Perception for Autonomous Systems 31392:

State Estimation - Histogram Filter

Lecturer: Evangelos Boukas—PhD

Cases of State Estimation

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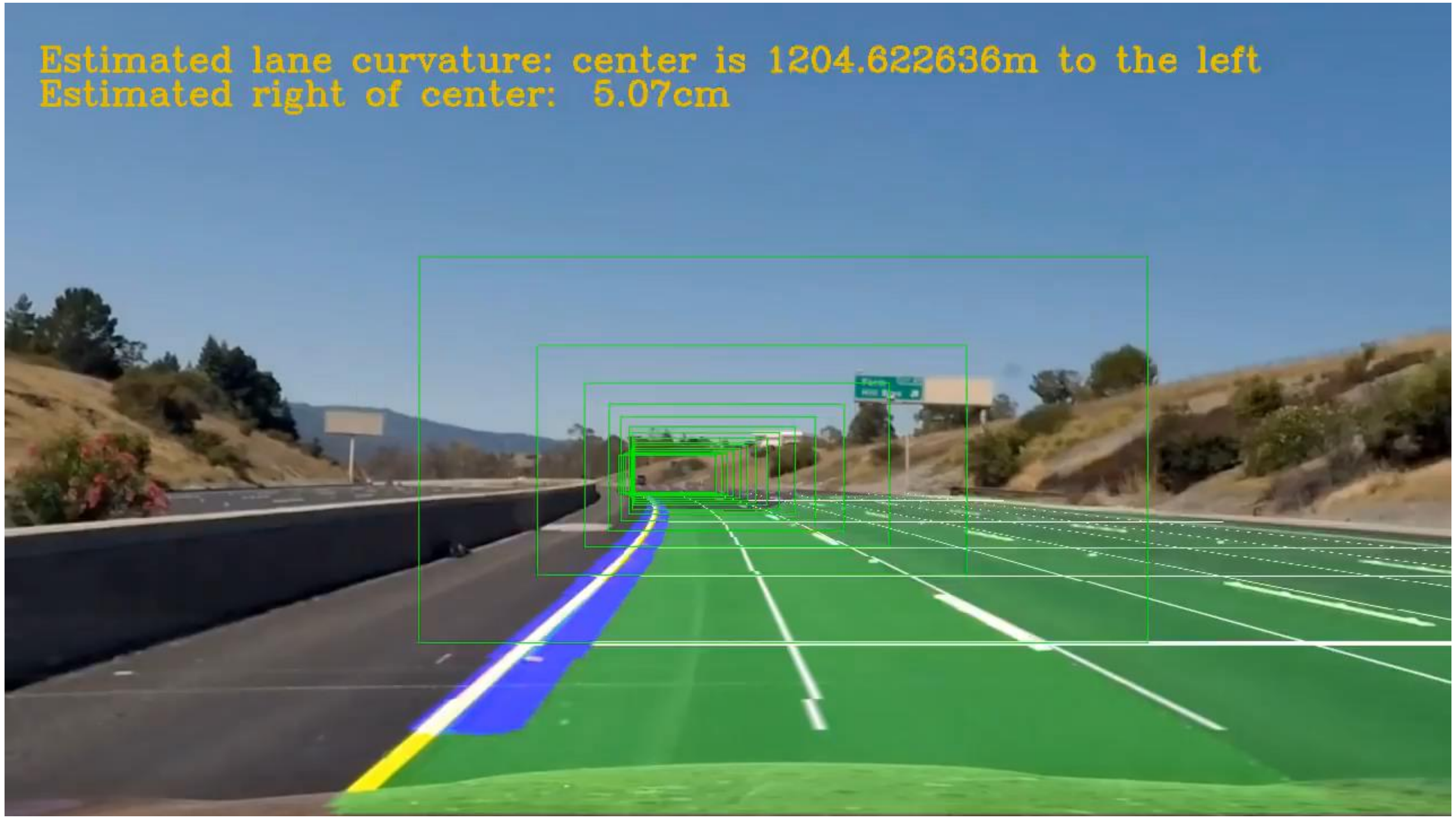

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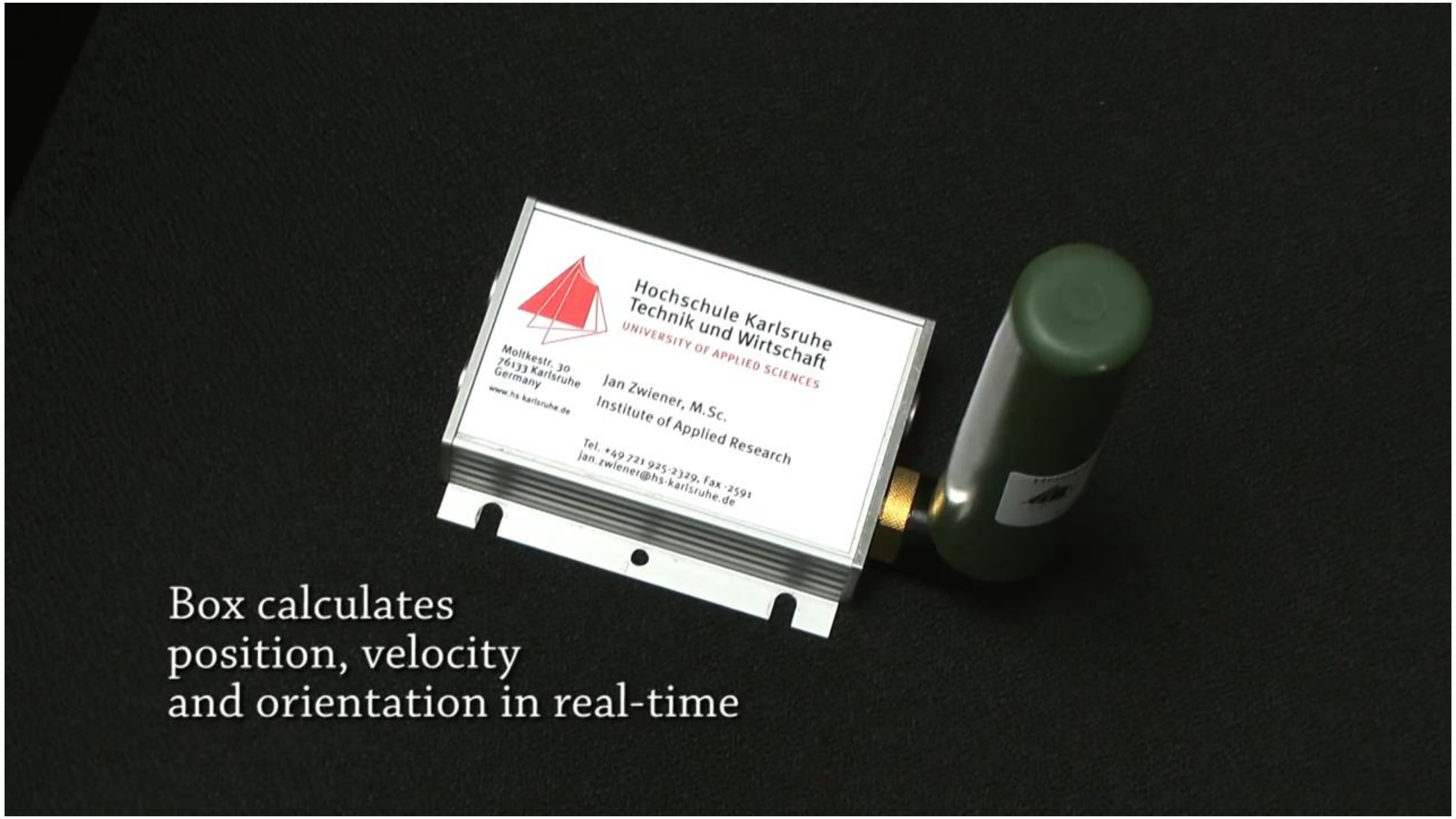


What is State Estimation

Goal:

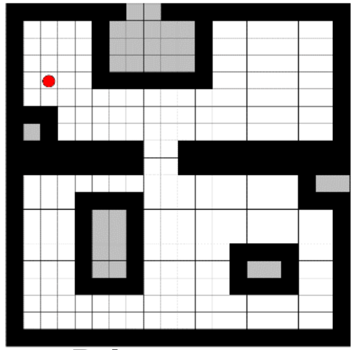
- Given a State Vector of a system
- Estimate over time the state using input of external sensors

Useful for:

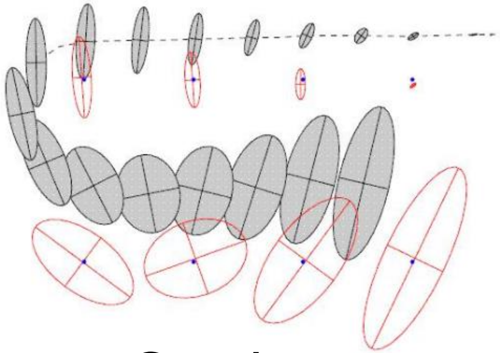
- Localization
- Tracking
- Prediction
- Sensor Fusion
- …

Continuous *vs* Discrete State
Unimodal *vs* Multimodal Distribution

- State:

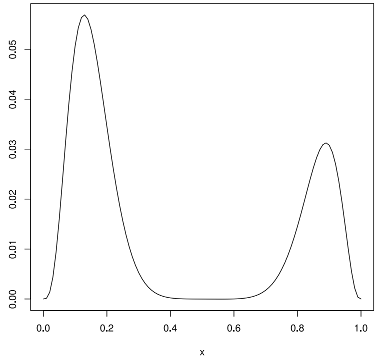


Discrete

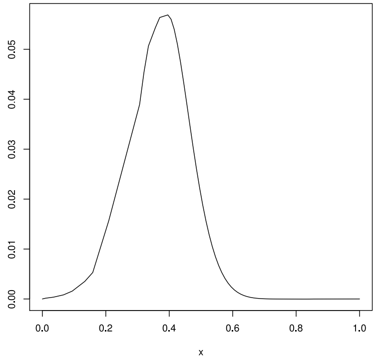


Continuous

- Distribution



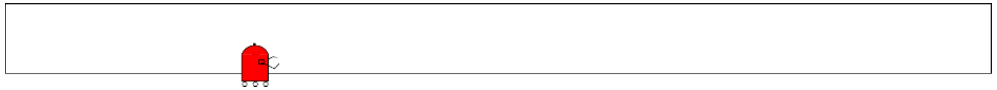
Multimodal



Unimodal

Simple State Estimation Example

- Assume a robot in an area like this one:



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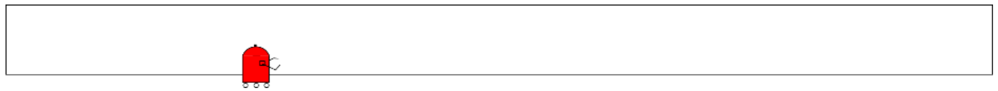
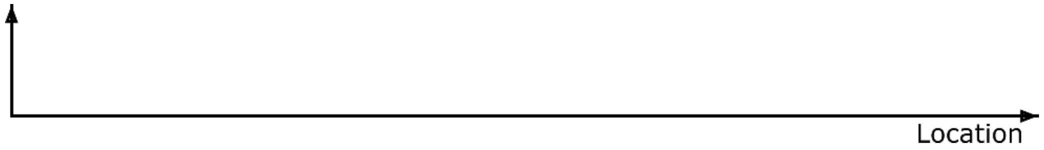


- We model the position of the robot as follows:



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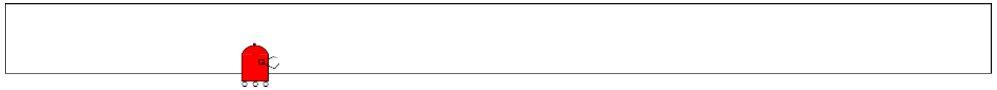

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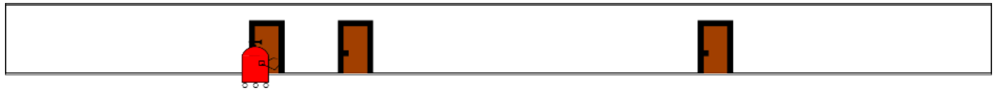

- We model the position of the robot as follows:

- The robot can distinguish (sense) a door vs “no-door”:

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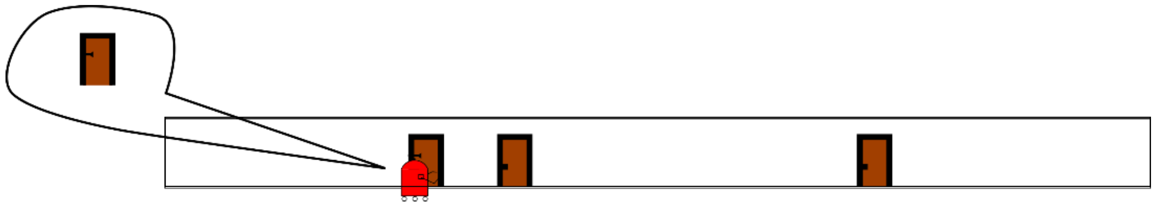

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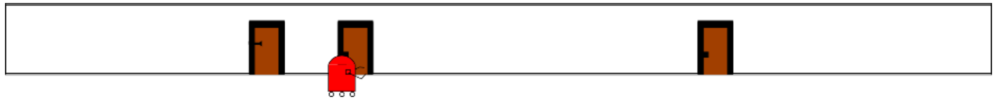

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- The robot moves to the right:

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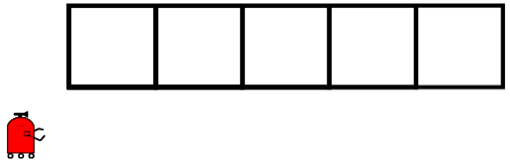
Sense, Let’s build it ourselves!

- This state estimation problem is called localization
- Assume a robot which can be in one of five blocks:



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X_1	X_2	X_3	X_4	X_5

- Without any info, What is the probability of the robot being in each cell?

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$$p(X_i) = \_\_\_\_$$
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X_1	X_2	X_3	X_4	X_5

- Without any info, What is the probability of the robot being in each cell?

$$p(X_i) = \underline{0.2}$$
$$\text{for } i \text{ in } (1 \ldots 5)$$

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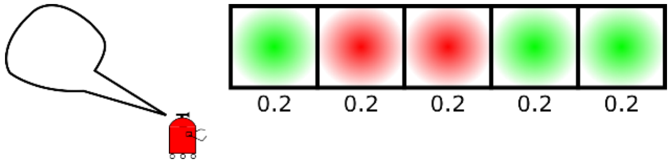
- Now our robot is allowed to sense:



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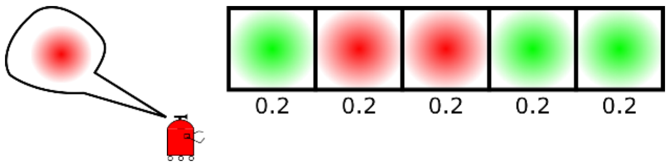

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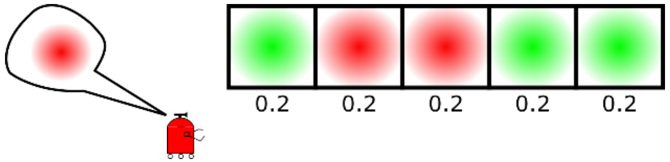

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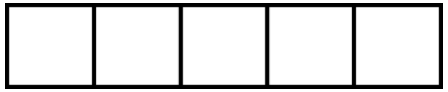

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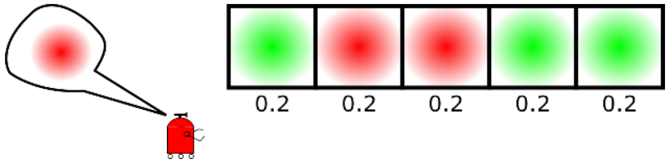

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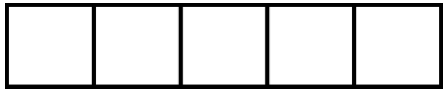

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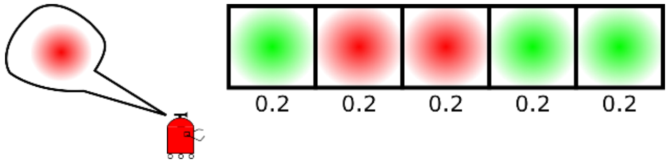

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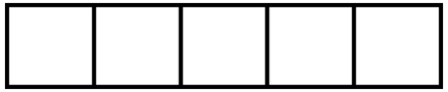

- Let’s start by arbitrarily multiplying any correct “sensing” with 0.6 & any incorrect with 0.2
- This is close to the belief. However, it is wrong, why?

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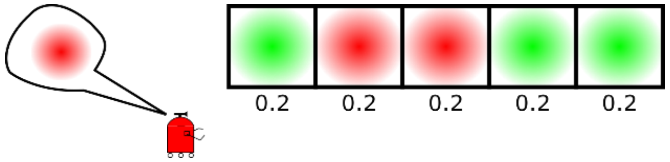

- What does that mean for our probability?

0.04	0.12	0.12	0.04	0.04

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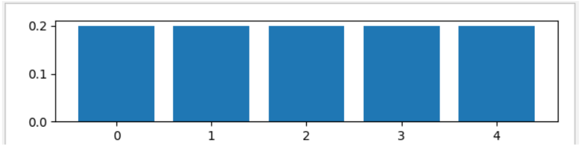

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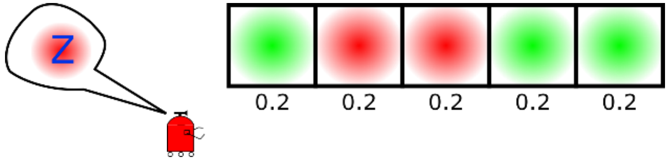
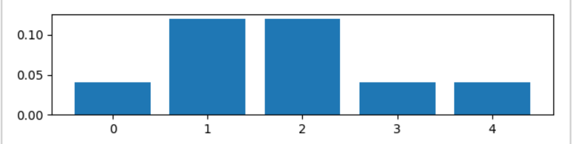
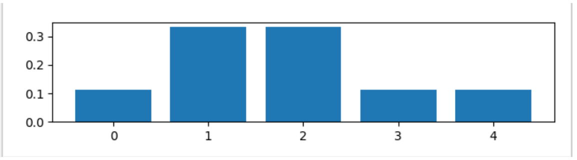




- What does that mean for our probability?

0.11	0.33	0.33	0.11	0.11



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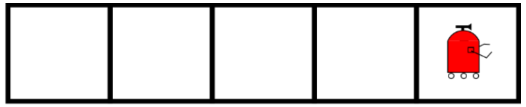
- This is close to the belief. However, it is wrong, why?
- It does not add up to one. How to do it?

$$p(X_i | Z)$$

Posterior Distibution

Motion, Let’s build it ourselves!

- Assume that the space is circular (i.e when moving right in the last cell you go to the first):



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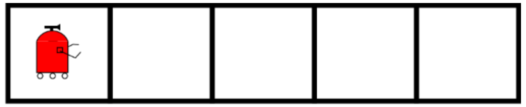

0.11	0.33	0.33	0.11	0.11

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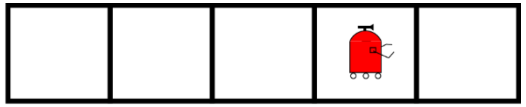

- What will happen with the posterior probability?

0.33	0.11	0.11	0.11	0.33

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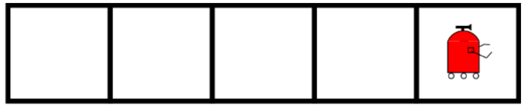

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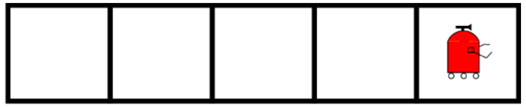

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1.0	0.0	0.0	0.0	0.0

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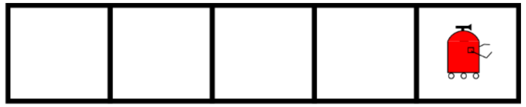

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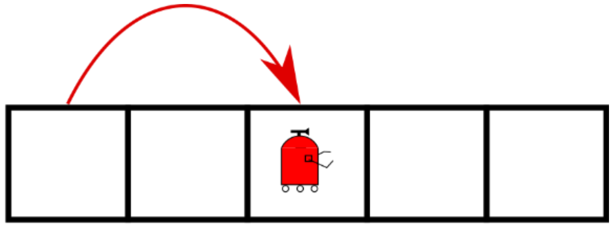

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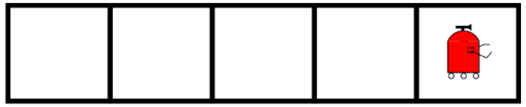

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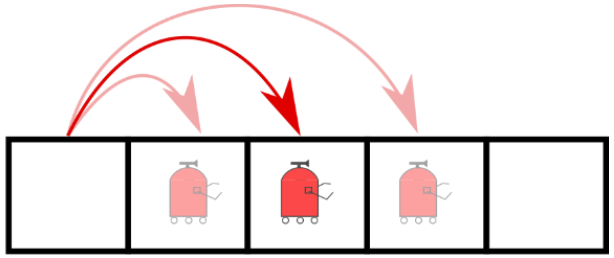

0.0	0.1	0.8	0.1	0.0

Uncertain Motion

$$p(X_{i+U-1}) = 0.1$$

$$p(X_{i+U}) = 0.8$$

$$p(X_{i+U+1}) = 0.1$$

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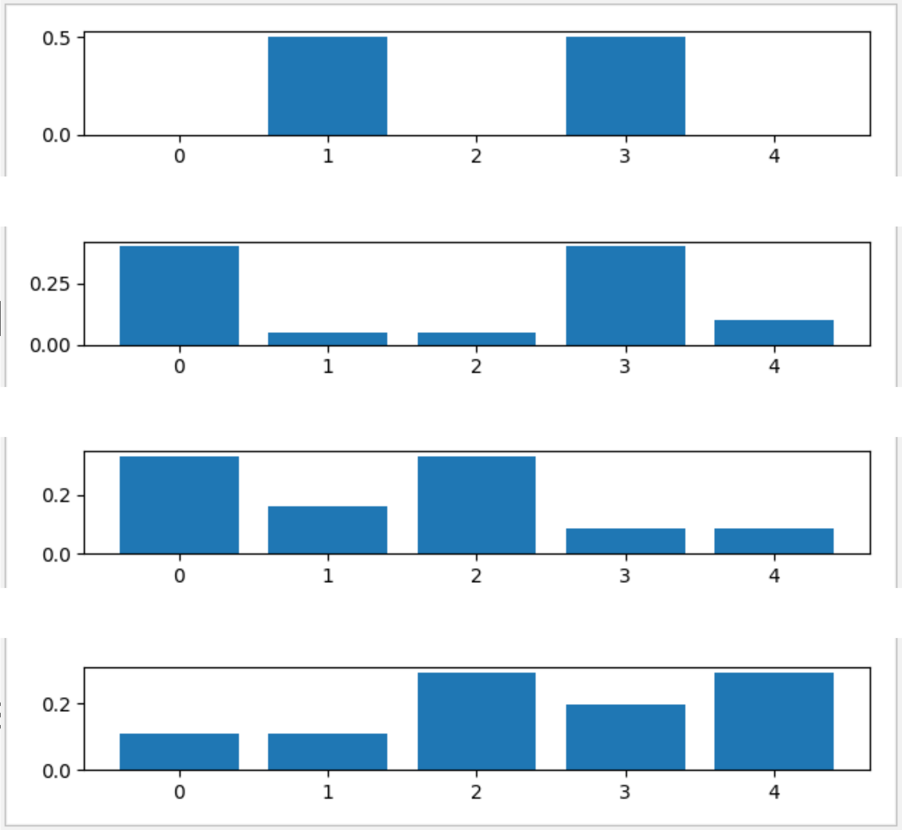

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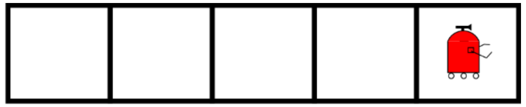

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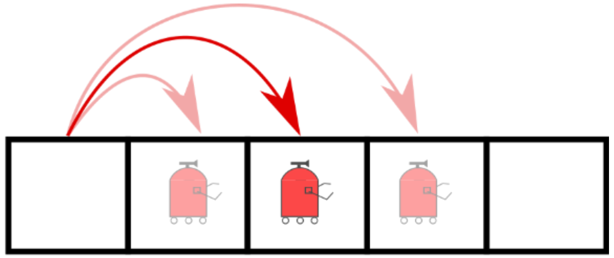

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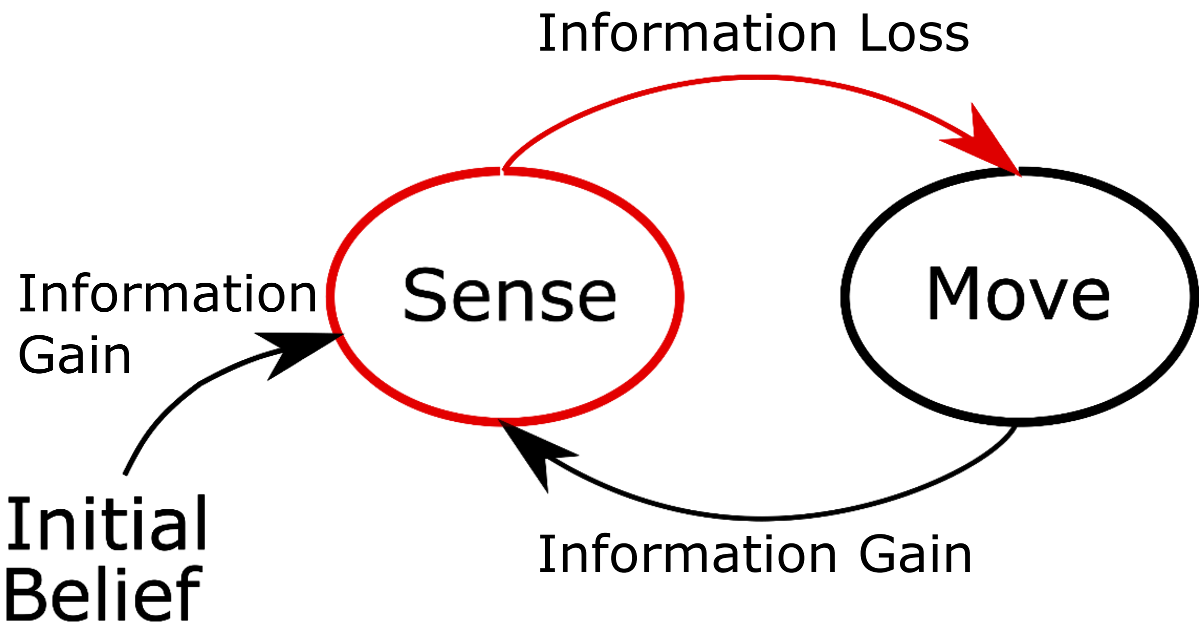

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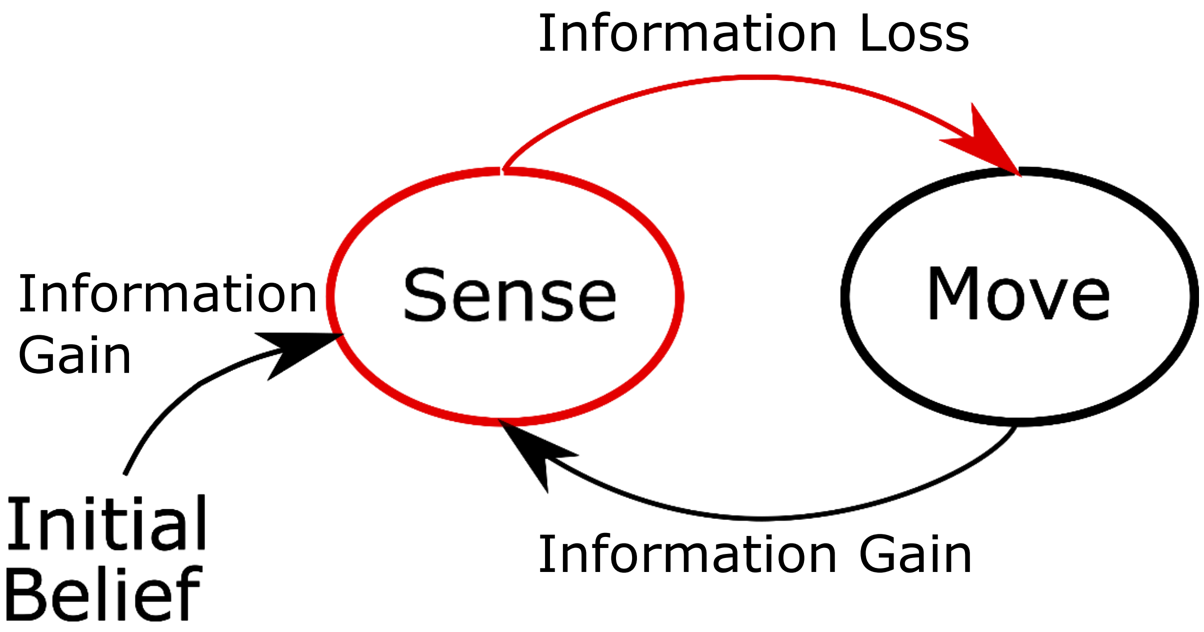

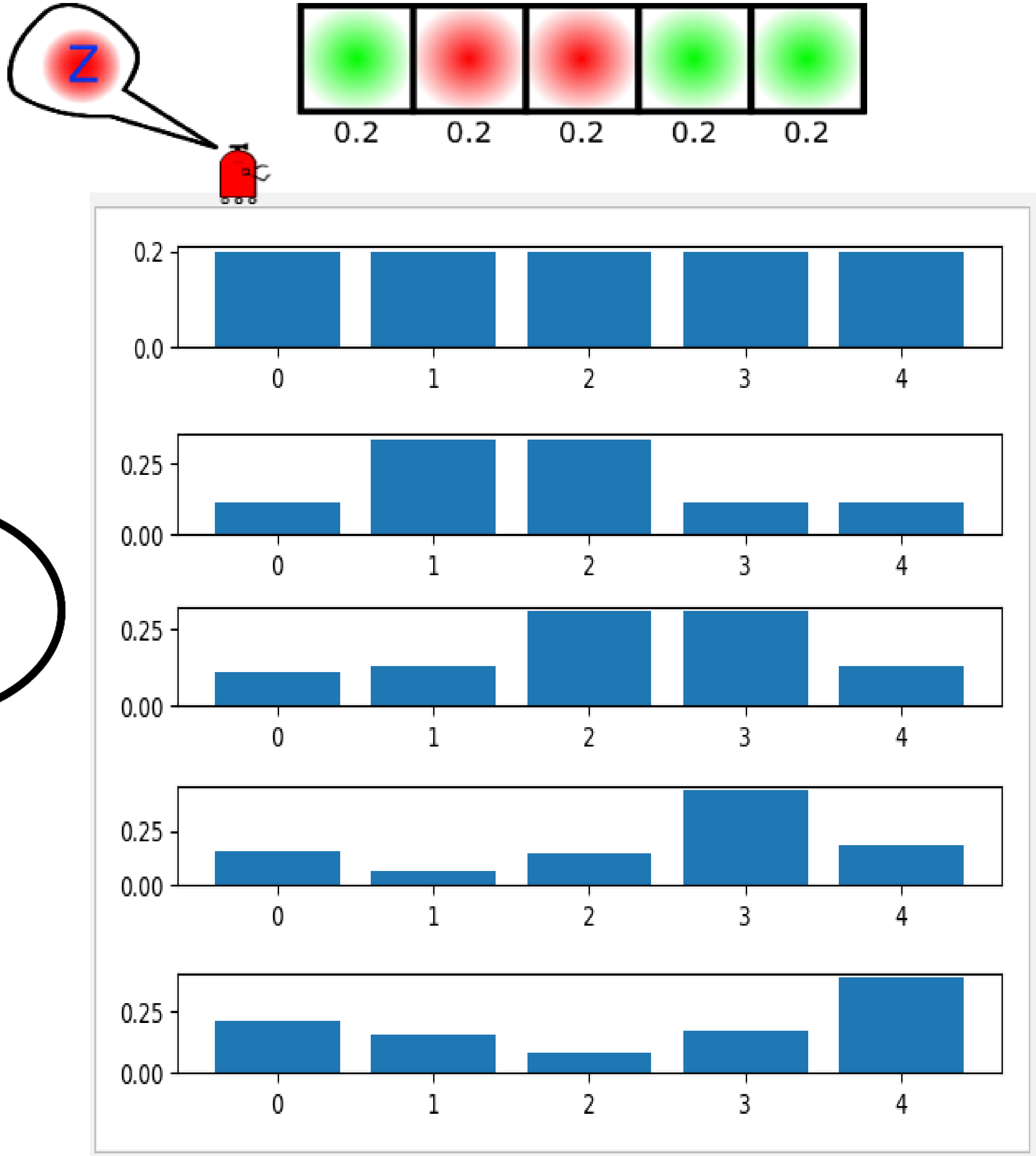

Sum Up Global Localization

- Belief → Probability
- Measurements → Multiplication followed by Normalization
- Moving → Convolution

Belief – Formal Definitions

- Probability:

$$0 \leqslant p(X) \leqslant 1$$

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$$p(x_1) = 0.2$$

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- Assuming 5 states:

0.1	0.1	0.1	0.1	

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$$0 \leqslant p(X) \leqslant 1$$

- Assuming 2 states:

$$p(x_1) = 0.2$$
$$p(x_2) = \underline{0.8}$$

- Assuming 5 states:

0.1	0.1	0.1	0.1	0.6

Measurement – Formal Definitions

- Bayes Rule
- Assuming a grid cell and the measurements:

X grid cell $\quad Z$ measurement

- The belief of the location given a measurement:

$$p(X_i \mid Z) = \frac{}{}$$

Measurement – Formal Definitions

- Bayes Rule
- Assuming a grid cell and the measurements:

X grid cell Z measurement

- The belief of the location given a measurement:

$$p(X_i \mid Z) = \frac{p(Z \mid X_i)\, p(X_i)}{p(Z)}$$

Measurement Probability *Prior*

- A product of the prior with the measurement probability
- The “probability of seeing a measurement independently of location” (normalizer...)

Movement – Formal Definitions

- This is a somewhat complicated formula:
- Notice:
  - Grid Location
  - Time

Time

Movement Probability

$$p(X_i^t) = \sum_j p(X_j^{t-1})\, p(X_i \mid X_j)$$

Prior Probability

Grid Location

- Here are the components:
  - Prior
  - Movement

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Prior Probability

Grid Location

- Here are the components:
  - Prior
  - Movement
- This is what is called Total Probability

$$\Pr(A) = \sum_n \Pr(A \mid B_n)\Pr(B_n),$$

Histogram-based State Estimation

Main histogram-based global localization problem (Markov Localization):

- Memory scaling is exponential
- So, it is unfeasible in large real world problems.

Sum-UP so far

- State Estimation
- Markov Localization
- Probability
- Bayes
- Total Probability

Coming UP:

- Kalman Filter

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