

Metric Prefixes

peta	P	10^{15}	1 000 000 000 000 000
tera	T	10^{12}	1 000 000 000 000
giga	G	10^9	1 000 000 000
mega	M	10^6	1 000 000
kilo	k	10^3	1 000
hecto	h	10^2	100
deca	da	10^1	10
one		10^0	1
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001
femto	f	10^{-15}	0.000 000 000 000 001

Ohm's Law $V = IR$, $I = \frac{V}{R}$, $R = \frac{V}{I}$

Complex Numbers

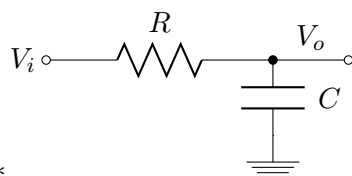
- $z = x + iy = re^{i\theta} = r[\cos(\theta) + i\sin(\theta)]$
- $[r(\cos(\theta) + i\sin(\theta))]^n = r^n[\cos(n\theta) + i\sin(n\theta)]$
- $z^n = (re^{i\theta}) = r^n e^{in\theta}$
- $\frac{1}{i} = -i$
- $\sqrt[n]{z} = \sqrt[n]{r} e^{\frac{\theta}{n} + \frac{2k\pi}{n}}$ for $n \in \mathbb{N}^*$ (ints ≥ 0)
- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$
- $\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$
- $\sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$
- normalized: $\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$
- $|\frac{a}{b}| = \frac{|a|}{|b|}$
- $\angle \frac{a}{b} = \angle a - \angle b$

Trig

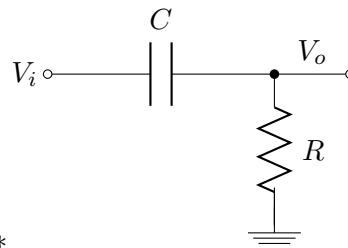
- $\cos^2(a) + \sin^2(a) = 1$
- $\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$
- $\sin(2a) = 2\sin(a)\cos(a)$
- $\cos^2(a) = \frac{1}{2}(1 + \cos(2a))$
- $\sin^2(a) = \frac{1}{2}(1 - \cos(2a))$

RC Filter

- Transmission Function: $T(s) = \frac{V_o(s)}{V_i(s)}$
- Corner frequency: frequency s at which $T(s) = \frac{1}{\sqrt{2}}$
- for simple circuit: ground \rightarrow source $\rightarrow R \rightarrow C \rightarrow$ ground
 - * $T(s) = \frac{1}{1+RCs}$
 - * $|T(j\omega)| = \frac{1}{\sqrt{1+R^2C^2s^2}}$
 - * $|\angle T(j\omega)| = \frac{1}{\sqrt{1+R^2C^2s^2}}$
- low pass



- * $T(s) = \frac{V_o}{V_i} = \frac{1}{1+RCs}$
- * corner frequency: $s = \frac{1}{RC}$ (also pole)
- * pole: $\frac{1}{RC}$
- high pass



- * $T(s) = \frac{V_o}{V_i} = \frac{RCs}{1+RCs}$
- * zero: $s = 0$, pole: $s = \frac{1}{RC}$

Bode Plots

- magnitude is plotted in dB:
 - * $|T(j\omega)|_{dB} = 20 \log_{10} |T(j\omega)|$
- starts on y-axis at DC offset with slope 0
- just add together the bode plots of each individual pole, zero, and the DC offset
- poles always slope down, zeros slope up (applies for both magnitude and phase)
- dec=decade, e.g. from 10^0 to 10^1
- magnitude:
 - * Pole/Zero at origin: constant slope $\pm 20\text{dB/dec}$ for all ω ; 0dB at $\omega = 10^0 = 1$
 - * Pole/Zero at ω_0 :
 - 0 for $\omega < \omega_0$
 - slope $\pm 20 \frac{\text{dB}}{\text{dec}}$ after
 - * Constant C: constant line at $20 \log_{10}(|C|)$
- phase:
 - * Pole at origin: constant $-\frac{\pi}{2}$ or -90°
 - * Zero at origin: constant $+\frac{\pi}{2}$ or $+90^\circ$
 - * Pole/Zero at ω_0 :
 - 0 for $\omega < \frac{\omega_0}{10}$
 - slope linearly ($\pm 45^\circ/\text{dec}$) until $10\omega_0$
 - 0 slope for $\omega > 10\omega_0$
 - * Constant C: no effect (0 for all ω)
- Prof wants us to actually show the -3dB drop curve, not just a straight intersection

Solving systems with Op Amps

- step 0: if the op amp is ideal, write out ideal properties:
 - * $V_+ = V_-$
 - * $I_- = 0, I_+ = 0$
 - * $A \approx \infty$
- avoid doing KCL/KVL directly on the output node of the op amp
- ignore resistors from a point at 0V to ground
- **Non-Ideal Op Amps**
 - still assume that current at input terminals is 0
 - $V_o = A(V_+ - V_-)$
 - * A: open-loop gain. Typically very large, 100,000 or more
 - open-loop gain dependent on frequency: $A(s) = \frac{A_0}{1 - \frac{s}{\omega_b}}$
 - * open-loop response drops off after ω_b (usually $2\pi \leq \omega_b \leq 2\pi 100$)
 - * A_0 : DC gain
 - * ω_t : unity gain frequency: $\text{dB}(T(\omega_t)) = 1$
 - $\omega_t \approx A_0 \omega_b$
 - AKA gain bandwidth product
 - * in this case, we still assume $I_- = I_+ = 0$ and $V_- = V_+$?
- slew rate
 - * max rate at which the output can change
 - * for a sinusoidal signal: (V_{pk} : peak voltage)
 - $SR > 2\pi f V_{pk}$ or $SR > \omega V_{pk}$
 - $\frac{dV_o}{dt}|_{MAX} < SR$

Op Amp Equations

• general form: $T(s) = \frac{K_0}{1 + \frac{s}{\omega_0}}$

* $T(0) = K_0$: DC offset. For these simple ones, it's equal to ideal response

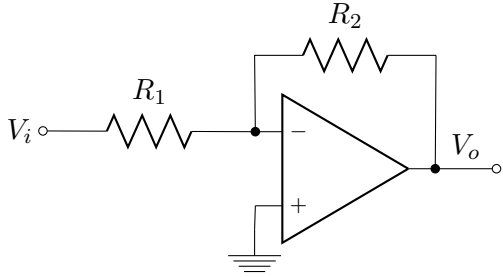
* $\omega_0 = \frac{\omega_t}{1 + R_2/R_1}$

• inverting op amp:

* ideal: $T(s) = \frac{V_o}{V_i} = -\frac{R_2}{R_1}$

* non-ideal:

$$T(s) = \frac{V_o}{V_i} = \frac{-R_2/R_1}{1 + \frac{1+R_2/R_1}{A(s)}} = \frac{-R_2/R_1}{1 + \frac{s}{\left(\frac{\omega_t}{1+R_2/R_1}\right)}} = \frac{-R_2/R_1}{1 + \frac{s}{\omega_0}}$$

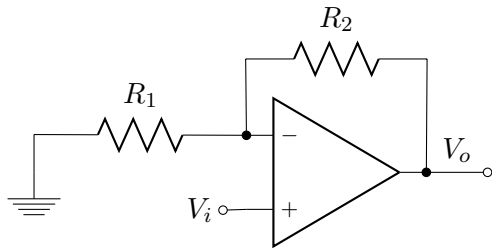


• non-inverting op-amp:

* ideal: $T(s) = \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$

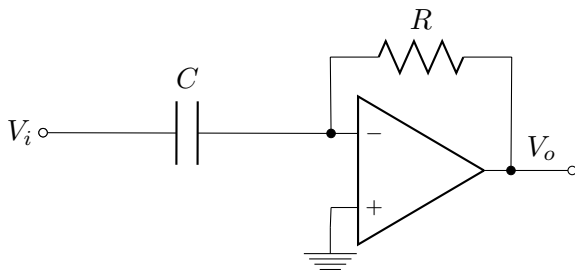
* non-ideal:

$$T(s) = \frac{V_o}{V_i} = \frac{1+R_2/R_1}{1 + \frac{1+R_2/R_1}{A(s)}} = \frac{1+R_2/R_1}{1 + \frac{s}{\left(\frac{\omega_t}{1+R_2/R_1}\right)}} = \frac{1+R_2/R_1}{1 + \frac{s}{\omega_0}}$$



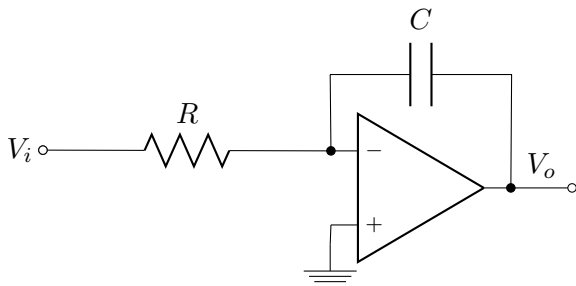
• integrating

* ideal: $V_o = -\int_0^t \frac{V_i}{RC} dt + C$
 $= C = V_o(t)$ at $t = 0$



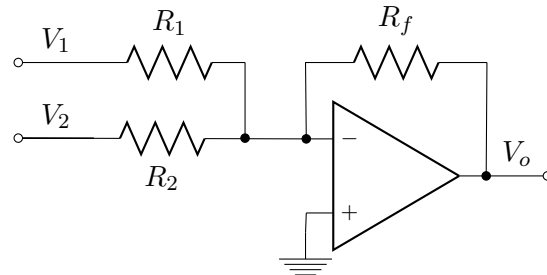
• differentiating

* ideal: $V_o = -RC \frac{dV_i}{dt}$



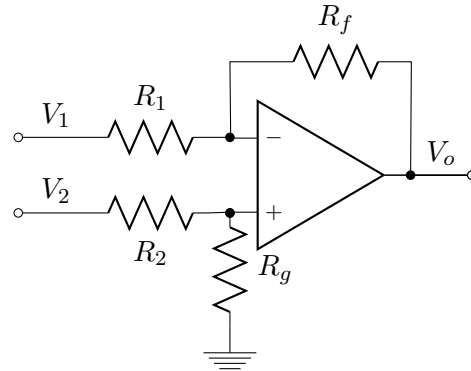
• summing

* ideal: $V_o = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$



• differential (subtracting)

* ideal: $V_o = \frac{(R_f + R_1)R_g}{(R_g + R_2)R_1} V_2 - \frac{R_f}{R_1} V_1$



Diodes

• $I_D = I_S \left(e^{\frac{V_D}{nV_T}} - 1 \right)$

* $I_S = 10^{-12} \text{ A}$

* $V_T = 25 \text{ mV}$

• small signal resistance: $R_D = \frac{nV_T}{I_D}$

* I_D : average (DC) current through diode (due to forward bias)

• bridge rectifier shape: all the diodes point toward the + end of the output (away from ground)

Design an (unregulated) AC adapter

• V_S : AC voltage input

• V_{S2} : output of transformer (still AC)

• V_p : peak DC output

* V_r : peak-to-peak ripple voltage (maximum variation)

• $V_p = V_{S2} - (\text{diode voltage})$

* diode voltage = 0.7V for single wave rectifier or center-tapped transformer

* diode voltage = 1.4V for full-wave rectifier

• turns ratio $= n = \frac{V_S}{V_{S2}}$

• apparent load resistance: $R_L = \frac{V_o}{I_L}$

• ripple voltage: $V_r = \frac{f}{R_L C}$

* f : actual ripple frequency (double for full-wave rectifier)

* C : filter cap size

• Peak Inverse Voltage: $PIV = V_{S2} - 0.7V$

• avg diode current: $I_{Davg} = I_L \left(1 + \pi \sqrt{\frac{2}{V_r}} \right)$

• max diode current: $I_{Dmax} = I_L \left(1 + 2\pi \sqrt{\frac{2}{V_r}} \right)$

* only difference is $\pi \rightarrow 2\pi$