ECEN321 - Lab 2
Noise

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1 Introduction

This report investigates noise and how it propagates through operations and ways at reducing the effect noise has on systems. To do this various applications where noise would be present will be simulated.

2 Applications

2.1 Amplifier

The first application being simulated is a voltage amplifier. This amplifier will have a voltage gain of 10 and a normally distributed signal of DC level $3V$ with variance $4V^2$ will be put through it.

The method of signal averaging will be applied to the original signal. For this 16 signals generated the same way will be averaged. Increasing the number of samples will decrease the standard deviation by $\sigma = \frac{\sigma}{\sqrt{N}}$, so by using 16 signals we expect an an approximate reduction of $\frac{1}{4}$.

2.2 AC Signal

Another application where noise is prevalent is alternating current. This can be represented for a low amplitude sinusoid with an amplitude of $1V$. For this application a large amount of noise with $\sigma^2 = 1$ will be applied as there are cases where the original signal is indistinguishable from what is desired. To compare how closely related these signals are, the covariance and correlation coefficients will be used.

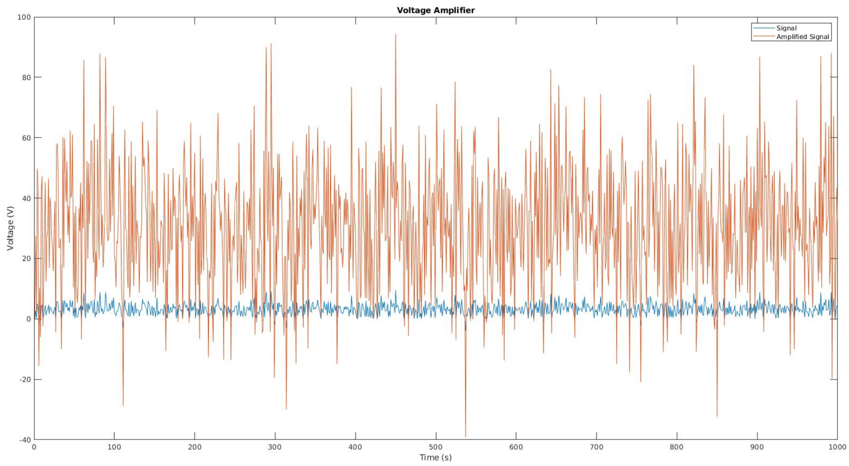

Figure 1: Voltage signals before and after amplification

2.3 Voltmeters

Voltmeters are commonly used measuring devices which, like all measuring devices have some uncertainty in their measurements. This is amplified when multiple signals are involved before a measurement has taken place. For this simulation two signals will be used which take the form $V_1 = 1.934 \pm 0.001$ and $V_2 = 2.53 \pm 0.01$; both signals are uniformly distributed. The resulting signal of $V_2 - V_1$ will be "measured", and the resulting PDF will be plotted.

3 Results and Discussion

3.1 Amplifier

The original signal had $\mu = 3.0418$ and $\sigma = 2.0041$. These values are fairly close to the set values of 3 and 2, and the signal can be seen in fig. 1. After amplifying this signal the follownig values were recorded: $\mu = 30.4184$ and $\sigma = 20.0405$. From this we can see that when a signal with uncertainty is multiplied by a constant both terms are multiplied. This has major implications on signal with noise as we now have a massive deviation from the target $30V$, even to the point where it's very negative. This is visible in fig. 1, and is not ideal for practical use.

As the above results are non-ideal, the noise should be reduced. As mentioned above, a method of doing so is to average many signals. A difference between the averaged signal and the original signals should be visible from

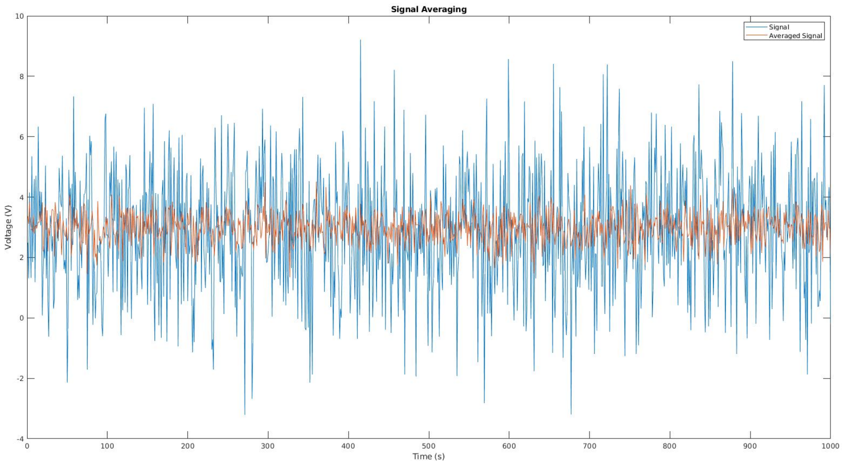

Figure 2: Voltage signals before and after averaging

viewing the mean and standard deviation of one of the used signals. One of these signals values is $\mu = 2.9038$ and $\sigma = 1.9926$. After averaging the new values are $\mu = 2.9809$ and $\sigma = 0.5017$. From this and fig. 2 we can see that the standard deviation (σ) has decreased by approximately a factor of 4 as hypothesized and that the mean has gotten closer to the true value. The change to the mean is likely from the removal of extreme values that were introduced by the larger noise. This emphasizes the importance of taking many measurements as precision is increased, and in extreme cases like this, the accuracy may be improved (or you'll get a better idea of the bias).

3.2 AC Signal

$$Cov = \begin{bmatrix} 0.4902 & 0.4828 \\ 0.4828 & 1.4863 \end{bmatrix}$$

After applying the noise to the sinusoid it can be seen in fig. 3 that the sinusoid is faintly visile in the scatter plot. The peaks and troughs are much lower and higher than expected though in some places. For comparing how closely related the new and old values are, the covariance and correlation coefficient are taken. The covariance matrix for the two datasets is above, the important value being the off diagonal (these are the same value), which is the covariance between the two sets. This value is 0.4828, which is positive indicating a positive relationship, is also small, indicating that the relationship is not strongly correlated. This indicates the noise on a signal can greatly distort it from being visible as the original signal. Because the time scale is

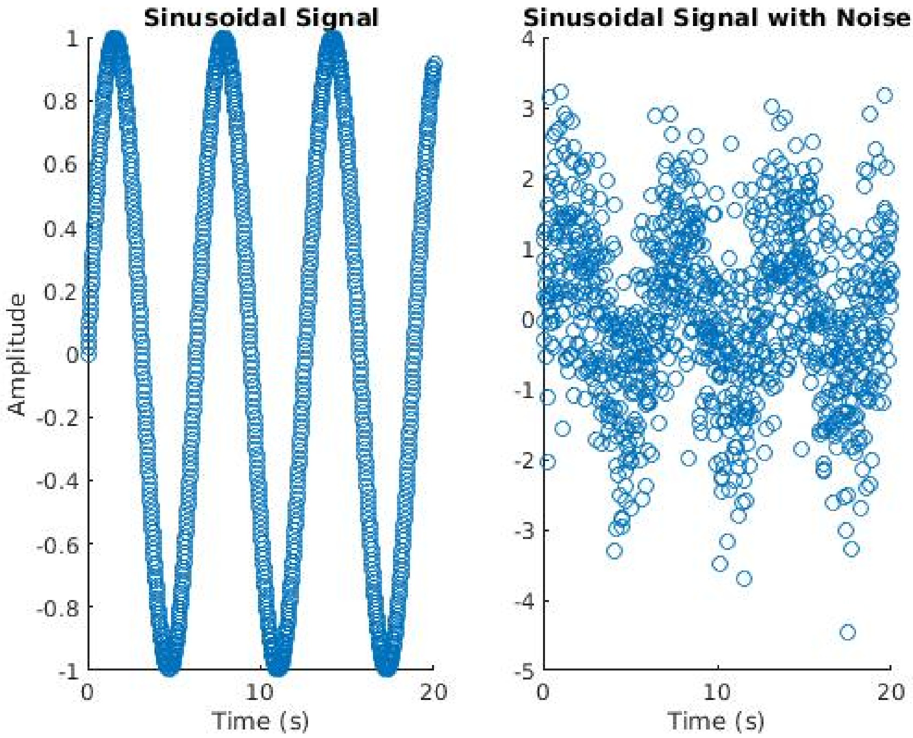

Figure 3: Sinusoidal signals before and after adding noise

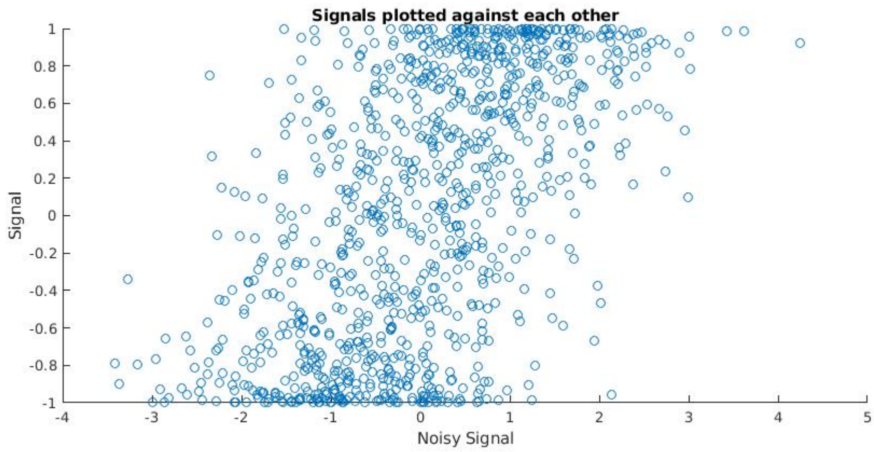

Figure 4: Sinusoidal signals with and without noise plotted against each other

tighter, it is possible to see the sinusoidal shape, but due to the large spread around the darker areas it is possible to understand how this can not be seen

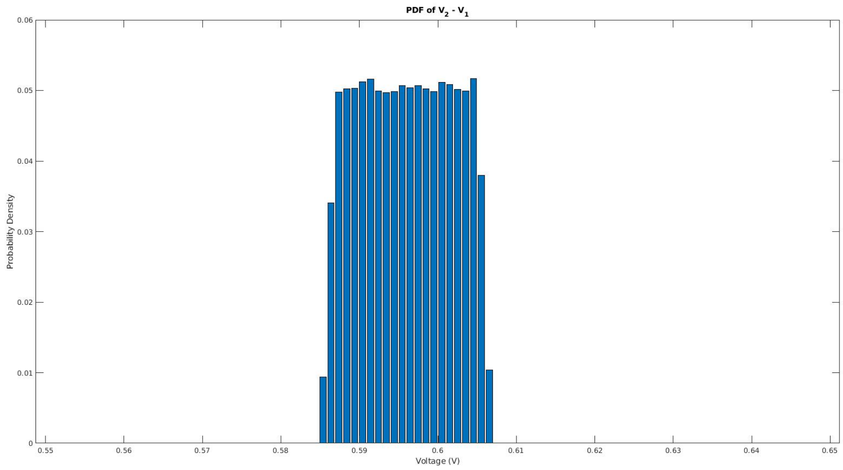

Figure 5: PDF of $V_2 - V_1$

as a sinusoid anymore, or close to.

The other variable gathered is the correlation coefficient which for this run of the program was $\rho = 0.5656$. This indicates that there is a moderate positive relationship [1], which can be seen when plotted against each other in fig. 4. Some correlation can be seen but there is lots of noise which make some of the data look essentially like noise.

3.3 Voltmeters

The final application explores the effect of two signals with noise being measured after one is subtracted from the other. By using two uniformly distributed signals we can see in fig. 5 that we get a uniformly distributed signal as a result. This signal has mean $\mu = 0.5960$ and std dev $\sigma = 0.0058$, and these values are consistent over multiple simulations. The mean is exactly the answer to $2.53 - 1.934$ and the standard deviation is close to the mean of the uncertainties, although this is less significant as the uncertainty is not the standard deviation. The final standard deviation is equal to the standard deviation of V_2 as the standard deviation of V_1 is on the order of 10^{-4} which matlab removes through rounding so we don't see it's effect initially. Multiplying by 1000 reveals that it's 0.0058051 which is the sum of the standard deviviations of both V_1 and V_2.

Additionally, it should be noted that the edges of the distribution are tapered and the distribution is not wholly uniform (fig. 5). This is a result of the nature of combining two distributions, being the convolution operator

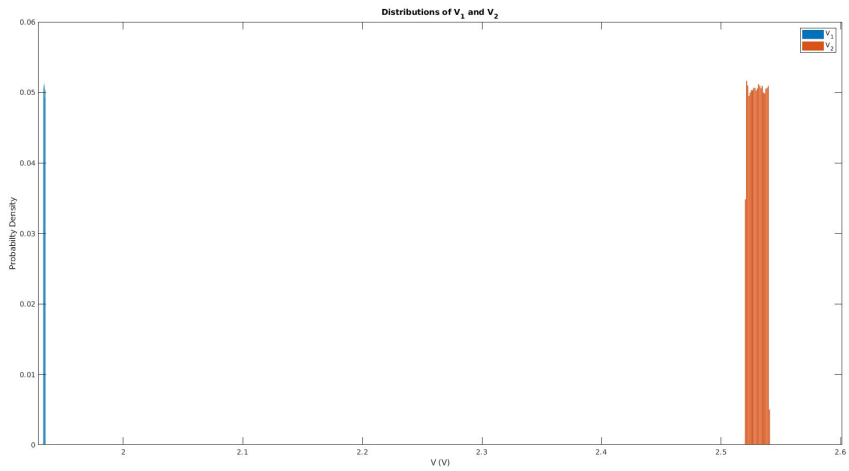

Figure 6: PDF of V_1 and V_2

[2].In fig. 6 we can see that V_1's distribution is a lot thinner than the other, so as this convolves with it, it will cause the edge to include some 0's in the sum, resulting in less included values and thus a lower bar.

4 Conclusion

The main take aways from this report are that, repeated measurements reduce the standard deviation by a facter of $\sqrt{N}$, increasing precision. This is important as any constant that multiplies a measurement with some uncertainty also multiplies the uncertainty, making it larger.

The addition of noise to a signal can make that signal much less distinguishable in shape from what it is. This only happens if the size of the noise is a significant percentage (including higher) than the amplitude of the wave as instead of following the signals shape it extends far past it. This happens to the point where it can be moderately correlated to an expected shape or totally uncorrelated which means we lose a lot of information about the signal or measurements.

Finally, when multiple independent measurements are combined, the distributions are convolved to get the resulting distributions. The means are the regular operation of that you are using, in this case, the difference.

References

[1] D. Rumsey, "How to interpret a correlation coefficient r - dummies," 2020. [Online]. Available: https://www.dummies.com/education/math/statistics/how-to-interpret-a-correlation-coefficient-r/

[2] S. Castrup, *Comparison of Methods for Establishing Confidence Limits and Expanded Uncertainties*. Integrated Sciences Group, 2020. [Online]. Available: https://www.isobudgets.com/pdf/papers/04_Confidence%20Limits%20%26%20Expanded%20Uncertainty.pdf

Appendices

A Q1 Code

```
clear

V = 3; %Mean Voltage level of the signal
N = 1000; %Number of samples in the signal
variance = 4; %Variance of the signal
sigma = sqrt(variance); %Std Dev of the signal
A_V = 10; %Gain of the amplifier
signal = V + sigma * randn(1,N); %Voltage Signal. DC Lvl V (3V), with noise that has mean 0V and stddev sigma (6V)

sample_mean = mean(signal)
sample_stddev = std(signal)

amplified_signal = A_V .* signal; %Pass the signal through an amplifier
amplified_sample_mean = mean(amplified_signal)
amplified_sample_stddev = std(amplified_signal)

figure(1)
plot(signal)
hold on
plot(amplified_signal)
xlabel("Time (s)")
```

```
ylabel("Voltage (V)")
title("Voltage Amplifier")
legend("Signal", "Amplified Signal")
```

B Q2 Code

```
clear

V = 3; % Mean Voltage level of the signal
N = 1000; % Number of samples in the signal
M = 16; % Number of signals to make
variance = 4; % Variance of the signal
sigma = sqrt(variance); % Std Dev of the signal
A_V = 10; % Gain of the amplifier
signals = V + sigma * randn(M,N); % Voltage Signal. DC
    Lvl V (3V), with noise that has mean 0V and stddev
    sigma (6V)

sample_mean = mean(signals(1,:)) % Mean of one of the
    signals
sample_stddev = std(signals(1,:)) % Std Dev of one of
    the signals

normalised_signal = sum(signals) ./ M;

normalised_sample_mean = mean(normalised_signal)
normalised_sample_std = std(normalised_signal)

figure(1)
plot(signals(1,:))
hold on
plot(normalised_signal)
xlabel("Time (s)")
ylabel("Voltage (V)")
title("Signal Averaging")
legend("Signal", "Averaged Signal")
```

C Q3 Code

```
clear

```

```
N = 1000;
x = linspace(0, 20, N);
y = sin(x);
variance = 1;
sigma = sqrt(variance);
noise = randn(1,N) * sigma;
noisy_signal = y + noise;

figure(1)
subplot(1,2,1)
scatter(x,y)
ylabel("Amplitude")
title("Sinusoidal Signal")

xlabel("Time (s)")
subplot(1,2,2)
scatter(x,noisy_signal)
xlabel("Time (s)")
title("Sinusoidal Signal with Noise")

figure(2)
scatter(noisy_signal, y)
xlabel("Noisy Signal")
ylabel("Signal")
title("Signals plotted against each other")

CC = cov(y, noisy_signal)
rho = CC(1,2)/sqrt(CC(1,1)*CC(2,2))
```

D Q4 Code

```
clear

N = 100000;
v1_val = 1.934;
v1_uncertainty = 0.001;
V_1 = unifrnd(v1_val-v1_uncertainty, v1_val+
    v1_uncertainty, 1, N);
v2_val = 2.53;
v2_uncertainty = 0.01;
V_2 = unifrnd(v2_val-v2_uncertainty, v2_val+
```

```
    v2_uncertainty, 1, N);

V_3 = V_2 - V_1;

dx = 0.01;
boundary = 1;
edges = linspace(0.55, 0.65, boundary / dx);

[histVals, bins] = hist(V_3, edges);
figure(1)
bar(edges, histVals/N);
sum(histVals/N)
title("PDF of V_2 - V_1")
xlabel("Voltage (V)")
ylabel("Probability Density")

std(V_1)
std(V_2)

mean(V_3)
std(V_3)*1000

figure(2)
edges = linspace(1.93, 1.94, boundary/dx);
[histV1, bins] = hist(V_1, edges);
bar(edges,histV1/N, 'EdgeColor','none')

hold on
edges = linspace(2.5, 2.6, boundary/dx);
[histV2, bins] = hist(V_2, edges);
bar(edges,histV2/N,'EdgeColor','none')
title("Distributions of V_1 and V_2");
xlabel("V (V)")
ylabel("Probabilty Density")
legend(["V_1","V_2"])
```