Proof Complexity and Solving LAB

Clause Learning

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https://github.com/JoshuaBlinkhorn/SAT-LAB

Goals

- Implemenatation of SAT solving algorithms
  - (a) 2-SAT (polynomial time)
  - (b) DPLL
  - (c) CDCL
    - watched literals
    - clause learning
    - decision heuristics
    - restart strategy
  - (d) QBF expansion..
- Practical programming experience
  - use your favourite language (Python, C, C++, Java, ..)
  - recommended: Python

CDCL Pseudocode

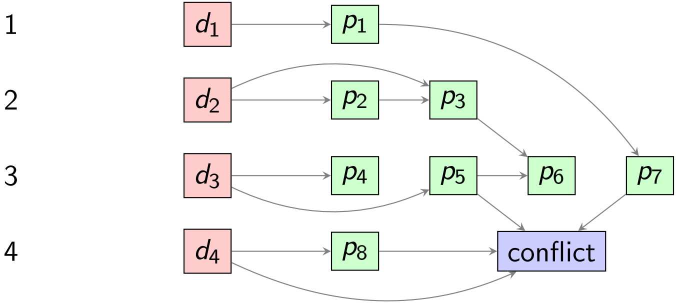
```
function CDCL-solver($\Phi$)                                  #assuming $\Phi$ is preprocessed
  decision-level $\leftarrow$ 0
  while there are unassigned variables
    decision-level++
    decide()                                                  #adds assignment to trail
    $C_{\text{conflict}} \leftarrow$ propagate()              #returns conflict clause or null

    while $C_{\text{conflict}}$ is not null
      if decision-level $= 0$ return UNSAT
      $C_{\text{learned}} \leftarrow$ analyse-conflict($C_{\text{conflict}}$)
      if $C_{\text{conflict}}$ is unit
        backtrack(0)
        assign unit literal
      else
        backtrack(asserting-level($C_{\text{learned}}$))      #changes trail and DL
        $\Phi \leftarrow \Phi \wedge C_{\text{learned}}$
      $C_{\text{conflict}} \leftarrow$ propagate()

    apply-restart-policy()
  return SAT
```

Resolution

$$(C \lor x), (D \lor \neg x) \quad \vdash \quad (C \lor D)$$

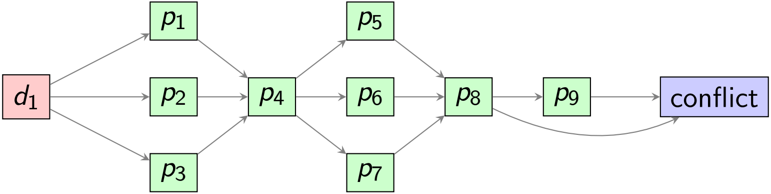
- $(C \lor D)$ is the resolvent
- x is the pivot
- logically correct rule: premises imply resolvent
- sound and complete proof system for UNSAT

Clause Learning - Cutting the Implication Graph



Unique Implication Points

- A unique implication point (UIP) is:
  - a node at the highest decision level
  - every path from highest decision to conflict passes through it



Common Implementation

- advantage: clauses learned from UIPs are always asserting
- asserting means 'becomes unit at a previous decision level'
- the asserting level is second highest decision level in learned clause
- hence: backtrack to asserting level ('backjumping')
- remember: we don't learn unit clauses – we make assignments instead

- easy implementation:
- resolve conflict clause with reason clauses until there is exactly one variable at highest decision level
- This is the 1UIP learning scheme

Thoughts

- Don't worry too much about why 1UIP schemes work well
- My opinion: the community has not agreed on a clear reason
- Theoreticians: SAT solver are automated decision procedures
- Practitioners: SAT solvers perform local search

Watched Literals Task

- implement a 1UIP clause learning scheme in your CDCL solver
- check correctness
- compare the solving time to DPLL-style learning