Assignment 6

Digital Signal Processing

AE4463P-23: Advanced Aircraft Noise

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Assignment 6

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by

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Style: TU Delft Report Style, with modifications by Daan Zwaneveld

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1 Test Setup and Methodology

Within this assignment an analysis will be performed on the characteristics of various circuits given an input signal. Five distinct circuit will be analysed for their effect on the phase and magnitude of the output signal with respect to the provided input signal. The following five circuits were included within the scope of analysis.

- Non-inverting amplifier (setup-1: gain = 101 ($R_2/R_1 = 100$), setup-2: gain = 6 ($R_2/R_1 = 5$))
- Buffer
- Low pass filter
- High pass filter
- Full combined circuit

To analyse the behaviour of the various circuits mentioned above the test setup found in Figure 1.1 was utilised. Here an input signal was provided trough the use of a function generator, which could be utilised to generate a select input signal. For all of the circuits analysed this signal was a sine wave, however the reference voltage was varied throughout the experiment based on the circuit being analysed. For the the full circuit a voltage 50mVpp was utilised, while for the high and low pass filter a voltage of 1Vpp was utilised. For the Non-inverting amplifier 50mVpp was utilised while for the buffer the 1Vpp was used. Here pp reference to peak to peak, and thus is double the amplitude of the sine wave.

Both the input an output could then be observed trough the use of digital oscilloscope. Here both the phase and magnitude difference could be measured. To then analyse the characteristics of the circuit, measurements were performed were the frequency of the input signal was varied. Trough this procedure it could then be analysed how to properties of the circuit are varied based on the frequency of the input signal. For all experiments except the following frequencies were used: 100Hz, 200Hz, 300Hz, 500Hz, 1kHz, 2kHz, 5kHz, 10kHz, 15kHz, 20kHz, 50kHz, 100kHz, 200kHz, 500kHz, and 1MHz. Afterwards utilising the measurements made it is possible to determined the phase and magnitude behaviour as a function of frequency for the circuits.

Figure 1.1: Test Setup extracted from the assignment discription

2
Experiment

Within this section the results obtain from the experiment described in chapter 1 will be presented. Furthermore a comparison will be made will be made will the results expected from theory. It must be noted that in all the derivations presented below $\omega = 2\pi f$.

2.1. Non-inverting amplifier

For ease of comparison, the 2 non-inverting amplifiers and the buffer have been combined into one bode plot. In the results the obtained experimental results as well as its theoretical counterpart. All measured outcomes, i.e. the amplitude of the signal, the period, the phase and the time interval, can be found in Appendix C for each data-point.

In order to obtain the theoretical prediction for both the phase and magnitude as a function of frequency, the following equations have been used.

$$A = \frac{A_0}{1 + j\omega\tau} \tag{2.1}$$

$$\tau = \frac{1}{2\pi f_c} \tag{2.2}$$

The non-inverting amplifier is characterised by the following transfer function (derived in part II of appendix B).

$$H(j\omega) = \frac{A(\frac{R_2}{R_1} + 1)}{(\frac{R_2}{R_1} + A)} \tag{2.3}$$

Utilising Equation 2.1 in the above equation:

$$H(j\omega) = \frac{\frac{A_0}{1+j\omega\tau}\left(\frac{R_2}{R_1} + 1\right)}{\left(\frac{R_2}{R_1} + 1\right) + \frac{A_0}{1+j\omega\tau}} \tag{2.4}$$

Simplifying the above equation the following can be observed:

$$H(j\omega) = \frac{A_0 R_2 + A_0 R_1}{R_2 + R_1 + R_1 A_0 + (R_1 + R_2) * j\omega\tau} \tag{2.5}$$

Utilising the above formulation both the phase and magnitude can be determined.

$$|H(j\omega)| = \sqrt{\frac{(A_0R_2 + A_0R_1)^2}{(R_2 + R_1 + R_1A_0)^2 + ((R_1 + R_2)\omega\tau)^2}} \tag{2.6}$$

$$\phi(j\omega) = -\arctan\left(\frac{(R_1 + R_2)\omega\tau}{R_2 + R_1 + R_1A_0}\right) \tag{2.7}$$

The values for τ and A_0 were determined trough data matching with the experimental output. For A_0 a fixed value of 100000 was utilised, while τ was varied until a sufficiently accurate match was found. A file value for τ of 0.01 was chosen within the scope of this project.

2.1.1. Buffer

Similarly to the previous derivation the bode plot can be constructed for the buffer utilising the procedure explain previously.

The transfer function for the buffer is described by Equation 2.8

$$H(j\omega) = \frac{1}{1 + j\omega\frac{\tau}{A_0}} \tag{2.8}$$

First converting the complex transfer function to the standard form of a + bi.

$$\begin{aligned} H(j\omega) &= \frac{1}{1 + j\omega\frac{\tau}{A_0}} = \frac{1}{1 + \frac{\omega\tau}{A_0}j} \\ &\Rightarrow \frac{1}{1 + \frac{\omega\tau}{A_0}j} * \frac{1 - \frac{\omega\tau}{A_0}j}{1 - \frac{\omega\tau}{A_o}j} \\ &\Rightarrow \frac{1}{1 + \left(\frac{\omega\tau}{A_0}\right)^2} - \frac{\frac{\omega\tau}{A_0}}{1 + \left(\frac{\omega\tau}{A_0}\right)^2}j \end{aligned} \tag{2.9}$$

Then the magnitude can be readily computed using the relation found below.

$$\Rightarrow |H(j\omega)| = \sqrt{\frac{1 + \left(\frac{\omega\tau}{A_0}\right)^2}{\left[1 + \left(\frac{\omega\tau}{A_0}\right)^2\right]^2}} = \sqrt{\frac{1}{1 + \left(\frac{\omega\tau}{A_0}\right)^2}} \tag{2.10}$$

For the phase any of the simplified transfer functions can be utilised. The phase can then be found by subtracting the phase of denominator from the numerator.

$$\begin{aligned} \frac{1}{1 + j\omega\frac{\tau}{A_0}} &\Rightarrow \phi = \arctan\left(\frac{0}{1}\right) - \arctan\left(\frac{\omega\frac{\tau}{A_0}}{1}\right) \\ &\Rightarrow -\arctan\left(\omega\frac{\tau}{A_0}\right) \end{aligned} \tag{2.11}$$

2.1.2. Results

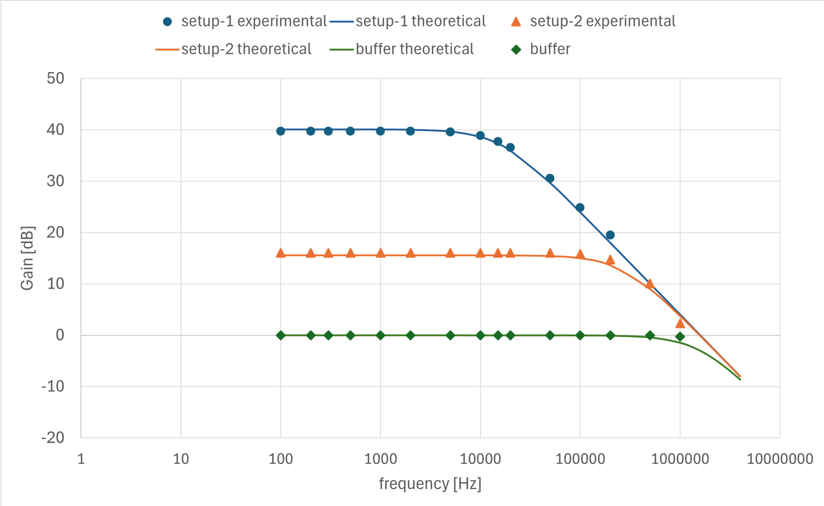


Figure 2.1: Magnitude Bode Plot showing theoretical vs experimental results of non-inverting amplifiers for setup-1 and setup-2, and for a buffer

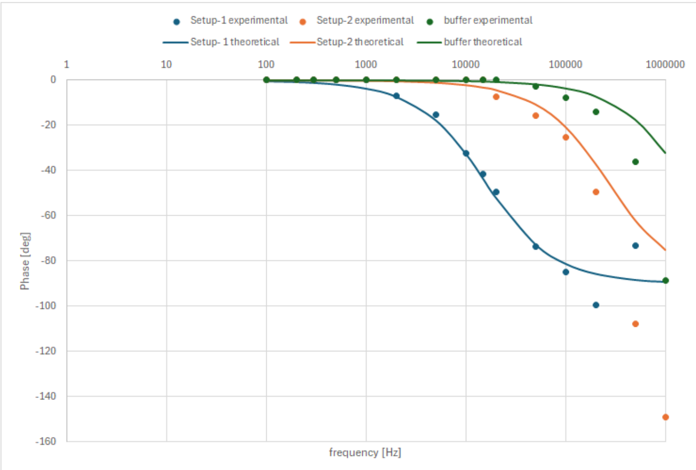


Figure 2.2: Phase Bode Plot showing theoretical vs experimental results of non-inverting amplifiers for setup-1 and setup-2, and for a buffer

Observing the two cases analysed of the non-inverting amplifier two notable difference can be observed. In case one, with $R_2/R_1 = 100$ the gain is high (40dB), however a low bandwidth is observed (20KHz). In case two, with $R_2/R_1 = 5$ the gain is lower (16dB), however the bandwidth has now increased to 300kHz. When utilising the non-decibel value for the gain it can be seen that the bandwidth expressed in frequency and the gain expressed in volt is constant for both. Therefore trough the use of different resistor ratios, it is possible to increase the bandwidth at the cost of the gain.
When observing the phase plot, it is noted that at higher frequencies the measured data does not fit the theory that well anymore, this is assumed to be due to the large fluctuations of the system at high frequencies making the measurements more inaccurate.

2.2. Low and High Pass Filters

For the low and high pass filters, the theoretical magnitude and phase as a function of frequency were determined using subsection 2.2.1 and subsection 2.2.2. For the low pass filter a capacitor of 48 nF and a resistor of 218 Ohm is used. While for high pass filter a capacitor of 108 nF and a resistor of 5.09 kOhm is utilised.

2.2.1. Low pass filter

Starting from the transfer function for low pass filter, the relation describing the magnitude and phase shift can be determined as a function of frequency.

First the complex form should be rewritten to the from of a + bj.

$$\begin{aligned} H(j\omega) &= \frac{v_{\text{out}}}{v_{\text{in}}} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC} \\ &= \frac{1}{1+\omega RC}\cdot\frac{1-j\omega RC}{1-j\omega RC} = \frac{1-\omega RCj}{1+(\omega RC)^2} = \frac{1}{1+(\omega RC)^2} - \frac{\omega RC}{1+(\omega RC)^2}j \end{aligned} \tag{2.12}$$

Then the magnitude can be readily computed using the relation found below.

$$\Rightarrow |H(j\omega)| = \sqrt{\frac{1+(\omega RC)^2}{[1+(\omega RC)^2]^2}} = \sqrt{\frac{1}{1+(\omega RC)^2}} \tag{2.13}$$

For the phase any of the simplified transfer functions can be utilised. The phase can then be found by subtracting the phase of denominator from the numerator

$$\begin{aligned} \frac{1-\omega RCj}{1+(\omega RC)^2} &\Rightarrow \phi = \text{arctan}\left(-\frac{\omega RC}{1}\right) - \text{arctan}\left(\frac{0}{1+(\omega RC)^2}\right) \\ &\Rightarrow \phi = -\text{arctan}\,(\omega R) \end{aligned} \tag{2.14}$$

2.2.2. High pass filter

Starting from the transfer function for high pass filter, the relation describing the magnitude and phase shift can be determined as a function of frequency.

First the complex form should be rewritten to the from of a + bj.

$$\begin{aligned} H(j\omega) &= \frac{v_{\text{out}}}{v_{\text{in}}} = \frac{R}{\frac{1}{j\omega C}+R} = \frac{j\omega RC}{1+j\omega RC} \\ &\frac{1}{1+\frac{1}{\omega RC}j} * \frac{1-\frac{1}{\omega RC}j}{1-\frac{1}{\omega RC}j} = \frac{1-\frac{1}{\omega Rc}j}{1+\left(\frac{1}{\omega Rc}\right)^2} = \frac{1}{1+\left(\frac{1}{\omega Rc}\right)^2} - \frac{\frac{1}{\omega Rc}}{1+\left(\frac{1}{\omega Rc}\right)^2}j \end{aligned} \tag{2.15}$$

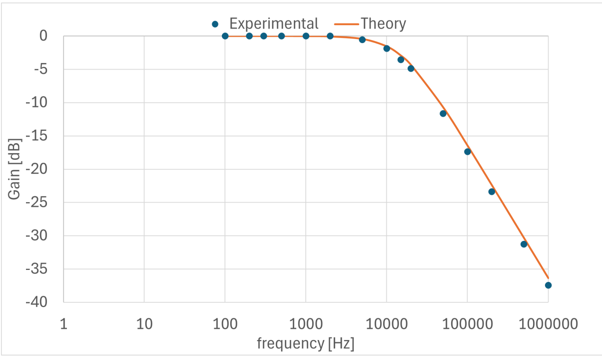
Then the magnitude can be readily computed using the relation found below.

$$\Rightarrow |H(j\omega)| = \sqrt{\frac{1}{1+\left(\frac{1}{\omega RC}\right)^2}} \tag{2.16}$$

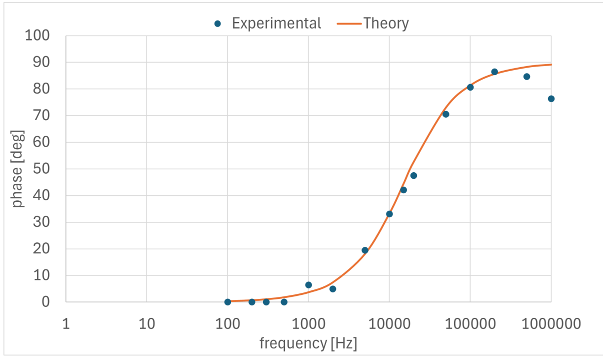
For the phase any of the simplified transfer functions can be utilised. The phase can then be found by subtracting the phase of denominator from the numerator.

$$\begin{aligned} \frac{j\omega RC}{1+j\omega RC} &\Rightarrow \phi = \text{arctan}\left(\frac{\omega RC}{0}\right) - \text{arctan}\left(\frac{\omega RC}{1}\right) \\ &\Rightarrow \phi = \frac{\pi}{2} - \text{arctan}\left(\omega RC\right) \end{aligned} \tag{2.17}$$

2.2.3. Results

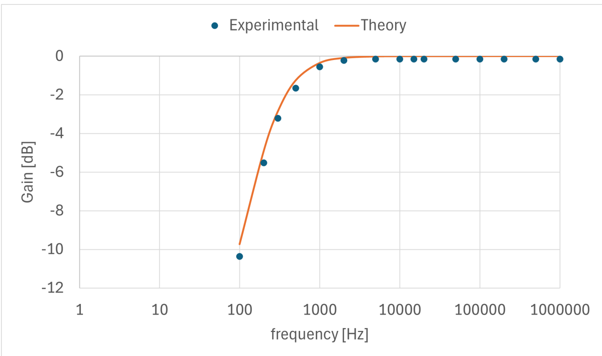


(a) Magnitude as a function of frequency

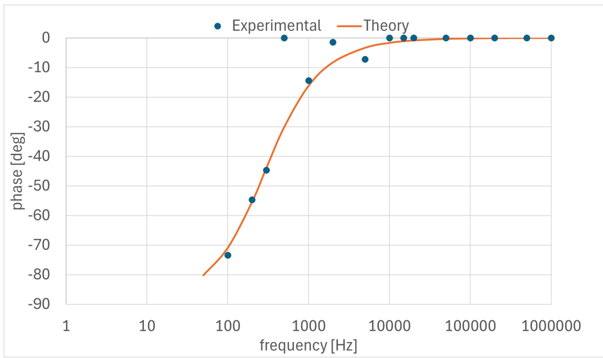


(b) Phase as a function of frequency

Figure 2.3: Bode plots for a low pass filter



(a) Magnitude as a function of frequency



(b) Phase as a function of frequency

Figure 2.4: Bode plots for a high pass filter

It can be observed that for both the high and low pass filter the expected behaviour from theory matches the acquired data. The low pass filter is characterised by the "zero" attenuation of signals for lower signals, however the larges frequency signals are heavily decrease in magnitude. Furthermore for the low frequencies the the phase difference is zero however for larges frequencies the lag tends to 90 degrees. For the high pass filter the exactly opposite behaviour is found. At low frequencies the signal is heavily decreased in magnitude, and lags behind the reference signal. While for the high frequencies this phase and magnitude disturbance is significantly decreased.

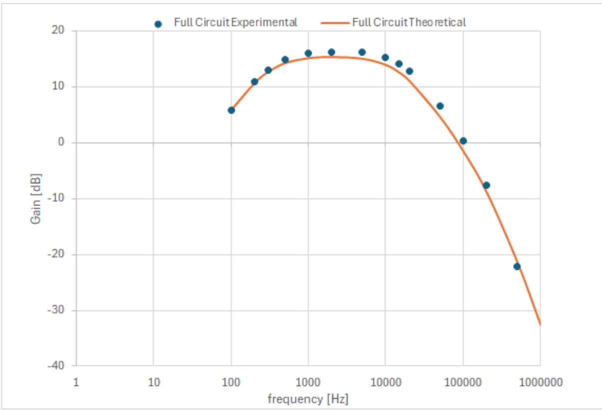
2.3. Full circuit

The non-inverting amplifier encompasses all filters e.i. the low pass filter, the high pass filter and the buffer. The magnitude can be found by multiplying all the previously found magnitudes, while the phase can be found by taking the sum.

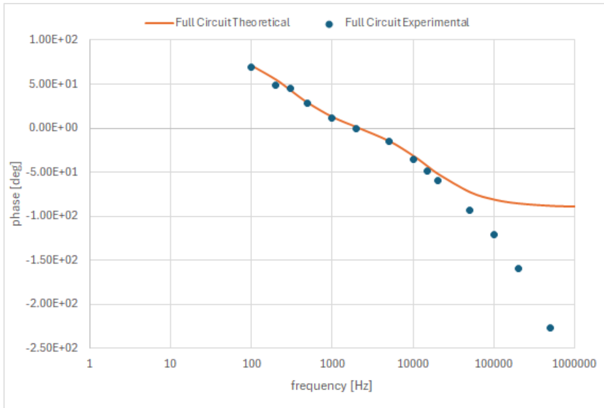
$$|H(j\omega)| = |H(j\omega)|_{Non-inverting} * |H(j\omega)|_{Low-pass} * |H(j\omega)|_{Buffer} * |H(j\omega)|_{High-pass} \tag{2.18}$$

$$\phi(j\omega) = \phi(j\omega)_{Non-inverting} + \phi(j\omega)_{Low-pass} + \phi(j\omega)_{Buffer} + \phi(j\omega)_{High-pass} \tag{2.19}$$

2.3.1. Results



(a) Magnitude as a function of frequency



(b) Phase as a function of frequency

Figure 2.5: Bode plots for a full circuit

In the above results it can be observed that the theoretical formulation strongly matches the values found trough during the experiments. This behaviour is seen for both the magnitude and phase. However for the high frequencies a increasing deviation starts forming for the phase. This deviation could be explained due to the less accurate nature of the data for the higher frequencies, since the reading became more distorted for the higher observed frequencies.

Furthermore it can be seen that utilising the described circuit it is possible to amplify a select band of the signal, while also keeping the phase difference low. While then also heavily attenuating the other parts of the signal, and therefore essentially removing the information stored within these parts of the frequencies.

References

[1] Prof. dr. ir. M. Snellen Prof. dr. D.G. Simons. *Digital Signal Processing an introduction*. Delft, Netherlands: TU Delft, Dec. 2023.

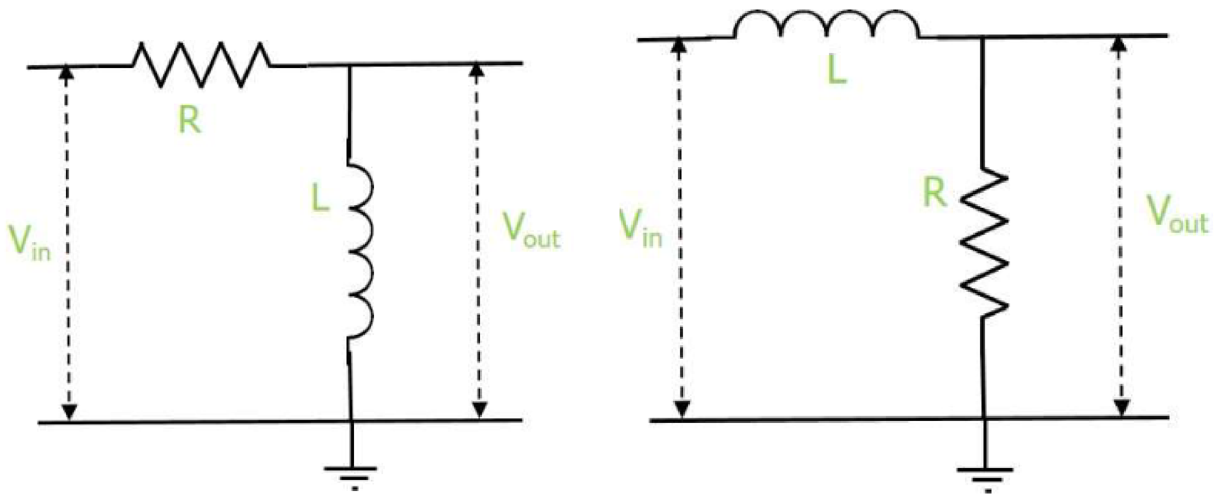
A
Matlab code

```
%% Assignment 6: Advanced Aircraft Noise
% By: Elisabeth and Joshua and Atze

clear;

%% Excersise 1

f = 0:10:100000;

R = 24;
L = 20e-3;

s = tf('s');    % s -> j * omega

H1 = (s*L) / (R + s*L);
H2 = R / (R + s*L);

% Bodeplot for first transfer function
% Identified to be High pass filter (HPF)
figure(1);
bp1 = bodeplot(H1);
setoptions(bp1,'FreqUnits','Hz');

% Bodeplot for second transfer function
% Identified to be Low pass filter (LPF)
figure(2);
bp2 = bodeplot(H2);
setoptions(bp2,'FreqUnits','Hz');

% Long methode to check bode plots
%c1 = 20*log10(R ./ sqrt(R^2 + (2*pi*f).^2 * L^2));
c2 = R ./ (R + 2*pi*f*L*1i);
c1 = (2*pi*f*L*1i) ./ (R + (2*pi*f*L*1i));

% Checker for phase

cp2 = 0 - atan((2*pi*f*L)./(R));
cp1 = pi/2 - atan((2*pi*f*L)./(R));

figure(3);
semilogx(f, 20*log10(abs(c1)))
hold on
yline(-3)
semilogx(f, 20*log10(abs(c2)))

figure(4);
semilogx(f, cp2*180/pi)
hold on
semilogx(f, cp1*180/pi)
```

B Preparation Exercises

B.1. Part I

In Figure B.1 two distinct filters can be observed, with Figure B.1a describes a high pass filter and Figure B.1b describes a low pass filter.



(a) High pass filter

(b) Low pass filter

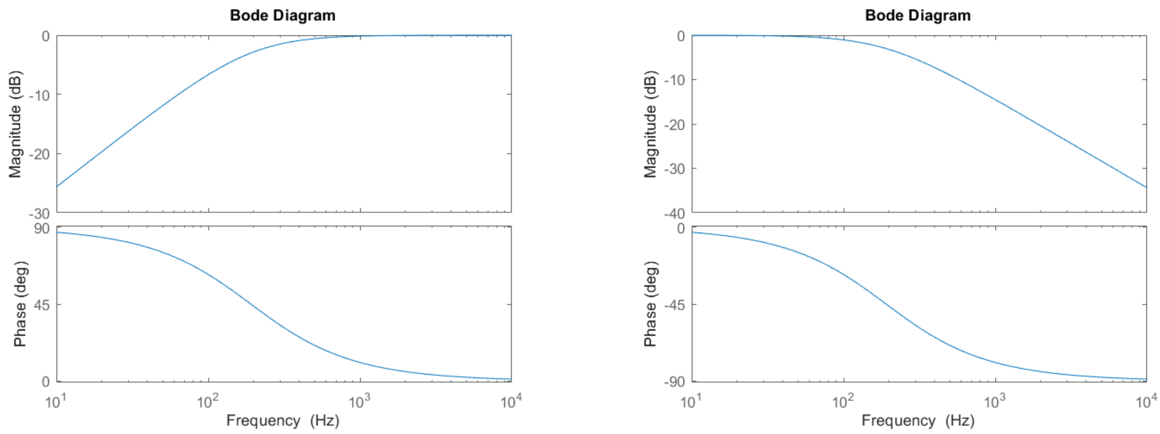
Figure B.1: Two filter extracted from [1]

For both diagrams the transfer function $H(j\omega)$ can be setup which can then be utilised to create the phase and magnitude diagrams as a function of frequency. Below in Equation B.3 the transfer function for the high pass filter in Figure B.1a can be observed. While in Equation B.4 the transfer function for the low pass filter in Figure B.1b can be observed

$$\mathbf{H}(j\omega) = \frac{v_{out}}{v_{in}} = \frac{j\omega L}{j\omega L + R} \tag{B.1}$$

$$\mathbf{H}(j\omega) = \frac{v_{out}}{v_{in}} = \frac{R}{j\omega L + R} \tag{B.2}$$

Where R = 24Ω and L = 20e-3mH. Since both of the equations described above a simple generic transfer functions the Matlab command "bodeplot" can be utilised to create both the phase and magnitude diagrams. Below in Figure B.2 the diagrams for both can be observed.



(a) Bode diagram for a high pass filter

(b) Bode diagram for a low pass filter

Figure B.2: Phase and magnitude described as a function of frequency for the two filter described in Figure B.1

It can be observed that filter one (Figure B.1a) heavily attenuates the lower frequencies, while the high frequencies are let trough without decay (High pass filter). While for the second filter seen in Figure B.1b the exact opposite can be seen, where the low frequencies are let trough.

Lastly for both can the cut of frequency can be determined. This is the frequency at which the magnitude equals exactly -3dB.

$$|\mathbf{H}\left(j\omega\right)_{high}| = |\frac{j\omega L}{j\omega L + R}| = -3dB \Rightarrow f_{c,high} = 190.99 \approx 190Hz \tag{B.3}$$

$$|\mathbf{H}\left(j\omega\right)_{low}| = |\frac{R}{j\omega L + R}| = -3dB \Rightarrow f_{c,low} = 190.99 \approx 190Hz \tag{B.4}$$

Below the analytical proof can be found. First the complex number is rewritten into the standard form (a + bi).

$$H(\omega i) = \frac{\omega L i}{\omega L i + R} = \frac{1}{1 + \frac{R}{\omega L} i} * \frac{1 - \frac{R}{\omega L} i}{1 - \frac{R}{\omega L} i} = \frac{1 - \frac{R}{\omega L} i}{1 + \left(\frac{R}{\omega L}\right)^2} = \frac{1}{1 + \left(\frac{R}{\omega L}\right)^2} - \frac{\frac{R}{\omega L} i}{1 + \left(\frac{R}{\omega L}\right)^2}$$

Taking the magnitude of the complex number using $\sqrt{a^2 + b^2}$:

$$\Rightarrow |H(\omega i)| = \sqrt{\frac{1 + \left(\frac{R}{\omega L}\right)^2}{\left[1 + \left(\frac{R}{\omega L}\right)^2\right]^2}} = \sqrt{\frac{1}{1 + \left(\frac{R}{\omega L}\right)^2}}$$

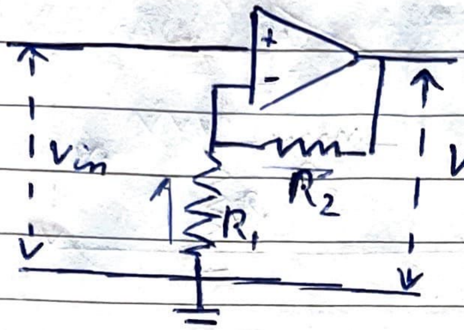
The cut of frequency is defined as the frequency when the magnitude is equal to -3dB or 1 over the square root of two.

$$\sqrt{\frac{1}{1 + \left(\frac{R}{2\pi f L}\right)^2}} = \frac{1}{\sqrt{2}} \rightarrow f_c = \frac{R}{2\pi L}$$

For the low pass filter the exact same procedure can be utilised, leading tho the exact same formulation of the cut of frequency.

B.2. Part II

Q2 Ans 6



$Z_f = R_f \qquad -\frac{R_f}{R_i}$

$Z_i = R_i$

Q: show here $Gain = 1 + \frac{R_2}{R_1}$

$I_2 = \frac{V_{out} - V^-}{R_2} \qquad I_1 = \frac{V^- - 0}{R_1} \qquad V^+ = V_{in}$

$\Rightarrow$ talking $I_2 = I_1 \Rightarrow \frac{V_{out} - V^-}{R_2} = \frac{V^-}{R_1}$

$\Rightarrow V^- \frac{R_2}{R_1} = V_{out} - V^-$

$\Rightarrow V^-\left(\frac{R_2}{R_1} + 1\right) = V_{out}$ (1) $\longrightarrow V^- = \frac{1}{\frac{R_2}{R_1} + 1} V_{out}$ (2)

$V_{out} = A(V^+ - V^-)$ (eq 19)

$\Rightarrow V_{out} = AV^+ - AV^-$

$\Rightarrow V_{out} = AV_{in} - AV^-$ using $V^+ = V_{in}$

$\Rightarrow V_{out} = AV_{in} - AV_{out}\frac{1}{\frac{R_2}{R_1} + 1}$ using (2)

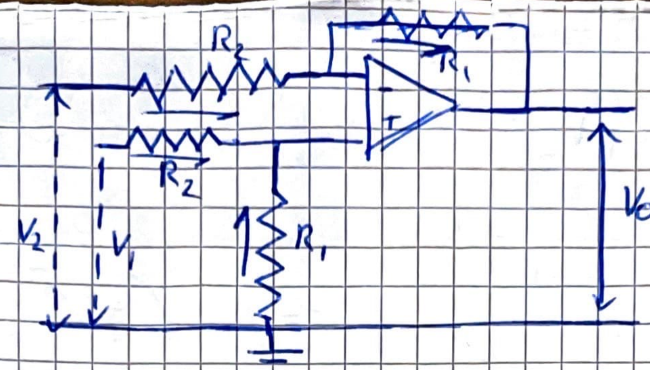
$\Rightarrow V_{out}\left(1 + A\frac{1}{\frac{R_2}{R_1} + 1}\right) = AV_{in}$

$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A}{1 + A\frac{1}{\frac{R_2}{R_1} + 1}} = \frac{A}{\frac{\frac{R_2}{R_1} + 1 + A}{\frac{R_2}{R_1} + 1}} = \frac{A\left(\frac{R_2}{R_1} + 1\right)}{\left(\frac{R_2}{R_1} + 1\right) + A}$

Assume open loop Gain A is ∞

$\frac{V_{out}}{V_{in}} = \lim_{A \to \infty} \frac{A\left(\frac{R_2}{R_1} + 1\right)}{\left(\frac{R_2}{R_1} + 1\right) + A} = \boxed{\frac{R_2}{R_1} + 1}$

<u>Q3 A036</u>



$V_{out} = A(V^+ - V^-)$

show: $\frac{R_1}{R_2}(V_1 - V_2) = V_{out}$

$I_2 = \frac{V^- - V_2}{R_2}$ $\quad I_1 = \frac{V_{out} - V^-}{R_1}$ $\quad I_1 R_1 + V^- = V_{out}$

$I_2 = \frac{V^+ - V_1}{R_2}$ $\quad I_1 = \frac{V^+ - 0}{R_1}$

$\frac{V^- - V_2}{R_2} = \frac{V_{out} - V^-}{R_1}$ $\quad \frac{V^+ - V_1}{R_2} = \frac{V^+}{R_1}$

$\Rightarrow V^- - V_2 = \frac{R_2}{R_1}(V_{out} - V^-)$ $\quad \Rightarrow V^+ - V_1 = \frac{R_2}{R_1} V^+$

$\Rightarrow V^- - V_2 - \frac{R_2}{R_1} V_{out} + \frac{R_2}{R_1} V^- = 0$ $\quad \Rightarrow V^+\left(1 + \frac{R_2}{R_1}\right) = V_1$

$\Rightarrow V^-\left(1 + \frac{R_2}{R_1}\right) = V_2 + \frac{R_2}{R_1} V_{out}$ $\quad \Rightarrow V^+ = \frac{1}{1 + \frac{R_2}{R_1}} V_1$

$\Rightarrow V^- = \frac{1}{\left(1 + \frac{R_2}{R_1}\right)} \cdot \left(V_2 + \frac{R_2}{R_1} V_{out}\right)$

$$V^- = \frac{1}{1+\frac{R_2}{R_1}} \cdot \left(V_2 + \frac{R_2}{R_1} \cdot V_{out}\right) \qquad U = IR$$

$$V^+ = \frac{1}{1+\frac{R_2}{R_1}} \cdot V_1 \qquad V_{out} = A\left(V^+ - V^-\right)$$

$$V_{out} = A \cdot \underbrace{\frac{1}{1+\frac{R_2}{R_1}}}\left[V_1 - V_2 - \frac{R_2}{R_1} V_{out}\right]$$

$$\Rightarrow V_{out} + A \frac{1}{1+\frac{R_2}{R_1}} \frac{R_2}{R_1} V_{out} = A \cdot \frac{1}{1+\frac{R_2}{R_1}}\left[V_1 - V_2\right]$$

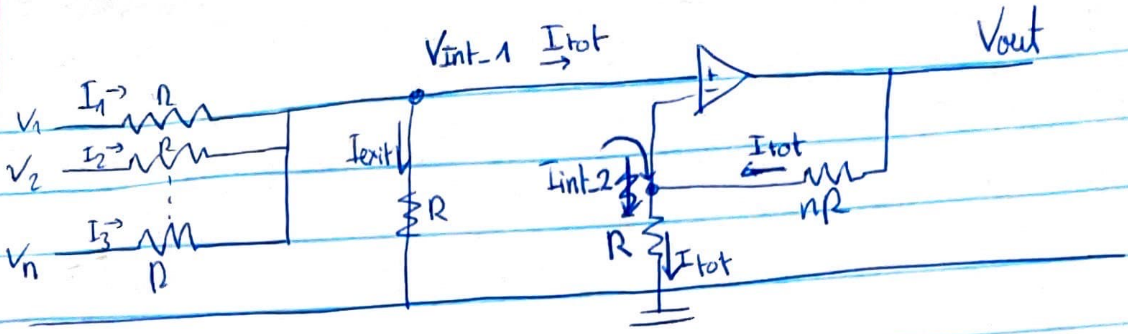
$$V_{out}\left[1 + A \frac{1}{1+\frac{R_2}{R_1}} \frac{R_2}{R_1}\right] = A \cdot \frac{1}{1+\frac{R_2}{R_1}}\left[V_1 - V_2\right]$$

$$V_{out}\left[1 + A \cdot \frac{1}{1+\frac{R_1}{R_2}}\right] = A \frac{1}{1+\frac{R_2}{R_1}}\left[V_1 - V_2\right]$$

$$\Rightarrow V_{out} = \frac{A \cdot \frac{1}{1+\frac{R_2}{R_1}}}{1 + A \cdot \frac{1}{1+\frac{R_1}{R_2}}}\left(V_1 - V_2\right) \Rightarrow \lim_{A \to \infty} \frac{A + \frac{1}{1+\frac{R_2}{R_1}}}{1 + A \frac{1}{1+\frac{R_1}{2}}} (V_2 - V_1$$

$$\cancel{= \frac{R_1}{R_2}(V_1 - V_2)} = \frac{\frac{1}{1+\frac{R_2}{R_1}}}{\frac{1}{1+\frac{R_1}{R_2}}}(V_2 - V_1) = \frac{1+\frac{R_1}{R_2}}{1+\frac{R_2}{R_1}}(V_2 - V_1)$$

$$= \frac{\frac{R_2+R_1}{R_2}}{\frac{R_1+R_2}{R_1}}(V_2 - V_1) = \frac{R_1}{R_2}(V_2 - V_1)$$



$$\bullet\ I_1 = \frac{V_1 - V_{int\_1}}{R} \ ; \ I_2 = \frac{V_2 - V_{int\_1}}{R} \ ; \ I_3 = \frac{V_3 - V_{int\_1}}{R}$$

$$\hookrightarrow \quad I_n = \frac{V_n - V_{int\_1}}{R}$$

$$\bullet\ I_{tot} = (I_1 + I_2 + I_3 + \ldots + I_n) - I_{exit}$$

$$\Leftrightarrow 0 = (I_1 + I_2 + I_3 + \ldots + I_n) - I_{exit}$$

$$\Leftrightarrow 0 = \left(\frac{V_1 - V_{int\_1}}{R} + \frac{V_2 - V_{int\_1}}{R} + \ldots + \frac{V_n - V_{int\_1}}{R}\right) - \frac{V_{int\_1}}{R}$$

$$\Leftrightarrow V_{int\_1} + n V_{int\_1} = V_1 + V_2 + \ldots + V_n \qquad (*)$$

$$\Leftrightarrow V_{int\_1} = \frac{(V_1 + V_2 + \ldots + V_n)}{(n+1)}$$

$$\bullet\ I_{tot} = \frac{V_{out} - V_{int\_2}}{nR} \ \ \& \ \ I_{tot} = \frac{V_{int\_2}}{R}$$

$$\Rightarrow \frac{V_{out} - V_{int\_2}}{n} = V_{int\_2}$$

$$\Leftrightarrow V_{out} = V_{int\_2}(n+1)$$

$V_{int\_2} = V_{int\_1}$, SO: $V_{out} = V_{int\_1}(n+1) \qquad (**)$

$(*)$ into $(**)$: $V_{out} = V_{int\_1}(n+1)$

$$\Leftrightarrow V_{out} = \frac{(V_1 + V_2 + \ldots + V_n)}{\cancel{(n+1)}} \cancel{(n+1)}$$

$$\Leftrightarrow \boxed{V_{out} = V_1 + V_2 + \ldots + V_n}$$

C

Excel data sheets

freq [Hz]	amplitude of signal Delta-y[V]	Delta x (us:E-6)	period (1/fr)	phase: deltaX/period*360	Gain in dB
100	4.86	0	0.01	0	39.7533253
200	4.86	0	0.005	0	39.7533253
300	4.86	0	0.003333333	0	39.7533253
500	4.86	0	0.002	0	39.7533253
1000	4.86	0	0.001	0	39.7533253
2000	4.86	9.6	0.0005	6.912	39.7533253
5000	4.76	8.4	0.0002	15.12	39.57273897
10000	4.38	9	0.0001	32.4	38.85008212
15000	3.84	7.7	6.66667E-05	41.58	37.7072244
20000	3.36	6.9	0.00005	49.68	36.54738546
50000	1.68	4.1	0.00002	73.8	30.52678555
100000	0.876	2.36	0.00001	84.96	24.87068204
200000	0.472	1.38	0.000005	99.36	19.49943989
500000	NA	0.408	0.000002	73.44	#VALUE!
1000000	NA	NA	0.000001	#VALUE!	#VALUE!

R2/R1 = 5					
freq [Hz]	amplitude of signal Delta-y[mV]	Delta x (us:E-6)	period (1/fr)	phase: deltaX/period*360	Gain in dB
100	318	0	0.01	0	16.06914231
200	318	0	0.005	0	16.06914231
300	318	0	0.003333333	0	16.06914231
500	318	0	0.002	0	16.06914231
1000	318	0	0.001	0	16.06914231
2000	318	0	0.0005	0	16.06914231
5000	318	0	0.0002	0	16.06914231
10000	318	0	0.0001	0	16.06914231
15000	318	0	6.66667E-05	0	16.06914231
20000	318	1	0.00005	7.2	16.06914231
50000	318	0.88	0.00002	15.84	16.06914231
100000	310	0.7	0.00001	25.2	15.84783379
200000	272	0.69	0.000005	49.68	14.71197799
500000	160	0.6	0.000002	108	10.10299957
1000000	65.2	0.414	0.000001	149.04	2.305551828

Buffer	R1:infinity, R2: 0=> gain=1				
freq [Hz]	amplitude of signal Delta-y[mV]	Delta x (us:E-6)	period (1/fr)	phase: deltaX/period*360	Gain
100	996	0	0.01	0	0
200	996	0	0.005	0	0
300	996	0	0.003333333	0	0
500	996	0	0.002	0	0
1000	996	0	0.001	0	0
2000	996	0	0.0005	0	0
5000	996	0	0.0002	0	0
10000	996	0	0.0001	0	0
15000	996	0	6.66667E-05	0	0
20000	996	0	0.00005	0	0
50000	996	0.16	0.00002	2.88	0
100000	996	0.22	0.00001	7.92	0
200000	996	0.196	0.000005	14.112	0
500000	996	0.2	0.000002	36	0
1000000	968	0.246	0.000001	88.56	-0.247679622

Ex 2: Low Pass Filter					
freq [Hz]	amplitude of signal Delta-y [mV]	Delta x (us:E-6)	period (1/frequency)	phase: deltaX/period*360	Gain in dB
100	996	0	0.01	0	0
200	996	0	0.005	0	0
300	996	0	0.003333333	0	0
500	996	0	0.002	0	0
1000	996	18	0.001	6.48	0
2000	996	6.8	0.0005	4.896	0
5000	936	10.8	0.0002	19.44	-0.539669794
10000	804	9.2	0.0001	33.12	-1.860065794
15000	660	7.8	6.66667E-05	42.12	-3.574308058
20000	568	6.6	0.00005	47.52	-4.878220054
50000	261	3.92	0.00002	70.56	-11.63237662
100000	135	2.24	0.00001	80.64	-17.3585114
200000	67.6	1.2	0.000005	86.4	-23.36625285
500000	27.2	0.47	0.000002	84.6	-31.27380869
1000000	13.4	0.212	0.000001	76.32	-37.4230908

EX 4: High Pass Filter					
freq [Hz]	amplitude of signal Delta-y[mV]	Delta x (ms:E-3)	period (1/fr)	phase: deltaX/period*360	Gain in dB
100	302	7.96	0.01	286.56	-10.36504791
200	528	4.24	0.005	305.28	-5.512508318
300	688	2.92	0.003333333	315.36	-3.213418004
500	824	2	0.002	360	-1.646642535
1000	936	0.96	0.001	345.6	-0.539669794
2000	972	0.498	0.0005	358.56	-0.21186147
5000	980	0.196	0.0002	352.8	-0.140665255
10000	980	0	0.0001	0	-0.140665255
15000	980	0	6.66667E-05	0	-0.140665255
20000	980	0	0.00005	0	-0.140665255
50000	980	0	0.00002	0	-0.140665255
100000	980	0	0.00001	0	-0.140665255
200000	980	0	0.000005	0	-0.140665255
500000	980	0	0.000002	0	-0.140665255
1000000	980	0	0.000001	0	-0.140665255

Ex 5: Non-Inverting Amplifier with All Filters (Full Circuit) - amplifier, low pass, high pass, buffer					
freq [Hz]	amplitude of signal Delta-y[mV]	Delta x (ms:E-3)	period (1/fr)	phase: deltaX/period*360	Gain in dB
100	94.8	8.08	0.01	290.88	-20.42902002
200	171	4.32	0.005	311.04	-15.30526456
300	216	2.92	0.003333333	315.36	-13.27611175
500	267	1.84	0.002	331.2	-11.43496154
1000	303	0.968	0.001	348.48	-10.3363342
2000	313	0	0.0005	0	-10.05430002
5000	310	0.008	0.0002	14.4	-10.13795289
10000	281	0.01	0.0001	36	-10.99106037
15000	244	0.009	6.66667E-05	48.6	-12.21739024
20000	209	0.0082	0.00005	59.04	-13.56226105
50000	103.2	0.0052	0.00002	93.6	-19.69159282
100000	50.4	0.00336	0.00001	120.96	-25.91657604
200000	20	0.00222	0.000005	159.84	-33.94458686
500000	3.8	0.00126	0.000002	226.8	-48.36951484
1000000	Signal not possible	na	na	na	#VALUE!