Perception for Autonomous Systems

Lecture 4 - Calibration and Stereo Vision 2 (22/02/2021)

Outline/Content:

- Camera matrix (Book A 2.1.5)
- Distortion coefficients (Book A 2.1.6)
- Calibration (Paper A)
- Undistortion
- Algebraic Derivation of Stereo Vision (Book B 7.1)
  - (Essential and) Fundamental Matrix
- Stereo Calibration and Rectification
  - Ranging

> Additional Reading Material: Paper A; Z. Zhang, "A flexible new technique for camera calibration," in IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 22, no. 11, pp. 1330-1334, Nov. 2000. doi: 10.1109/34.888718

Epipolar Geometry: General Case

In the general case, the following assumptions can be made for the determination of the components in the epipolar geometry:

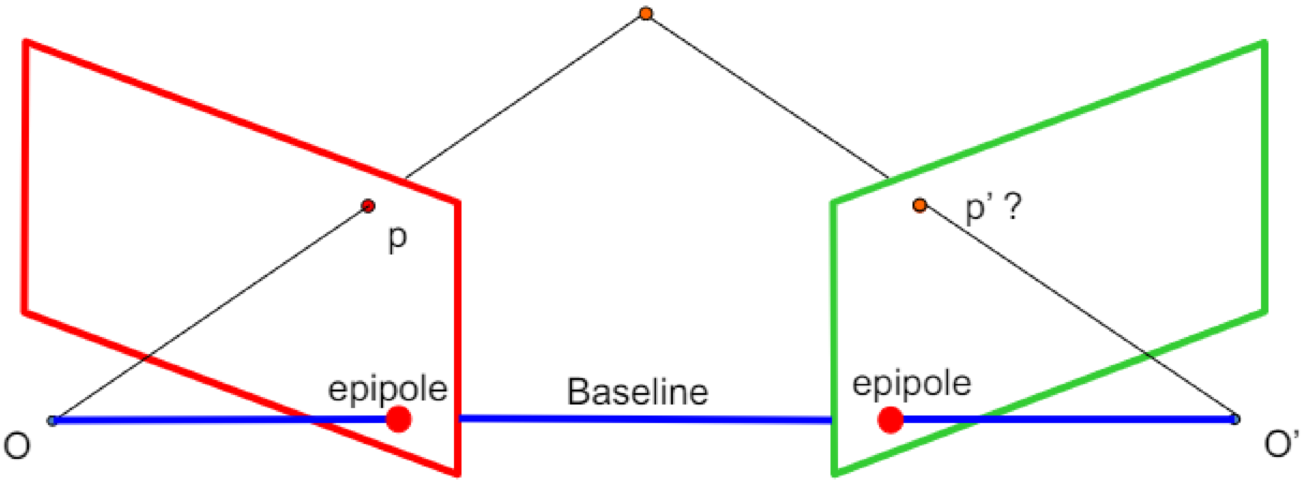
- 2 **Angled** camera views
- Rays passing through the camera center

> NOTE: If the camera views are parallel, we don't

With this knowledge, we can easily obtain the following two components (as shown in the next figure):

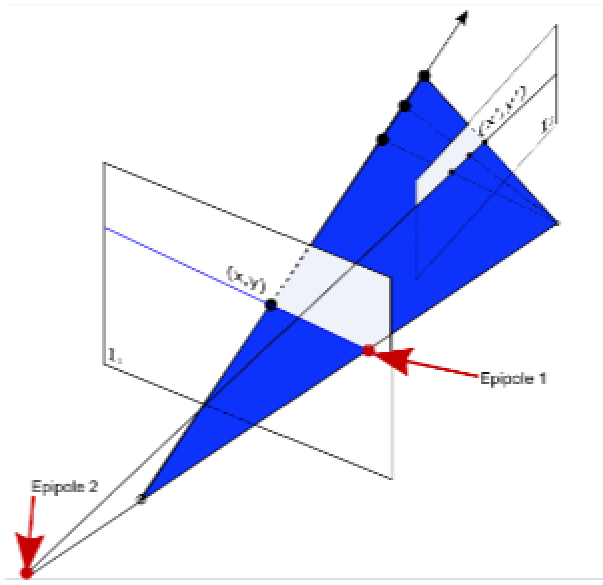
- Baseline
- Epipolar lines
- Epipolar plane
- Epipoles

> Necessary knowledge from the previous lecture:
>
> - *Baseline*: The line connecting the two camera centers.
> - *Epipole*: Point of intersection of baseline with the image plane.
> - *Epipolar plane*: The plane that contains the two camera centers and a 3D point in the world.
> - *Epipolar line*: Intersection of the epipolar plane with each image plane.

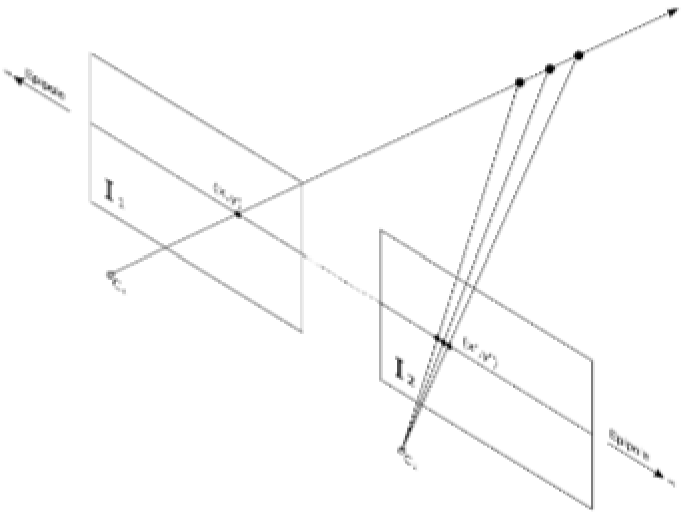


Epipolar Geometry: Special Case

For special cases it's not always possible to determine the epipolar geometry comonents.

Example 1: 

In the first case, we have two staggered images. We're able to determine the **Baseline**, **Epipolar plane**, **Epipolar lines** and the **Epipoles**. However, Due to the placement of the cameras it's noticeable that one of the epipolar poles is not part of the epipolar plane.



Example 2:

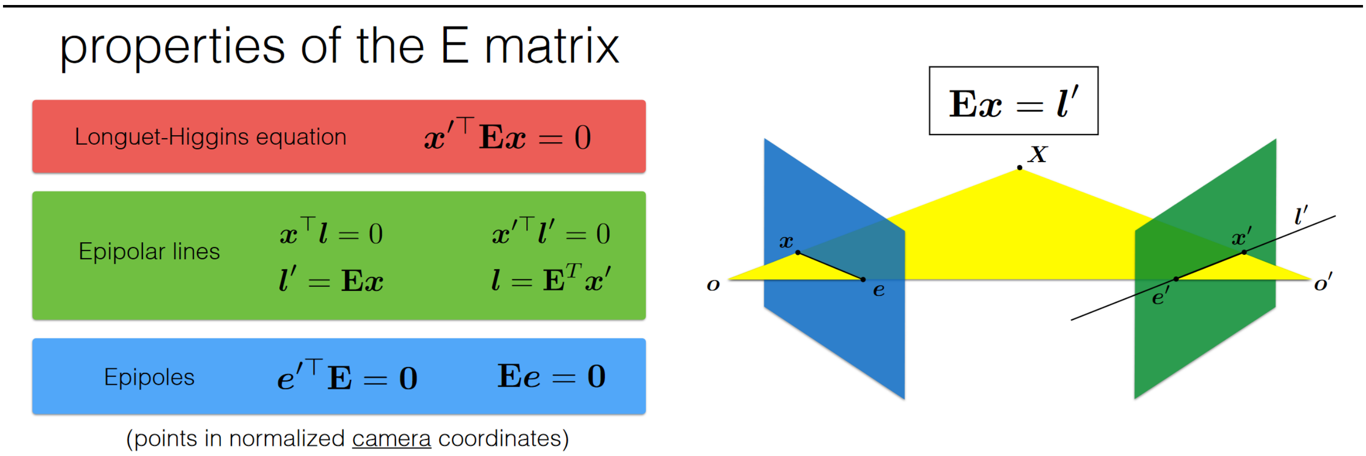
In the second case we have vertically aligned images. We can determine the **baseline**, **epipolar plane**, **epipolar lines** but NOT the **epipoles**. This is because the alignment of the two images makes them parallel to the baseline. Therefore, an intersection between the baseline and the epipolar line is not possible.

> This case can be useful by using the scanline to calculate the disparity

Essential Matrix Lecture Slides - Carnegie University

The Essential Matrix E is a 3x3 matrix, and it relates corresponding image points between both cameras, given the rotation and translation.

> 1. Given a point in one image, multiplying by the essential matrix will tell us the epipolar line in the second view.
> 2. It's assumed that both cameras are calibrated. For uncalibrated cameras, see section Fundamental Matrix.
> 3. See the link for the lecture slides of the Carnegie Mellon university, as it is perfectly explained, but unfeasible to include into this section.

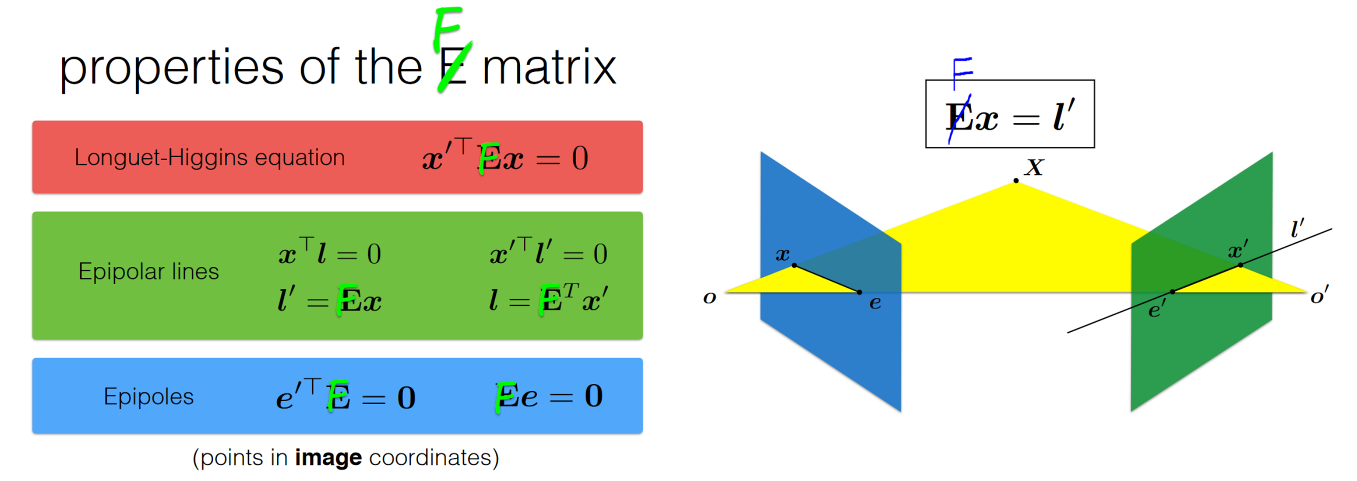


Characteristics:

– $E x'$ is the epipolar line associated with x' ($l = E x'$)
– $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
– $E e' = 0$ and $E^T e = 0$
– E is singular (rank two)
– E has five degrees of freedom

Fundamental Matrix [Lecture Slides - Carnegie University]

The 3x3 Fundamental matrix is a generalization of the Essential matrix, where the assumption of calibrated cameras is removed.



Characteristics:

– $F x'$ is the epipolar line associated with x'
– $F^T x$ is the epipolar line associated with x
– $F e' = 0$ and $F^T e = 0$
– F is singular (rank two): det(F)=0
– F has seven degrees of freedom:

> See the link for the lecture slides of the Carnegie Mellon university, as it's well explained, but unfeasible to include into this section.

How to compute the Fundamental Matrix [Lecture Slides - Carnegie University]

The method for computing the Fundamental Matrix is called the 8 point algorithm, as 8 points are needed to solve it.

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$xx'f_{11} + xy'f_{12} + xf_{13} + yx'f_{21} + yy'f_{22} + yf_{23} + x'f_{31} + y'f_{32} + f_{33} = 0$$

$$Af = \begin{bmatrix} x_1x_1' & x_1y_1' & x_1 & y_1x_1' & y_1y_1' & y_1 & x_1' & y_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_ny_v' & x_ny_n' & x_n & y_nx_n' & y_ny_n' & y_n & x_n' & y_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

- Homography (No Translation)
- Correspondence Relation

  $$\mathbf{x}' = \mathbf{H}\mathbf{x} \Rightarrow \mathbf{x}' \times \mathbf{H}\mathbf{x} = \mathbf{0}$$

1. Normalize image coordinates

   $$\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x} \quad \tilde{\mathbf{x}}' = \mathbf{T}'\mathbf{x}'$$

2. RANSAC with 4 points
   - Solution via SVD
3. De-normalize:

   $$\mathbf{H} = \mathbf{T}'^{-1}\tilde{\mathbf{H}}\mathbf{T}$$

- Fundamental Matrix (Translation)
- Correspondence Relation

  $$\mathbf{x}'^T \mathbf{F}\mathbf{x} = 0$$

1. Normalize image coordinates

   $$\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x} \quad \tilde{\mathbf{x}}' = \mathbf{T}'\mathbf{x}'$$

2. RANSAC with 8 points
   - Initial solution via SVD
   - Enforce $\det(\tilde{\mathbf{F}}) = 0$ by SVD
3. De-normalize:

   $$\mathbf{F} = \mathbf{T}'^T\tilde{\mathbf{F}}\mathbf{T}$$

> 3. See the link for the lecture slides of the Carnegie Mellon university, as it is perfectly explained, but unfeasible to include into this section.

Epipolar Geometry to Rectified Epipolar Geometry

Necessary steps:

1. Match some points
2. Calculate the fundamental matrix
3. Calculate the rectified images