

# Abstract Data Types

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# 1 Encapsulation

## 1.1 In Object-Oriented Programming

- In object-oriented programming, encapsulation separates the interface from the implementation.
- The implementation is encapsulated and can be changed without affecting the usage of the class.
- The interface consists of the set of public methods.

## 1.2 In Abstract Data Types

- Abstract Data Types (ADTs) are similar to interfaces but are independent of programming languages.
- An ADT specifies the operations through which a data structure can be accessed.
- The purpose of an ADT is to allow for the declaration of intentions, abstracting away the implementation details.

# 2 Stacks

## 2.1 Description

- Last In First Out (LIFO) Memory
- Only allowed to add and remove items in the collection in a very limited way
- Can only see the very top item
- Can only add and remove from the top
- Implemented using an array or *linked list*

## 2.2 Standard Operations

- **Push** - Address an element to the top of the stack
- **Peek** - Allows to see the top element of the stack
- **Pop** - Removes the item from the top of the stack and returns it
- **IsEmpty** - Boolean determining whether the stack is empty

## 2.3 Array Implementation

- Making a stack with at most  $n$  elements using array  $S[1..n]$
- `top` is the top index of the stack

### 2.3.1 Push

```
1  if (top < n) {  
2      top = top + 1  
3      S[top] = x  
4  }
```

### 2.3.2 Pop

```
1  if (top==0){  
2      error  
3  } else{
```

```
4     top = top - 1
5     return S[top+1] // Doing this after to allow the decrement
6 }
```

### 2.3.3 Peek

```
1  if (top==0){
2      error
3  } else{
4      return S[top]
5  }
```

### 2.3.4 Is-Empty

```
1  return top==0
```

## 2.4 Why use a Stack

- Provides a very simple interface
- Limits memory access to become non-random
- Sufficient for many applications
- Prevents programmers from using memory that may break existing code

## 2.5 Evaluating Arithmetic Expressions Using Stacks

For a previously fully-bracketed expression

1. Ignore any opening brackets
2. Repeat, moving left-to-right, until reaching the end:
  - (a) Repeat until reaching a closing bracket:
    - i. Add numbers to number stack
    - ii. Add operators to operator stack
  - (b) Pop the operator from the operator stack
  - (c) Pop the required number of operands from the number stack
  - (d) Apply the operator to the operands
  - (e) Push the result back onto the number stack
3. The result will be the only number left in the number stack

## 2.6 Example

Consider the expression  $(3 + (2 \times 5))$ :

1. Ignore the opening bracket '('
2. Add 3 to the number stack
3. Add '+' to the operator stack
4. Ignore the opening bracket '('
5. Add 2 to the number stack

6. Add '\*' to the operator stack
7. Add 5 to the number stack
8. Encounter closing bracket ')':
  - (a) Pop '\*' from the operator stack
  - (b) Pop 5 and 2 from the number stack
  - (c) Calculate  $2 \times 5 = 10$
  - (d) Push 10 onto the number stack
9. Encounter closing bracket ')':
  - (a) Pop '+' from the operator stack
  - (b) Pop 10 and 3 from the number stack
  - (c) Calculate  $3 + 10 = 13$
  - (d) Push 13 onto the number stack
10. The result is 13

## 3 Queues

### 3.1 Description

- First-in-first-out (FIFO) memory model
- Acts like a queue of people, the first one entering the line will be the first to reach the end at all times

### 3.2 Standard Operations

#### 3.2.1 Enqueue

- Adds an element to the end of the queue.

#### 3.2.2 Peek

- Allows to see the front element of the queue without removing it.

#### 3.2.3 Dequeue

- Removes the front element from the queue and returns it.

#### 3.2.4 isEmpty

- Boolean to define if the queue is empty

### 3.3 Array Implementation

#### 3.3.1 Enqueue

- Using an array `Q[1..n]` with `front` and `rear` indices.

```
1  if (rear < n) {  
2      rear = rear + 1;  
3      Q[rear] = x;  
4  } else {  
5      error("Queue is full");  
6  }
```

**Big Theta:**  $\Theta(1)$  - Enqueue operation is constant time as it involves a simple increment and assignment.

### 3.3.2 Peek

- Returns the element at the **front** of the queue.

```
1  if (front == rear) {  
2      error("Queue is empty");  
3  } else {  
4      return Q[front + 1];  
5  }
```

**Big Theta:**  $\Theta(1)$  - Peek operation is constant time as it involves accessing an element at a specific index.

### 3.3.3 Dequeue

- Removes and returns the element at the **front** of the queue.

```
1  if (front == rear) {  
2      error("Queue is empty");  
3  } else {  
4      front = front + 1;  
5      return Q[front];  
6  }
```

**Big Theta:**  $\Theta(1)$  - Dequeue operation is constant time as it involves a simple increment and access.

### 3.3.4 isEmpty

- Checks if the queue is empty.

```
1  return front == rear;
```

**Big Theta:**  $\Theta(1)$  - isEmpty operation is constant time as it involves a simple comparison.

## 4 Priority Queues

### 4.1 Description

- A priority queue is an abstract data type where each element has a priority.
- Elements with higher priority are dequeued before elements with lower priority.
- If two elements have the same priority, they are dequeued according to their order in the queue.

### 4.2 Standard Operations

#### 4.2.1 Enqueue

- Adds an element to the queue with an associated priority.

#### 4.2.2 Peek

- Allows to see the element with the highest priority without removing it.

### 4.2.3 Dequeue

- Removes and returns the element with the highest priority.

### 4.2.4 isEmpty

- Boolean to define if the priority queue is empty.

## 4.3 Implementation

- Implemented most efficiently using a **heap**
- Can also be implemented using a linked list or a binary tree

# 5 Lists

## 5.1 Description

- An ordered collection of elements.
- Elements can be accessed by their position (index) in the list.

## 5.2 Standard Operations

### 5.2.1 Add

- Adds an element to the end of the list.

### 5.2.2 Get

- Retrieves the element at a specified index.

### 5.2.3 Remove

- Removes the element at a specified index.

### 5.2.4 isEmpty

- Boolean to define if the list is empty.

## 5.3 Array Implementation

### 5.3.1 Add

```
1  if (size < n) {  
2      list[size] = x;  
3      size = size + 1;  
4  } else {  
5      error("List is full");  
6  }
```

### 5.3.2 Get

```
1  if (index >= 0 && index < size) {  
2      return list[index];  
3  } else {  
4      error("Index out of bounds");  
5  }
```

### 5.3.3 Remove

```
1  if (index >= 0 && index < size) {
2      for (int i = index; i < size - 1; i++) {
3          list[i] = list[i + 1];
4      }
5      size = size - 1;
6  } else {
7      error("Index out of bounds");
8  }
```

### 5.3.4 isEmpty

```
1  return size == 0;
```

## 6 Sets

### 6.1 Description

- An unordered collection of unique elements.
- Does not allow duplicate elements.

### 6.2 Standard Operations

#### 6.2.1 Add

- Adds an element to the set if it is not already present.

#### 6.2.2 Contains

- Checks if the set contains a specified element.

#### 6.2.3 Remove

- Removes a specified element from the set.

#### 6.2.4 isEmpty

- Boolean to define if the set is empty.

### 6.3 Array Implementation

#### 6.3.1 Add

```
1  if (!contains(set, x)) {
2      if (size < n) {
3          set[size] = x;
4          size = size + 1;
5      } else {
6          error("Set is full");
7      }
8  }
```



### 6.3.2 Contains

```
1 for (int i = 0; i < size; i++) {  
2     if (set[i] == x) {  
3         return true;  
4     }  
5 }  
6 return false;
```

### 6.3.3 Remove

```
1 for (int i = 0; i < size; i++) {  
2     if (set[i] == x) {  
3         for (int j = i; j < size - 1; j++) {  
4             set[j] = set[j + 1];  
5         }  
6         size = size - 1;  
7         return;  
8     }  
9 }  
10 error("Element not found");
```

### 6.3.4 isEmpty

```
1 return size == 0;
```

## 7 Maps

### 7.1 Description

- A collection of key-value pairs.
- Each key is unique and maps to a specific value.

### 7.2 Standard Operations

#### 7.2.1 Put

- Adds a key-value pair to the map.

#### 7.2.2 Get

- Retrieves the value associated with a specified key.

#### 7.2.3 Remove

- Removes the key-value pair associated with a specified key.

#### 7.2.4 ContainsKey

- Checks if the map contains a specified key.

#### 7.2.5 isEmpty

- Boolean to define if the map is empty.

## 7.3 Array Implementation

### 7.3.1 Put

```
1  for (int i = 0; i < size; i++) {
2      if (map[i].key == key) {
3          map[i].value = value;
4          return;
5      }
6  }
7  if (size < n) {
8      map[size].key = key;
9      map[size].value = value;
10     size = size + 1;
11 } else {
12     error("Map is full");
13 }
```

### 7.3.2 Get

```
1  for (int i = 0; i < size; i++) {
2      if (map[i].key == key) {
3          return map[i].value;
4      }
5  }
6  error("Key not found");
```

### 7.3.3 Remove

```
1  for (int i = 0; i < size; i++) {
2      if (map[i].key == key) {
3          for (int j = i; j < size - 1; j++) {
4              map[j] = map[j + 1];
5          }
6          size = size - 1;
7          return;
8      }
9  }
10 error("Key not found");
```

### 7.3.4 ContainsKey

```
1  for (int i = 0; i < size; i++) {
2      if (map[i].key == key) {
3          return true;
4      }
5  }
6  return false;
```

**7.3.5 isEmpty**

```
1 return size == 0;
```