite > 10 ~~but~~ close. so can't use normal dist approximation.

~~use~~ to find $P(X \geq 90) = P(X=90) + P(X=91) + \dots + P(X=99) + P(X=100)$

$P(X=k) = \frac{N!}{k!(N-k)!} p^k (1-p)^{N-k}$ ~~$N = 100$~~

~~$k = [90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100]$~~

$P(X=90) = 0.1317$
$P(X=91) = 0.1325$
$P(X=92) = 0.11859$
$P(X=93) = 0.0934$
$P(X=94) = 0.06363$
$P(X=95) = 0.03678$
$P(X=96) = 0.0175$
$P(X=97) = 0.00662$
$P(X=98) = 0.00185$
$P(X=99) = 0.000343$
$P(X=100) = 0.00003137$

$P(X \geq 90) = 0.6029.$

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2 $\mu = 40$ hours $\sigma = 5$ hours.

$\bar{X}$ is mean of ~~RV X~~ ~~of~~ RS of size $N=100$

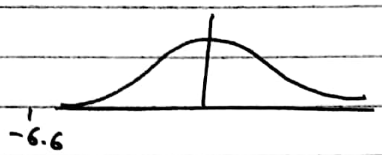
$\cancel{Z = \frac{387-40}{5}}$

=

$Z_{\bar{x}} = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$

$= \frac{36.7 - 40}{5/\sqrt{100}}$

$= -6.6$



≈ 0.5 a $Z = 3.8$ from table

$P(\hat{X} \leq 36.7) = P(Z \leq -6.6)$

≈ 0

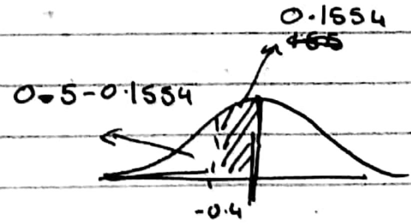
b Yes. as there is a small (very) that it's ~~a~~ less than this. let alone this time as the above prob is all values up to, and values below have less chance

c given that it's more than 6 ~~sd~~ $\sigma_{\bar{x}}$ away if the sample mean were lower than the true mean, the claim would seem inplausible.

d ~~$\bar{X}$ ... $\mu_{\bar{x}}$~~ $\bar{x} = 39.8$ $\mu = 40$ $\sigma = 5$ $N = 100$

~~$P(\hat{X} \leq 39.8) =$~~ $Z = \frac{39.8 - 40}{5/10}$

$= -0.4$



$P(\hat{X} \leq 39.8) = P(Z \leq -0.4)$

$= \cancel{3}\ 0.3446.$

e no

f yes. because 40 is within ~~30~~ 34.46% of 39.8 hours.

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3 a $Z_{\alpha/2} = 1.96$

$CI = 0.4750 \times 2$

$= 0.95$

$= 95\%$

b $Z_{\alpha/2} = 2.17$

$CI = 0.4850 \times 2$

$= 0.97$

$= 97\%$

c $Z_{\alpha/2} = 1.28$

$CI = 0.3997 \times 2$

$= 0.7994$

$= 79.94\%$

d $Z_{\alpha/2} = 3.28$

$CI = 0.4995 \times 2$

$= 0.999$

$= 99.9$

4 $N = 50$

$\mu = 654.1$

$\sigma = 311.7$

a 95% CI is $\bar{X} \pm 1.96\, \sigma_{\bar{x}}$

$= 654.1 \pm 1.96 \times \frac{311.7}{\sqrt{50}}$

$= 654.1 \pm 86.40$

(567.70, 740.50)

b $Z_{\alpha/2} = 2.33$

98% CI $= 654.1 \pm 2.33 \times \frac{311.7}{\sqrt{50}}$

(551.39, 756.81)

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4c $\cancel{\frac{726.6 - 581.6}{2}}$ $\cancel{852}$ = E

$\cancel{E = 72.5}$

$\cancel{= Z_{\alpha/2} \frac{\sigma}{\sqrt{50}}}$

$\cancel{Z}$

$\cancel{Z_{\alpha/2} = \frac{311.7}{72.5\sqrt{50}}}$

$= 0.608$

$$Z = \frac{581.6 - 654.1}{311.7/\sqrt{50}}$$

$= -1.64$

$$Z = \frac{726.6 - 654.1}{311.7/\sqrt{50}}$$

$= 1.64$

$CI = 2 \times 4495$

$= 0.899$

$= 89.9\%$ CI

d for 95% CI $Z_{\alpha/2} = 1.96$

$\cancel{E = \bar{x} - \mu}$

at $E = Z_{\alpha/2} \sigma_{\bar{x}}$

$$50 = 1.96 \times \frac{311.7}{\sqrt{N}}$$

$$N = \left(\frac{1.96 \times 311.7}{50}\right)^2$$

$= 149.3$

$= 150$ samples.

e $Z = 2.33$ $N = \left(\frac{2.33 \times 311.7}{50}\right)^2$

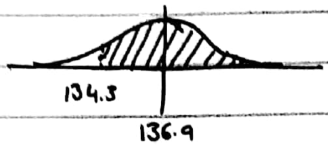
$= 210.98$

$= 211.$

$Z_\alpha =$

5f 98% lower Bound: $136.9 - 2.33\left(\frac{22.6}{\sqrt{123}}\right) = 132.151988$

g



$$Z = \frac{134.3 - 136.9}{\frac{22.6}{\sqrt{123}}}$$

$= -1.275$

$P(\bar{X} > 134.3) = P(Z > -1.275)$

$= P(-1.275 < Z < 0) + P(Z > 0)$

$= 0.39885 + 0.5$

$= 0.899.$

$= 89.9\%$

with 90% confidence.