System Simulation Homework

January 9, 2021

General Homework Guidelines

Submission of homework will be done electronically via the course's Brightspace page. Each problem will have a submission folder. The required pieces for each simulation are

(A) a .pdf file containing all plots for the simulations,

(B) an .m file containing only the code used to perform the simulation,

(C) an .m file containing whatever other code was used in the problem, and

(D) a .pdf file of a scan of any work done by hand.

Problem 01: Simulation of a Difference Equation

Implement the difference equation given by

$$\begin{aligned} x[0] &= 0.11 \\ x[k+1] &= \alpha\,(1 - x[k])\,x[k], \end{aligned}$$

for at least seven values of gain α. Six of the values must be $\alpha = 0.80$, $\alpha = 1.35$, $\alpha = 2.75$, $\alpha = 3.20$, $\alpha = 3.52$, and $\alpha = 4.0$. Then choose a value of $\alpha \in (3.57, 4.0)$. Create a plot of $x[k]$ versus k that includes at least 100 samples, but not so many that the system dynamics are obscured.

Please upload to the course Brightspace the required documents.

It is highly recommended that you use MATLAB for this assignment, as you are expected to use MATLAB for the assignments for the rest of the semester. If you wish to use another program, please discuss it with me before doing so.

The purpose of this assignment is for you to refresh yourself on how to write an m-file in MATLAB and plot the results. You will be doing this for one or two problems a week for the rest of the semester.

The MATLAB commands that I used are

- Assignment and arithmetic commands
- $zeros$
- $linspace$
- for
- $plot$
- $title$, $xlabel$, and $ylabel$

Problem 02: SR-71 Supersonic Inlet

The SR-71 was a supersonic aircraft that could fly at speeds in excess of $Mach$ 3.2. Its propulsion system used two Pratt & Whitney J-58 turbo-ramjet engines. Essential to the performance of these engines is the supersonic inlet on their front. These inlets convert fast cold air into slow hot air through a normal shock wave that sits just downstream of the inlet throat. The position of the shock wave determines the efficiency of the energy conversion. The closer the shock is to the throat, the more efficient the conversion, however, if the shock moves up past the throat, the engine will unstart in milliseconds, potentially causing a catastrophic failure. The shock position is controlled by linearly opening and closing downstream bypass doors. The transfer function for $Mach$ 2.5 operation is

$$H(s) = \frac{X_s(s)}{W_{BP}(s)} = \frac{50e^{-0.015s}}{s + 2 + 50e^{-0.02s}}, \tag{1}$$

which can be approximated by

$$H(s) \approx \frac{50(-s^2 + 33.3333s + 13333.3)}{s^3 + 185.333s^2 + 12133.3s + 693333}. \tag{2}$$

Simulate the approximated transfer function given in Equation 2 using the Madwed (see page 31 of the text) operational substitution technique to obtain a difference equation. Provide an impulse response and a step response from your simulation, along with your analysis, your programs, and the plots. Note that for the area of the "continuized" discrete-time (Kronecker) delta to be one, the height of the discrete-time delta must be $\frac{1}{T}$, where T is the time-step.

Problem 03: Buck Converter - Transfer Function Approach

A DC-DC Buck Converter is a circuit that is used to convert voltages and/or currents. Currently, buck converters are seeing use as regulators for photovoltaic cell outputs. An ideal Buck converter

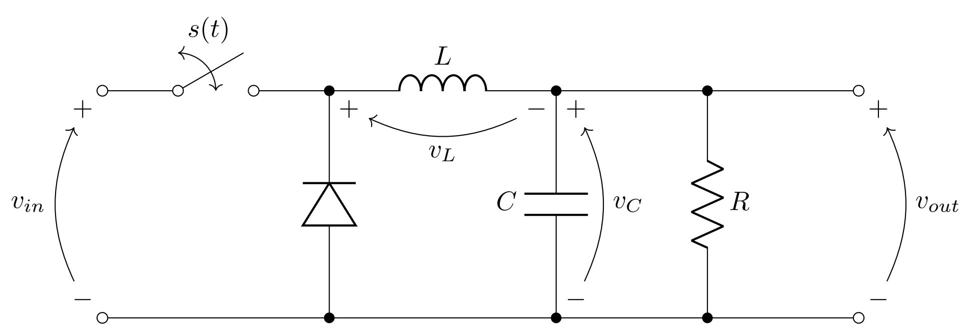

Figure 1: Ideal DC-DC Buck Converter.

is shown in Figure 1. The switch on the input voltage makes this a non-linear system; additionally, both the input voltage and the switching duty cycle can be considered inputs to the system. A technique called state-space averaging is used to obtain transfer functions from each input v_{in}(t) and $s(t)$ to the output $v_{out}(t)$ by assuming that $s(t)$ is a constant equal to its duty cycle and $V_{in}(t)$ is a constant voltage, respectively. These transfer functions are

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{4}{3} \times 10^7}{s^2 + 250s + 3.33 \times 10^7}$$

and

$$\frac{V_{out}(s)}{S(s)} = \frac{4 \times 10^8}{s^2 + 250s + 3.33 \times 10^7}.$$

(A) Simulate the response of $v_{out}(t)$ to $v_{in}(t)$ if $v_{in}(t) = 12\ \mathcal{U}_s(t)$ using the Tustin operational substitution technique to obtain a difference equation.

(B) Simulate the response of $v_{out}(t)$ to $s(t)$ if $s(t) = 0.4\ \mathcal{U}_s(t)$ using the Tustin operational substitution technique to obtain a difference equation.

Be sure to include your analysis, your programs, and the plots.

Problem 04: State-Space Representation for an RLC Circuit

Consider the circuit shown in Figure 2.

(A) Obtain a state-space representation for the RLC circuit using the following steps. Note that your state-space representation should be an expression in terms of the component values R_1, R_2, R_3, C_1, C_2, C_3, and L.

(a) Assign each capacitor voltage and the inductor current to a state. It is probably a good idea to have v_ℓ assigned to x_ℓ.

(b) Write the sum of the currents into each of the nodes at v_1, v_2, and v_3 in terms of the state variables.

(c) Write the sum of the voltages in the loop that includes the input and capacitor C_1 in terms of the state variables.

(d) Combine the resulting equations into a single matrix equation for the state-update equation.

(e) If each of the capacitor voltage is considered an output, explain why

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

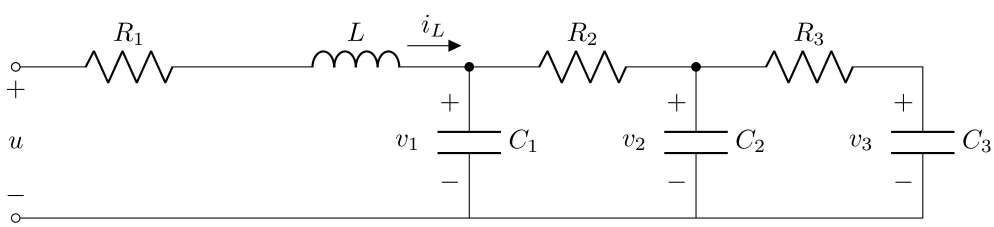

Figure 2: RLC Circuit for Problem 04.

(B) Using the component values shown in Table 1, write the numeric state-space representation.

(C) Assume that only the voltage across capacitor C_3 is the output and write the numeric state-space representation using the component values in Table 1. Then determine the associated transfer function (you may use the MATLAB command $ss2tf$). Verify that the eigenvalues of A and the poles of the transfer function are the same.

Component	Value
R_1	$500\ \Omega$
R_2	$1\ k\Omega$
R_3	$1\ k\Omega$
C_1	$4.7\ \mu F$
C_2	$4.7\ \mu F$
C_3	$4.7\ \mu F$
L	$2\ H$

Table 1: Component Values for RLC Circuit.