ECEN325 Ref Sheet © Josh Wright April 25, 2017

Metric Prefixes

peta	P	10^{15}	1 000 000 000 000 000
tera	T	10^{12}	1 000 000 000 000
giga	G	10^{9}	1 000 000 000
mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1 000
hecto	h	10^{2}	100
deca	da	10^{1}	10
one		10^{0}	1
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001
femto	f	10^{-15}	0.000 000 000 000 001

Ohm's Law $V = IR, \quad I = \frac{V}{R}, \quad R = \frac{V}{I}$

Battery Symbol

The side with the longer line is the positive side

Complex Numbers

- $z = x + iy = re^{i\theta} = r[\cos(\theta) + i\sin(\theta)]$
- $[r(\cos(\theta) + i\sin(\theta))]^n = r^n[\cos(n\theta) + i\sin(n\theta)]$
- $z^n = (re^{i\theta}) = r^n e^{in\theta}$
- $\frac{1}{i} = -i$
- $\sqrt[n]{z} = \sqrt[n]{r} e^{\frac{\theta}{n} + \frac{2k\pi}{n}}$ for $n \in N^*$ (ints ≥ 0)
- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$ | $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$
 $\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$ | $\sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$
- normalized: $sinc(t) = \frac{\sin(\pi t)}{\pi t}$
- $|\frac{a}{b}| = \frac{|a|}{|b|}$ $\qquad \angle\frac{a}{b} = \angle a - \angle b$

Trig

$\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$

$\cos^2(a) + \sin^2(a) = 1$ | $\sin(2a) = 2\sin(a)\cos(a)$

$\cos^2(a) = \frac{1}{2}(1 + \cos(2a))$ | $\sin^2(a) = \frac{1}{2}(1 - \cos(2a))$

Voltage Division between two non-zero points

$V_{DD} \to R_1 \to V_1 \to R_2 \to V_{EE}$ (superposition)

$V_1 = V_{DD}\frac{R_2}{R_1+R_2} + V_{EE}\frac{R_1}{R_1+R_2}$

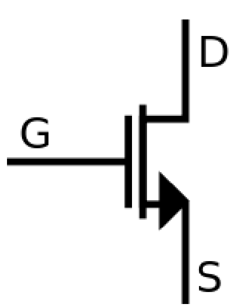
AC Resistors and Capacitors

$$R||\frac{1}{Cs} = \frac{R\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{R}{RCs+1}$$

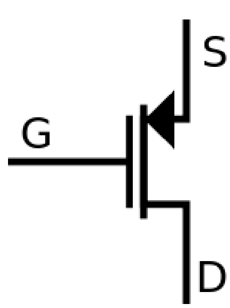
$$\frac{R}{R+\frac{1}{Cs}} = \frac{RCs}{RCs+1} = \frac{s}{s+\frac{1}{RC}}$$

$$\frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} = \frac{1}{RCs+1}$$

MOS



NMOS



PMOS

MOS DC Biasing

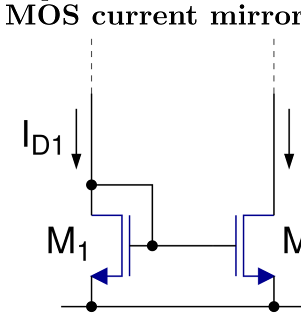
- this is all for NMOS. PMOS is backward
- $\beta = k_n'(\frac{W}{L})$
- cutoff: $V_{GS} < V_{th}$
  - $I_D = 0$
- triode (linear): $V_{DS} < V_{GS} - V_{th}$
  - $I_D = k_n' \frac{W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right)$
- active (saturation): $V_{DS} > V_{GS} - V_{th}$
  - $I_D = \frac{k_n'}{2}\frac{W}{L}(V_{GS} - V_{th})^2$
  - overdrive voltage $V_{ov} = V_{GS} - V_{th}$
- to show it's active: show that $V_{DS} > V_{ov}$ (or $V_{DS} > V_{GS} - V_{th}$)

MOS Small Signal

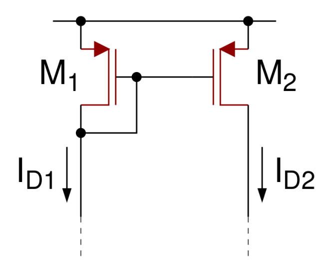
	$\frac{V_o}{V_g}$	R_i	R_o
CS	$-\frac{R_D\|\|R_L}{\frac{1}{g_m}+R_S}$	$R_{G1}\|\|R_{G2}$	R_D
CG	$\frac{R_D\|\|R_L}{R_i}$	$\frac{1}{g_m}$	R_D
CD	$\frac{R_L}{\frac{1}{g_m}+R_L}$	$R_{G1}\|\|R_{G2}$	$\frac{1}{g_m}$

- $g_m = k'\frac{W}{L}(V_{GS} - V_t) = k'\frac{W}{L}(V_{ov}) = \sqrt{2k'\frac{W}{L}I_D}$
- CS: Common Source
- CG: Common Gate
- CD: Common Drain (buffer)
- apparently, when calculating the max gain of the frequency response, it is OK to just use the part of he equation that doesn't depend on s

MOS current mirrors



NMOS



PMOS

- where M1 is the transistor with base shorted to ground (the reference branch)
- $\frac{I_{D1}}{I_{D2}} = \frac{(W/L)_1}{(W/L)_2}$ or $I_{D2} = \frac{(W/L)_2}{(W/L)_1}I_{D1}$

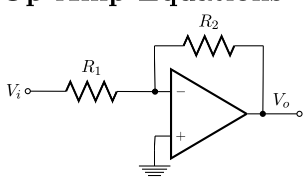
Bode Plots

- magnitude is plotted in dB: $|T(j\omega)|_{dB} = 20\log_{10}|T(j\omega)|$
- starts on y-axis at DC offset with slope 0
- just add together the bode plots of each individual pole, zero, and the DC offset
- poles always slope down, zeros slope up (applies for both magnitude and phase)
- dec=decade, e.g. from 10^0 to 10^1
- magnitude:
  - Pole/Zero at origin: constant slope $\pm 20db/dec$ for all ω; $0dB$ at $\omega = 10^0 = 1$
  - Pole/Zero at ω_0: 0 for $\omega < \omega_0$ slope $\pm 20\frac{db}{dec}$ after

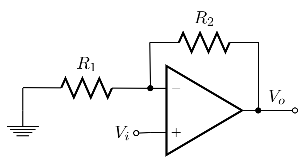
∗ Constant C: constant line at $20\log_{10}(|C|)$
• phase:
∗ Pole at origin: constant $-\frac{\pi}{2}$ or $-90°$
∗ Zero at origin: constant $+\frac{\pi}{2}$ or $+90°$
∗ Pole/Zero at ω_0:
0 for $\omega < \frac{\omega_0}{10}$
slope linearly ($\pm 45°/dec$) until $10\omega_0$
0 slope for $\omega > 10\omega_0$
∗ Constant C: no effect (0 for all ω)
• Prof wants us to actually show the -3dB drop curve, not just a straight intersection
• on the x-axis of the bode plot you need to plot frequency in Hz, so take $s = j\omega$ and divide by 2π

Solving systems with Op Amps
• only applies if the op-amp has feedback
• step 0: if the op amp is ideal, write out ideal properties:
∗ $V_+ = V_-$
∗ $I_- = 0, I_+ = 0$
∗ $A \approx \infty$
• avoid doing KCL/KVL directly on the output node of the op amp
• ignore resistors from a point at $0V$ to ground

Op Amp Equations



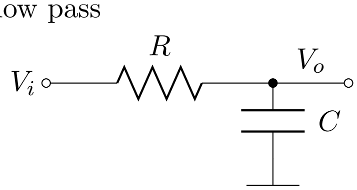
Inverting Amplifier



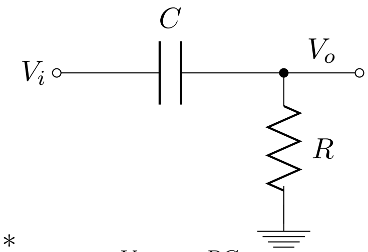
Non-Inverting Amplifier

• ideal open-loop behavior: $(V_p - V_n) > 0 \to V_o = V_{DD}$
$(V_p - V_n) < 0 \to V_o = -V_{DD}$
• general form: $T(s) = \frac{K_0}{1+\frac{s}{\omega_0}}$
∗ $T(0) = K_0$: DC offset. For these simple ones, it's equal to ideal response
∗ $\omega_0 = \frac{\omega_t}{1+R_2/R_1}$
• inverting op amp:
∗ ideal: $T(s) = \frac{V_o}{V_i} = -\frac{R_2}{R_1}$
∗ non-ideal:
$T(s) = \frac{V_o}{V_i} = \frac{-R_2/R_1}{1+\frac{1+R_2/R_1}{A(s)}} = \frac{-R_2/R_1}{1+\frac{s}{\left(\frac{\omega_t}{1+R_2/R_1}\right)}} = \frac{-R_2/R_1}{1+\frac{s}{\omega_0}}$
• non-inverting op-amp:
∗ ideal: $T(s) = \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$
∗ non-ideal:
$T(s) = \frac{V_o}{V_i} = \frac{1+R_2/R_1}{1+\frac{1+R_2/R_1}{A(s)}} = \frac{1+R_2/R_1}{1+\frac{s}{\left(\frac{\omega_t}{1+R_2/R_1}\right)}} = \frac{1+R_2/R_1}{1+\frac{s}{\omega_0}}$

RC Filter
• Transmission Function: $T(s) = \frac{V_o(s)}{V_i(s)}$
• Corner frequency: frequency s at which $T(s) = \frac{1}{\sqrt{2}}$
• for simple circuit: ground→source→ $R \to C \to$ground
∗ $T(s) = \frac{1}{1+RCs}$
$|T(j\omega)| = \frac{1}{\sqrt{1+R^2C^2s^2}}$
$|\angle T(j\omega)| = \frac{1}{\sqrt{1+R^2C^2s^2}}$
• low pass



∗
∗ $T(s) = \frac{V_o}{V_i} = \frac{1}{1+RCs}$
∗ corner frequency: $s = \frac{1}{RC}$ (also pole)
∗ pole: $\frac{1}{RC}$
• high pass



∗
∗ $T(s) = \frac{V_o}{V_i} = \frac{RCs}{1+RCs}$
∗ zero: $s = 0$, pole: $s = \frac{1}{RC}$