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1 Design of a Cantilever Beam

All units are [**SI**]

The analysis and design of a cantilever beam with a range of assumptions and scenarios, multiple variations are included for comparison.

1.1 Definition

The problem as stated is a cantilevered beam is to be designed to support a 10kN load acting at 1.5m from the fixed wall it is attached to. An optimized solution is to be found and a range of cross sectional geometries and materials are to be compared and considered.

The primary goals are to design a beam that, with no more than 2mm deflection, can support the 10kN load.
Secondary goals include cost and weight considerations, and simulations to be carried out using Solid works simulations and Ansys finite element packages,
I will also endevour to produce a calculator for the relevant formulas using Excel.

1.2 Assumptions made

It is to be assumed for ease of calculation and within the scope of this project that the beam will be of uniform geometry and uniform material, however the geometry and material chosen will be selected after a range of comparisons.

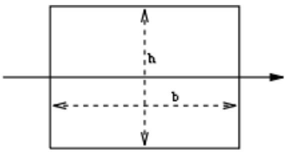
1.3 Summary of findings

Please read on for detailed analysis.

2 Comparison of various geometries

2.1 Cantilever beam of square cross section

For square geometry second moment of area is equal to $\frac{bd^3}{12}$ however for a square b=d, thus second moment of area $I = \frac{d^4}{12}$



2.1.1 Material 1: Steel BS 4360 grade 43A

Dimensions

In units of $\frac{N}{mm^2}$

Using Steel BS 4360 grade 43A with a strength of 255 N/mm^2 up to 25mm thick and 245 N/mm^2 over 50mm thick.

$\sigma = \frac{My}{I}$ with y being $\frac{d}{2}$ and I being $\frac{d^4}{12}$
thus simplifying to $\sigma = \frac{6M}{d^3}$

$M = 1.5 \times 10 \times 10^3 = 15000Nm$ $\sigma = 245 \times 10^6 \frac{N}{m^2}$

Thus $d = \sqrt[3]{\frac{6M}{\sigma}} = \sqrt[3]{\frac{6 \times 15000}{245 \times 10^6}} = 71.62mm$

Therefore a load of 15kN acting at 1.5m for the fixing point of the cantilever beam must have dimensions of cross section of 71.62mm high and wide i.e square.

For a safety factor of 2 assume σ is now half i.e 122.5N/mm, repeating the calculation yields d to be 90.23mm

Selected safety factors are given in the table below:

Safety factor	value of d(mm)	volume mm^3	Cost($)	Mass(kg)
1	71.62	7.69×10^6	39.24	60.36
2	90.23	1.22×10^7	62.25	95.77
3	103.29	1.60×10^7	81.64	125.6
4	113.69	1.94×10^7	98.99	152.29

Density $= 7.85 \times 10^{-6} \frac{kg}{m^3}$, Price = $650 per Ton (1000kg), i.e $0.65 per kg. As you can see the safety factor dimensions are non-linear and it may be cost effective to implement a factor of 3 or even 4 given the difference in material costs.

Deflection

Specification dictates a limiting stiffness of 2mm, in order to calculate deflection this equation is used: $\delta = \frac{Fl^3}{3EI}$ for our scenario this becomes a rearrangement for d following on from the fact I is $\frac{d^4}{12}$

Thus we get: $d = \sqrt[4]{\frac{4FL^3}{E\delta}}$

$$d = \sqrt[4]{\frac{4 \times 10000 \times 1.5^3}{200 \times 10^9 \times 2 \times 10{-}3}} \times 1000 = 135.54mm$$

Which gives a design stress safety factor of 6.78.

Safety factor	value of d(mm)	volume mm^3	Cost($)	Mass(kg)
6.78	135.54	2.76×10^7	140.83	140.83

2.1.2 Material 2: Alloy Steel B.S. CDS 110

Dimensions

Using Steel BS 4360 grade 43A with a strength of 1034 N/mm^2

$\sigma = \frac{My}{I}$ with y being $\frac{d}{2}$ and I being $\frac{d^4}{12}$
thus simplifying to $\sigma = \frac{6M}{d^3}$

$M = 1.5 \times 10 \times 10^3 = 15000Nm$ $\sigma = 1034 \times 10^6 \frac{N}{m^2}$
Thus $d = \sqrt[3]{\frac{6M}{\sigma}} = \sqrt[3]{\frac{6 \times 15000}{1034 \times 10^6}} = 44.32mm$

Therefore a load of 15kN acting at 1.5m for the fixing point of the cantilever beam must have dimensions of cross section of 44.32mm high and wide i.e square.

For a safety factor of 2 assume σ is now half i.e 517 N/mm^2, repeating the calculation yields d to be 55.84mm

Selected safety factors are given in the table below:

Safety factor	value of d(mm)	volume mm^3	Cost($)	Mass(kg)
1	44.32	2.95×10^6	39.24	60.36
2	55.84	4.67×10^6	62.25	95.77
3	63.92	6.13×10^6	81.64	125.6
4	70.34	7.42×10^6	98.99	152.29

Density = $8.03 \times 10^{-6} \frac{kg}{m^3}$, Price = $4000 per Ton (1000kg), i.e $4 per kg. As you can see the safety factor dimensions are non-linear and it may be cost effective to implement a factor of 3 or even 4 given the difference in material costs.

Deflection

Specification dictates a limiting stiffness of 2mm, in order to calculate deflection this equation is used: $\delta = \frac{Fl^3}{3EI}$ for our scenario this becomes a rearrangement for d following on from the fact I is $\frac{d^4}{12}$

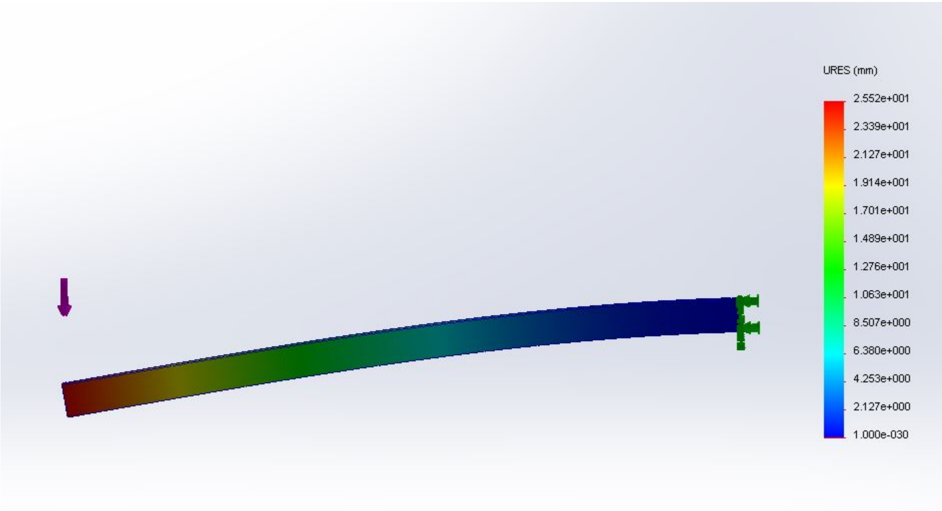
Thus we get: $d = \sqrt[4]{\frac{4FL^3}{E\delta}}$

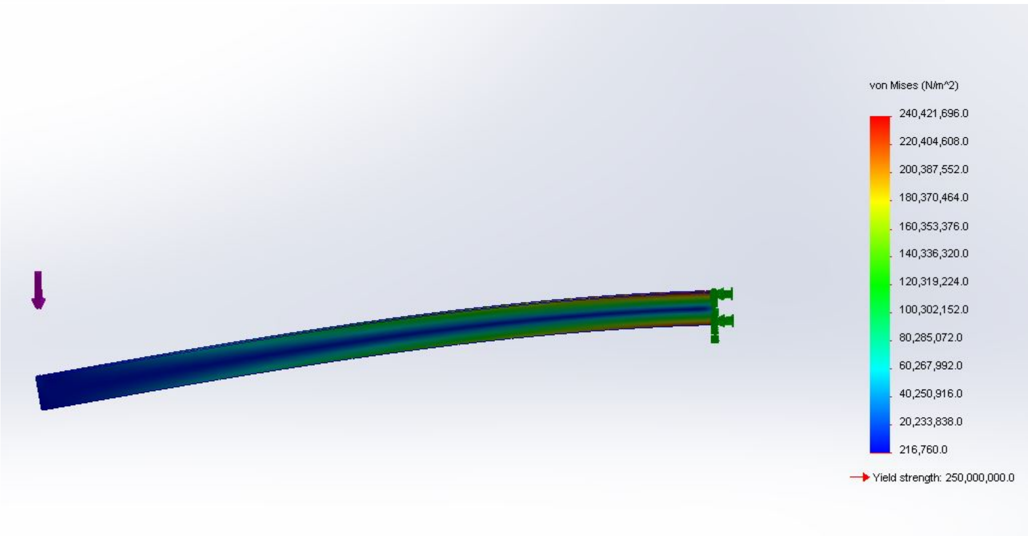
$$d = \sqrt[4]{\frac{4 \times 10000 \times 1.5^3}{200 \times 10^9 \times 2 \times 10^-3}} \times 1000 = 135.54mm$$

Which gives a design stress safety factor of 6.78.

Safety factor	value of d(mm)	volume mm^3	Cost($)
6.78	135.54	2.76×10^7	

3 Solidworks simulation





4 Appendix 1

4.1 Calculators in Excel

Cantilever Beam Calculator

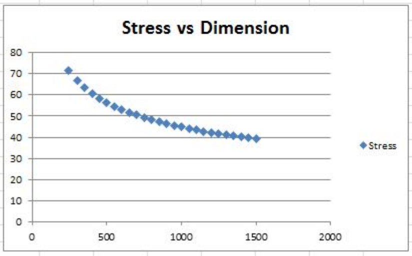
Geometry 1: square cross section. **Bending moment**

$\sigma = \frac{My}{I}$ with y being $\frac{d}{2}$ and I being $\frac{d^4}{12}$

thus simplifying to $\sigma = \frac{6M}{d^3}$

M =	**15000**
d for various σ	

$$d = \sqrt[3]{\frac{6M}{\sigma}}$$



Material Strength vs dimensions needed to support 10 kN

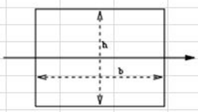
σ	d			
245	71.61854			
300	66.9433	For BS CDS 110	517	55.8364
350	63.59039		1034	44.3174
400	60.82202			
450	58.48035			
500	56.46216			
550	54.69655			
600	53.13293			
650	51.73404			
700	50.47173			
750	49.32424			
800	48.27447			
850	47.30872			
900	46.41589			
950	45.58686			
1000	44.81405			
1050	44.09111			
1100	43.41268			
1150	42.77417			
1200	42.17163			
1250	41.60168			
1300	41.06133			
1350	40.54801			
1400	40.05944			
1450	39.59359			
1500	39.14868			

Cantilever Beam Calculator

Geometry 1: square cross section. **Deflection**

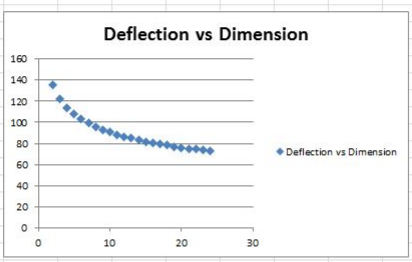
$$d = \sqrt[4]{\frac{4FL^3}{E\delta}}$$

F = 10000	
L=1.5	
E=200*10^9	
δ = 2mm	



δ	d (m)	mm
2	0.13554	135.5403
3	0.12247	122.4745
4	0.11398	113.9754
5	0.10779	107.7912
6	0.10299	102.9884
7	0.09909	99.09493
8	0.09584	95.84147
9	0.09306	93.06049
10	0.09064	90.64126
11	0.08851	88.50703
12	0.0866	86.60254
13	0.08489	84.88679
14	0.08333	83.32857
15	0.0819	81.90363
16	0.08059	80.59274
17	0.07938	79.38048
18	0.07825	78.25423
19	0.0772	77.2036
20	0.07622	76.21991
21	0.0753	75.29586
22	0.07443	74.42525
23	0.0736	73.60274
24	0.07282	72.82377

Deflection vs dimensions



4.2 Equations Used

Engineers theory of bending equation:

$$\sigma = \frac{My}{I}$$

Deflection equation:

$$\delta = \frac{Fl^3}{3EI}$$

Table of symbols:

- M= Bending Moment
- y = geometric depth /2
- I = second moment of area
- F = Force
- E = Youngs modulus
- σ = Yield Stress/Strength
- δ = Deflection