HW1

1. [0.5p] What is the angle (in radians) between two 3D vectors [3,2,1] and [0,-1,-1]? Hint: use dot product in your calculations.

Ans:

$$\vec{a} = (3, 2, 1)$$
$$\vec{b} = (0, -1, -1)$$
$$a \cdot b = |a||b| \cos\theta$$
$$\theta = 2.1736\pi$$

2. [0.5p] What is the normalized vector that is perpendicular to both [3,2,1] and [0,-1,-1] (i.e., the normal of the plane formed by the two vectors)? Hint: use cross product in your calculations.

Ans:

$$\vec{a} = (3, 2, 1)$$
$$\vec{b} = (0, -1, -1)$$
$$\vec{a} \times \vec{b} = (-1, 3, -3)$$
$$\vec{n} = (\frac{-1}{\sqrt{19}}, \frac{3}{\sqrt{19}}, \frac{-3}{\sqrt{19}})$$

3. [1p] What is the coordinate of 3D point [1,6,8] using a new basis {(0,0,-1), (-1,0,0), (0,1,0)}? Show your deduction process as a sequence of matrix-vector multiplications.

Ans:

$$\vec{b}_1 = (0, 0, -1)$$
$$\vec{b}_2 = (-1, 0, 0)$$
$$\vec{b}_3 = (0, 1, 0)$$
$$P = (1, 6, 8)$$
$$P = -1 * \vec{b}_1 + -8 * \vec{b}_2 + 6 * \vec{b}_3$$
$$Therefore, the\ cordinate\ of\ P = (-1, -8, 6)$$

4. By the pinhole camera model (e.g., page 6-8 in Geometry#1) with focal length $f = 1$, what are the projected 2D positions (of the three vertices) of a 3D triangle with vertices [0,0,-2], [2,-4,-4], and [3,3,-6]? [1p] What are the areas of the original triangle in 3D and the projected 2D triangle? [1p] Hint: search online for the area formula of arbitrary 3D triangles.

Ans:

$$v_1 = (0, 0, -2)$$
$$v_2 = (2, -4, -4)$$
$$v_3 = (3, 3, -6)$$
$$The\ area = 14.248$$

$$Projection\ equation = \begin{cases} x' = f\frac{x}{z} \\ y' = f\frac{y}{z} \end{cases}$$

Therefore, the 2D positions of a 3D triangle

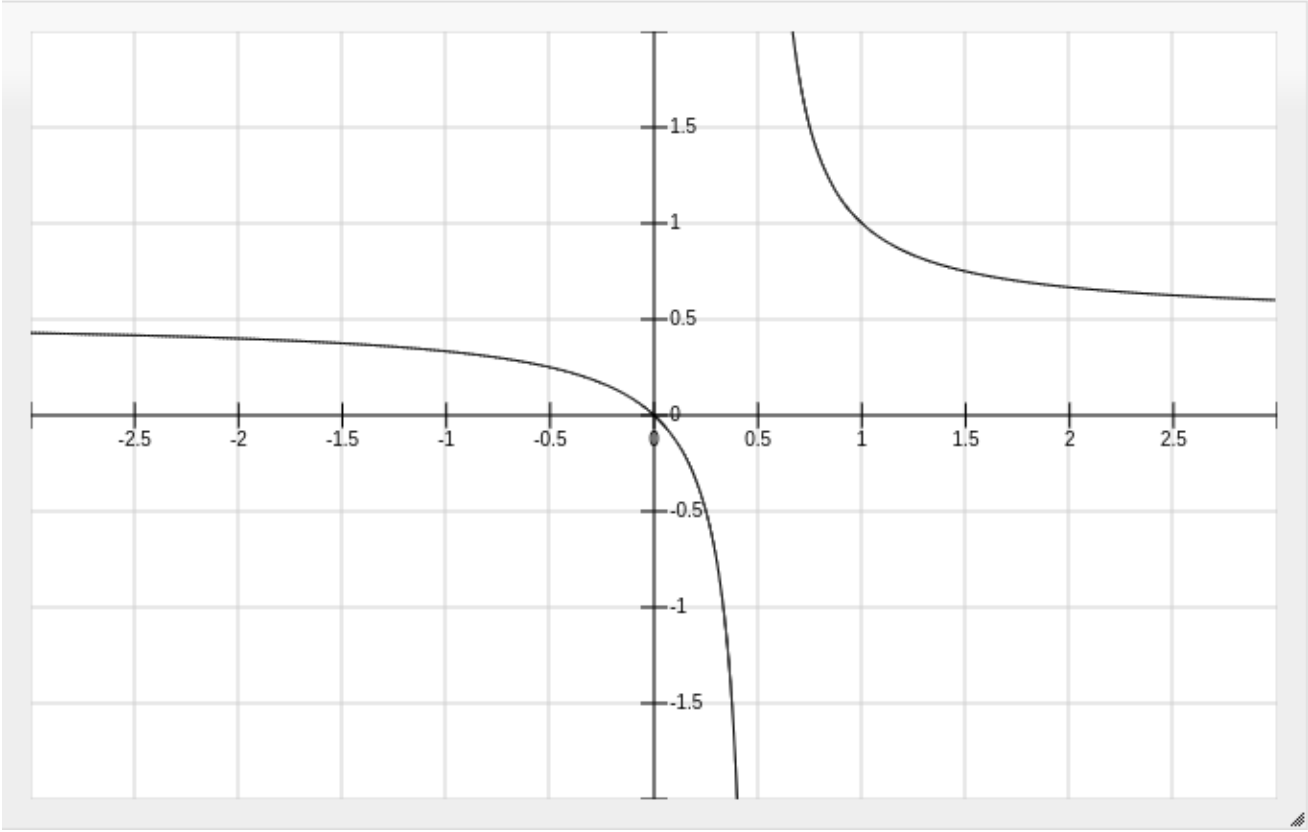
$$v_1' = (0,0)$$
$$v_2' = (-2,1)$$
$$v_3' = (-1/2,-1/2)$$
$$The\ area\ = 0.75$$

5. By the lens formula (e.g., page 13-15 in Geomtry#1) with focal length $f = 1/2$ and focus distance $D' = 2/3$, what is the value of the in-focus object distance D ? [0.5p] If we change the camera design so that the focus distance is twice ($D' = 4/3$), what is the new in-focus object distance? [0.5p] Show a plot of the in-focus object distance as a function of the focus distance (with focal length = $1/2$) using fooplot.com (with x range from -3 to 3 and y range from -2 to 2). [0.5p] What is the smallest possible value of the in-focus object distance (with focal length = $1/2$)? [0.5p]

Ans:

$$Lens\ formula: \frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

1. $$\frac{3}{2} + \frac{1}{D} = 2$$
$$D = 2$$
2. $$\frac{3}{4} + \frac{1}{D} = 2$$
$$D = \frac{4}{5}$$
3. $$D = \frac{1}{2 - \frac{1}{D'}}$$



4.

$$\frac{1}{D'} + \frac{1}{D} = 2$$
$$if\ D' \to \infty,\ \frac{1}{D'} \to 0$$
$$D \to 1/2$$

The smallest possible value of the D is 1/2.

6. For a lens-formula camera with focal length = $1/4$, focus distance = $1/2$, and **full** sensor size = 1, what is the FOV in degree(度)? [0.5p] To make the FOV 120 degrees, but we cannot change the sensor size, what would be new in-focus object distance? [0.5p]

Ans:

1.

$$FOV = \theta = 2atan(s, D')$$
$$\theta = 2 * atan(0.5, 0.5)$$
$$\theta = \pi/2$$

2.

$$FOV = \deg 120$$
$$60 = atan(0.5, D')$$
$$D' = \frac{2}{\sqrt{3}}$$
$$D = \frac{1}{2 - \frac{1}{D'}}$$
$$Therefore\ D = \frac{2}{4 - \sqrt{3}}$$

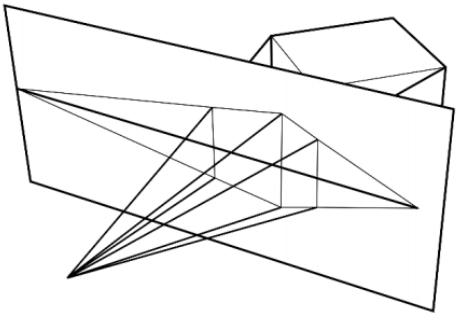
7. Show the deduction process as a vector-matrix multiplication form of the pinhole camera / perspective projection model to find the projected 2D position of a 3D point [2,-4,-4] with focus length = 2. [0.5p] What is the result in 2+1D homogeneous coordinate and 2D Euclidean coordinate? [0.5p]

Ans:

8. What is the camera matrix with focus length = 2, scaling factor = 0.01 for both x and y, 2D offset = (30,20), and a skew parameter = 1.2? [1p]

Ans:

9. Someone drew a cube in a “two-point perspective” as below:



Dude! where is the third vanishing point(s)? [1p]

Ans:

There are two vanish points in the inf field parallel to vertical axis.

10. Give a case when the angles between two 3D lines remain the same in a perspective projection. [1p]

Ans :

In the codition that the plane contain two 3D lines is parallel to the image plane, The angles between those lines whold ramain the same.

11. Which one of the two photos below is more like rectilinear/perspective projection? Give your reasons. [1p]

(a)

(b)

Ans:

(a) - A rectilinear projection will preserve straight lines but not correct angles between straight lines. But I am not sure about that. Because maybe buildings above were designed by Antoni Gaudi.

12. The main difference between the classic “camera matrix” perspective projection model in CV and the “MVP” 3D-to-2D projection pipeline in CG is that the projected coordinates in the former is 2D and in the latter it is 3D! What does the third value (i.e., z) in the 3D projected coordinates mean? and why do we need it? [0.5p+0.5p]

Ans: