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1a $X \sim N(2, 9)$

$\mu = 2$

$\sigma^2 = 9$

Using Z table in lecture slide. which contains probabilities up to the ~~end~~ Z score from the center.

$\sigma = 3$

$Z = \frac{x - \mu}{3}$

for $P(X \geq 2)$

$Z = \frac{2 - 2}{3}$

$= 0$

$\therefore$

$P(X \geq 2) = P(Z \geq 0)$ center of dist

$= 0.5$

b $Z_{x=1} = \frac{1-2}{3}$ $Z_{x=7} = \frac{7-2}{3}$

$= -1/3$ $= 5/3$

$P(1 \leq X \leq 7) = P(-1/3 \leq Z \leq 5/3) \approx P(-0.33 \leq Z \leq 1.67)$

$\approx 0.1293 + 0.4525$

≈ 0.5818

c $P(-2.5 \leq X \leq -1)$

$Z_{x=-2.5} = \frac{-2.5-2}{3}$ $Z_{x=-1} = \frac{-1-2}{3}$

$= \frac{-4.5}{3}$ $= -1$

$= -1.5$

$\therefore$

$P(-1.5 \leq Z \leq -1) = 0.4332 - 0.3413$

$= 0.0919$

1d $P(-3 \leq (X-2) < 3) = P(-1 \leq X < 5)$

$Z_{X=-1} = \frac{-1-2}{3}$ $\quad Z_{X=5} = \frac{5-2}{3}$

$= -1$ $\quad = 1$

$P(-1 \leq Z < 1) = \cancel{0.3413\times2}$ 0.3413

$= 0.6826$

2a $M = 0.5X + Y$ as X and Y are Normal, M is also normal

$M \sim N(0.5 \times 0.45 + 1 \times \overset{0.25}{\cancel{0.05}}, 0.5^2 \times 0.05^2 + 1^2 \times 0.025^2)$

$M \sim N(0.475, 1.25 \times 10^{-3})$

M is normal with

$\mu = 0.475$

$\sigma^2 = 1.25 \times 10^{-3}$

$\sigma = 0.354$

b $P(M > 0.5) = \cancel{P(1\ P(Z<0)}$

$= 1 - P(0 > Z) - P(0 < Z < 0.5)$

$= 0.5 - P(0 < Z < 0.5)$

$= 0.5 - P(Z < 0.5)$

$Z = \frac{0.5 - 0.475}{0.354}$

$= 0.7071$

$= 0.71$

$P(M > 0.5) = 0.5 - \cancel{0.2580}$ 0.2611

$= \cancel{0.242}$

$= 0.2389$

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3 $\mu = 3$

a $\lambda = 1/3$

this is the mean of the poisson distribution and indicates that there are 1/3 flaws per meter

b $P(X = 2) = \frac{\lambda^k e^{-\lambda}}{k!}$

where $\lambda = 5 \times \mu_{poisson}$ and $k = 2$

$= 5/3$

$P(X = 2) = \frac{5/3^2 \, e^{-5/3}}{2}$

$= 0.2623$

4a b

5 Bias is difference between mean of the estimator and the true value.

$Bias = \mu_{\hat{\sigma}^2_k} - \sigma^2$

s^2 is unbiased.

so $\mu_{s^2} = \sigma^2$

$= \mu_{\frac{N-1}{k} s^2}$ using rule $A\mu_x = \mu_{Ax}$

$= \frac{N-1}{k} \mu_{s^2} - \sigma^2$

$= \frac{N-1}{k} \sigma^2 - \sigma^2$

$= \sigma^2 \left(\frac{N-1}{k} - 1\right)$

$= \sigma^2 \left(\frac{N-1-k}{k}\right)$

5b hint: $\sigma^2_{s^2} = \frac{2\sigma^4}{N-1}$ $\hat{\sigma}^2_k = \frac{(N-1)s^2}{k}$

find $\sigma^2_{\hat{\sigma}^2_k}$

$$\sigma^2_{(\hat{\sigma}^2_k)} = \sigma^2_{\left(\frac{N-1}{k}\right)s^2} \qquad a^2\sigma^2_x = \sigma^2_{ax}$$

$$= \left(\frac{N-1}{k}\right)^2 \sigma^2_{s^2}$$

$$= \left(\frac{N-1}{k}\right)^2 \times \frac{2\sigma^4}{N-1}$$

$$= \frac{(N-1)\cancel{2} \times 2\sigma^4}{k^2}$$

c find $MSE_{\hat{\sigma}^2_k}$

let $\theta = \hat{\sigma}^2_k$ our parameter

$$MSE_{\hat{\theta}} = (\mu_{\hat{\theta}} - \theta)^2 + \sigma^2_{\hat{\theta}}$$

$$= Bias^2 + \sigma^2_{\hat{\theta}}$$

$$= \left(\frac{\sigma^2(N-k-1)}{k}\right)^2 + \frac{2(N-1)\sigma^4}{k^2}$$

$$= \frac{\sigma^4}{k^2}(N-k-1)^2 + \frac{\sigma^4}{k^2}2(N-1)$$

$$= \frac{\sigma^4}{k^2}\left(N^2 - kN - N - kN + k^2 + k - N + k + 1 + 2N - 2\right)$$

$$= \frac{\sigma^4}{k^2}\left(N^2 - 2kN + k^2 + 2k - 1\right)$$

5d $\frac{d\,mse}{dk} = \sigma^4 \frac{d}{dk}\left(\frac{N^2}{k^2} - \frac{2N}{k} + 1 + \frac{2}{k} - \frac{1}{k^2}\right)$

$= \sigma^4\left(\frac{-\cancel{3}^{2} N^2}{k^3} + \frac{\cancel{4}^{2} N}{k^2} - \frac{2}{k^2} + \frac{2}{k^3}\right)$

$= 0$

$\frac{-2N^2}{k^3} + \frac{2N}{k^2} - \frac{2}{k^2} + \frac{2}{k^3} = 0$

$\cancel{-2kN^{2}\,+}$

$-2N^2 + 2kN - 2k + 2 = 0$

$2kN - 2k = 2N^2 - 2$

$(2N-2)k = 2N^2 - 2$

$k = \frac{2N^2 - 2}{2N-2}$ } This minimises the ~~E~~MSE

6 $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$f(x_1 \dots x_n) = \prod_{i=1}^{n} f(x_i; \mu, 1)$

$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2}}$

$= \frac{e^{\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}\right)}}{(2\pi)^{n/2}}$ ← e to the power of this.

$\ln f(x_1 \dots x_n) = -\frac{n}{2}\ln(2\pi) - \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}$

6 cont

$$\ln\left(f(x_1 \dots x_n)\right) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{i=1}^{n}(x_i - \mu)^2$$

$$= -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\left(\sum_{i=1}^{n} x_i^2 - 2\mu\sum_{i=1}^{n} x_i + N\mu^2\right)$$

$$\frac{\partial \ln\left(f(x_1 \dots x_n)\right)}{\partial \mu} = \frac{-1}{2}\left(-2\sum_{i=1}^{n} x_i + 2N\mu\right)$$

$$= 0$$

$$0 = -2\sum_{i=1}^{n} x_i + 2N\mu$$

$$2\sum_{i=1}^{n} x_i = 2N\mu$$

$$\hat{\mu} = \sum_{i=1}^{n} x_i$$

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