Deflection Of Framework

Structural Mechanics Laboratory

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Summary

In summary we have carefully modeled our framework deflection experiment using standard techniques and compared the experimental with the theoretical, given that the difference between them is % it seems they are in agreement (the difference being only in experimental error and devations in the material properties).

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1 Introduction

In practise structural design of trusses involves a configuration of beams orientated at various angles, and joined in complex ways. In reality this is not the case however for engineering analysis it is assumed that these beams may be joined at their ends using frictionless pins (*1*) and at simple angles. In which a factor of safety is incorporated, the accuracy of this assumtion in modeling a problem is sufficient.

A pin joint allows the joined members to swivel as opposed to a rigid joint that does not. A rigid joint may be welded but a pin joint may be a bolt, a rivet or any form of swivel pin.

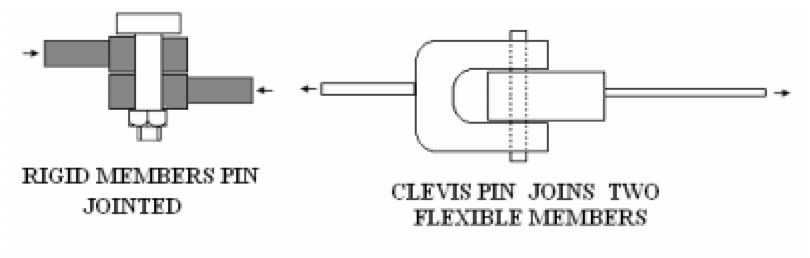


Figure 1

The important points about a pin joint are:

- The connected members are free to rotate.
- The force in the member can only pull or push along the line of the member.

In the study of structural mechanics, an important part of engineering science, the deflection of structural members and systems is of importance when designing any system required to bear a load, statically or dynamically. In our treatment of this property we will measure the effect of a static load on a truss structure, and derive the equations governing it's response to loading, so that we may predict the behaviour and design new trusses for other applications.

A later section of this report will be devoted to the difference between the theoretical and experimental modeling of a structure, and the real world implications. Further research may show that empirical more accurate models exist, however for the bulk of this lab report and it's scope, we will explore pin-jointed frameworks, and always assume a statically determinate configuration that is always in static equalibrium ($\Sigma F = 0$ for all points in the system).

2 Experimental Test

2.1 Experimental Objectives

"To load the Framework Structure, measure the deflection and compare with theory.[1]"

To gain an understanding and appreciation of the deflection experienced by loading a pin jointed frame, and explore the real world analogy of it's application.

2.2 Description of Apparatus

1. Pin-jointed truss structure: A four-Bay, statically derterminant pin jointed framework, composed of rigid spring members.[1]
2. Assorted weights and standard masses, the range used was from 100g to 1000g, in 100g increments, the system was allowed to return to equalibrium after any dynamic loading was experienced due to applying the increment before deflection measurment was taken.

2.3 Experimental Procedure

1. Apply a small amount of preload at the loading point to absorb the "slack" in the framework.
2. Apply the load W, in increments and measure the deflection of the frame after each increment.
3. Plot a graph of deflection against load for each point.Then determine the slope.
4. Determine the deflection per unit load, using the strain energy method.
5. Compare experimental results with theoretical predictions.

2.4 Experimental Test Results

LOAD (kg)	LOAD (N)	DEFLECTION (mm)
0.100	0.981	0.57
0.200	1.962	1.17
0.300	2.943	1.86
0.400	3.924	2.41
0.500	4.905	2.98
0.600	5.886	3.49
0.700	6.867	4.05
0.800	7.848	4.61
0.900	8.829	5.14
1.000	9.81	5.72
1.100	10.791	6.32

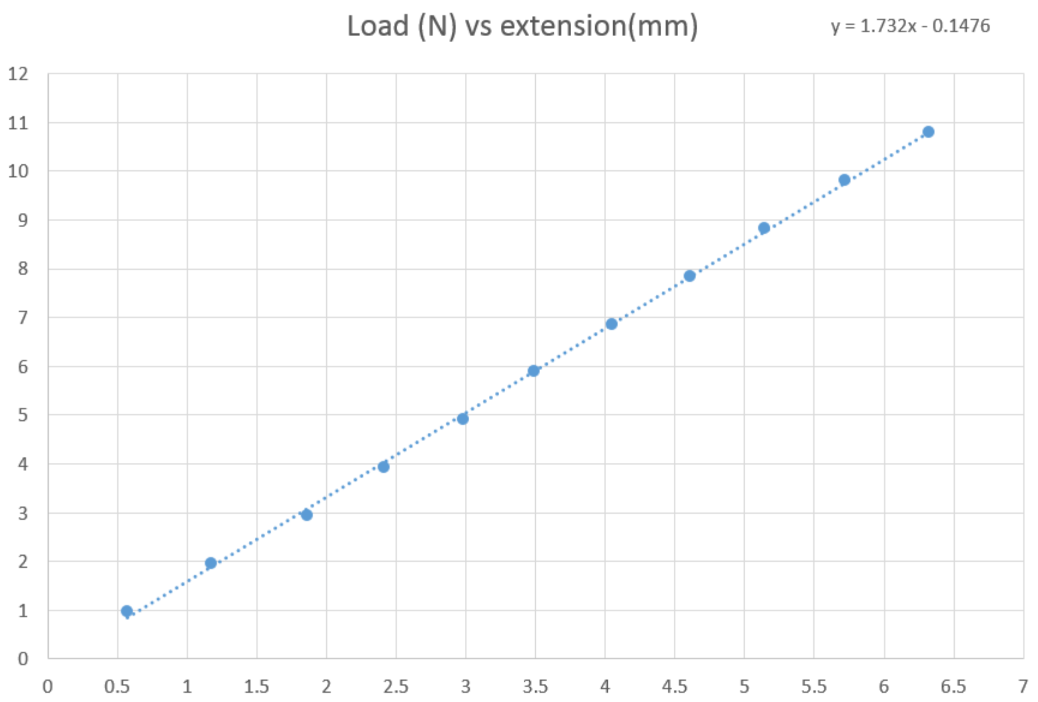


Figure 1: Experimental Results (N per mm)

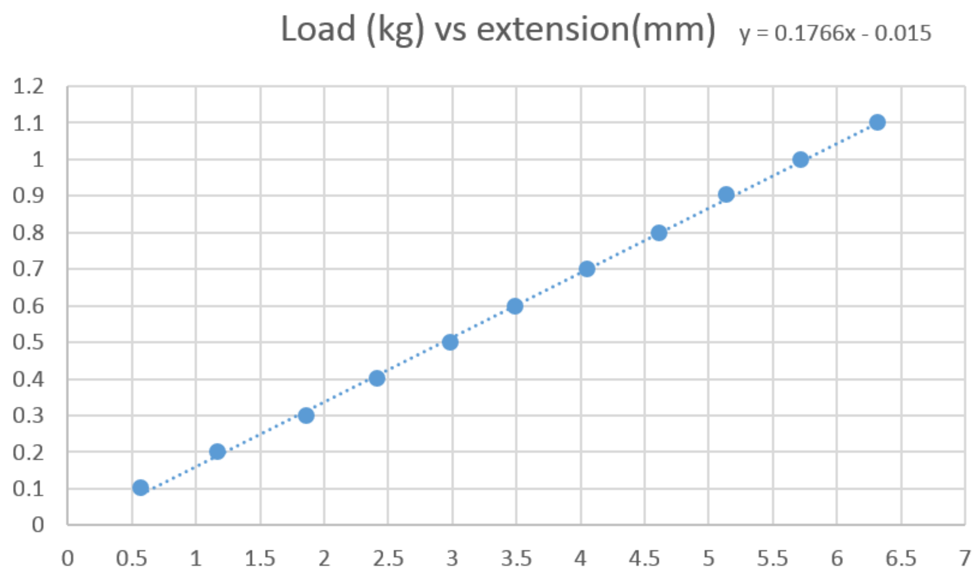


Figure 2: Experimental Results (kg per mm)

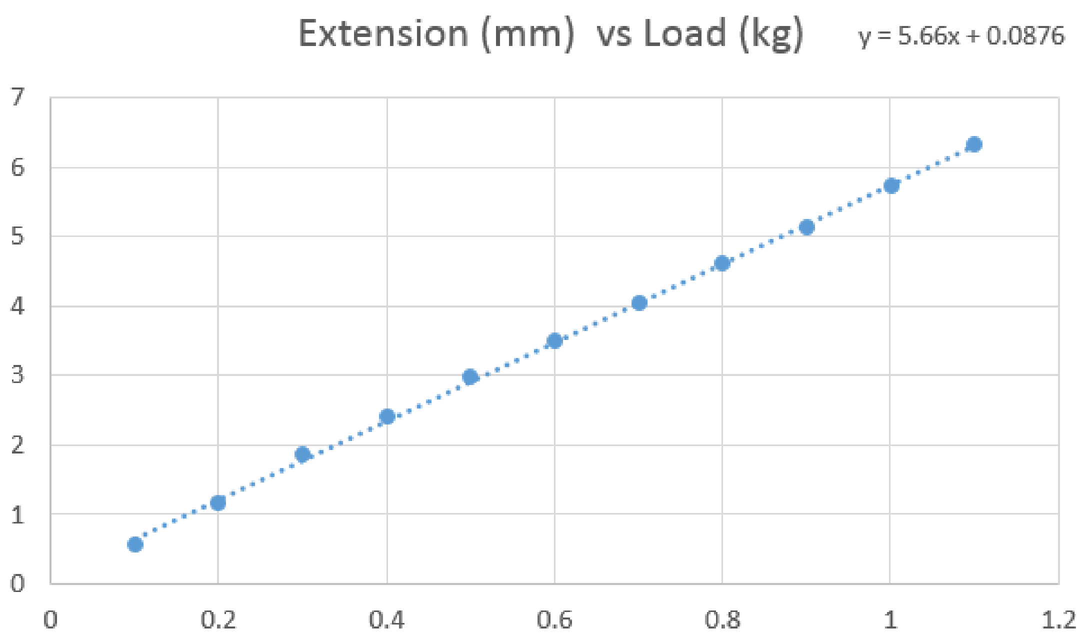


Figure 3: Experimental Results (mm per kg)

From the two graphs the systems response to loading is a fairly linear one, using the addition of a trend line the experimental rate of deflection is 1.732 N/mm or 0.1766 kg/mm. *(From the derivatives of the graphs trend line equation)*

For the energy in the system U,

$$U = \int_{0.57}^{6.32} 1.732x - 0.1476 \ \mathrm{d}x$$

Which evaluates to U being equal to 33.46 Nmm.

From figures 2 and 3 we can see that the experimental value of $\frac{\delta}{W}$ is 5.66 mm/kg, which is indeed the reciprocal of 0.1766kg/mm from figure 2.

2.5 Individual deflections (Experimental)

Calculated using the ratio of P_n = some ratio of W from the analysis.

Member Force	Value
P_1	$\frac{5W}{6}$
P_2	$\frac{5W}{6}$
P_3	$\frac{2W}{3}$

w(kg)	P1(mm)	P2(mm)	P3(mm)
0.100	0.475	0.475	0.380
0.200	0.975	0.975	0.780
0.300	1.550	1.550	1.240
0.400	2.008	2.008	1.607
0.500	2.483	2.483	1.987
0.600	2.908	2.908	2.327
0.700	3.375	3.375	2.700
0.800	3.842	3.842	3.073
0.900	4.283	4.283	3.427
1.000	4.767	4.767	3.813
1.100	5.267	5.267	4.213

3 Theory and Theoretical Results

3.1 Analysis

3.1.1 Trusses

Trusses consist of straight, slender members whose ends are connected at joints. Two-dimensional plane trusses carry loads acting in their planes and are often connected to form three-dimensional space trusses. Two typical trusses are shown in figure 4.

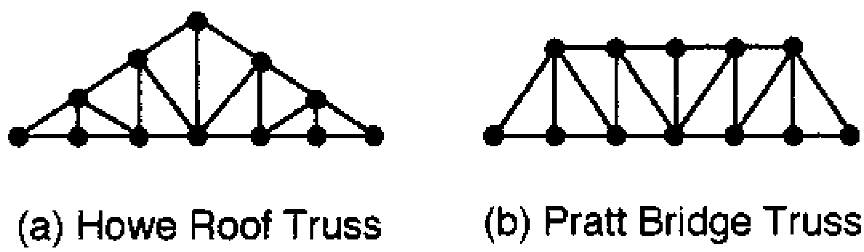


Figure 4: Schematic examples of trusses

3.1.2 External Forces:

$R_1 + R_2 = W$, $R_1 = R_2$ by symmetry
$R_1 = R_2 \therefore \frac{W}{2}$ By this symmetry we can see that the truss system will be symmetrical and solving of only one side is necessary, in keeping with the convention of the notes we will solve R_1 and it's related truss system.
Determine; P_1, P_{14}, P_{34} (Using P_3 and P_2, in terms of W.)

3.1.3 Strain Energy Method

Work done, W.D. = Strain Energy = $\frac{W\delta}{2}$, (From the integral of the Load-Extension graph) Also, Strain Energy = $\Sigma U = W$ that is the work done in deflecting the truss is stored as elastic energy in the spring loaded members. In an individual spring the strain energy $U = \frac{kx^2}{2}$. From the integral of the Spring force equation $F = kx$.

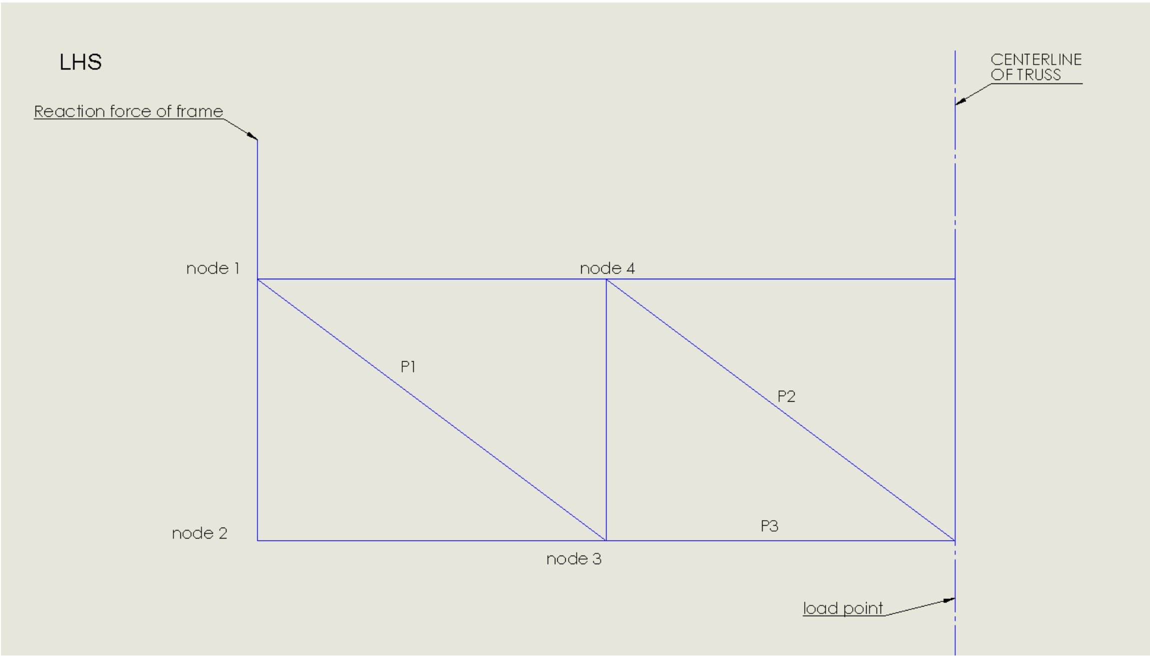
If $P = kx$, $U = \frac{Px}{2} = \frac{kx^2}{2}$

It follows that if $x = \frac{P}{k}$ then $U = \frac{p^2}{2k}$ For the whole W.D. of the system =

$$\frac{W\delta}{2} = \sum \frac{P^2}{2k}$$

(The sum of the spring tensions.)

Solving the forces about node 1:
$\sum Fy, Fx = 0$
$sin(\theta) = \frac{4}{5}$, $cos(\theta) = \frac{3}{5}$ Spring constant K =$0.5 \frac{\text{kg}_f}{mm}$



Reaction force $= \frac{w}{2} = P_1$ by symmetry,
between nodes 1 and 4, $\dfrac{W}{2} = P_1 cos(\theta) = \dfrac{4 \times P_1}{5}$
we get $W = \dfrac{6P_1}{5}$ conversely $P_1 = \dfrac{5W}{6}$
$P_{12}, P_{23} = 0$

$P_{14} = P_1 sin(\theta) = \dfrac{2W}{3}$

Between nodes 3 and 4: $P_{34} = P_1 cos(\theta) = P_1 \dfrac{3}{5} = \dfrac{5W}{6} \times \dfrac{3}{2} = \dfrac{W}{2}$
so $P_{34} = \dfrac{W}{2}$ and $P_{23} = 0$

$P_3 = P_1 sin(\theta) = \dfrac{5W}{6} \times \dfrac{4}{5}$, therfore, $P_3 = \dfrac{2W}{3}$

About node 4:
$P_{34} = P_2 Cos(\theta)$

$\dfrac{W}{2} = \dfrac{3P_2}{5}$ therefore $P_2 = \dfrac{5W}{6}$

Member Force	Value
P_1	$\frac{5W}{6}$
P_2	$\frac{5W}{6}$
P_3	$\frac{2W}{3}$

3.1.4 Calculation

$$\frac{W\delta}{2} = 2\left(\frac{P_1^2}{2k} + \frac{P_2^2}{2k} + \frac{P_3^2}{2k}\right)$$

$$\frac{W\delta}{2} = \frac{\left(\frac{5W}{6}\right)^2}{2k} + \frac{\left(\frac{5W}{6}\right)^2}{2k} + \frac{\left(\frac{2W}{3}\right)^2}{2k}$$

since K = 0.5, 2k = 1:

$$\frac{W\delta}{2} = 2W^2 \times (\frac{25}{36} + \frac{25}{36} + \frac{4}{9})$$

$$\delta W = 4W^2 \times \frac{11}{6}$$

$$\delta = 4W \times \frac{11}{6}$$

$$\frac{d\delta}{dW} = 4 \times \frac{11}{6}$$

3.1.5 Theoretical Results

LOAD (kg)	LOAD (N)	DEFLECTION (mm)
0.100	0.981	0.733
0.200	1.962	1.467
0.300	2.943	2.2
0.400	3.924	2.933
0.500	4.905	3.667
0.600	5.886	4.400
0.700	6.867	5.133
0.800	7.848	5.867
0.900	8.829	6.600
1.000	9.81	7.333
1.100	10.791	8.067

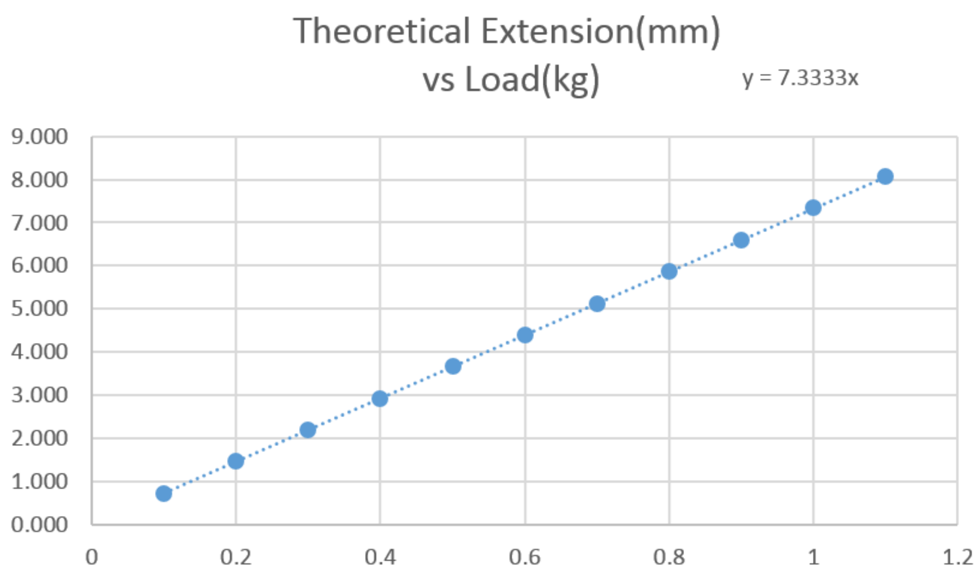


Figure 5: Theoretical results Results (mm per kg)

3.2 Individual deflections (Theoretical)

Calculated using the ratio of P_n = some ratio of W from the analysis.

Member Force	Value
P_1	$\frac{5W}{6}$
P_2	$\frac{5W}{6}$
P_3	$\frac{2W}{3}$

w(kg)	P1(mm)	P2(mm)	P3(mm)
0.1	0.611	0.611	0.488
0.2	1.222	1.222	0.977
0.3	1.833	1.83	1.466
0.4	2.444	2.444	1.955
0.5	3.055	3.055	2.444
0.6	3.666	3.666	2.933
0.7	4.277	4.277	3.422
0.8	4.888	4.888	3.911
0.9	5.500	5.500	4.400
1	6.111	6.111	4.889
1.1	6.722	6.722	5.377

4 Discussions

From the experimental results we have determined the ratio of $\frac{\delta}{W}$ to be equal to 5.66 mm/kg, the theoretical results show this value to be 7.33 mm/kg, a 29.5% difference this is indeed a large variation from the theoretical value, however logically one would expect it to be less for the reasons discussed.

This real world value compared to the theoretical value differs for many reasons, the physical set up of the framework is such that there is friction between the pin joints and any displacement theoretically, would be lessened because some force is lost in overcoming the friction, also the geometry of the real life system can only be approximated, there is always error in the measurement of angles and lengths and thus the frameworks configuration cannot be modeled with 100% accuracy, the cross sectional area of the beams also factors in the real life performance, however is not factored into our analysis.

Human error is a factor, in recording the results, and also the value of the measurement may be erroneous due to the dial indicator gauge measuring equipment used.

The physical condition of the pin jointed frame, for experimentation should be cleaned and all joints lubricated to best mimic the frictionless pins also it is assumed all the deflection is experienced in the axis of operation as indicated by the diagram and the forces act solely in line with the members, in reality all forces and displacements act in 3 directions, and the torques imposed act in 3 dimensions also, this contributes to the differences in measurement and predicted results.

For practical purposes the beams would not be loaded with springs and instead the natural elasticity of the material would be used to derive an effective spring constant, for structural steels this is likely much much higher than the value used for the experiment however it is assumed the experimentation techniques, and the theoretical calculations would scale well with size however this is undertermined.

Potential method of computing the Spring constant of a member
For a cantilevered beam the Spring constant K is expressed as

$$K = \frac{F}{\delta} = \frac{Ewt^3}{4L^3}$$

cited here: [3]

5 Conclusion

In conclusion we have modeled our situation adequately, and compared it to an expected theoretical data set, in comparison we have explain why there may exist differences in the results, for generallised applications the theoretical values are recommended, however for applications involving the specific framework the experimental values are recommended.

Some applications of this framework if such a structure were to be designed could include supporting the load of a roof, or bridge.

To improve the design of the framework for practical purposes obviously the members would not be spring loaded unless for a specific application, in general a rigid strong beam would be used as the members of the frame work.

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6 References

[1] Notes given out during the lab session, WS Jouri, Jan 2014

[2] INTRODUCTION TO UNDERGRADUATE REPORT WRITING, WS Jouri acessed Jan 2014

[3] Hool, George A.; Johnson, Nathan Clarke (1920). "Elements of Structural Theory - Definitions". Handbook of Building Construction (Google Books). vol. 1 (1st ed.). New York: McGraw-Hill. p. 2. Retrieved 2008-10-01. "A cantilever beam is a beam having one end rigidly fixed and the other end free."

[4] TRUSSES,David Roylance,Department of Materials Science and Engineering,Massachusetts Institute of Technology, Cambridge, MA 02139, June 8, 2000

A Auxillary theory and Definitions

In general:

	If		The structure is
number of unknowns	<	number of equations	Unstable
number of unknowns	=	number of equations	Stable & Determinate
number of unknowns	>	number of equations	Indeterminate

Figure 6: An illustration of determinancy conditions

In three dimensions, our treatment of structural mechanics is as such:

$$\sum F_x = 0 \qquad \sum M_{xy} = 0$$

$$\sum F_y = 0 \qquad \sum M_{xz} = 0$$

$$\sum F_z = 0 \qquad \sum M_{yz} = 0$$

[4]

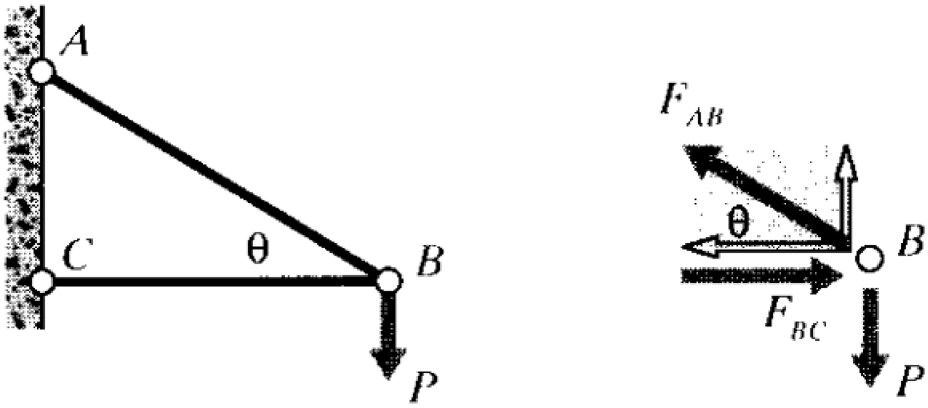

Figure 7: A two element truss [4]