System Simulation

- Homework 3: Buck Converter- Transfer Function Approach
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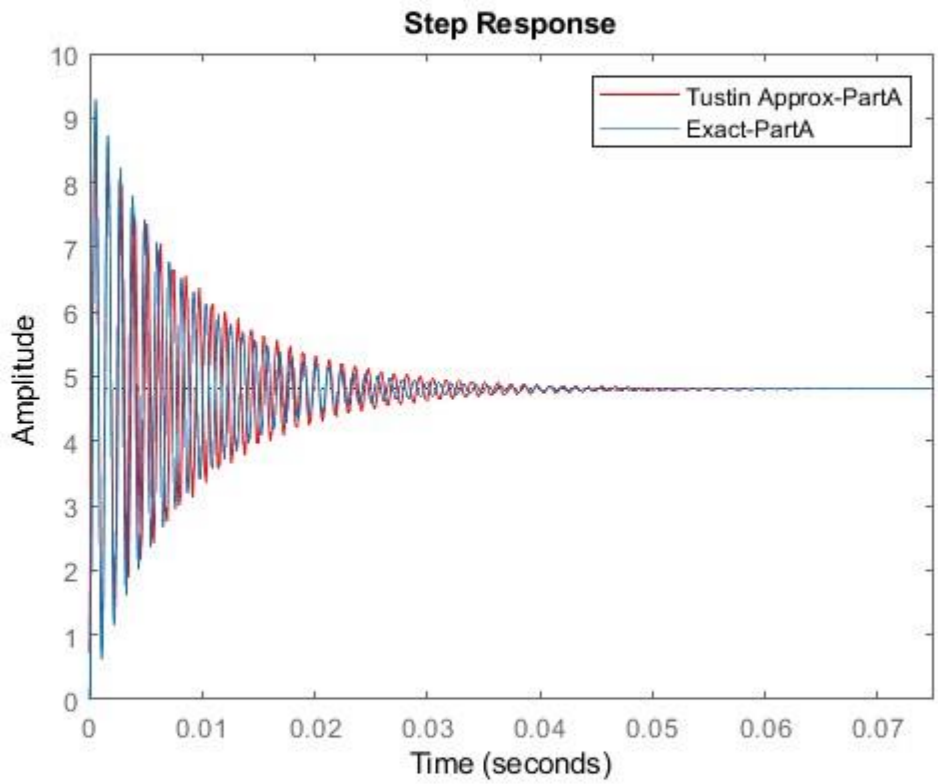
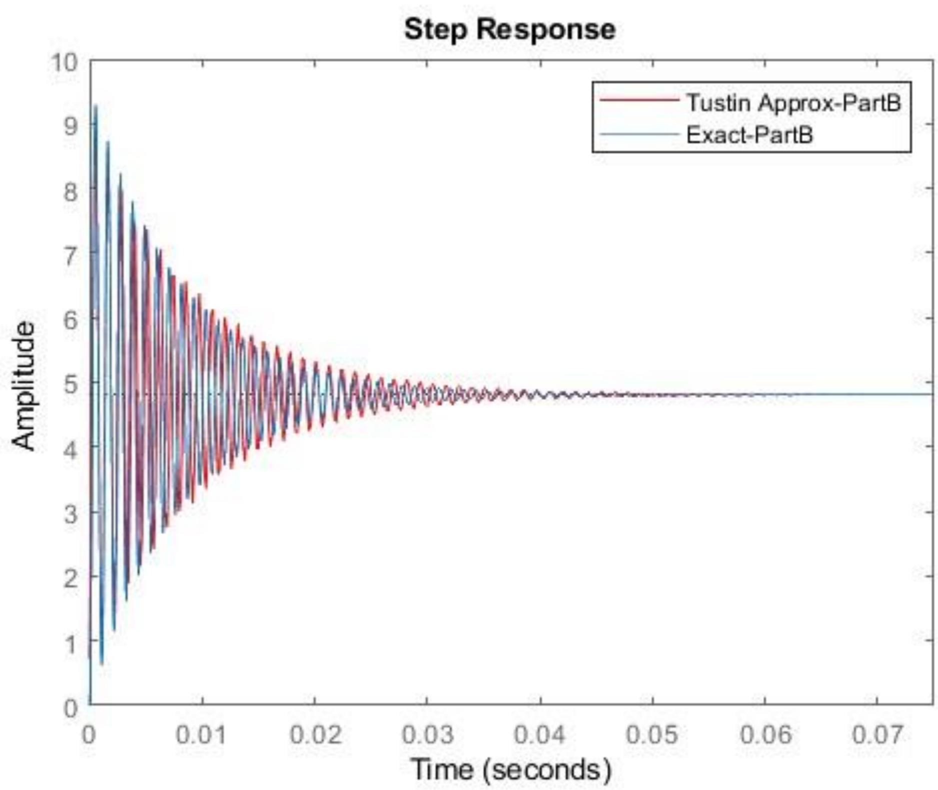
```
clc; clear; close all;

timeSample = .00015; % 2/|-125 - 5769.26*i = T < .000347
tfinal=.075; Tvec= [-2*timeSample:timeSample:tfinal];
N= length(Tvec);

%Initialized Systems
sys1 = zeros(1,N);
sys2 = zeros(1,N);

%Coefficients
A1=4/3*1e7;
A2=4*1e8;
B=1;
C=250;
D=3.33*1e7;
z= tf('z');

%Exact Function
SysExact1 = tf(A1, [B C D]);
SysExact2 = tf(A2, [B C D]);

%Unit Steps
uA=12*ones(1,N);
uA(1)=0; uA(2)=0; uB
= .4 * ones(1,N);
uB(1) =0; uB(2) =0;

%Numerators num1=
A1*timeSample^2; numB1=
A2*timeSample^2; num2=
2*A1*timeSample^2; numB2=
2*A2*timeSample^2; num3=
A1*timeSample^2; numB3=
A2*timeSample^2;

%Denominators
Den1 = (4+2*C*timeSample+D*timeSample^2);
Den2 = (-8+2*D*timeSample^2);

Den3 = 4-2*C*timeSample+D*timeSample^2;
```

```
%Looping for k=1: N-2      sys1(k+2) = (-(Den2*sys1(k+1) +
(Den3)*sys1(k))+(num1)*uA(k
+2)+(num2)*uA(k+1)+(num3)*uA(k))/(Den1);      sys2(k+2) = (-
(Den2*sys2(k+1)  +  (Den3)*sys2(k))+(numB1)*uB(k
+2)+(numB2)*uB(k+1)+(numB3)*uB(k))/(Den1); end
```

Plots

```
figure hold on xlim([0 tfinal]) plot(Tvec,
sys1, 'r') step(12*SysExact1, tfinal)
legend('Tustin Approx-PartA', 'Exact-PartA')
hold off

figure hold on xlim([0 tfinal]) plot(Tvec,
sys2, 'r') step(0.4*SysExact2, tfinal)
legend('Tustin Approx-PartB', 'Exact-PartB')
hold off
```


Step Response
Tustin Approx-PartA
Exact-PartA
Amplitude
Time (seconds)
0
1
2
3
4
5
6
7
8
9
10
0.01
0.02
0.03
0.04
0.05
0.06
0.07


Step Response
Tustin Approx-PartB
Exact-PartB
Amplitude
Time (seconds)
0
1
2
3
4
5
6
7
8
9
10
0.01
0.02
0.03
0.04
0.05
0.06
0.07

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Handwritten Analysis

Part A:

$$V_{out} = \frac{4/3 \times 10^7}{S^2 + 250s + 3.33\times 10^7} = \frac{A}{BS^2 + Cs + D}\left(\frac{1/s^2}{1/s^2}\right)$$

$$= \frac{\frac{A}{S^2}}{1 + \frac{C}{S} + \frac{D}{S^2}} \Rightarrow \frac{\frac{A * t^2(z+1)^2}{4(z-1)^2}}{1 + \frac{Ct(z+1)}{2(z-1)} + \frac{Dt^2(z+1)^2}{4(z-1)^2}}\left(\frac{4(z-1)^2}{4(z-1)^2}\right)$$

$$= \frac{At^2(z+1)^2}{4(z-1)^2 + 4Ct(z+1)(z-1) + Dt^2(z+1)^2}$$

$$= \frac{At^2(z^2+2z+1)}{(4z^2-8z+4) + 2Ct(z^2-1) + Dt^2(z^2+2z+1)}$$

$$= \frac{z^2(At^2) + 2z(At^2) + At^2}{z^2(4+2Ct+Dt^2) + z(-8+2Dt^2) + (4-2Ct+Dt^2)}$$

$$V_{out}(z^2) = \frac{\text{Numerator}}{4+2Ct+Dt^2} - \frac{\text{Denominator}}{4+2Ct+Dt^2}$$

$$\left(z^2(4+2Ct+Dt^2) + z(-8+2Dt^2) + (4-2Ct+Dt^2)\right)V_{out}$$

$$V_{out} * z^2(4+2Ct+Dt^2) + V_{out} * z(-8+2Dt^2) + V_{out}(4-2Ct+Dt^2)$$

$$= z^2(At^2)V_{in} + z(At^2)V_{in} + V_{in}(At^2)$$

$$V_{out} * z^2(4+2Ct+Dt^2) = -\left[V_{out} * z(-8+2Dt^2) + V_{out} * (4-2Ct+Dt^2)\right]$$

$$\frac{+ z(At^2)V_{in} + V_{in}(At^2)}{4+2Ct+Dt^2}$$

$$V_{out} * z^2 = \frac{-\left[z(-8+2Dt^2)V_{out} + V_{out}(4-2Ct+Dt^2)\right] + z(At^2)V_{in} + V_{in}(At^2)}{4+2Ct+Dt^2}$$

$$V_{out}(k+2) = \frac{-\left[(-8+2Dt^2)V_{out}(k+1) + (4-2Ct+Dt^2)V_{out}(k)\right] + (At^2)V_{in}(k+1) + (2At^2)V_{in}(k) + (At^2)V_{in}(k+2)}{4+2Ct+Dt^2}$$

Part B:

Part B

$$G(s) = \frac{4\times10^{8}}{s^2+250s+3.33\times10^{7}}$$

$A = 4\times10^{8}$
$B = 250$
$C = 3.33\times10^{7}$

$$= \frac{A}{s^2+Bs+C}\left(\frac{1/s^2}{1/s^2}\right) = \frac{\frac{A}{s^2}}{1+\frac{B}{s}+\frac{C}{s^2}}$$

* Tustin Substitutions *

$$= \frac{\frac{At^2(z+1)^2}{4(z-1)^2}}{1+\frac{Bt(z+1)}{(z-1)}+\frac{Ct^2(z+1)^2}{4(z-1)^2}} * \left(\frac{4(z-1)^2}{4(z-1)^2}\right)$$

$$= \frac{At^2(z^2+2z+1) \Rightarrow z^2(At^2)+z(2At^2)+At^2}{z^2(4+4Bt+Ct^2)+z(-8+2Ct^2)+(4-2Bt+Ct^2)}$$

* this is the Same As A, but a different Coefficient for variable A!!