System Simulation

- Homework 4: State-Space Representation for an RLC Circuit
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```
clc; clear;

%Component values
R1= 500;
R2= 1000;
R3= 1000;
C1= 4.7*1e-6;
C2= 4.7*1e-6;
C3= 4.7*1e-6;
L= 2;

%Establishing component values into our matrices
A = [ -1/(R2*C1), 1/(R2*C1), 0, 1/C1;...
        1/(R2*C2), -(1/(R2*C2)+1/(R3*C2)), 1/(R3*C2), 0;...
        0, 1/(R3*C3), -1/(R3*C3), 0;...
        -1/L, 0, 0, -R1/L];

B = [ 0; 0; 0; 1/L];

C = [ 0 0 1 0];
D= 0;

% State-Space Equation to Transfer Function
[b,a] = ss2tf(A, B, C, D); transferFunc =
tf(b,a)
```

```
transferFunc =
                        4.816e09
  ---------------------------------------------------
s^4 + 1101 s^3 + 4.55e05 s^2 + 1.019e08 s + 4.816e09
Continuous-time transfer function.
```

```
%Finding Eigen Values and Poles of Transfer Function
eigenValues = eig(transferFunc) Poles =
pole(transferFunc)
```

```
eigenValues =
```

```
   1.0e+02 *

  -6.0605 + 0.0000i
  -2.1649 + 2.8500i
  -2.1649 - 2.8500i
  -0.6204 + 0.0000i


Poles =

   1.0e+02 *

  -6.0605 + 0.0000i
  -2.1649 + 2.8500i
  -2.1649 - 2.8500i
  -0.6204 + 0.0000i
```

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Handwritten Analysis

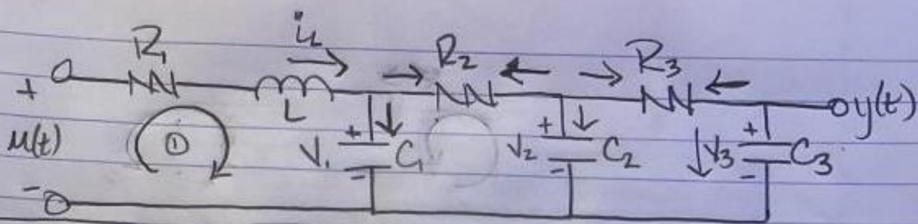
A) ①
② All Same from Part C, I did C before A.
③
④

$$y = \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix} x$$

$\dot{x}$ = SAME AS A

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x$$

c)



B) ① $-i_L + C_1\dot{V}_1 + \frac{V_1 - V_2}{R_2} = 0$

② $\frac{V_2 - V_1}{R_2} + \frac{V_2 - V_3}{R_3} + C_2\dot{V}_2 = 0$

③ $\frac{V_3 - V_2}{R_3} + C_3\dot{V}_3 = 0$

A= $V_1 = X_1$
$V_2 = X_2$
$V_3 = X_3$

C) KVL $0 = -u + R_1 i_L + L\frac{di_L}{dt} + V_1$

$\frac{L\dot{i}_L}{L} = \frac{u - R_1 i_L - V_1}{L} \Big\{ \dot{i}_L = \frac{u - R_1 i_L - V_1}{L}$

$\dot{X}_4 = -\frac{1}{L}X_1 - \frac{R_1}{L}X_4 + \frac{1}{L}u$

① $\frac{C_1\dot{V}_1}{C_1} = i_L - \frac{V_1 - V_2}{R_2} \Rightarrow \dot{V}_1 = \frac{i_L}{C_1} - \frac{V_1 - V_2}{C_1 R_2}$

② $\frac{C_2\dot{V}_2}{C_2} = -\left(\frac{V_2 - V_1}{R_2}\right) - \left(\frac{V_2 - V_3}{R_3}\right) \Rightarrow \dot{V}_2 = \frac{V_1 - V_2}{R_2 C_2} + \frac{V_3 - V_2}{R_3 C_2}$

③ $\frac{\cancel{C_3}\dot{V}_3}{\cancel{C_3}} = \frac{V_2 - V_3}{R_3} \Rightarrow \dot{V}_3 = \frac{V_2 - V_3}{R_3 C_3}$

④ $y = V_3$

① $\dot{x}_1 = \frac{-1}{R_2C_1}x_1 + \frac{1}{R_2C_1}x_2 + \frac{1}{C_1}x_4$

② $\dot{x}_2 = \frac{1}{R_2C_2}x_1 - \left(\frac{1}{R_2C_2+} + \frac{1}{R_3C_2}\right)x_2 + \frac{1}{R_3C_2}x_3$

③ $\dot{x}_3 = \frac{1}{C_3R_3}x_2 - \frac{1}{R_3C_3}x_3$

$y = V_3$

④ $\dot{x}_4 = -\frac{1}{L}x_1 - \frac{R_1}{L}x_4 + \frac{1}{L}u$

D)

$$\dot{x} = \begin{pmatrix} -1/R_2C_1 & 1/R_2C_1 & 0 & 1/C_1 \\ 1/R_2C_2 & -(1/R_2C_2 + 1/R_3C_2) & 1/R_3C_2 & 0 \\ 0 & 1/R_3C_3 & -1/R_3C_3 & 0 \\ -1/L & 0 & 0 & -R_1/L \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1/L \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} x$$

E) Because if C_1, C_2, C_3 were all outputs

$$y = V_1 + V_2 + V_3 \Rightarrow x_1 + x_2 + x_3$$

$$\Downarrow$$

that would make $C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$