MATH3024:Complex Systems
Conways game of life on a network

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Contents

1 Introduction

Billy Corgan the creator and primary song writer for the Smashing pumpkins once wrote "Despite all my rage, I am still just a rat in a cage". Although Corgan is well known for his insightful lyrics, I have purposefully over-analysed this by considering this platitude as a global metaphor for geo-political life and death, the question of how to model the relations of trapped or inhibited nations, and to understand properties of long-term survival.

1.1 Overview of the Model

Conway's game of life; a famous 2D-cellular automata, in which a grid of dead and alive tiles evolve through rule sets based on there locally adjacent, and diagonal tiles.

The primary model behind the "game of life" is how it represents social overpopulation, death, reproduction, development and growth. By generalising the model to a network, I believe we can investigate further interesting qualities of this social system such as the 'cut off nature' of development. Be it the physical geographic barriers, political milieu or social conditions, each social group can be cutoff in some sense from another.

The relevant cliques and clustering of the network can represent interesting geo-politics and modern sociological conditions. In the original "game of life" the game is played on a 2D grid, which is the same as a 8-regular graph(known as a moore neighborhood) this is to say each node has an equal 8 opportunities of an alive neighbors. By investigating how the properties of asymmetric graphs which unevenly distribute the neighbors of each node, can affect the stability and evolution of "the game of life" we can investigate the effects of inequality and isolation of societies on global development and progress.

1.2 Literature Review

There are 3 main papers which have influenced my model;

Conway's game of life[2]

Conway's Game of Life, first introduced by John Conway in 1970 in his paper Mathematical Games in Scientific American. At the time this paper was semi-revolutionary for mathematical modeling.

From this paper comes the original process of alive and dead, although in my system the topology of the 'space' form which the automation occurs will be less symmetrical.

Dynamical Phase Transitions in Graph Cellular Automata [1]

In this 2020 paper, Miguel Aguilera and Enrique Muñoz explore how Conway's Game of Life can be extended from the traditional 2D grid to random graph structures.

From this paper came the use of random graphs to design the model, as well as certain techniques used in the paper.

References

[1] F. Behrens, B. Hudcová, and L. Zdeborová. Dynamical phase transitions in graph cellular automata. *Physical Review E*, 109(4), Apr. 2024.

[2] J. Conway. Conway's game of life. *Scientific American*, 1970.

2 Model Description

We first formally define our model mathematically as well as all the variables we wish to investigate.

State Space

We define the space of all possible states of the system or state space, as a non-directed graph

$$G = \{V, B, E\}$$

- **V** - set of vertices/nodes on graph
- $\mathbf{B} \subseteq \mathbf{V}$ - set of black or dead nodes(implied that all not dead are alive)
- $\mathbf{E} = \mathbf{V^2}$ - Edges on the graph(determined by binary-sets of vertices)

alive/dead neighbourhood of a node 'n' - the number of alive/dead adjacent nodes, depicted as $\mathbf{a_n}, \mathbf{d_n}$ respectively.

dead/alive - we say a node is dead if it is an element of B, otherwise it is alive, we refer to alive and dead nodes as 1/0 respectively.

Initialisation

How we initialise is especially important to what kind of emergence and development we get within the graph. To simplify the initialisation into 3 variables, we define each initialisation as the Erdos-Renyi type of random graph with parameters n,p,pb

- **n** - the number of nodes
- **p** - probability that any edge exists
- **pb** - probability that any node is initially black

Rule Set

Originally, with Conway's Game of Life, the rules are as follows: given a state, we get to the next state by these conditions. Given a node-tile as n, we give the mapping:

$$m(n) = \begin{cases} a_n > 3, a_n < 2 & \rightarrow dead \quad (over/underpopulation) \\ a_n = 3 & \rightarrow alive \quad (just\ right) \\ a_n = 2 & \rightarrow unchanged \end{cases}$$

We follow this convention by considering percentages, so instead of 2 or less, we consider less than 2/8 neighbours are alive. Let d = degree of n, we have

$$m(n) = \begin{cases} a_n/d > 0.4, a_n/d < 0.125 & \rightarrow dead \quad (over/underpopulation) \\ a_n/d \in (0.25, 0.4) & \rightarrow alive \quad (just\ right) \\ a_n/d \in (0.125, 0.25) & \rightarrow unchanged \end{cases}$$

Parameters

Our main measurements are

- **p** - edros-renyi edge probability (representing the globalisation or generally connectiveness of a environment)
- **pb** - probability of any node initialising as dead (representing the initial state of the world)

It is by investigating the relationship between "globalisation" and ""

3 Methods

3.1 data extraction

We use the file named bootleg.py to run a high frequency of trials. We standardise the number of nodes at 50 for 2 reasons, 1. we need the number of nodes to be high enough as to stabilise as the erdos percent and percent of initial dead nodes are independent of the number of nodes, 2. computationally the number of nodes cannot be too high as the model will not be able to process.

For each trial we randomise the p,pb parameters, as to gain a stochastic population of sample data from which to analyse. From this the data was loaded into a data.csv file and used in other files to infer properties of the model.

The csv has the following collumns

- **trial** - the trial number
- **edos renyi p**
- **poss of initial dead** - pb
- **steps till all dead** - number of steps before graph is completely dead
- **clustering** - average clustering
- **assortativity coefficient**

3.2 Quantitative Results

3.2.1 Relationship between P,PB on longevity of model

The following graphs are made in stat1.py
Each point on the graph showcases a model that survived more than 0 steps.

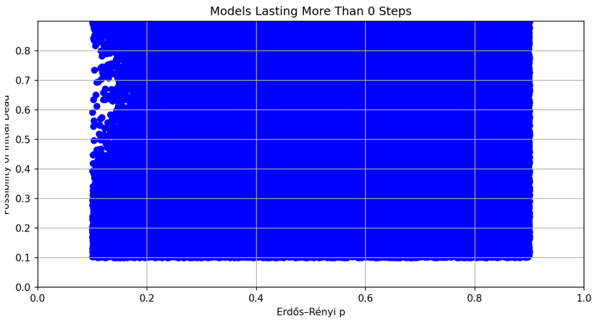

Figure 1: scatter plot, models more than 0 steps

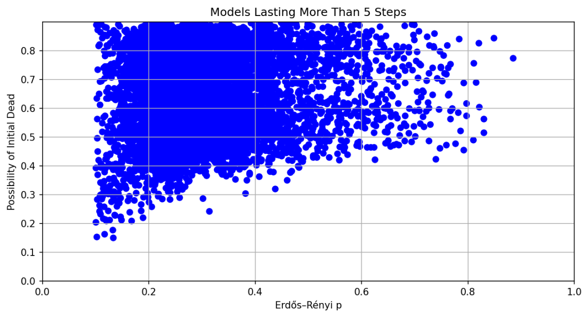

Figure 2: scatter plot, models more than 5 steps

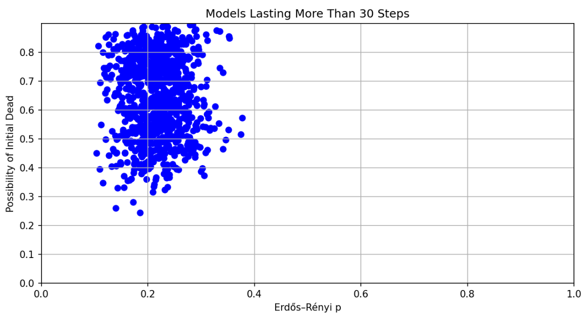

Figure 3: scatter plot, models more than 30 steps

The graphs here point towards a relationship between p,pb and the longevity of a model. Clearly the models survive longer when we have roughly a 0.2 erdos renyi p value and from 0.4 to 0.8 pb value.

3.2.2 Relationship for Assortativity and survival

This graph was produced by stat2.py

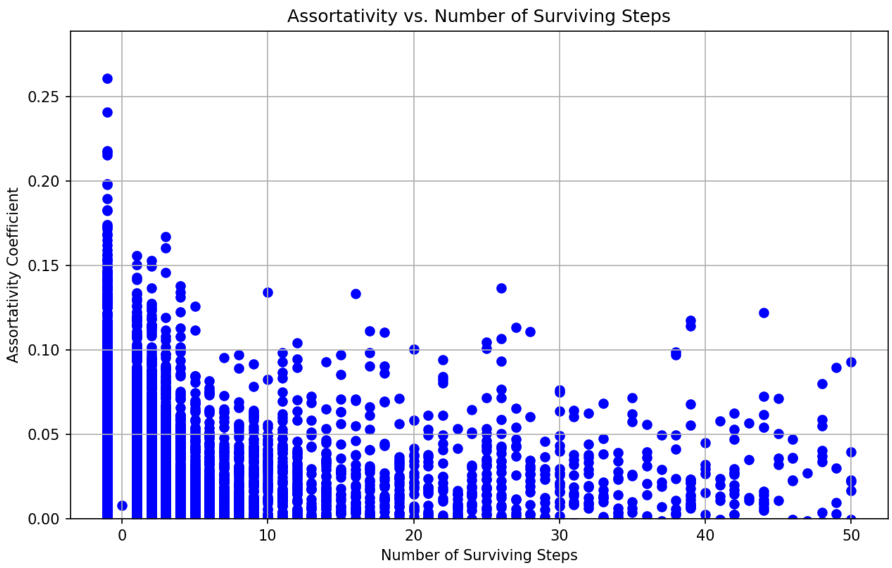

Figure 4: Assortativity vs survival

The graph implies a subtle relationship between the survival of the graph and the loss of assortativity.

4 Discussion

4.1 Results in numbers

Firstly in 1 we see the random population scattered over the graph randomly, but as we look at the simulations that survived longer and longer as in 3, we see a the trajectory of the data, that they tend to surround the mark where the p = 0.2 and pb is from 0.4 to 0.8.

We Also see in 4 that as the longevity of the graph increases the assortativity decreases.

4.2 Interpretation of results

The first point implies that generally graphs that have lower or around 0.2% of its edges, or that each node has a degree of 0.2(n-1), and if the initial dead nodes are above 0.4% survive longer.

The second point implies an inverse relationship between assortativity and longevity. Assortativity in graphs refers to the tendency of nodes to connect to other nodes that are similar in some way. This implies that longer lasting models are such that the edges are less alike in the sense of similarity of connections.

4.3 Model implications

Going back to the original interpretation, such that the simulations survival implies the survival of a society, not so much in the sense of death but in the sense of culture or power. The data points in our model therefore implies that when systems of societies showcase relatively low connective ness or globalisation, and relatively average to high amount of conformity(in the sense of appealing to the majority opinion) than the system survives longer than otherwise. This could be implicative of how globalisation can lead to greater conflict and periodisation as well as the machiavellian thought that for the system as a whole to survive, most of the societies must start off "dead" or antagonistic.

The relationship between assortativity and survival also implies a virtuous condition to global sociology, that if every society by means of its own individuality and integrity infer its own relations rather than conforming to others which would imply a a higher assortativity, there is a greater likely hood of "surviving".

5 Conclusion

The findings suggest that societies exhibit greater longevity when they demonstrate low connectivity or globalisation, coupled with a moderate to high level of conformity to majority opinions. This implies that excessive globalisation may incite conflict and division