

Abstract Data Types

Josh Wilcox (jw14g24@soton.ac.uk)

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Contents

1	Encapsulation	3
1.1	In Object-Oriented Programming	3
1.2	In Abstract Data Types	3
2	Stacks	3
2.1	Description	3
2.2	Standard Operations	3
2.3	Array Implementation	3
2.3.1	Push	3
2.3.2	Pop	3
2.3.3	Peek	4
2.3.4	Is-Empty	4
2.4	Why use a Stack	4
2.5	Evaluating Arithmetic Expressions Using Stacks	4
2.6	Example	4
3	Queues	5
3.1	Description	5
3.2	Standard Operations	5
3.2.1	Enqueue	5
3.2.2	Peek	5
3.2.3	Dequeue	5
3.2.4	isEmpty	5
3.3	Array Implementation	5
3.3.1	Enqueue	5
3.3.2	Peek	6
3.3.3	Dequeue	6
3.3.4	isEmpty	6
4	Priority Queues	6
4.1	Description	6
4.2	Standard Operations	6
4.2.1	Enqueue	6
4.2.2	Peek	6
4.2.3	Dequeue	7
4.2.4	isEmpty	7
4.3	Implementation	7
5	Lists	7
5.1	Description	7
5.2	Standard Operations	7
5.2.1	Add	7
5.2.2	Get	7
5.2.3	Remove	7
5.2.4	isEmpty	7

5.3	Array Implementation	7
5.3.1	Add	7
5.3.2	Get	7
5.3.3	Remove	8
5.3.4	isEmpty	8
6	Sets	8
6.1	Description	8
6.2	Standard Operations	8
6.2.1	Add	8
6.2.2	Contains	8
6.2.3	Remove	8
6.2.4	isEmpty	8
6.3	Array Implementation	8
6.3.1	Add	8
6.3.2	Contains	9
6.3.3	Remove	9
6.3.4	isEmpty	9
7	Maps	9
7.1	Description	9
7.2	Standard Operations	9
7.2.1	Put	9
7.2.2	Get	9
7.2.3	Remove	9
7.2.4	ContainsKey	9
7.2.5	isEmpty	9
7.3	Array Implementation	10
7.3.1	Put	10
7.3.2	Get	10
7.3.3	Remove	10
7.3.4	ContainsKey	10
7.3.5	isEmpty	11

1 Encapsulation

1.1 In Object-Oriented Programming

- In object-oriented programming, encapsulation separates the interface from the implementation.
- The implementation is encapsulated and can be changed without affecting the usage of the class.
- The interface consists of the set of public methods.

1.2 In Abstract Data Types

- Abstract Data Types (ADTs) are similar to interfaces but are independent of programming languages.
- An ADT specifies the operations through which a data structure can be accessed.
- The purpose of an ADT is to allow for the declaration of intentions, abstracting away the implementation details.

2 Stacks

2.1 Description

- Last In First Out (LIFO) Memory
- Only allowed to add and remove items in the collection in a very limited way
- Can only see the very top item
- Can only add and remove from the top
- Implemented using an array or *linked list*

2.2 Standard Operations

- **Push** - Address an element to the top of the stack
- **Peek** - Allows to see the top element of the stack
- **Pop** - Removes the item from the top of the stack and returns it
- **IsEmpty** - Boolean determining whether the stack is empty

2.3 Array Implementation

- Making a stack with at most n elements using array $S[1..n]$
- top is the top index of the stack

2.3.1 Push

```
1 if (top < n) {  
2     top = top + 1  
3     S[top] = x  
4 }
```

2.3.2 Pop

```
1 if (top==0){  
2     error  
3 } else{
```

```

4     top = top - 1
5     return S[top+1] // Doing this after to allow the decrement
6 }
```

2.3.3 Peek

```

1 if (top==0){
2     error
3 } else{
4     return S[top]
5 }
```

2.3.4 Is-Empty

```
1 return top==0
```

2.4 Why use a Stack

- Provides a very simple interface
- Limits memory access to become non-random
- Sufficient for many applications
- Prevents programmers from using memory that may break existing code

2.5 Evaluating Arithmetic Expressions Using Stacks

For a previously fully-bracketed expression

1. Ignore any opening brackets
2. Repeat, moving left-to-right, until reaching the end:
 - (a) Repeat until reaching a closing bracket:
 - i. Add numbers to number stack
 - ii. Add operators to operator stack
 - (b) Pop the operator from the operator stack
 - (c) Pop the required number of operands from the number stack
 - (d) Apply the operator to the operands
 - (e) Push the result back onto the number stack
3. The result will be the only number left in the number stack

2.6 Example

Consider the expression $(3 + (2 \times 5))$:

1. Ignore the opening bracket '('
2. Add 3 to the number stack
3. Add '+' to the operator stack
4. Ignore the opening bracket '('
5. Add 2 to the number stack

6. Add '*' to the operator stack
7. Add 5 to the number stack
8. Encounter closing bracket ')':
 - (a) Pop '*' from the operator stack
 - (b) Pop 5 and 2 from the number stack
 - (c) Calculate $2 \times 5 = 10$
 - (d) Push 10 onto the number stack
9. Encounter closing bracket ')':
 - (a) Pop '+' from the operator stack
 - (b) Pop 10 and 3 from the number stack
 - (c) Calculate $3 + 10 = 13$
 - (d) Push 13 onto the number stack
10. The result is 13

3 Queues

3.1 Description

- First-in-first-out (FIFO) memory model
- Acts like a queue of people, the first one entering the line will be the first to reach the end at all times

3.2 Standard Operations

3.2.1 Enqueue

- Adds an element to the end of the queue.

3.2.2 Peek

- Allows to see the front element of the queue without removing it.

3.2.3 Dequeue

- Removes the front element from the queue and returns it.

3.2.4 isEmpty

- Boolean to define if the queue is empty

3.3 Array Implementation

3.3.1 Enqueue

- Using an array `Q[1..n]` with `front` and `rear` indices.

```
1  if (rear < n) {  
2      rear = rear + 1;  
3      Q[rear] = x;  
4  } else {  
5      error("Queue is full");  
6  }
```

Big Theta: $\Theta(1)$ - Enqueue operation is constant time as it involves a simple increment and assignment.

3.3.2 Peek

- Returns the element at the `front` of the queue.

```
1 if (front == rear) {  
2     error("Queue is empty");  
3 } else {  
4     return Q[front + 1];  
5 }
```

Big Theta: $\Theta(1)$ - Peek operation is constant time as it involves accessing an element at a specific index.

3.3.3 Dequeue

- Removes and returns the element at the `front` of the queue.

```
1 if (front == rear) {  
2     error("Queue is empty");  
3 } else {  
4     front = front + 1;  
5     return Q[front];  
6 }
```

Big Theta: $\Theta(1)$ - Dequeue operation is constant time as it involves a simple increment and access.

3.3.4 isEmpty

- Checks if the queue is empty.

```
1 return front == rear;
```

Big Theta: $\Theta(1)$ - isEmpty operation is constant time as it involves a simple comparison.

4 Priority Queues

4.1 Description

- A priority queue is an abstract data type where each element has a priority.
- Elements with higher priority are dequeued before elements with lower priority.
- If two elements have the same priority, they are dequeued according to their order in the queue.

4.2 Standard Operations

4.2.1 Enqueue

- Adds an element to the queue with an associated priority.

4.2.2 Peek

- Allows to see the element with the highest priority without removing it.

4.2.3 Dequeue

- Removes and returns the element with the highest priority.

4.2.4 isEmpty

- Boolean to define if the priority queue is empty.

4.3 Implementation

- Implemented most efficiently using a **heap**
- Can also be implemented using a linked list or a binary tree

5 Lists

5.1 Description

- An ordered collection of elements.
- Elements can be accessed by their position (index) in the list.

5.2 Standard Operations

5.2.1 Add

- Adds an element to the end of the list.

5.2.2 Get

- Retrieves the element at a specified index.

5.2.3 Remove

- Removes the element at a specified index.

5.2.4 isEmpty

- Boolean to define if the list is empty.

5.3 Array Implementation

5.3.1 Add

```

1  if (size < n) {
2      list[size] = x;
3      size = size + 1;
4  } else {
5      error("List is full");
6 }
```

5.3.2 Get

```

1  if (index >= 0 && index < size) {
2      return list[index];
3  } else {
4      error("Index out of bounds");
5 }
```

5.3.3 Remove

```
1 if (index >= 0 && index < size) {  
2     for (int i = index; i < size - 1; i++) {  
3         list[i] = list[i + 1];  
4     }  
5     size = size - 1;  
6 } else {  
7     error("Index out of bounds");  
8 }
```

5.3.4 isEmpty

```
1 return size == 0;
```

6 Sets

6.1 Description

- An unordered collection of unique elements.
- Does not allow duplicate elements.

6.2 Standard Operations

6.2.1 Add

- Adds an element to the set if it is not already present.

6.2.2 Contains

- Checks if the set contains a specified element.

6.2.3 Remove

- Removes a specified element from the set.

6.2.4 isEmpty

- Boolean to define if the set is empty.

6.3 Array Implementation

6.3.1 Add

```
1 if (!contains(set, x)) {  
2     if (size < n) {  
3         set[size] = x;  
4         size = size + 1;  
5     } else {  
6         error("Set is full");  
7     }  
8 }
```

6.3.2 Contains

```
1 for (int i = 0; i < size; i++) {  
2     if (set[i] == x) {  
3         return true;  
4     }  
5 }  
6 return false;
```

6.3.3 Remove

```
1 for (int i = 0; i < size; i++) {  
2     if (set[i] == x) {  
3         for (int j = i; j < size - 1; j++) {  
4             set[j] = set[j + 1];  
5         }  
6         size = size - 1;  
7         return;  
8     }  
9 }  
10 error("Element not found");
```

6.3.4 isEmpty

```
1 return size == 0;
```

7 Maps

7.1 Description

- A collection of key-value pairs.
- Each key is unique and maps to a specific value.

7.2 Standard Operations

7.2.1 Put

- Adds a key-value pair to the map.

7.2.2 Get

- Retrieves the value associated with a specified key.

7.2.3 Remove

- Removes the key-value pair associated with a specified key.

7.2.4 ContainsKey

- Checks if the map contains a specified key.

7.2.5 isEmpty

- Boolean to define if the map is empty.

7.3 Array Implementation

7.3.1 Put

```
1 for (int i = 0; i < size; i++) {
2     if (map[i].key == key) {
3         map[i].value = value;
4         return;
5     }
6 }
7 if (size < n) {
8     map[size].key = key;
9     map[size].value = value;
10    size = size + 1;
11 } else {
12     error("Map is full");
13 }
```

7.3.2 Get

```
1 for (int i = 0; i < size; i++) {
2     if (map[i].key == key) {
3         return map[i].value;
4     }
5 }
6 error("Key not found");
```

7.3.3 Remove

```
1 for (int i = 0; i < size; i++) {
2     if (map[i].key == key) {
3         for (int j = i; j < size - 1; j++) {
4             map[j] = map[j + 1];
5         }
6         size = size - 1;
7         return;
8     }
9 }
10 error("Key not found");
```

7.3.4 ContainsKey

```
1 for (int i = 0; i < size; i++) {
2     if (map[i].key == key) {
3         return true;
4     }
5 }
6 return false;
```

7.3.5 isEmpty

```
1 return size == 0;
```