
DTU

Perception for Autonomous Systems 31392:

Fitting

Lecturer: Evangelos Boukas—PhD

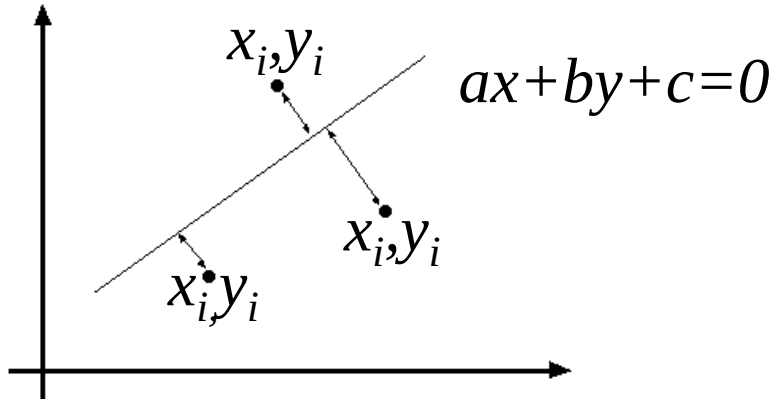
Fitting Data to a Model (Handling Outliers)

- Let’s work with the line example
  - Fitting a model without (or with minimum) outliers data points
  - Fitting data through voting (Hough Transform)
  - RANdom SAmple Consensus (RANSAC)
- What about data not in a line?

Least Squares

- Let the line depicted here be described by:
  ax+by+c =0
- Then the distance of a point x_i,y_j is defined as:

$$|ax_i + by_i + c|$$



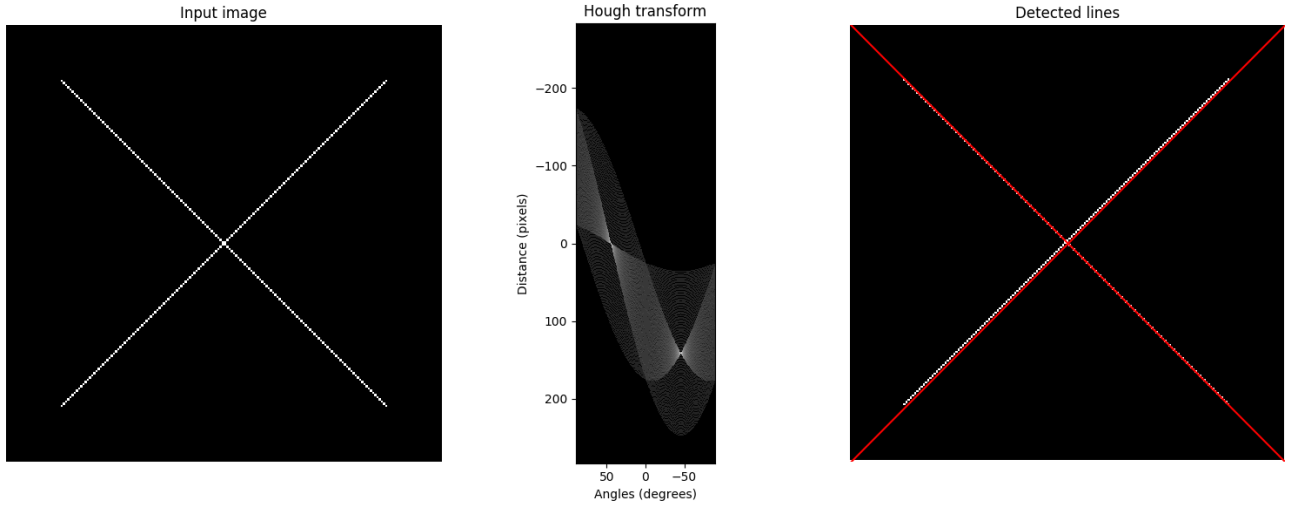
- Therefore, we can find the line that best matches our data by minimizing the following function:

$$E = \sum_{i=1}^{n} (a x_i + b y_i + c)^2$$

- However, in the presence of a lot of outlier data this problem is not directly solvable (in a closed form solution)

Hough Transform

- Assuming we want to fit a line in our data we can use the Hough transform as follows:
  - Formulate the problem as a bounded one
  - Create a grid of parameter values
  - Each data point votes on the grid
  - Find *local*-maxima in the grid and track back to lines in image



Hough Transform (1/4)

- Lets consider the line equation:

  Y=aX+b

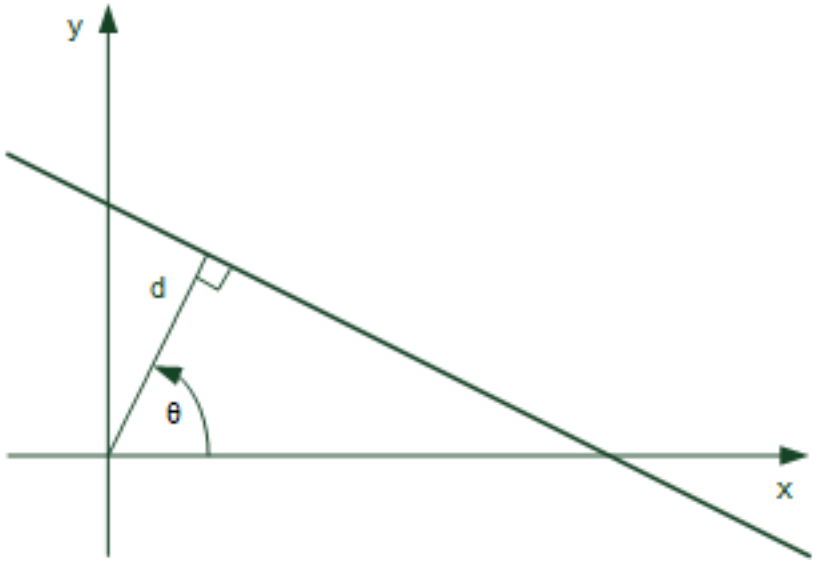
- The problem with the above equation is that a,b are unbounded,

Therefore we consider the following formulation (polar transformation):

$$x\cos\theta - y\sin\theta = d$$

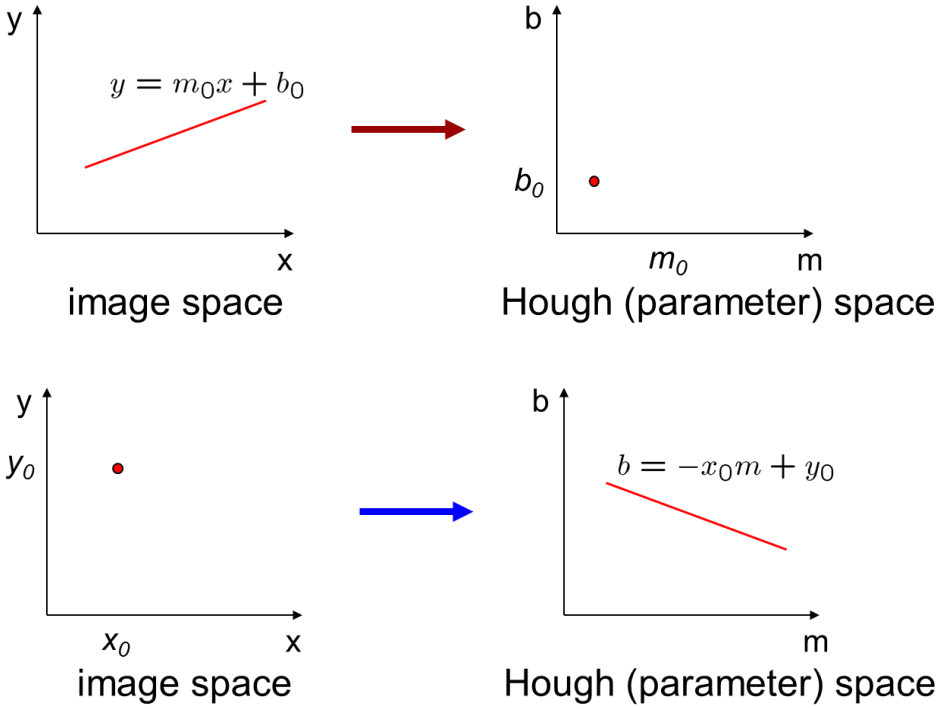
d : perpendicular distance from line to origin

θ : angle the perpendicular makes with the x-axis

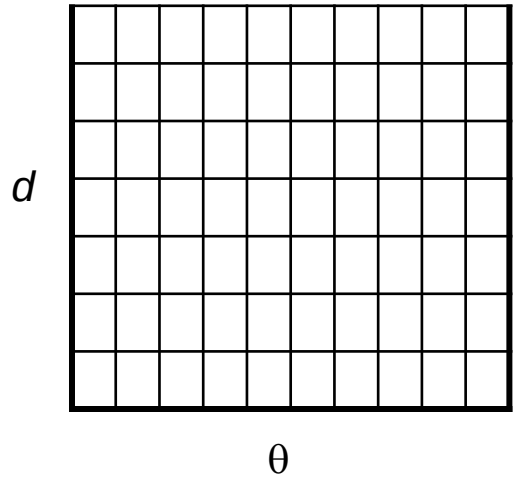


Hough Transform (2/4)

- Next step:
  - Initialize the grid using (d,θ)

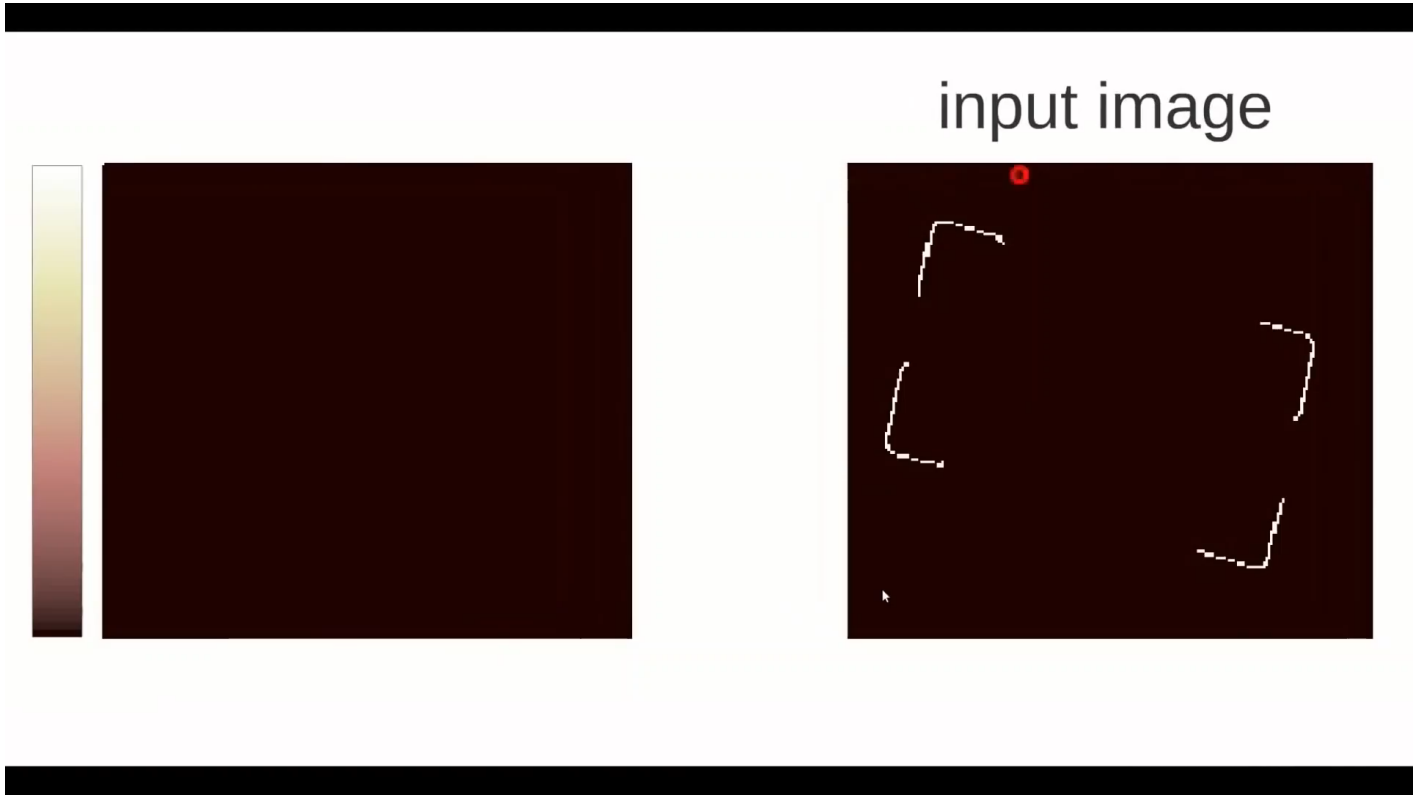


H: accumulator array (votes)



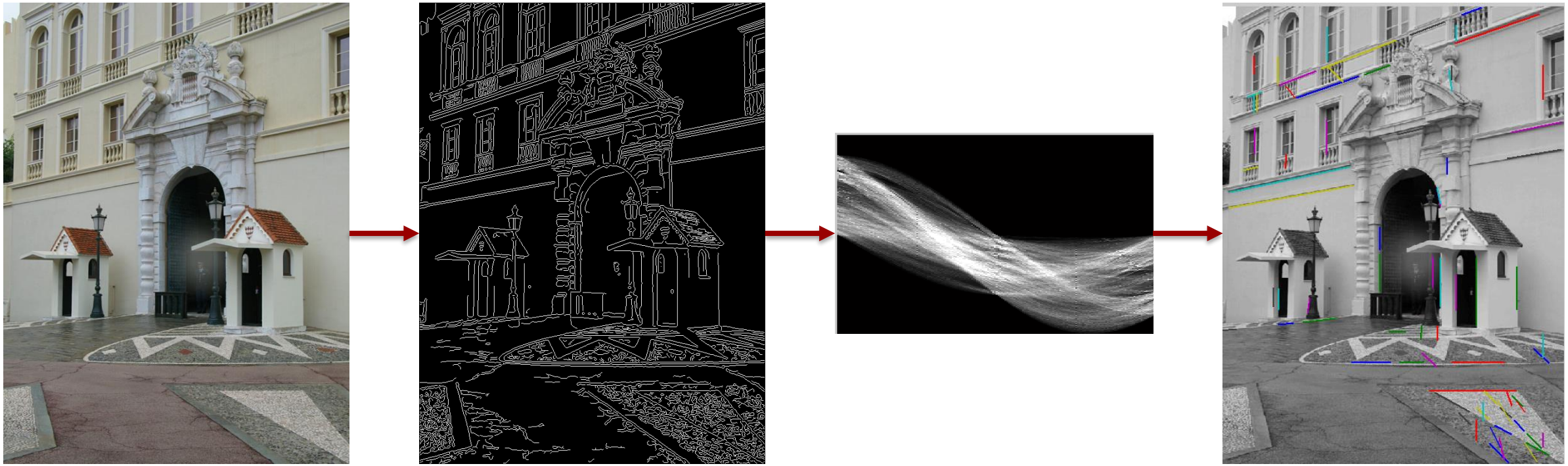
Hough Transform (3/4)

- Populate the grid by passing through the whole image and adding votes.
- See the following video:



Hough Transform (4/4)

- Identify maxima and track lines back to image:



RANdom SAmple Consensus (RANSAC)

Algorithm:

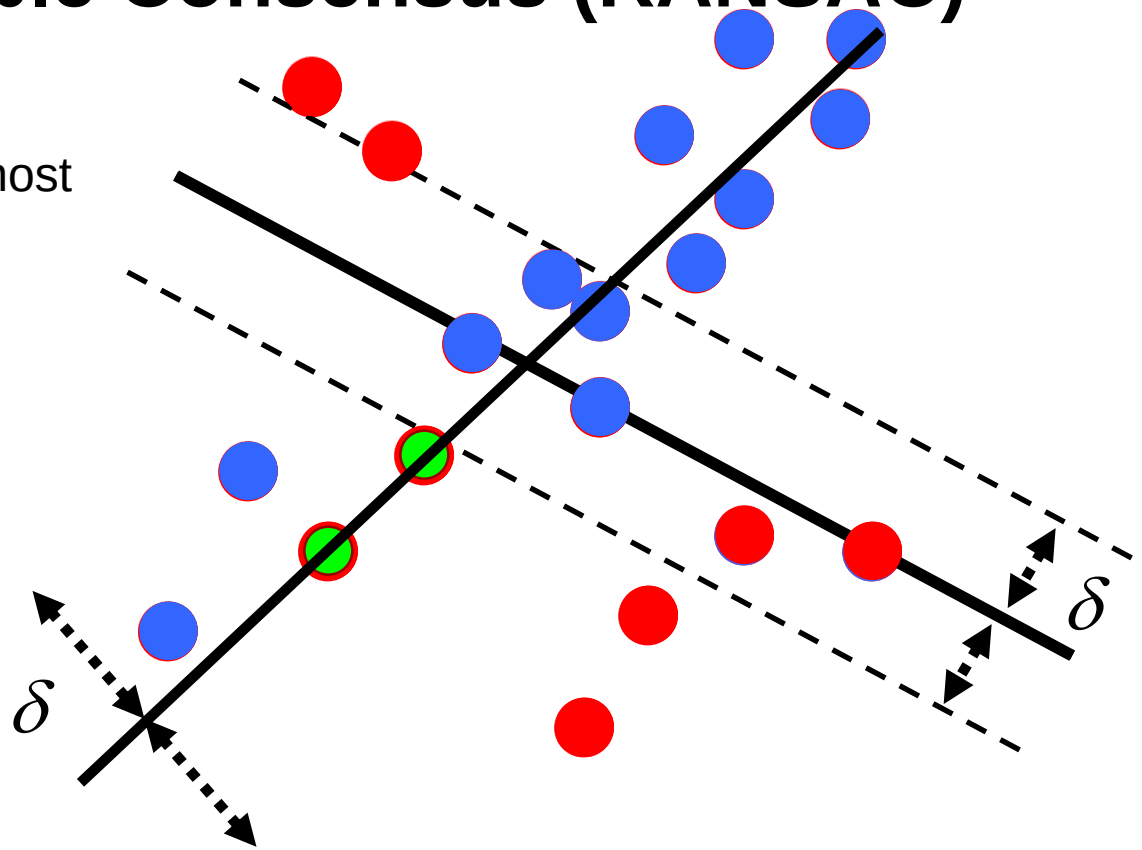
1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

- **Repeat** 1-3 until the best model is found with high confidence

- δ- is the threshold upon whitch a sample is considered to not fit to the selected model

RANdom SAmple Consensus (RANSAC)

- Select the models with most inliers to create lines



What about data not in a line?

- Same approach is followed for more complex models eg: circle model is:

$$(x - x_0)^2 + (y - y_0)^= r^2$$

which requires 3 parameters in the Hough grid

- However the complexity grows exponentially (usually up to 4 parameters is advised)

- Ransac can handle higher order models.

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