

General:

- `newtype Parser a = P (String -> [(a,String)])`
- Predicate: a function that takes one argument and returns a boolean
 - * if `pred x == True` then `x` satisfies predicate `pred`
- function composition:

```
-- the . operator composes functions:
(f . g) x == f (g x)
```

useful library functions:

```
-- Data.List
nubBy :: (a -> a -> Bool) -> [a] -> [a]
nubBy pred xs = -- unique elements only from xs as
                -- determined by pred

nub :: Eq a => [a] -> [a]
nub xs = nubBy (==) a -- unique elements from xs

--
words :: String -> [String]
words xs = -- list of whitespace-separated
           -- words from xs

--
-- concatenate container of lists
concat :: Foldable t => t [a] -> [a]
-- or for list-of-lists specifically:
concat :: [[a]] -> [a]
concat xs = foldl (++) [] xs

--
-- like concat, but use a function to get the inner lists
concatMap :: (a -> [b]) -> [a] -> [b]
concatMap f xs = foldr ((++) . f) [] xs

--
-- get the longest prefix of xs for which pred is true
-- and also return the rest of the list
span :: (a -> Bool) -> [a] -> ([a], [a])
span pred xs = (takeWhile pred xs, dropWhile pred xs)

--
-- repeat a = infinite list of a
repeat :: a -> [a]
repeat x = map (\_ -> x) [1..]
repeat x = [ x | _ <- [1..] ]

-- replicate n a = list of length n repeating a
replicate :: Int -> a -> [a]
replicate n x = map (\_ -> x) [1..n]
replicate n x = [ x | _ <- [1..n] ]

--
-- folds (works on any foldable, not just lists)
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z [a,b,c] = a 'f' (b 'f' (c 'f' z))
foldr f z [a,b,c] = f a $ f b $ f c z
-- combines into z from right to left
-- can potentially work on an empty list if one of the folds
-- does not evaluate it's second argument
foldl :: (b -> a -> b) -> b -> [a] -> b
foldl f z [a,b,c] = ((z 'f' a) 'f' b) 'f' c
foldl f z [a,b,c] = f (f (f z a) b) c
-- evaluates from right to left
-- will not work on infinite list because it must start at
-- the end of the list
--
-- these are the same as above, except they take the first
-- two elements for the first application of f
foldr1 :: (a -> a -> a) -> [a] -> a
foldl1 :: (a -> a -> a) -> [a] -> a
```

Parsing.hs:

- `sat :: (Char -> Bool) -> Parser Char`
 - * returns a character if that character satisfies the predicate
- `digit, letter, alphanum :: Parser Char`
 - * parses a digit, letter, or alpha-numeric letter respectively
- `char :: Char -> Parser Char`
 - * `char 'a'` parses exactly the character `'a'`
- `item :: Parser Char`
 - * parses any character
- similar to above: `digit letter alphanum lower upper string`
- `many :: Parser a -> Parser [a]`
 - * parses 0 or more instances of `a` and collects them into a list
- `many1 :: Parser a -> Parser [a]`
 - * same as `many`, but
- `(+++)` choice:
 - * parse first argument if possible, else parse second argument
 - * first successfully parsed argument is returned

```
(+++) :: Parser a -> Parser a -> Parser a
p +++ q = P (\inp -> case parse p inp of
                    [] -> parse q inp
                    [(v,out)] -> [(v,out)])
```

- `((>=>))` sequential composition
 - * `a >=> b` unboxes monad `a` into an output `a0` and then unboxes monad `b` with input `a0`

```
type Parser a = String -> [(a, String)]
-- implementation for in-class mostly-complete
-- parser 'monads'
(>=>) :: Parser a -> (a -> Parser b) -> Parser b
(>=>) p1 p2 = \inp -> case parse p1 inp of
                    [] -> []
                    [(v, out)] -> parse (p2 v) out
```

* usage:

```
doubleDigit :: Parser [Char]
doubleDigit =
    digit >=> \a ->
    digit >=> \b ->
    return [a,b]
-- is equivalent to
doubleDigit' :: Parser [Char]
doubleDigit' = do
    a <- digit
    b <- digit
    return [a,b]
```

- * `(>>)` is the same except that it discards the result of the first monad (thus it has signature `(>>) :: Parser a -> Parser b -> Parser b`)

Parsing Examples:

- bind and lambda method of parsing:
 - * parse a number:
- parse arithmetic expressions using do syntax:

```
expr :: Parser Int
expr = do t <- term
        do {char '+'
           ;e <- expr
           ;return (t + e)}
        +++ return t
term :: Parser Int
term = do f <- factor
        do char '*'
           t <- term
           return (f * t)
        +++ return f
factor :: Parser Int
factor = do d <- digit
           return (digitToInt d)
        +++ do char '('
           e <- expr
           char ')'
           return e
eval :: String -> Int
eval xs = fst (head (parse expr xs))
```

Trees:

- represent either a leaf node or some kind of internal node
- arithmetic tree declaration:

```
data Expr = Val Int
          | Neg Expr
          | Add Expr Expr
          | Mul Expr Expr
```

- how to fold over a tree:

```
-- exprFold valF negF addF mulF
exprFold :: (Int->b) -> (b->b) -> (b->b->b) -> (b->b->b) -> b->b
-- mulF input output
(b->b->b) -> Expr -> b
exprFold valF _ _ (Val i) = valF i
exprFold valF negF addF mulF (Neg e)
    = negF (exprFold valF negF addF mulF e)
exprFold valF negF addF mulF (Add s1 s2)
    = addF (exprFold valF negF addF mulF s1)
           (exprFold valF negF addF mulF s2)
exprFold valF negF addF mulF (Mul s1 s2)
    = mulF (exprFold valF negF addF mulF s1)
           (exprFold valF negF addF mulF s2)
```

- * basically, just collect values into some type `b` and use supplied functions at each node to fold into single value
- * useful for evaluating simple things like:

```
-- evaluate an expression
evalExpr' = exprFold id (\x -> 0 - x) (+) (*)
id -- integers map to integers
(\x -> 0 - x) -- negation
```

```

-- everything else is just simple numeric operators
-- count leaves in a tree
countLeaves' = exprFold (\_ -> 1) id (+) (+)
(\_ -> 1) -- leaf integer node is one node
id -- negation node has only one child, pass on count
(+) (+) -- nodes with two children: add number
-- of leaf grandchildren

```

HW2: Water Gates:

```

waterGate :: Int -> Int
waterGate n =
  length -- number of True's
  $ filter id -- filter just True's
  $ waterGate' n initial -- initial call to helper
  where
    -- start with all gates closed
    initial = replicate n False
    -- flip states
    waterGate' 1 state = map not state
    -- base case: flip every state
    waterGate' n state = flip n $ waterGate' (n-1) state
    -- otherwise, first get the state for (n-1) and then flip every (n-1)th state
    -- flip every nth gate
    flip :: Int -> [Bool] -> [Bool]
    flip 1 xs = map not xs -- flip every gate
    -- flip only gates which index are multiples of n
    flip nth xs = [ if (i `mod` nth == 0) then not x else x
                    | (x,i) <- (zip xs [1..]) ]

```

HW2: Goldbach's Other Conjecture:

```

-- check if a number is prime
primeTest :: Integer -> Bool
primeTest 1 = False
primeTest t = and [ (gcd t i) == 1 | i <- [2..t-1]]
-- all numbers less than n that are double a square
twiceSquares :: Integer -> [Integer]
twiceSquares n = takeWhile (<n) [ 2 *x^2 | x <- [1..]]
-- list of odd numbers
oddList = map (\x -> 2*x + 1) [0..]
-- all odd numbers that are composite (not prime)
allOddComp = [ o | o <- (drop 1 oddList)
                , not (primeTest o) ]
-- if a number satisfies conditions for conjecture
-- method: for enough square nubmers, check if n-(that number)
-- is prime
satsConds n = or [ primeTest k |
                   k <- map (\x->(n-x)) (twiceSquares n) ]
-- find the first number
goldbachNum = head [ x | x <- allOddComp, not (satsConds x) ]

```

HW4: Sets:

```

type Set a = [a]
a = mkSet [1,2,3,4,5]
b = mkSet [1,2,3]
addToSet :: Eq a => Set a -> a -> Set a
addToSet s a | a `elem` s = s
              | otherwise = a : s
mkSet :: Eq a => [a] -> Set a
mkSet lst = foldl addToSet [] lst
isInSet :: Eq a => Set a -> a -> Bool
isInSet [] _ = False
isInSet [a] b = a == b
isInSet (x:xs) b | x == b = True
                  | otherwise = isInSet xs b
subset :: Eq a => Set a -> Set a -> Bool
subset sub super = and [ isInSet super x | x <- sub ]
setEqual :: Eq a => Set a -> Set a -> Bool
setEqual a b = subset a b && subset b a
-- instance (Eq a) => Eq (Set a) where
--   a == b = subset a b && subset b a
setProd :: Set a -> Set a -> [(a,a)]
setProd a b = [ (ai,bj) | ai <- a
                        , bj <- b ]

```

Prev Exam: Run Length Encoding:

```

import Parsing
import Data.Char
q4 = do
  d <- sat isUpper
  e <- char (toLower d)
  f <- many item
  return [d,e]
ones = (map (\_ -> 1) [1..])

```

```

myRLE [] = []
myRLE ls = myhelper (zip ones ls)
myhelper [(n,c)] = [(n,c)]
myhelper ((n,c):(m,d):rest)
  | (d == c) = myhelper ((n+m),c):rest
  | otherwise = (n,c):myhelper ((m,d):rest)

```

Rock Paper Scissors:

```

data RPS = Rock | Paper | Scissors
  deriving (Eq, Show)
rps :: RPS -> RPS -> Int
rps a b | a == b = 0
rps Rock    Scissors = 1
rps Paper    Rock     = 1
rps Scissors Paper    = 1
rps _        _        = 2
rps2 :: RPS -> RPS -> Int
rps2 a b =
  if a == b then 0 else case (a,b) of
    (Rock,    Scissors) -> 1
    (Paper,    Rock)    -> 1
    (Scissors, Paper)   -> 1
    _              -> 2

```

99 problems:

```

-- 9. pack consecutive duplicates into sublists
pack (x:xs) = let (first,rest) = span (==x) xs
               in (x:first) : pack rest
pack [] = []
-- example:
pack [1,2,3,2,2,3] == [[1,1],[2],[3],[2,2],[3]]

```