

Euler Project 65

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Problem Statement

The first ten terms in the sequence of convergents for e are:

$$2, 3, \frac{8}{3}, \frac{11}{4}, \frac{19}{7}, \frac{87}{32}, \frac{106}{39}, \frac{193}{71}, \frac{1264}{465}, \frac{1457}{536}, \dots$$

The sum of digits in the numerator of the 10th convergent is $1 + 4 + 5 + 7 = 17$.

Find the sum of digits in the numerator of the 100th convergent of the continued fraction for e .

Answer

272

Idea

A simple piece-wise function exists to calculate the numerators of the convergents of e . I used this function to calculate the 100th term of the sequence and then summed the digits. I started my sequence from 1, 1 instead 2, 3 as the problem suggests so I actually calculated the 102nd term of my sequence.

Python Code

```
def sequence(n):
    a = [1, 1]
    for i in range(2, n + 1):
        remainder = i % 3
        if remainder == 0:
            a.append(a[i - 1] + a[i - 2])
        elif remainder == 1:
            a.append(2 * (i - 1) // 3 * a[i - 1] + a[i - 2])
        elif remainder == 2:
            a.append(a[i - 1] + a[i - 2])
    #print(a)
    #print(a[101])
    return a[101]
```

```
number = sequence(101)
result = 0
while number > 0:
    result += number % 10
    number //= 10
print(result)
```