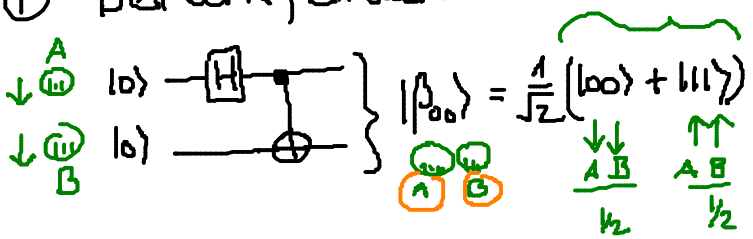
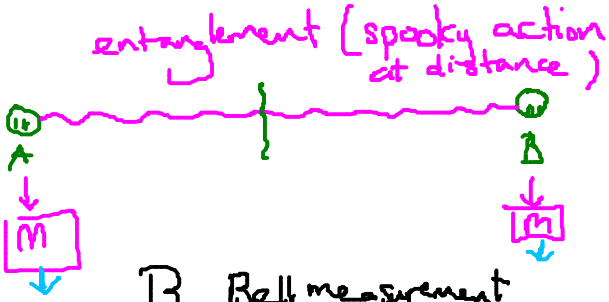
Gottesman & Chuang (1999): teleportation for computation

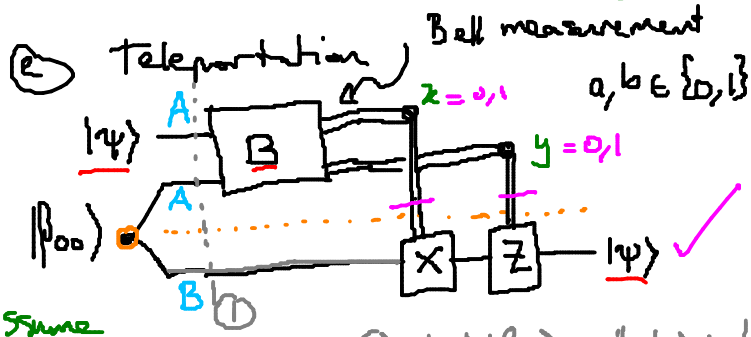
Recap: EPR: Einstein-Podolsky-Rosen (1935)

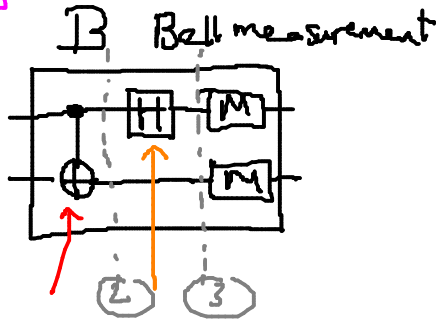
① Bell (EPR) circuit:





② Teleportation





assume $|\psi\rangle = a|0\rangle + b|1\rangle$

$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

① $|\psi\rangle|\beta_{00}\rangle = (a|0\rangle + b|1\rangle)\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(a|0\rangle(|00\rangle + |11\rangle) + b|1\rangle(|00\rangle + |11\rangle))$

② $\frac{1}{\sqrt{2}}[a|0\rangle(|00\rangle + |11\rangle) + b|1\rangle(|10\rangle + |01\rangle)]$

$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

③ $\frac{1}{2}[a(|0\rangle+|1\rangle)(|00\rangle+|11\rangle) + b(|0\rangle-|1\rangle)(|10\rangle+|01\rangle)]$

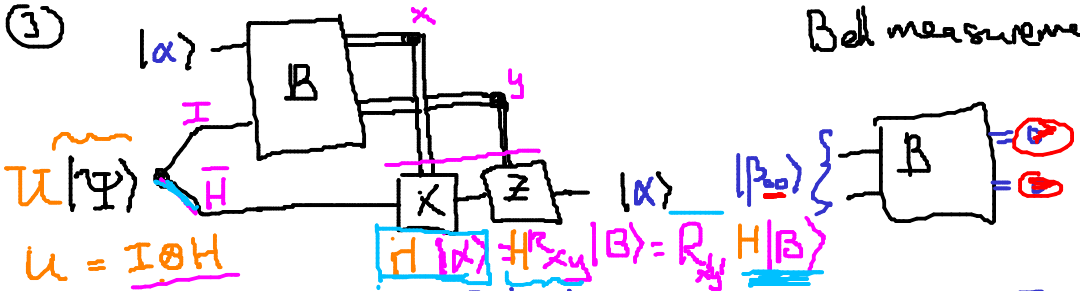
$= \frac{1}{2}[|00\rangle(a|0\rangle+b|1\rangle) + |01\rangle(a|1\rangle+b|0\rangle) + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|1\rangle - b|0\rangle)]$

$|\psi\rangle$ $\quad X|\psi\rangle$ $\quad Z|\psi\rangle$ $\quad XZ|\psi\rangle$

$X^2 = I = Z^2$

$\begin{bmatrix} a \\ b \end{bmatrix} \xrightarrow{Z} \begin{bmatrix} a \\ -b \end{bmatrix} \xrightarrow{X} \begin{bmatrix} -b \\ a \end{bmatrix}$

③



Bell measurement

4 Bell states: $|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle$

$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

$|01\rangle \to |01\rangle + |11\rangle \to |01\rangle + |10\rangle$

$|10\rangle \to |00\rangle - |10\rangle \to |00\rangle - |11\rangle$

$|11\rangle \to |01\rangle - |11\rangle \to |01\rangle - |10\rangle$

Universal

Quantum Models of Computation

1. Circuit (gate) model ✓
2. Adiabatic model ✓
3. Topological model ?
4. Measurement-based model
5. Quantum Walk

Hamiltonian

$U(t) = e^{-iHt}$ unitary matrix

$|\psi(t)\rangle = U(t)|\psi(0)\rangle$

$|z_0\rangle, \lambda_0 \leftarrow H_0$ initial Hamiltonian

$|z_1\rangle, \lambda_1 \leftarrow H_1$ target

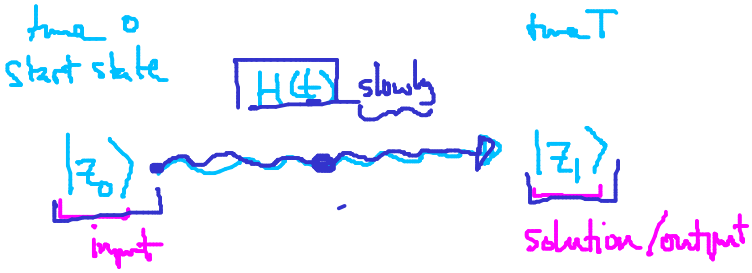
ground state of Hamiltonians

eigenvalues of H = energy levels

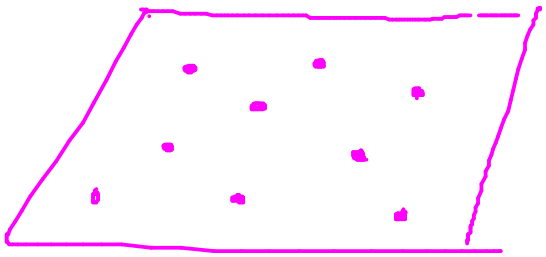
$H(t) = (1-\alpha(t))H_0 + \alpha(t)H_1$

$\alpha(0) = 0 \quad 0 \le \alpha(t) \le 1$

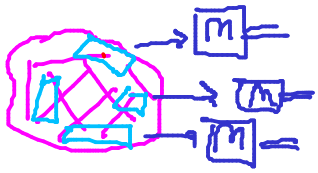
Adiabatic Theorem



(3) Topological: [Kitaev] Freedman — Particles + anyons (fault tolerance)



(4) MBQC [Briegel et al]

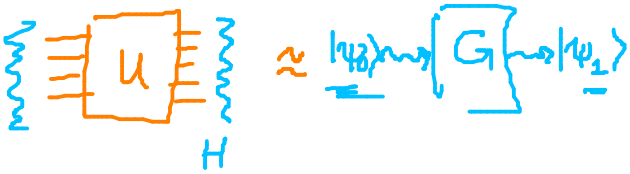


output

$|\Psi\rangle$?

(5) Quantum walk [Childs]

optics simulation of quantum walk ?



$e^{-iA(G)t}$