ECEN325 Ref Sheet © Josh Wright February 28, 2017

Metric Prefixes

peta	P	10^{15}	1 000 000 000 000 000
tera	T	10^{12}	1 000 000 000 000
giga	G	10^{9}	1 000 000 000
mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1 000
hecto	h	10^{2}	100
deca	da	10^{1}	10
one		10^{0}	1
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001
femto	f	10^{-15}	0.000 000 000 000 001

Ohm's Law $V = IR, \quad I = \frac{V}{R}, \quad R = \frac{V}{I}$

Complex Numbers

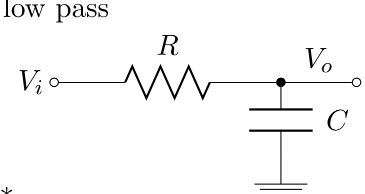
- $z = x + iy = re^{i\theta} = r[\cos(\theta) + i\sin(\theta)]$
- $[r(\cos(\theta) + i\sin(\theta))]^n = r^n[\cos(n\theta) + i\sin(n\theta)]$
- $z^n = (re^{i\theta}) = r^n e^{in\theta}$
- $\frac{1}{i} = -i$
- $\sqrt[n]{z} = \sqrt[n]{r} e^{\frac{\theta}{n} + \frac{2k\pi}{n}}$ for $n \in N^*$ (ints ≥ 0)
- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$
- $\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$
- $\sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$
- normalized: $sinc(t) = \frac{\sin(\pi t)}{\pi t}$
- $|\frac{a}{b}| = \frac{|a|}{|b|}$
- $\angle\frac{a}{b} = \angle a - \angle b$

Trig

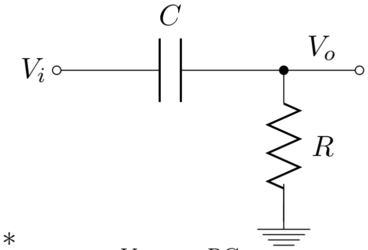
- $\cos^2(a) + \sin^2(a) = 1$
- $\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$
- $\sin(2a) = 2\sin(a)\cos(a)$
- $\cos^2(a) = \frac{1}{2}(1 + \cos(2a))$
- $\sin^2(a) = \frac{1}{2}(1 - \cos(2a))$

RC Filter

- Transmission Function: $T(s) = \frac{V_o(s)}{V_i(s)}$
- Corner frequency: frequency s at which $T(s) = \frac{1}{\sqrt{2}}$
- for simple circuit: ground→source→ $R \to C \to$ground
  - $T(s) = \frac{1}{1+RCs}$
    $|T(j\omega)| = \frac{1}{\sqrt{1+R^2C^2s^2}}$
    $|\angle T(j\omega)| = \frac{1}{\sqrt{1+R^2C^2s^2}}$
- low pass



  *
  - $T(s) = \frac{V_o}{V_i} = \frac{1}{1+RCs}$
  - corner frequency: $s = \frac{1}{RC}$ (also pole)
  - pole: $\frac{1}{RC}$
- high pass



  *
  - $T(s) = \frac{V_o}{V_i} = \frac{RCs}{1+RCs}$
  - zero: $s = 0$, pole: $s = \frac{1}{RC}$

Bode Plots

- magnitude is plotted in dB:
  $|T(j\omega)|_{dB} = 20\log_{10}|T(j\omega)|$
- starts on y-axis at DC offset with slope 0
- just add together the bode plots of each individual pole, zero, and the DC offset
- poles always slope down, zeros slope up (applies for both magnitude and phase)
- dec=decade, e.g. from 10^0 to 10^1
- magnitude:
  - Pole/Zero at origin:
    constant slope $\pm 20db/dec$ for all ω; $0dB$ at $\omega = 10^0 = 1$
  - Pole/Zero at ω_0:
    0 for $\omega < \omega_0$
    slope $\pm 20\frac{db}{dec}$ after
  - Constant C: constant line at $20\log_{10}(|C|)$
- phase:
  - Pole at origin: constant $-\frac{\pi}{2}$ or $-90°$
  - Zero at origin: constant $+\frac{\pi}{2}$ or $+90°$
  - Pole/Zero at ω_0:
    0 for $\omega < \frac{\omega_0}{10}$
    slope linearly ($\pm 45°/dec$) until $10\omega_0$
    0 slope for $\omega > 10\omega_0$
  - Constant C: no effect (0 for all ω)
- Prof wants us to actually show the -3dB drop curve, not just a straight intersection

Solving systems with Op Amps

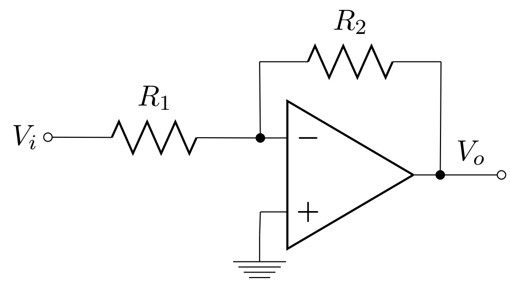
- step 0: if the op amp is ideal, write out ideal properties:
  - $V_+ = V_-$
  - $I_- = 0, I_+ = 0$
  - $A \approx \infty$
- avoid doing KCL/KVL directly on the output node of the op amp
- ignore resistors from a point at $0V$ to ground

Non-Ideal Op Amps

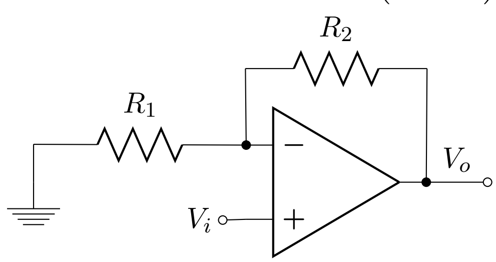
- still assume that current at input terminals is 0
- $V_o = A(V_+ - V_-)$
  - A: open-loop gain. Typically very large, 100,000 or more
- open-loop gain dependent on frequency: $A(s) = \frac{A_0}{1 - \frac{s}{\omega_b}}$
  - open-loop response drops off after ω_b (usually $2\pi \leq \omega_b \leq 2\pi 100$)
  - A_0: DC gain
  - ω_t: unity gain frequency: $dB(T(\omega_t)) = 1$
    $\omega_t \approx A_o\omega_b$
    AKA gain bandwidth product
  - in this case, we still assume $I_- = I_+ = 0$ and $V_- = V_+$?
- slew rate
  - max rate at which the output can change
  - for a sinusoidal signal: (V_{pk}: peak voltage)
    $SR > 2\pi f V_{pk}$ or $SR > \omega V_{pk}$
    $\frac{dV_o}{dt}|_{MAX} < SR$

Op Amp Equations

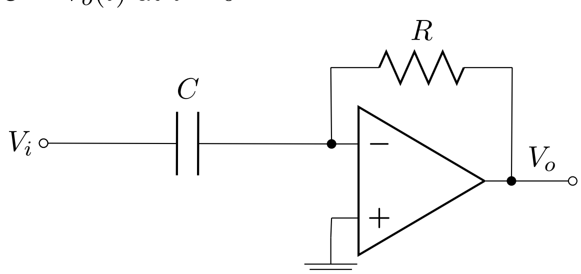
- general form: $T(s) = \frac{K_0}{1+\frac{s}{\omega_0}}$
  - $*\,T(0) = K_0$: DC offset. For these simple ones, it's equal to ideal response
  - $*\,\omega_0 = \frac{\omega_t}{1+R_2/R_1}$
- inverting op amp:
  - $*$ ideal: $T(s) = \frac{V_o}{V_i} = -\frac{R_2}{R_1}$
  - $*$ non-ideal:
    $T(s) = \frac{V_o}{V_i} = \frac{-R_2/R_1}{1+\frac{1+R_2/R_1}{A(s)}} = \frac{-R_2/R_1}{1+\frac{s}{\left(\frac{\omega_t}{1+R_2/R_1}\right)}} = \frac{-R_2/R_1}{1+\frac{s}{\omega_0}}$



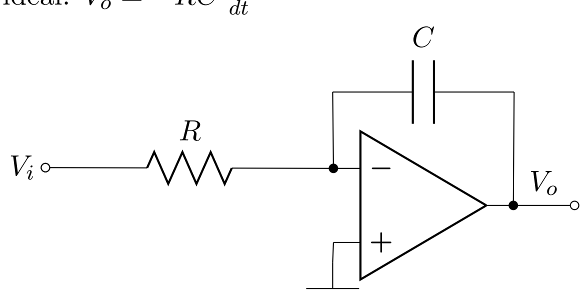
- non-inverting op-amp:
  - $*$ ideal: $T(s) = \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$
  - $*$ non-ideal:
    $T(s) = \frac{V_o}{V_i} = \frac{1+R_2/R_1}{1+\frac{1+R_2/R_1}{A(s)}} = \frac{1+R_2/R_1}{1+\frac{s}{\left(\frac{\omega_t}{1+R_2/R_1}\right)}} = \frac{1+R_2/R_1}{1+\frac{s}{\omega_0}}$



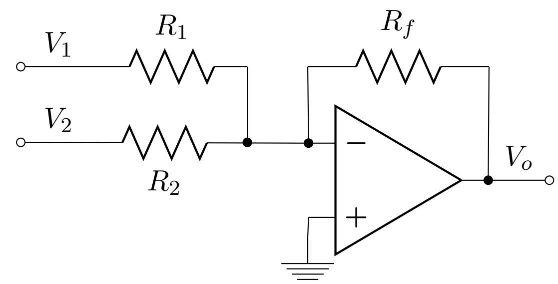
- integrating
  - $*$ ideal: $V_o = -\int_0^t \frac{V_i}{RC} dt + C$
    % $C = V_o(t)$ at $t = 0$



- differentiating
  - $*$ ideal: $V_o = -RC\frac{dV_i}{dt}$



- summing
  - $*$ ideal: $V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$



- differential (subtracting)
  - $*$ ideal: $V_o = \frac{(R_f+R_1)R_g}{(R_g+R_2)R_1}V_2 - \frac{R_f}{R_1}V_1$