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Ecen 321 Test 1 Joshua Benfell 29/04/2020 Page 1

1a $\bar{x} = \frac{8 \times 3 + 5 + 4 + 9}{6}$

$= \frac{42}{6}$

$= 7$

$\bar{y} = \frac{18 + 22 + 16 + 9 + 25 + 12}{6}$

$= \frac{102}{6}$

$= 17$

b $S = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}$

$S_x = \sqrt{\frac{1}{6-1}\left((8-7)^2 + (8-7)^2 + (8-7)^2 + (5-7)^2 + (9-7)^2 + (4-7)^2\right)}$

$= \sqrt{\frac{1}{5}(1+1+1+4+4+9)}$

$= 2$

$Sy = \sqrt{\frac{1}{5}\left((18-17)^2 + (22-17)^2 + (16-17)^2 + (9-17)^2 + (25-17)^2 + (12-17)^2\right)}$

$= \sqrt{\frac{1}{5}(1 + 25 + 1 + 64 + 64 + 25)}$

$= 6$

Q1 c $\text{Cov}(X,Y) = \cancel{E\{(X}$

$$= E\{(X-\bar{x})(y-\bar{y})\} = \frac{\sum(\cancel{x}x-\bar{x})(y-\bar{y})}{N-1}$$

$$= \frac{(8-7)(18-17)+(5-7)(22-17)+(9-\cancel{16}7)(16-17)+(4-7)(25-17)+(8-7)(12-17)+(8-7)(9-17)}{\cancel{6}\ \cancel{N}\ 5}$$

$$= \frac{1 + -2\times 5 + 2\times -1 + -3\times 8 + 1\times -5 + 1\times -8}{\cancel{6}\ 5}$$

$$= \frac{-48}{5}$$

$$= -9.6$$

$\cancel{\rho(x,y)} =$

$$\rho_{x,y} = \frac{\text{Cov}(X,Y)}{S_x\ S_y}$$

$$= \cancel{92}\ \frac{-9.6}{2\times 6}$$

$$= -0.8$$

Scanned with CamScanner

Q2 $P(I_1 | F) = 0.9$ $P(I_2 | F) = 0.7$

a $P(I_1 | F \cap I_2 | F) = 0.9 \times 0.7$
$= 0.63$

b $P(I_1 | F \cup I_2 | F) = P(I_1 | F) + P(I_2 | F) - P(I_1 | F \cap I_2 | F)$
$= 0.9 + 0.7 - 0.63$
$= 0.97$

c Inspector 2 is no longer independent of inspector 1 as ~~A~~ They will only have passed which alters the items they ~~othe~~ will be examining.

percentage of items being viewed.

$P(I_1^c | F \cap I_2 | F) = 0.1 \times 0.7$
$= 0.07$

d Find $P(F | I_1^c)$

$$P(F | I_1^c) = \frac{\overbrace{P(I_1^c | F)}^{= 1 - P(I_1 | F) = 0.1} \ \underbrace{P(F)}_{= 0.1}}{P(I_1^c)}$$

Bayes Rule.

$P(I_1^c) = P(I_1^c \cap F) + P(I_1^c \cap F^c)$
$= P(I_1^c | F) P(F) + P(I_1^c | F^c) P(F^c)$
$= 0.1 \times 0.1 + 0.9 \times P(I_1^c | F^c)$
$= 0.01 + 0.9 \times P(I_1^c | F^c)$

No way of finding this.

$\therefore$

$$P(F | I_1^c) = \frac{0.01 \times 0.1}{0.01 + 0.9 \times P(I_1^c | F^c)}$$

$$= \frac{0.01}{0.01 + 0.9 \times P(I_1^c | F^c)}$$

lets assume that there are no false positives and that the inspectors know ~~the~~ their stuff for 2d and 2e so $P(I_1^c | F^c)$ and $P(I_2^c | F^c) = 1$.

$\therefore P(F | I_1^c) = \frac{0.01}{0.01 + 0.9 \times 1}$

$= 0.010989$

Q2 e ~~Find $P((I_1^c \cap I_2^c))$~~

Find $P(F \mid (I_1^c \cap I_2^c)) = \dfrac{P((I_1^c \cap I_2^c \mid F) P(F)}{P(I_1^c \cap I_2^c \mid F) P(F) + P(I_1^c \cap I_2^c \mid F^c) P(F^c)}$

$= \dfrac{P(I_1^c \mid F) P(I_2^c \mid F) P(F)}{P(I_1^c \mid F) P(I_2^c \mid F) P(F) + P(I_1^c \mid F^c) P(I_2^c \mid F^c) P(F^c)}$

$P(I_1^c \mid F) = 1 - 0.9 = 0.1$

$P(I_2^c \mid F) =$ ~~a~~ $1 - 0.7 = 0.3$

$P(F) = 0.1$

$P(F^c) = 1 - 0.1 = 0.9$

I can't find $P(I_1^c \mid F^c)$ and $P(I_2^c \mid F^c)$ so it will equal

$P(F \mid (I_1^c \cap I_2^c)) = \dfrac{0.1 \times 0.3 \times 0.1}{0.1 \times 0.3 \times 0.1 + 0.9 \times P(I_1^c \mid F^c) P(I_2^c \mid F^c)}$

$= \dfrac{3 \times 10^{-3}}{3 \times 10^{-3} + 0.9 \times P(I_1^c \mid F^c) P(I_2^c \mid F^c)}$

using the assumption stated in 2d

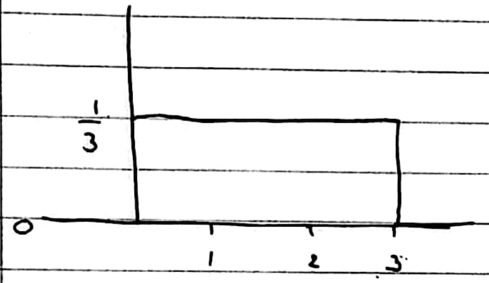
$P(F \mid (I_1^c \cap I_2^c)) = \dfrac{3 \times 10^{-3}}{3 \times 10^{-3} + 0.9 \times 1 \times 1}$

$= 3.32 \times 10^{-3}$

Q3 a $\displaystyle \int_y \int_x f_{XY}(x,y)\,dx\,dy = 1$

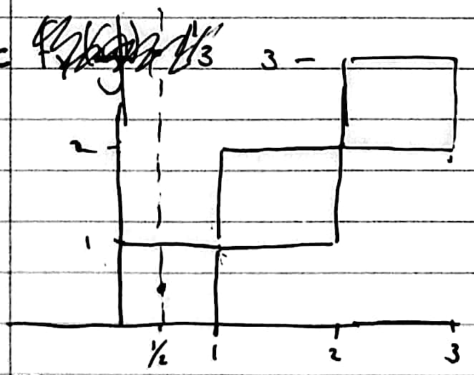
There are 3 equally sized regions so $c = \frac{1}{3}$.

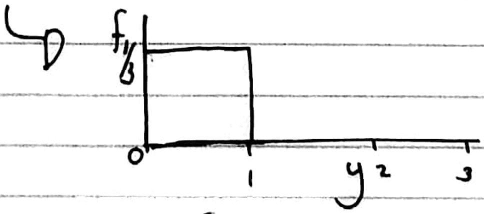
b $f_X(x) = \begin{cases} 1/3, & 0 \leq x \leq 3 \\ 0, & \text{otherwise.} \end{cases}$



as we integrate on all y values essentially viewing the density from the x axis side

c





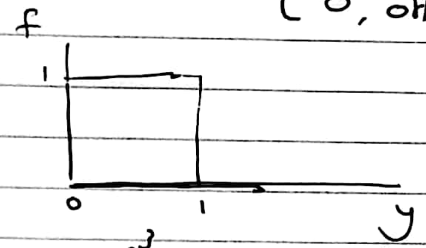
$f_{Y,\frac{1}{2}}(y, \tfrac{1}{2}) = \begin{cases} 1/3, & 0 < y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$

Q3 d

$$f_{Y|X}(y \mid 1/2) = \frac{f_{Y,X}(y, 1/2)}{f_X(1/2)}$$

$$= \frac{\frac{1}{3},\ 0 \le y \le 1}{1/3}$$

$$= \begin{cases} 1, & 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$



e $\cancel{E\{X\} = \int_0^3 x \tfrac{1}{3}\, dx}$

$\cancel{= \frac{1}{6} x^2 \Big|_0^3}$

$\cancel{= 4 \ \frac{9}{6}}$

$= 4\ 3/2$

$$E\{X\} = \int_x x f_X(x)\, dx$$

$$= \int_0^3 x \times \frac{1}{3}\, dx$$

$$= \frac{1}{6}\left[x^2\right]_0^3$$

$$= \frac{9}{6} - 0$$

$$= 3/2$$

f $E\{XY\} = \int_x \int_y xy\, f_{XY}(x,y)\, dx\, dy$

$$= \int_0^1 \int_0^1 xy \times 1/3 \; dx\,dy + \int_1^2 \int_1^2 xy \times 1/3 \; dx\,dy + \int_2^3 \int_2^3 xy \times 1/3 \; dx\,dy$$

$$\int_a^b \int_a^b xy \; \cancel{x\,dx\,dy} = \int_a^b y \int_a^b x \, dx\, dy$$

$$= \int y \times \frac{x^2}{2}\Big|_a^b dy$$

$$= \frac{y^2}{2}\Big|_a^b \times \frac{x^2}{2}\Big|_a^b = \frac{1}{4} \times y^2\Big|_a^b \times x^2\Big|_a^b$$

$$E\{XY\} = \frac{1}{12}\left(y^2\Big|_0^1 \times x^2\Big|_0^1 + y^2\Big|_1^2 \times x^2\Big|_1^2 + y^2\Big|_2^3 \times x^2\Big|_2^3 \right)$$

$$= \frac{1}{12}\Big((1-0)(1-0) + (4-1)(4-1) + (9-4)(9-4) \Big)$$

$$= \frac{1}{12}\left(1 + 9 + 25 \right)$$

$$= \frac{35}{12} = 2.9167$$

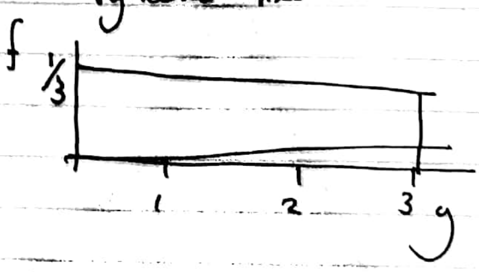
g $Cov(X,Y) = \mu_{XY} - \mu_X \mu_Y$

$$= E\{XY\} - E\{X\}E\{Y\}$$

$$= \frac{35}{12} - \left(\frac{3}{2}\right)^2$$

$$= \frac{2}{3}$$

$E\{X\} = E\{Y\}$
as
f_Y looks like



Q4

$$f(x,y) = \begin{cases} \frac{1}{6}e^{-\frac{x}{2}}e^{-\frac{y}{3}} & \cancel{x \geq 0}\ x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{elsewise} \end{cases}$$

a $P(X \leq 2 \text{ and } Y \leq 3) = \frac{1}{6}\int_0^3 \int_0^2 e^{-\frac{x}{2}} e^{-\frac{y}{3}}\, dx\, dy$

$$\int_0^2 \frac{1}{6} e^{\frac{-x}{2}} e^{\frac{-y}{3}}\, dx = \frac{1}{6} e^{\frac{-y}{3}} \int_0^2 e^{\frac{-x}{2}}\, dx$$

$$= \frac{1}{6} e^{\frac{-y}{3}} \left[-2e^{\frac{-x}{2}} \right]_0^2$$

$$= \frac{-1}{3} e^{\frac{-y}{3}} \left(e^{-1} - 1 \right)$$

$$= \cancel{\frac{-1}{3}e^{-1}}$$

$$\int_0^3 \frac{-1}{3} e^{\frac{-y}{3}} \left(e^{-1} - 1\right) dy = \frac{-1}{3}\left(e^{-1}-1\right) \int_0^3 e^{\frac{-y}{3}}\, dy$$

$$= \frac{-1}{3}\left(e^{-1}-1\right) \left[-3e^{\frac{-y}{3}} \right]_0^3$$

$$= 1 \times \left(e^{-1} - 1\right)\left(e^{-1} - 1\right)$$

$$= e^{-2} - e^{-1} - e^{-1} + 1$$

$$= e^{-2} - 2e^{-1} + 1$$

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Qu b $P(X \geq 3 \text{ and } Y \geq 3) = \int_3^\infty \int_3^\infty \frac{1}{6} e^{-\frac{x}{2}} e^{-\frac{y}{3}} \, dy \, dx$

$\frac{1}{6} e^{-\frac{y}{3}} \int_3^\infty e^{-\frac{x}{2}} dx = -\frac{1}{3} e^{-\frac{y}{3}} \left[e^{-\frac{x}{2}} \right]_3^\infty$

$= \frac{-1}{3} e^{-\frac{y}{3}} \left(0 - e^{-\frac{3}{2}} \right)$

$\frac{-1}{3} \left(e^{-\frac{3}{2}} \right) \int_3^\infty e^{-\frac{y}{3}} dy = \left(e^{-\frac{3}{2}} \right) \left(e^{-\frac{y}{3}} \right) \Big|_3^\infty$

$= e^{-\frac{3}{2}} e^{-1}$

$= e^{-\frac{5}{2}}$

c ~~$f_x(x)$~~

$f_x(x) = \int_0^\infty \frac{1}{6} e^{-\frac{x}{2}} e^{-\frac{y}{3}} dy$

$= \frac{1}{6} e^{-\frac{x}{2}} \int_0^\infty e^{-\frac{y}{3}} dy$

$= -\frac{1}{2} e^{-\frac{x}{2}} \left(e^{-\frac{y}{3}} \right) \Big|_0^\infty$

$= -\frac{1}{2} e^{-\frac{x}{2}} (0 - 1) = \frac{1}{2} e^{-\frac{x}{2}}$

d ~~$f_x(x)$~~ $f_Y(y) = \int_0^\infty \frac{1}{6} e^{-\frac{x}{2}} e^{-\frac{y}{3}} dx$

$= \frac{1}{6} e^{-\frac{y}{3}} \int_0^\infty e^{-\frac{x}{2}} dx$

$= -\frac{1}{3} e^{-\frac{y}{3}} \left[e^{-\frac{x}{2}} \right]_0^\infty$

$= \frac{-1}{3} e^{-\frac{y}{3}} [0 - 1]$

$= \frac{1}{3} e^{-\frac{y}{3}}$

Q4 e Yes as:

$$f(x,y) = f(x)\,f(y)$$

$$= \frac{1}{2}e^{-\frac{x}{2}} \times \frac{1}{3}e^{-\cancel{x}\frac{y}{3}}$$

$$= \frac{1}{6}e^{-\frac{x}{2}}e^{-\cancel{x}\frac{y}{3}}$$

$$= f(x,y).$$

and

$$f_{\cancel{x}Y|X}(\cancel{x}y|x) = f_Y(y)$$

$$\frac{f(x,y)}{f(x)} = f(y)$$

$$\frac{1/6\, e^{-x/2}e^{-y/3}}{1/2\, e^{-\frac{x}{2}}} = 1/3\, e^{-y/3}$$

$\therefore$ These variables are independent