

## 7SENG010W Data Structures & Algorithms

### Week 1 Tutorial Exercises Solutions

These exercises cover: Abstract Data Types (ADTs), Big-O Complexity, Timing an Algorithm

#### Exercise 1 Solution

See the TubeStation and Testing class code for a sample solution.

#### Exercise 2 Solution

Complete the table for the quadratic growth rate running time equation  $T(N) = 2N^2 + 3N + 4$ . This gives an idea why when calculating the Big-O for a  $T(N)$  that only the value of the *dominant term*, i.e.  $N^2$ , determines its order of complexity.

Values of N	$T(N) = 2N^2 + 3N + 4$			
	$2N^2$	$3N$	4	$T(N)$
1000	2,000,000	3,000	4	2,003,004
2000	8,000,000	6,000	4	8,006,004
4000	32,000,000	12,000	4	32,012,004
8,000	128,000,000	24,000	4	128,024,004
16,000	512,000,000	48,000	4	512,048,004

Note that when  $N = 16,000$  that the percentage of  $T(16000)$  accounted for by the  $2N^2$  term is  $(512,000,000 / 512,048,004) * 100 = 99.99\%$ .

So this illustrates why when calculating the Big-O for an algorithm's  $T(N)$ , it ignores all but the "dominant term" in the  $T(N)$  function.

### Exercise 3 Solution

Completed the B-g-O values table.

Big-O	Values of N				
	20	50	100	1000	100,000
O(1)	1	1	1	1	1
O(N)	20	50	100	1000	100,000
O(N <sup>2</sup> )	400	2500	10,000	1,000,000	10,000,000,000
O(N <sup>3</sup> )	8000	125,000	1,000,000	1,000,000,000	1×10 <sup>15</sup>
O( log <sub>2</sub> (N) )	5 (4.3219280948874)	6	7	10	17
O( N log <sub>2</sub> (N) )	100	300	700	10,000	1,700,000
O( 2 <sup>N</sup> )	1048576	1.125899907×10 <sup>15</sup>	1.2676506×10 <sup>30</sup>	1.071508607×10 <sup>301</sup>	Largest N = 1342: 2 <sup>1342</sup> = 9.599623077×10 <sup>403</sup>
O( N! )	2.432902008×10 <sup>18</sup>	3.04140932×10 <sup>64</sup>	9.332621544×10 <sup>157</sup>	Largest N = 212: 212! = 4.733702183×10 <sup>402</sup>	Too big!

Note that when we deal with order of complexity expressions that involve log<sub>2</sub>(N) the result is very rarely a whole number, so the standard practice is to take the smallest whole number that is greater than the fractional value, this is called the “ceiling” in maths e.g. ceiling( 4.3219280948874 ) = 5.

The comparison given for the rough age of the Universe as approximately 13.5 billion years, & in seconds that is: (13.5 × 10<sup>9</sup>) × ( 365×24×3600 ) = 4.25736×10<sup>17</sup>.)

Again for comparison if for a particular algorithm it has an order of complexity of O( 2<sup>N</sup> ), then if the algorithm was applied to 59 data items, i.e. N = 59 then its expected “execution time” in time units would be:

$$O( 2^N ): 2^{59} = 5.764607523 \times 10^{17}$$

which is longer than the age of the Universe!

### Exercise 4 Solution

See the linear searching program class code for a sample solution.

On my laptop with a quite good intel i7 processor it too well into the 10s of millions to get even close to a second.