Proof Complexity and Solving LAB

New Year Recap

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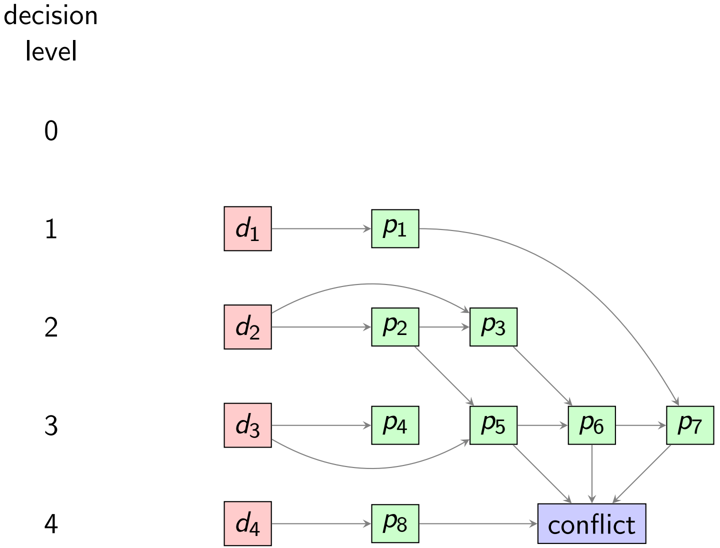
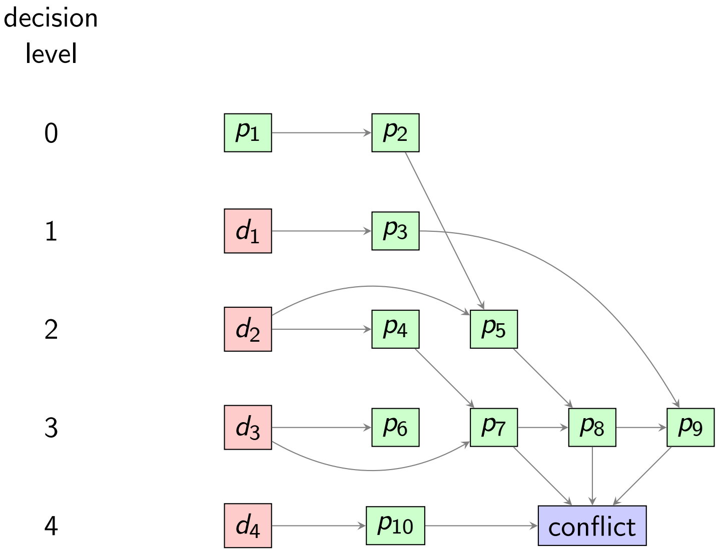
Goals

- Implemenatation of SAT solving algorithms
  (a) 2-SAT (polynomial time)
  (b) DPLL
  (c) CDCL
    - watched literals
    - clause learning
    - decision heuristics
    - restart strategy
  (d) QBF expansion..
- Practical programming experience
  - use your favourite language (Python, C, C++, Java, ..)
  - recommended: Python

Reaching the First Conflict



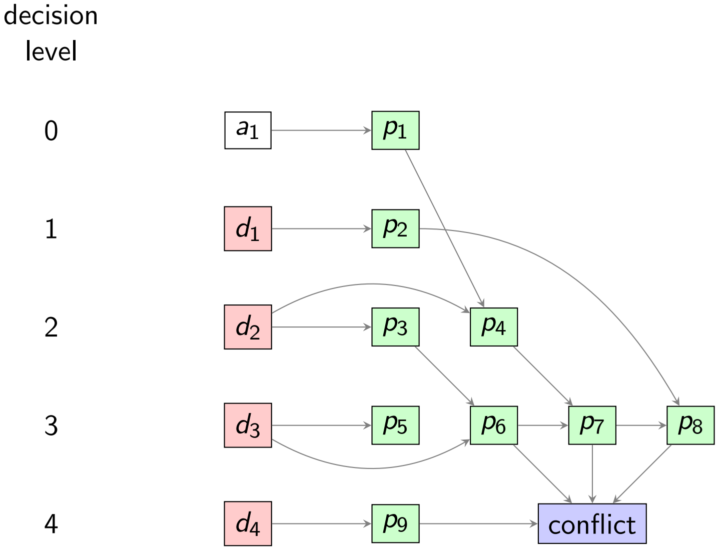
Reaching Later Conflicts



Learning Unit Clauses (!?)

- a learned unit clause should propagate:
  (a) immediately
  (b) forever
- result: adding learned unit clauses is unnecessary ..
- .. just make the assignment at decision level 0 instead
- decision level 0 is never undone
- watched literals do not work with unit clauses

Reaching Later Conflicts (Revisited)



CDCL Pseudocode

```
function CDCL-solver(Φ)                                  #assuming Φ is preprocessed
  decision-level ← 0
  while there are unassigned variables
    decision-level++
    decide()                                             #adds assignment to trail
    C_conflict ← propagate()                             #returns conflict clause or null

    while C_conflict is not null
      if decision-level = 0 return UNSAT
      C_learned ← analyse-conflict(C_conflict)
      if C_conflict is unit
        backtrack(0)
        assign unit literal
      else
        backtrack(asserting-level(C_learned))            #changes trail and DL
        Φ ← Φ ∧ C_learned
      C_conflict ← propagate()

    apply-restart-policy()
  return SAT
```

Watched Literals

- when searching for conflict, we only care about unit clauses
- a clause becomes unit when:
  - it has exactly one unassigned literal, and
  - all other literals are falsified
- sufficient to watch just two literals in every clause
- maintain this invariant for each clause:
  - either both watched literals are unassigned
  - or at least one watched literal is satisfied
- important: if both watched literals are assigned, and one is falsifed: its decision level should be no lower than the satisfied one

How is it done?

- many options - here's one:

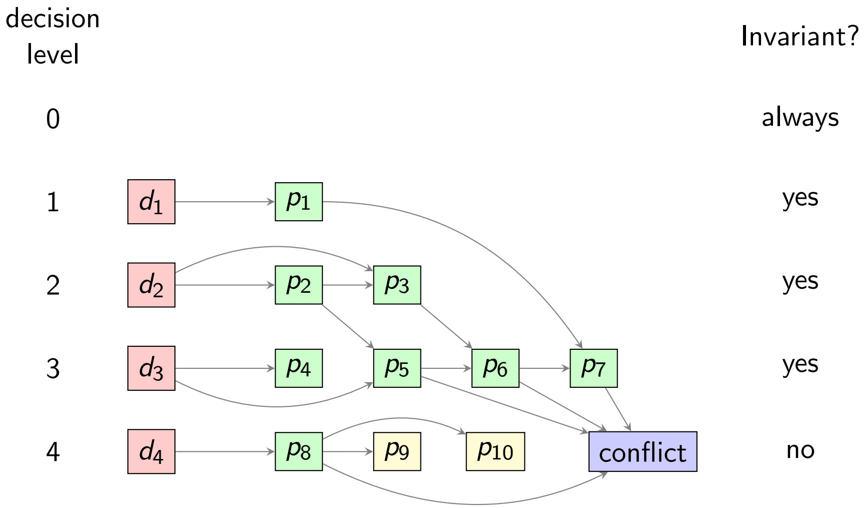
(a) maintain a list of 'watched clauses' for each literal

(b) process a variable assignment by:

  1. visit watched clauses for the falsified literal in order
  2. make sure the invariant holds
     - you may need to 'swap the watch'
  3. if clause becomes unit, add unit assignment to trail
     - note: in this case, both watched literals have the same decision level
     - so there is no need to swap the watch

(c) if the invariant cannot be maintained, we reach conflict

Conflicts and Backtracking



Swapping the Watch

$$\boxed{x_1 \mapsto 0} \quad \boxed{x_2 \mapsto 1} \quad \boxed{x_3 \mapsto 0} \quad \overset{\downarrow}{\boxed{x_4 \mapsto 1}} \quad \boxed{x_5 \mapsto 1}$$

trivial case: $(\overset{\downarrow}{x_6} \vee \overset{\downarrow}{\overline{x_7}} \vee \overline{x_4}) \quad \rightarrow \quad (\overset{\downarrow}{x_6} \vee \overset{\downarrow}{\overline{x_7}} \vee \overline{x_4})$

sat case: $(\overset{\downarrow}{\overline{x_1}} \vee \overset{\downarrow}{\overline{x_4}} \vee \overline{x_2}) \quad \rightarrow \quad (\overset{\downarrow}{\overline{x_1}} \vee \overset{\downarrow}{\overline{x_4}} \vee \overline{x_2})$

sat-swap case: $(\overset{\downarrow}{\overline{x_4}} \vee \overset{\downarrow}{\overline{x_6}} \vee x_3 \vee x_2) \quad \rightarrow \quad (\overline{x_4} \vee \overset{\downarrow}{\overline{x_6}} \vee x_3 \vee \overset{\downarrow}{x_2})$

swap case: $(\overset{\downarrow}{\overline{x_4}} \vee \overset{\downarrow}{\overline{x_6}} \vee x_3 \vee x_7) \quad \rightarrow \quad (\overline{x_4} \vee \overset{\downarrow}{\overline{x_6}} \vee x_3 \vee \overset{\downarrow}{x_7})$

Unit Clauses and Conflicts

$$\begin{array}{ccccc} & & & \downarrow & \\ \boxed{x_1 \mapsto 0} & \boxed{x_2 \mapsto 1} & \boxed{x_3 \mapsto 0} & \boxed{x_4 \mapsto 1} & \boxed{x_5 \mapsto 1} \end{array}$$

unit case: $(\overset{\downarrow}{\overline{x_4}} \vee \overset{\downarrow}{\overline{x_6}} \vee x_3) \quad \rightarrow \quad (\overset{\downarrow}{\overline{x_4}} \vee \overset{\downarrow}{\overline{x_6}} \vee x_3)$

add assignment: $x_6 \mapsto 0$

conflict case: $(\overset{\downarrow}{\overline{x_4}} \vee \overset{\downarrow}{\overline{x_5}} \vee x_3) \quad \rightarrow \quad (\overset{\downarrow}{\overline{x_4}} \vee \overset{\downarrow}{\overline{x_5}} \vee x_3)$

try to add assignment: $x_5 \mapsto 0$

$\Longrightarrow$ CONFLICT

Watched Literals Task

- implement unit propagation with watched literals in your CDCL solver
- ignore pure literal elimination
- check correctness
- compare the solving time to naive propagation

Clause Learning - Cutting the Implication Graph