DTU

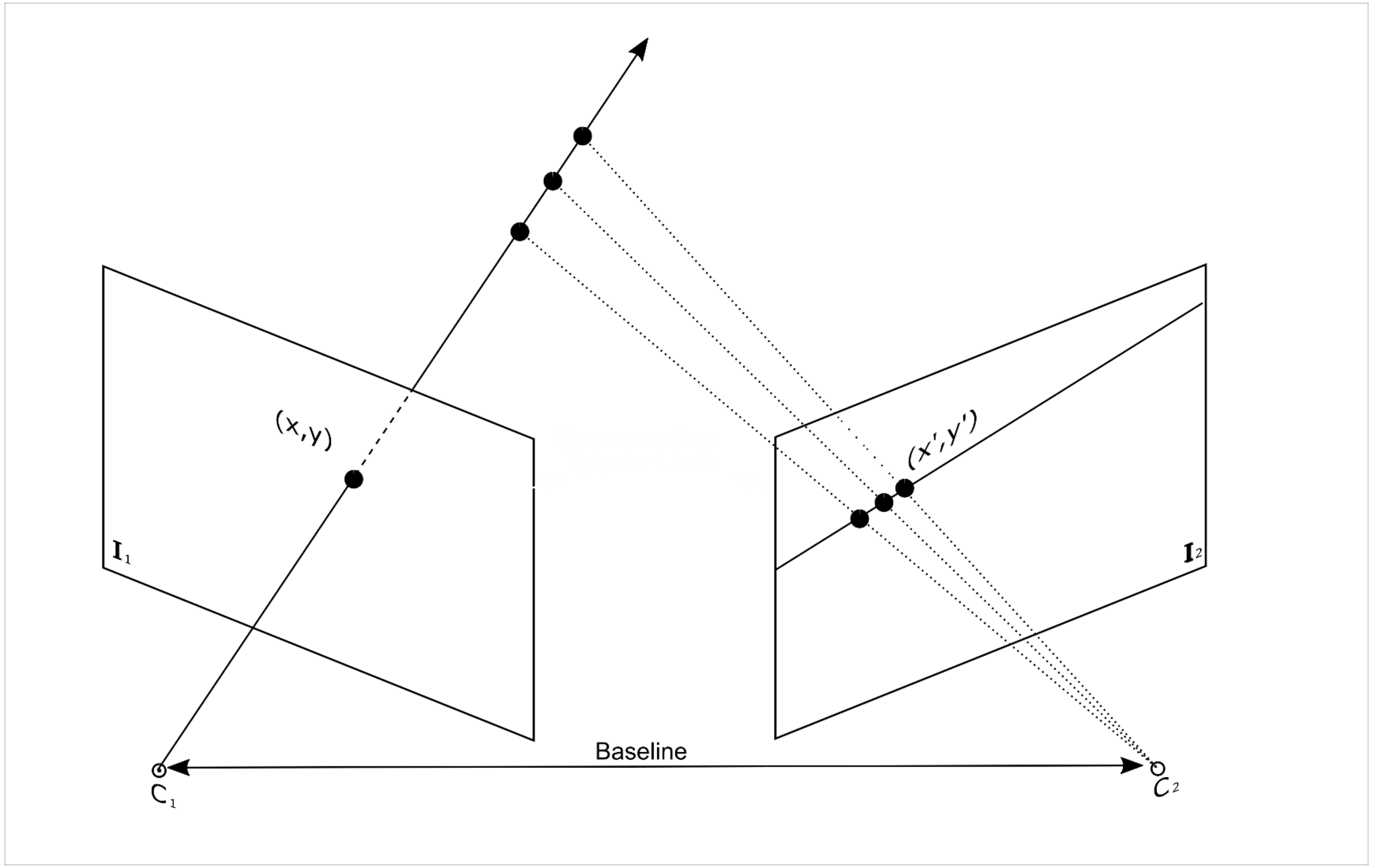
Perception for Autonomous Systems 31392:

Epipolar Geometry and the Fundamental Matrix

Lecturer: Evangelos Boukas—PhD

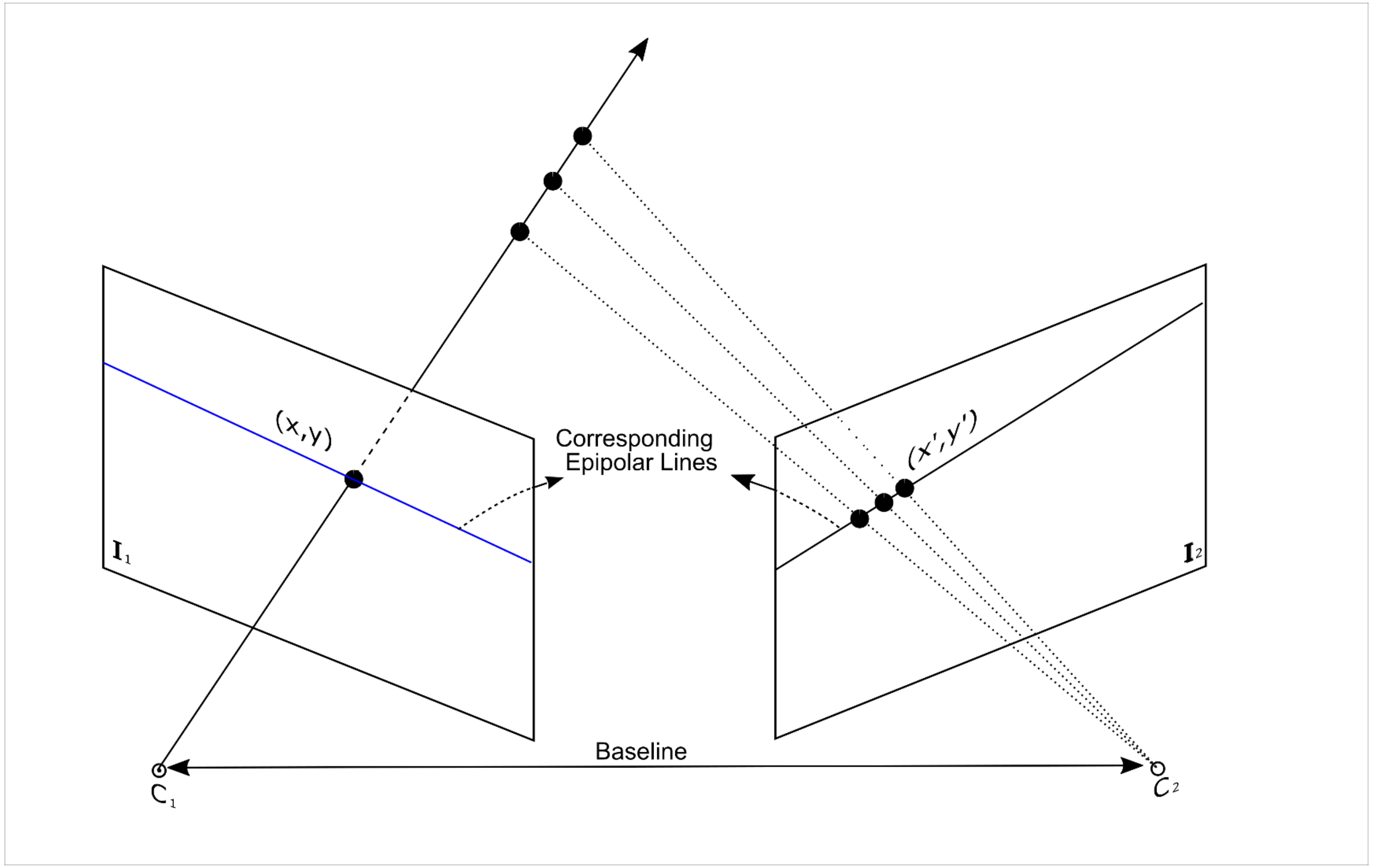
Epipolar Geometry: General Case

- Assuming:
  - 2 Camera Views
  - A ray passing through the camera center



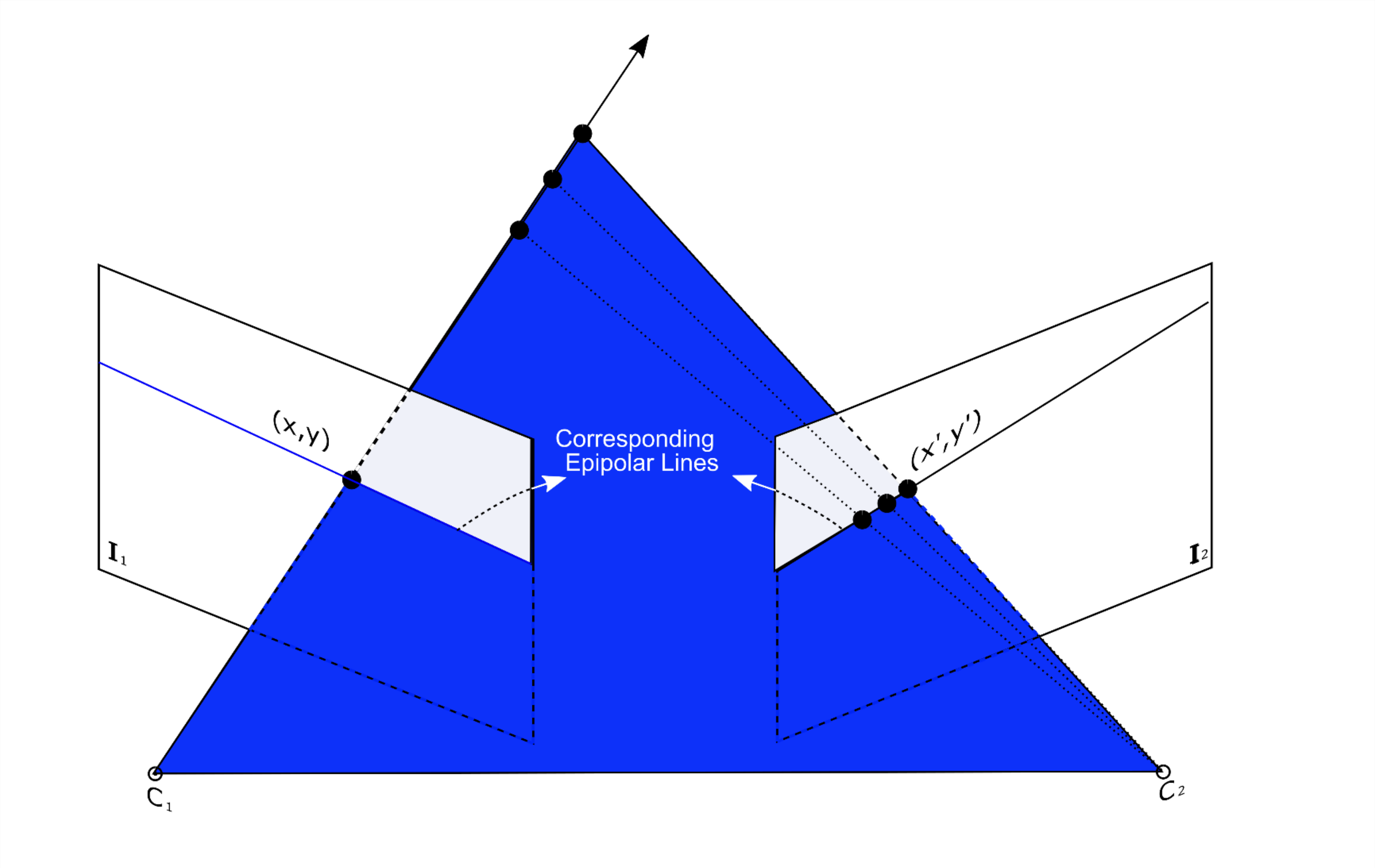
Epipolar Geometry: General Case

- Assuming:
  - 2 Camera Views
  - A ray passing through the camera center
- We can find the:
  - baseline and
  - the epipolar lines



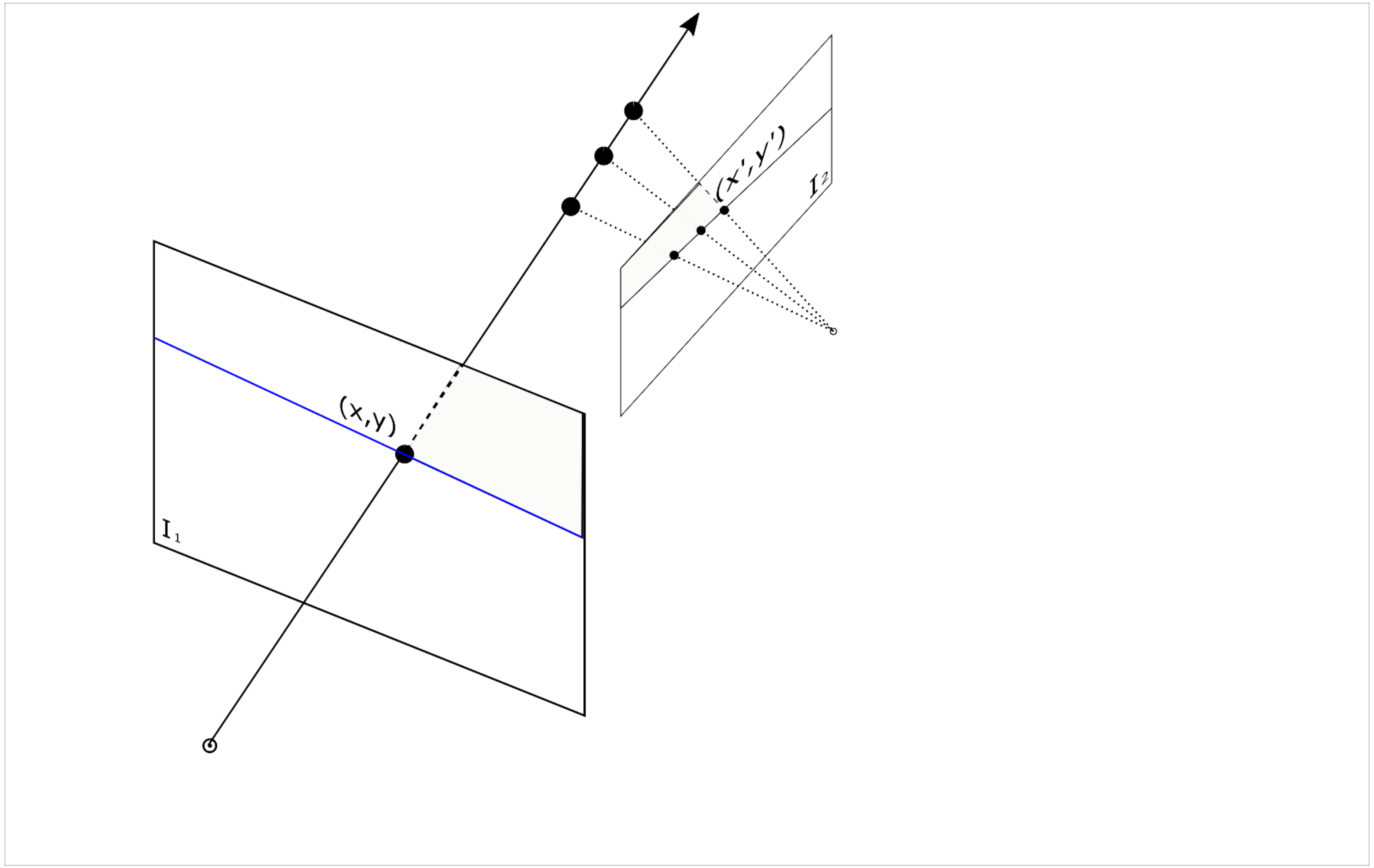
Epipolar Geometry: General Case

- Assuming:
  - 2 Camera Views
  - A ray passing through the camera center
- We can find the:
  - baseline and
  - the epipolar lines
  - Through the epipolar plane



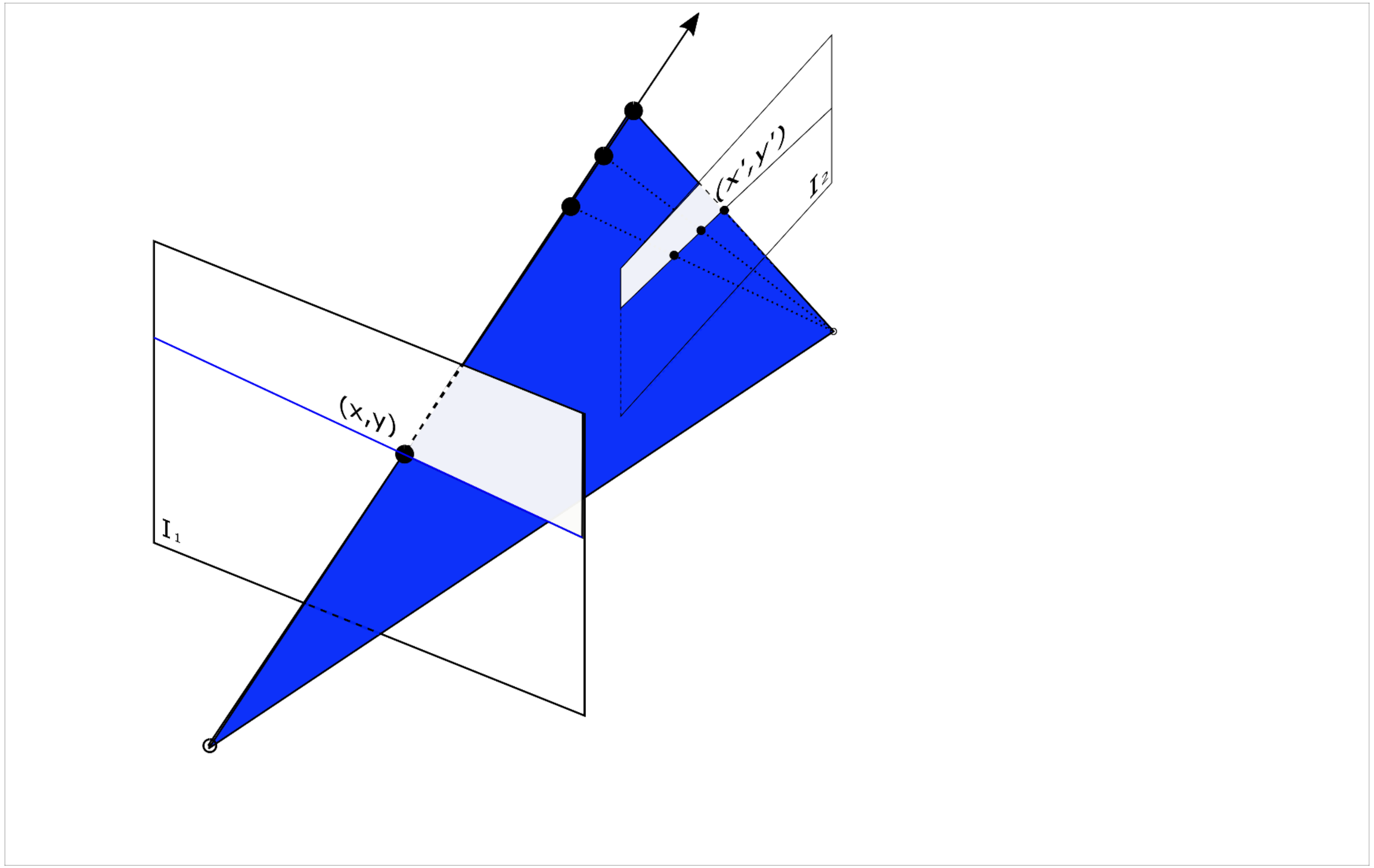
Epipolar Geometry

- What about this case?



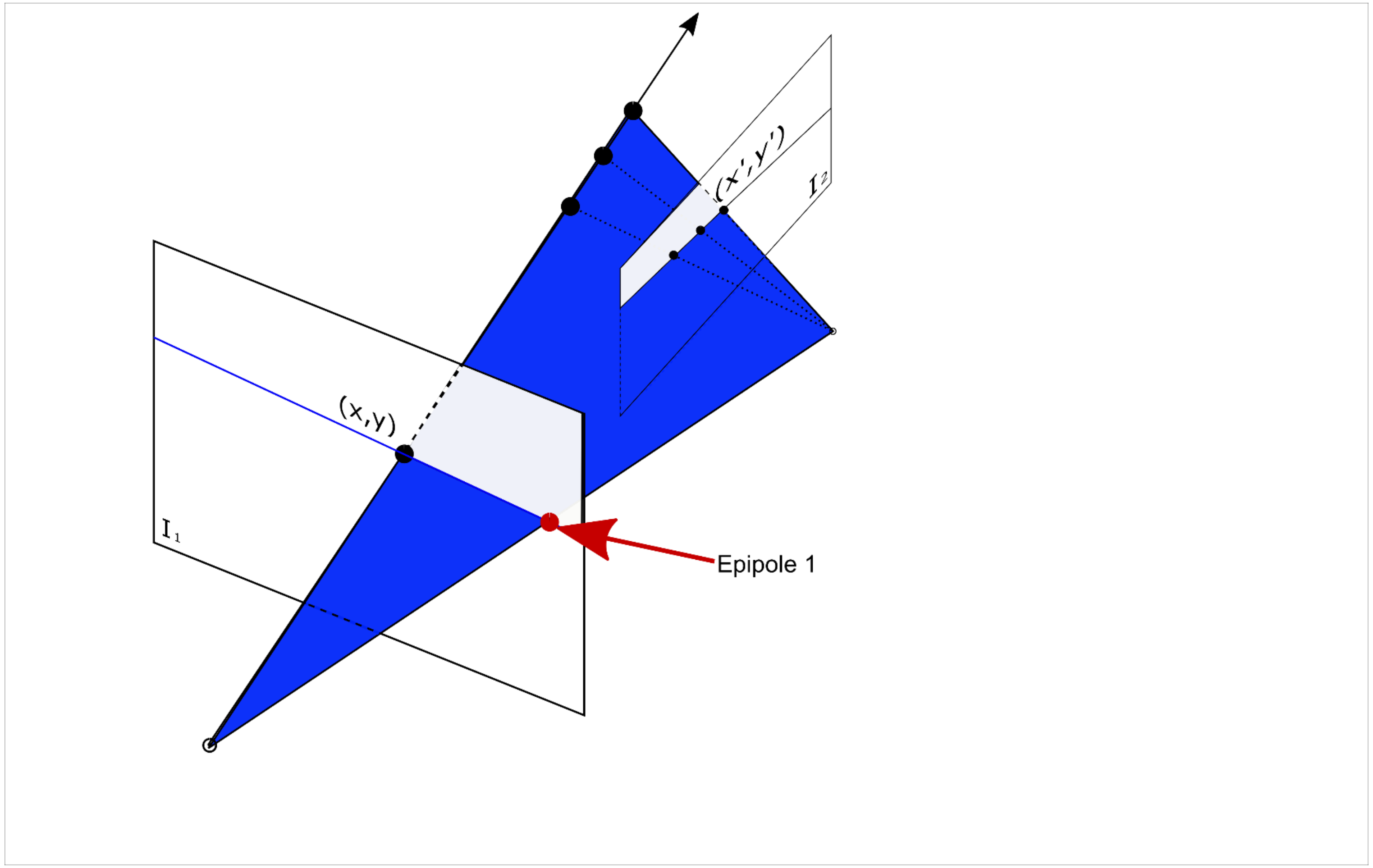
Epipolar Geometry

- What about this case?
- What’s the epipolar plane?



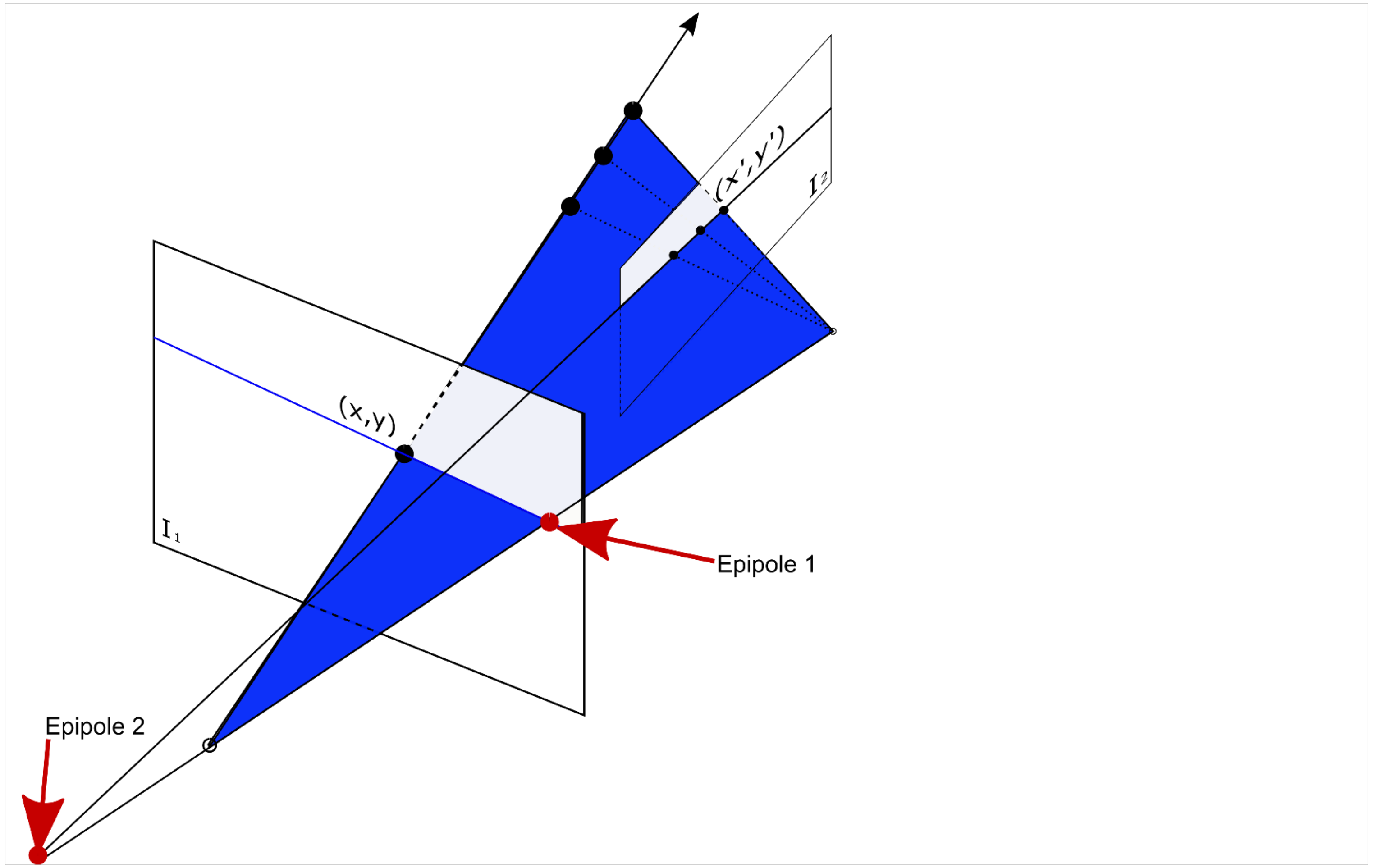
Epipolar Geometry

- What about this case?
- What’s the epipolar plane?
- Can we find the epipole of I_1?



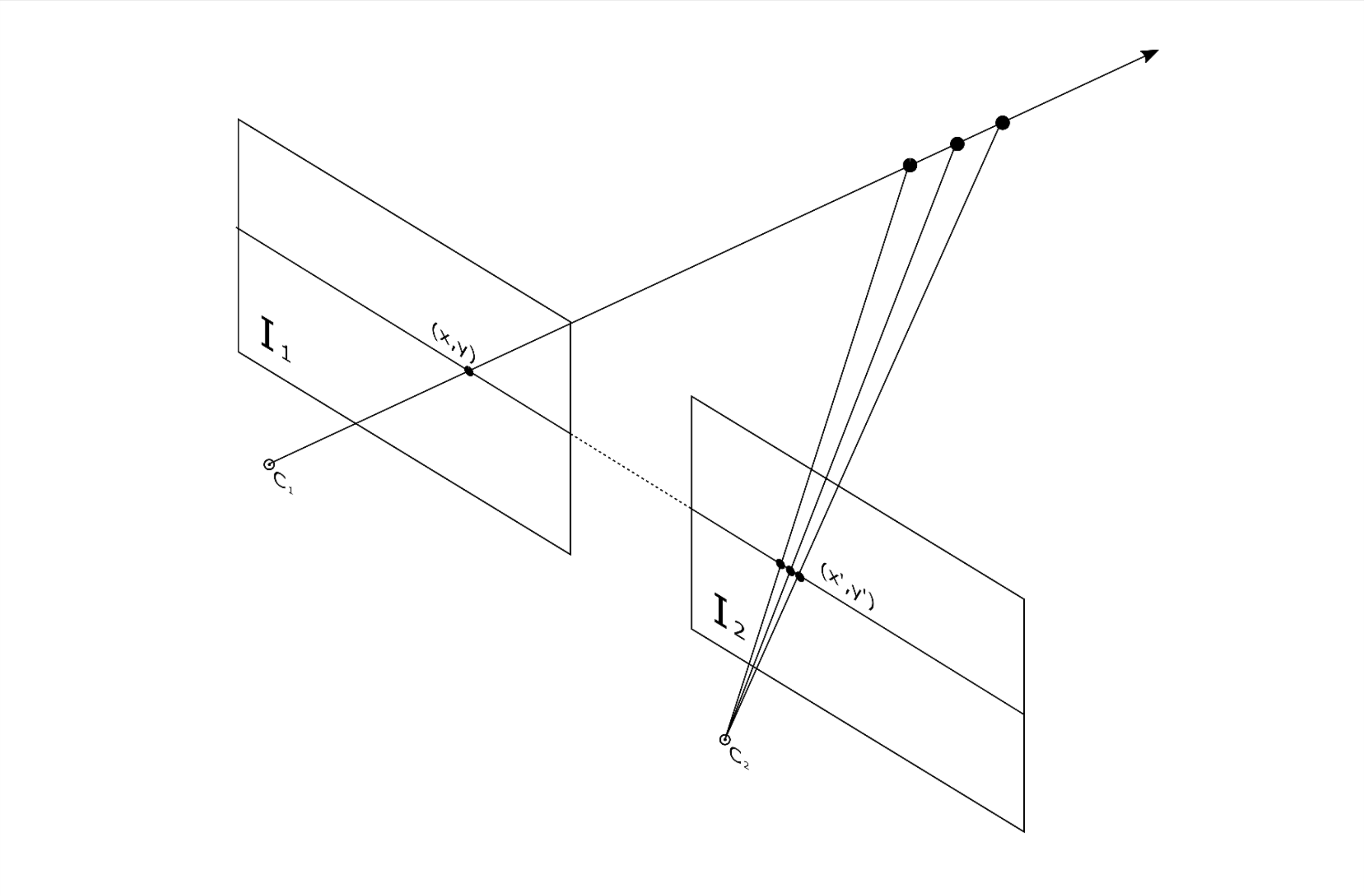
Epipolar Geometry

- What about this case?
- What's the epipolar plane?
- Can we find the epipole of I_1?
- What about the epipole of I_2?



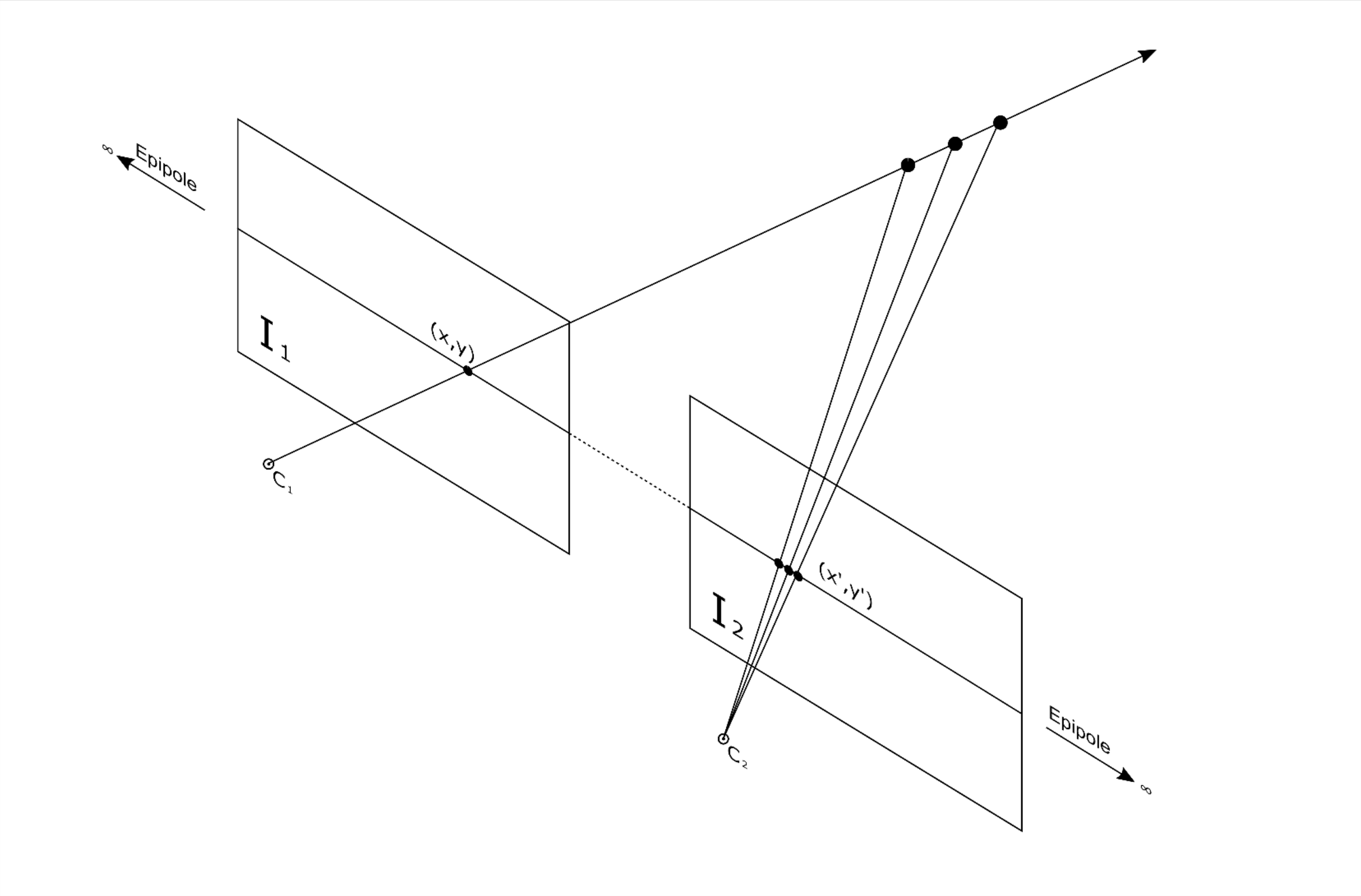
Epipolar Geometry:

- How about this case?



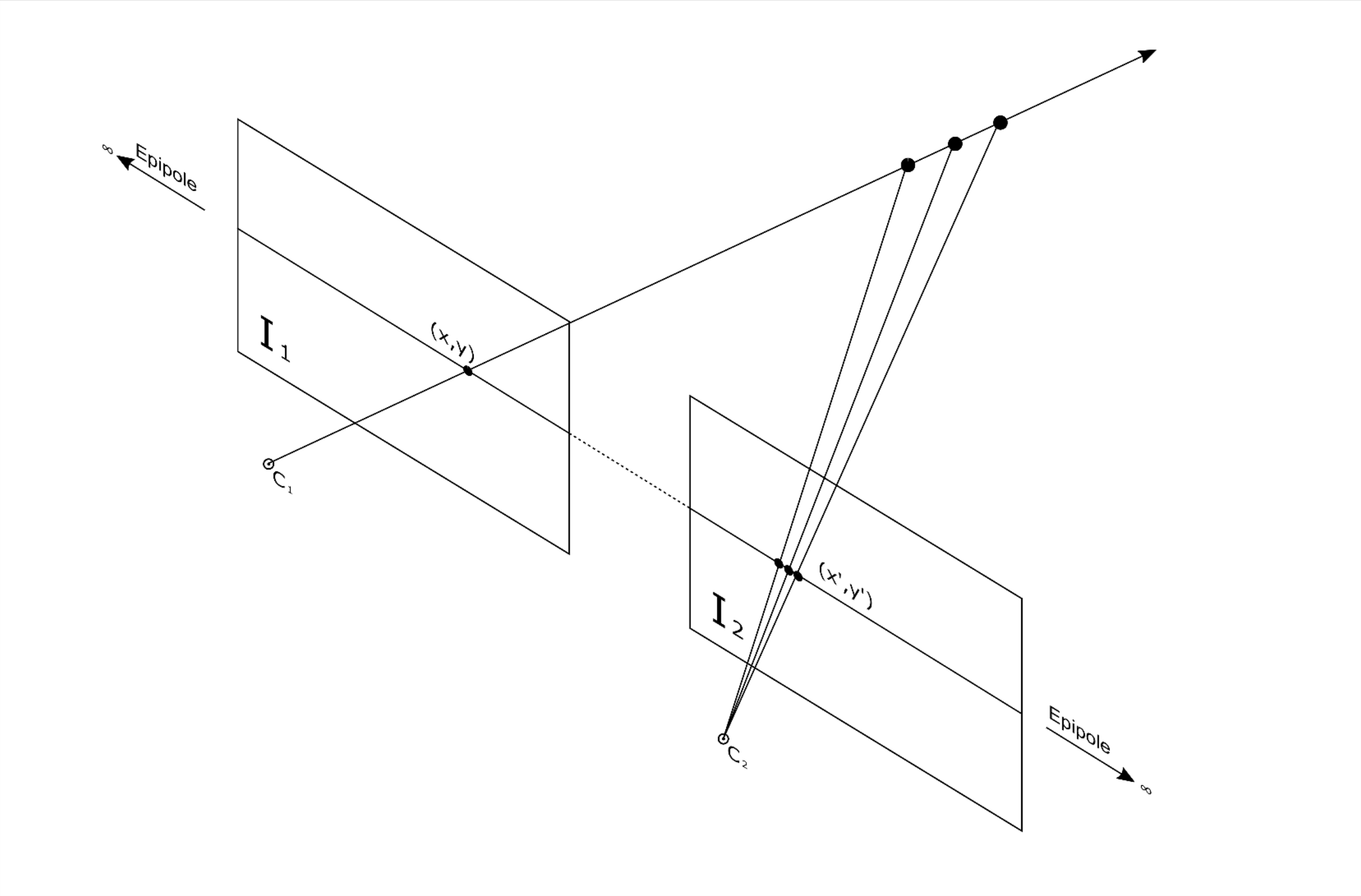
Epipolar Geometry:

- How about this case?
- Where are the epipoles?



Epipolar Geometry:

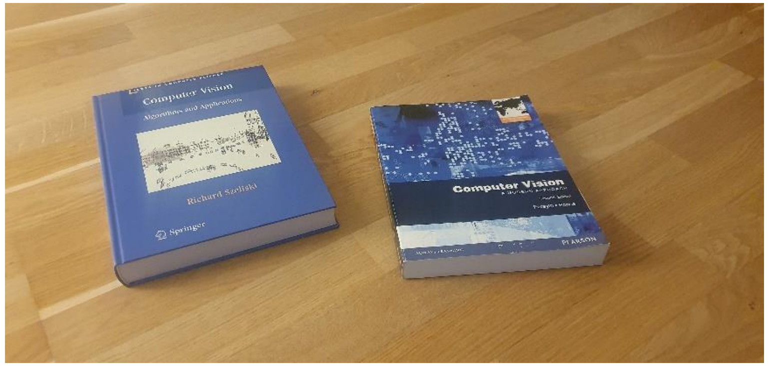
- How about this case?
- Where are the epipoles?



- How would such a case be usefull
  - The scanline is used in Disparity calculation

Example of Epipolar Lines “in the wild”

- Let’s think of this example:



- Where should the epipole be?

Example of Epipolar Lines “in the wild”

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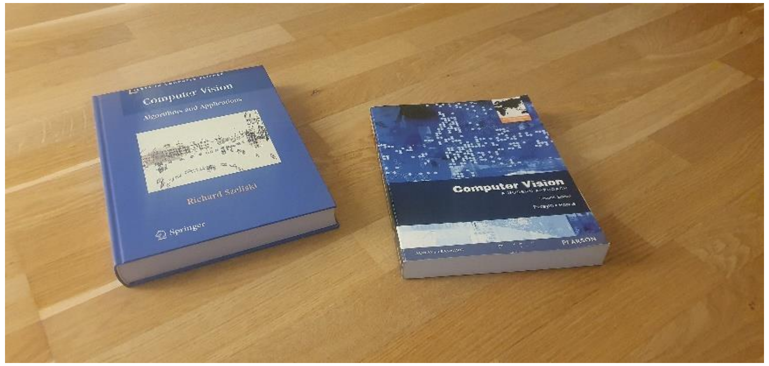
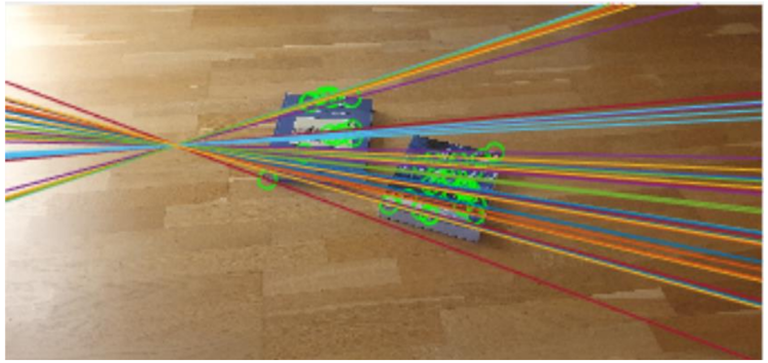


- Where should the epipole be?



Example of Epipolar Lines “in the wild”

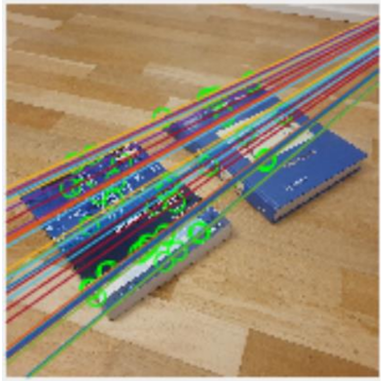
- What about this:

- How should the epipolar Lines look like?

Example of Epipolar Lines “in the wild”

- What about this:

- How should the epipolar Lines look like?

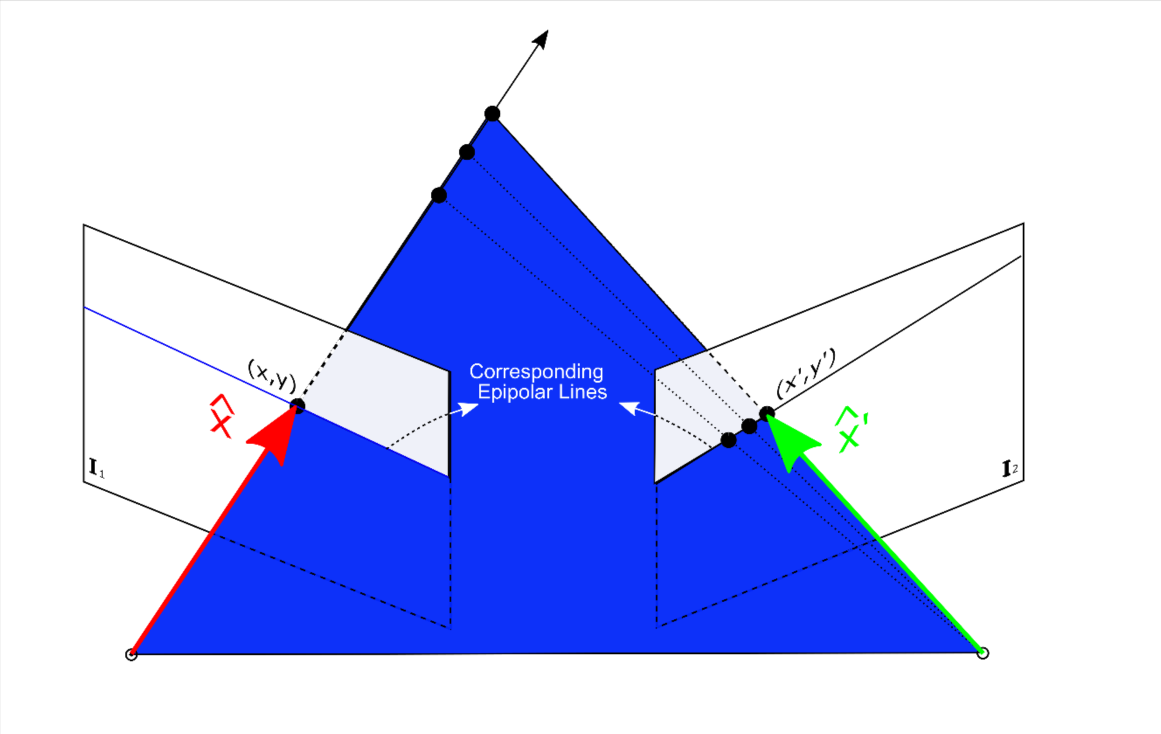


Essential Matrix

- Assuming two calibrated stereo pairs:
- We can express the points x, y from the image plane to homogeneous coordinates $\hat{x}$ and $\widehat{x'}$ using the inverse of the camera matrix

$$\hat{x} = K^{-1}x = X$$

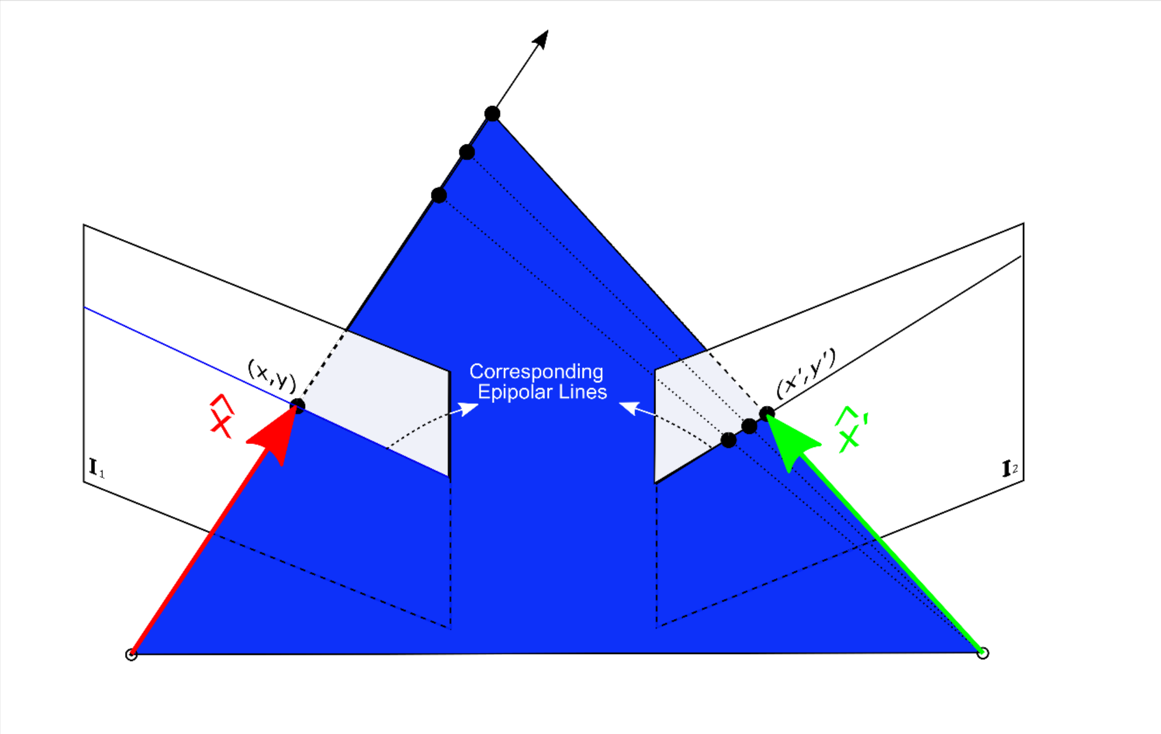
$$\hat{x}' = K'^{-1}x' = X'$$



Essential Matrix

- We can express the left homogeneous point to the right one:
- $\hat{x} = \mathrm{R} * \widehat{x'} + T$, where R is the Rotation Matrix and T the translation vector
- The we can prove that there is a Matrix connecting the two points $\hat{x}$, $\widehat{x'}$
- Trying to eliminate the left side by Applying cross product and then dot product

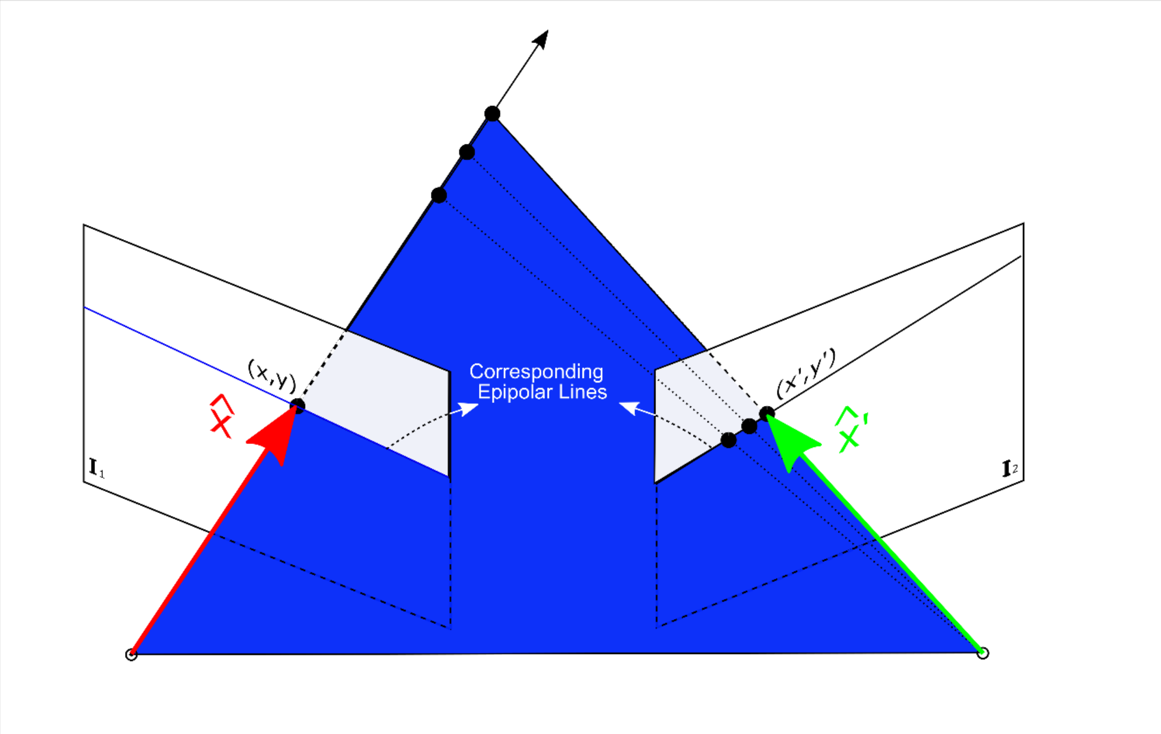
$$\mathrm{T}\,\mathbf{x}\,\hat{x} = \mathrm{T}\,\mathbf{x}\,\mathrm{R} * \widehat{x'} + T\,\mathbf{x}\,T$$
$$\hat{x}\,.\,\mathrm{T}\,\mathbf{x}\,\hat{x} = \hat{x}\,.\,\mathrm{T}\,\mathbf{x}\,\mathrm{R} * \widehat{x'} + 0$$
$$\underline{0 = \hat{x}\,.\,\mathrm{T}\,\mathbf{x}\,\mathrm{R} * \widehat{x'}}$$



- Or we can write it in matrix form: $\hat{x}^T E \hat{x}' = 0$ with $E = [t]_\times R$, where $[t]_x$ is the skew symmetric matrix

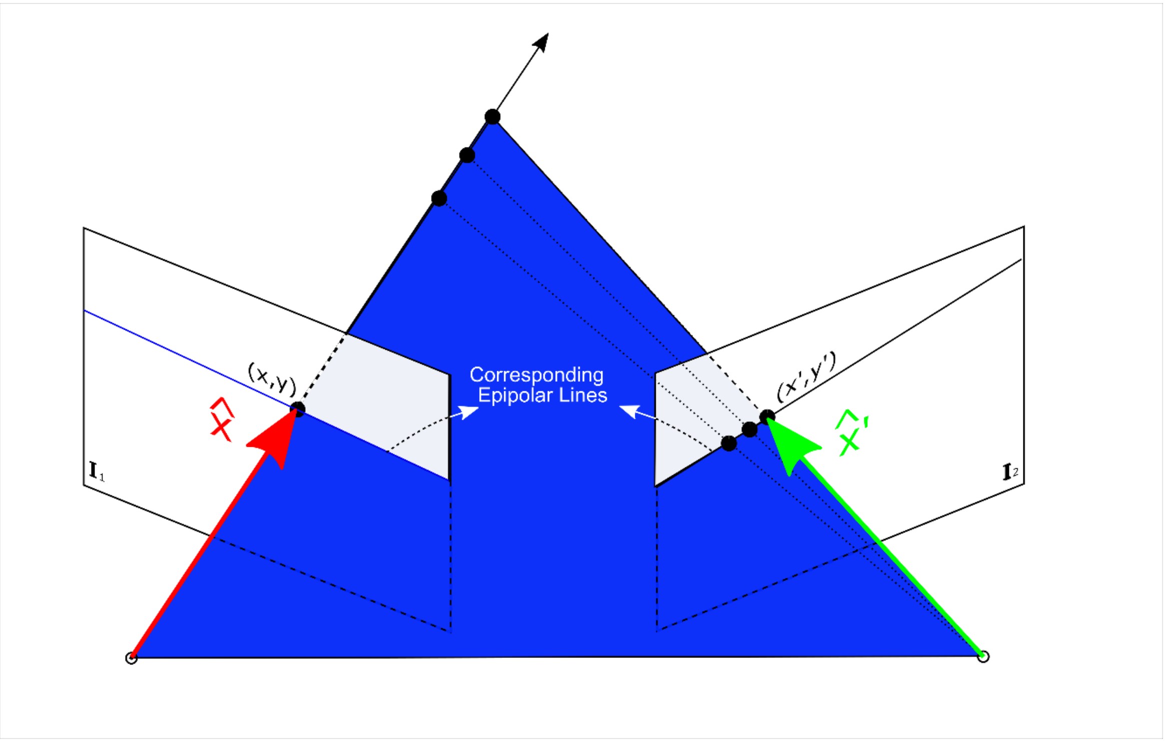
Essential Matrix

- The essential matrix $E = [t]_{\times} R$ is a 3x3 matrix, for which:
  - $E\,x'$ is the epipolar line associated with x' ($l = E\,x'$)
  - $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
  - $E\,e' = 0$ and $E^T e = 0$
  - E is singular (rank two)
  - E has five degrees of freedom



Fundamental Matrix

- We know how to get from a homogeneous point in one camera to another
- How can we get directly from one image to another?



$$\hat{x}^T E \hat{x}' = 0$$

$$\hat{x} = K^{-1} x$$

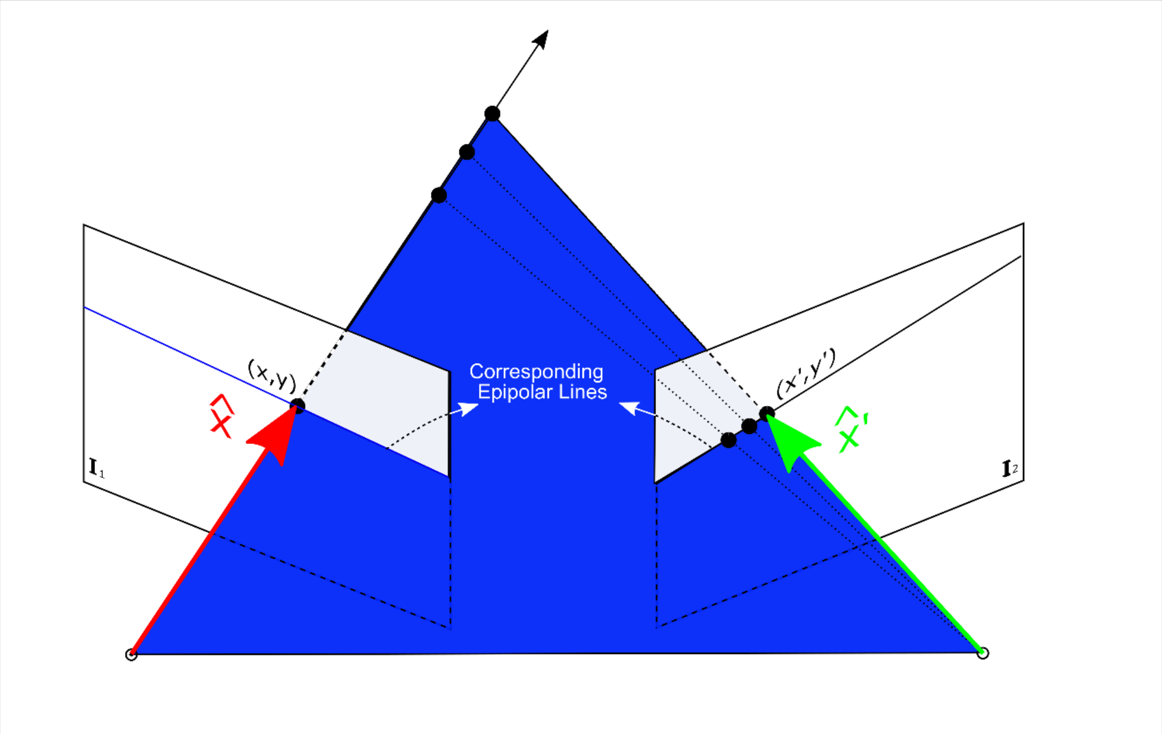
$$\hat{x}' = K'^{-1} x'$$

$$\Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

Which is the fundamental matrix

Fundamental Matrix

- The fundamental matrix $F = K^{-T} E K'^{-1}$ is a 3x3 matrix, for which:
  - $F\,x'$ is the epipolar line associated with x'
  - $F^T x$ is the epipolar line associated with x
  - $F\,e' = 0$ and $F^T e = 0$
  - F is singular (rank two): det(F)=0
  - F has seven degrees of freedom:



How to compute the ~~Homography~~ Fundamental Matrix

- Similar to DLT method as before

It's called the 8 point algorithm as we need 8 points to solve it

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$xx'f_{11} + xy'f_{12} + xf_{13} + yx'f_{21} + yy'f_{22} + yf_{23} + x'f_{31} + y'f_{32} + f_{33} = 0$$

$$\mathrm{A}\boldsymbol{f} = \begin{bmatrix} x_1x_1' & x_1y_1' & x_1 & y_1x_1' & y_1y_1' & y_1 & x_1' & y_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_ny_v' & x_ny_n' & x_n & y_nx_n' & y_ny_n' & y_n & x_n' & y_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

How to compute the Fundamental Matrix

- Homography (No Translation)
- Correspondence Relation

$$\mathbf{x}' = \mathbf{H}\mathbf{x} \Rightarrow \mathbf{x}' \times \mathbf{H}\mathbf{x} = \mathbf{0}$$

1. Normalize image coordinates

$$\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x} \quad \tilde{\mathbf{x}}' = \mathbf{T}'\mathbf{x}'$$

2. RANSAC with 4 points
   - Solution via SVD
3. De-normalize:

$$\mathbf{H} = \mathbf{T}'^{-1}\tilde{\mathbf{H}}\mathbf{T}$$

- Fundamental Matrix (Translation)
- Correspondence Relation

$$\mathbf{x}'^T \mathbf{F}\mathbf{x} = 0$$

1. Normalize image coordinates

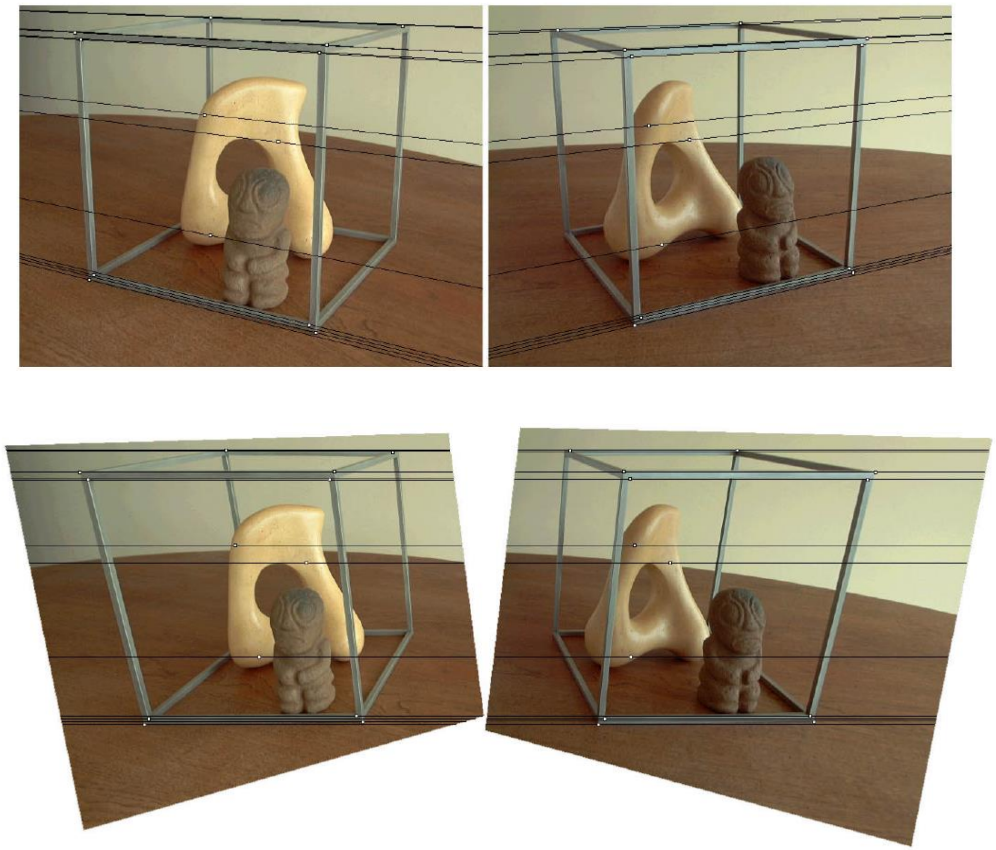
$$\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x} \quad \tilde{\mathbf{x}}' = \mathbf{T}'\mathbf{x}'$$

2. RANSAC with 8 points
   - Initial solution via SVD
   - Enforce $\det\left(\tilde{\mathbf{F}}\right) = 0$ by SVD
3. De-normalize:

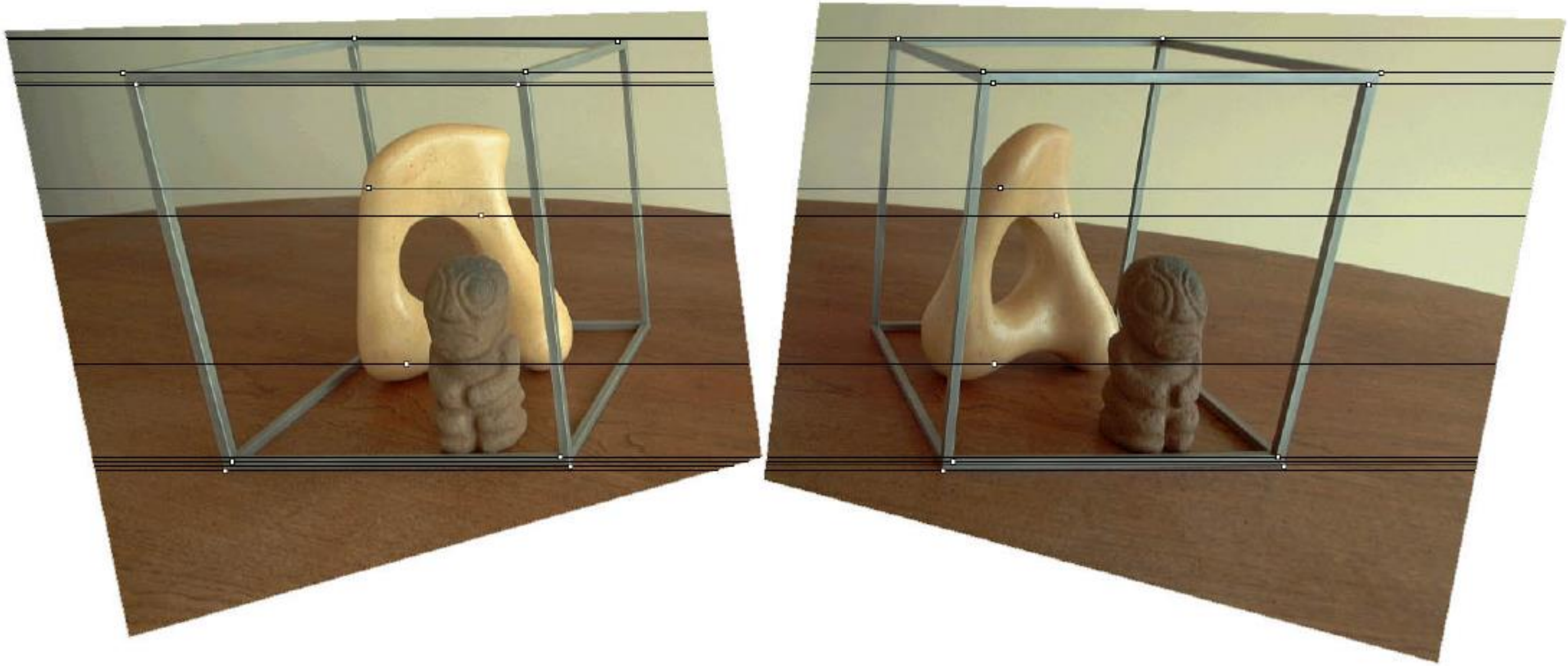
$$\mathbf{F} = \mathbf{T}'^T \tilde{\mathbf{F}}\mathbf{T}$$

Epipolar Geometry to Rectified Epipolar Geometry

- To finally go full circle:
  - To be able to use the stereo in a block matching algorithm to produce 3D points we must first **convert to rectified stereo**
  - **How would you do that?**



Rectified Epipolar Geometry

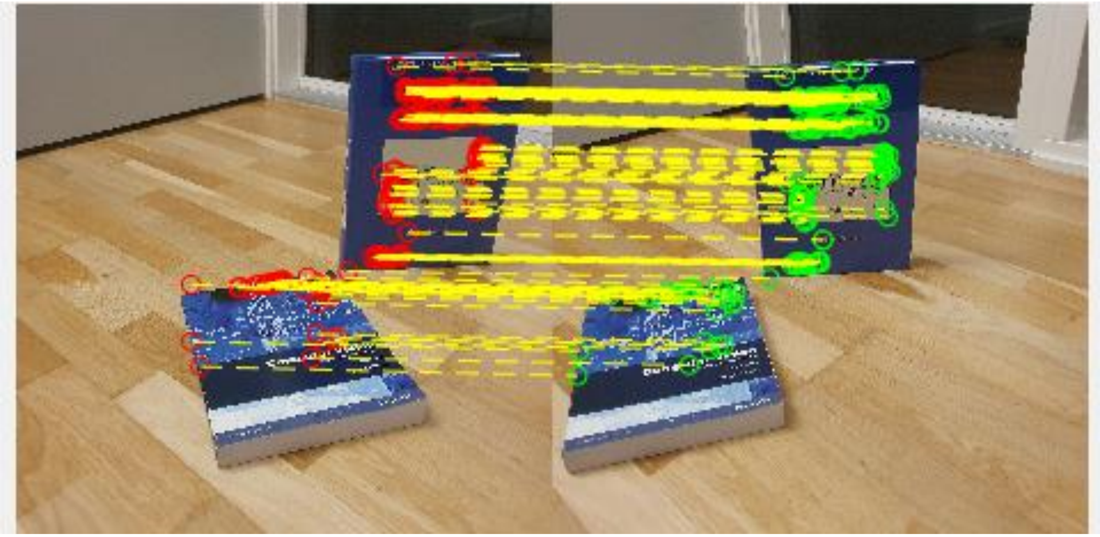


An Example

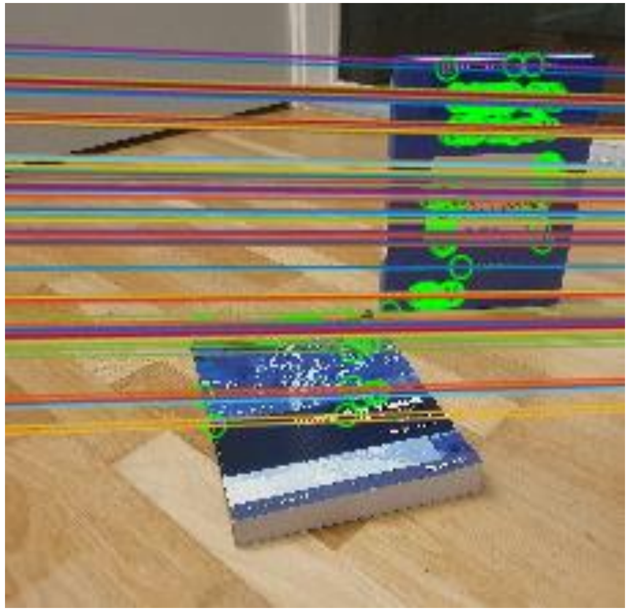
- Assume these two images

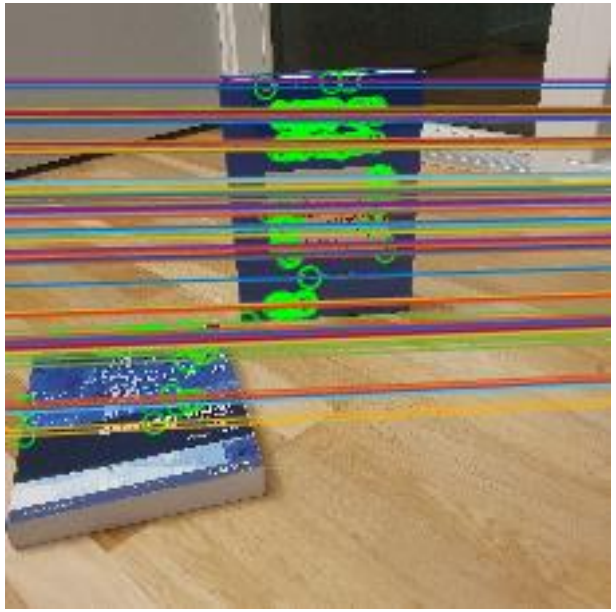
An Example

- First step is to match some points



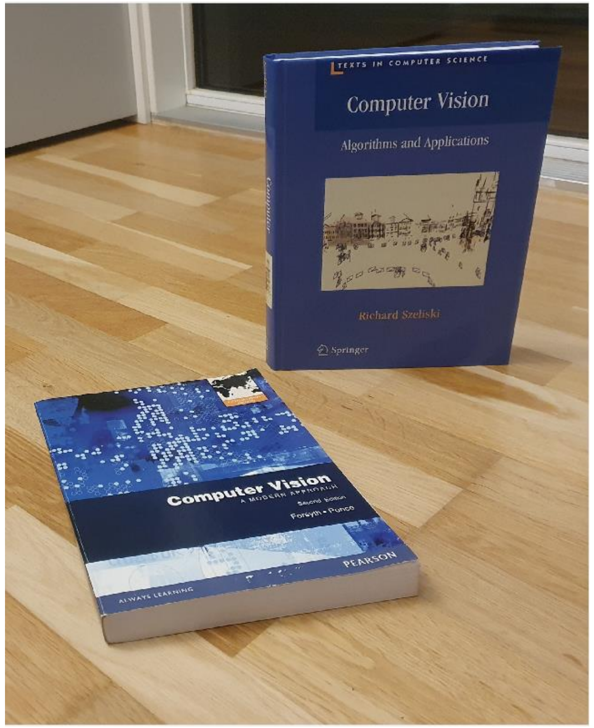
- Next, calculate the fundamental matrix

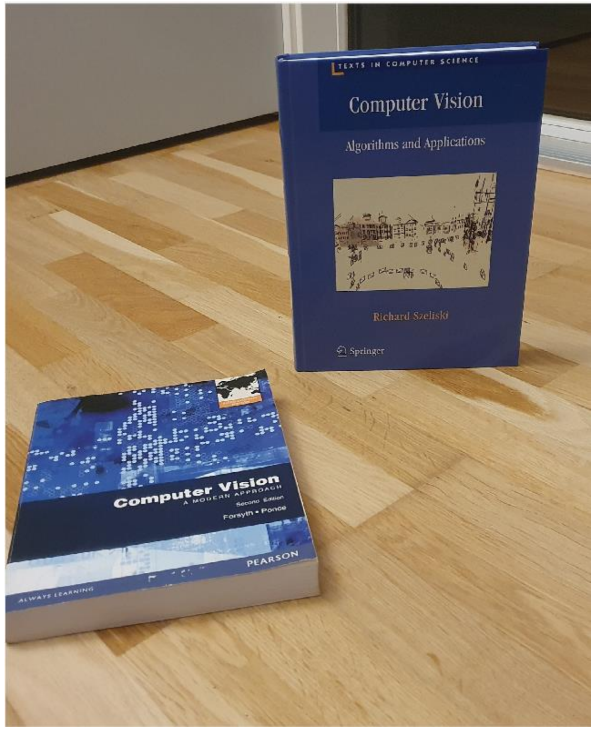




An Example

- Finally calculate the rectified images





- Notice how the images are now scan line

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