Assignment 2

Digital Signal Processing

AE4463P-23: Advanced Aircraft Noise

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Assignment 2

Digital Signal Processing

by

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1
Reader exercises (ch4, ex1)

1.1. 1a

Problem statement: A continuous signal x(t) with bandwidth 450 Hz is sampled with F = 1000 Hz. We want to make a plot of |X(f)| , using 0.25 seconds of the signal, with a frequency step of 1 Hz. Find the required length N of your DFT.
It is given that sample frequency (f_s) is $1000\,$Hz, furthermore the time resolution T is provided to be $0.25\,$s. Ignoring the frequency resolution it can be found that N is 250. This can be done by utilising the following relations: $T = N * \frac{1}{f_s}$.

$$N = T * f_s = 0.25 * 1000 = 250 \tag{1.1}$$

However it can be noticed that the above N will result in a frequency resolution which does not satisfy the requirement. The frequency resolution which is defined as one over T (snapshot length), is $\frac{1}{0.25\,\text{s}} = 4\,$Hz. Therefore an alternative approach needs to utilised to guarantee a frequency resolution of $1\,$Hz.

From the frequency resolution it can be computed that a snapshot length of $1\,$s is required ($\delta_f = \frac{1}{T} \Leftrightarrow T = \frac{1}{\delta_f} = \frac{1}{1} = 1$). Then using Equation 1.1 the amount of samples can be computed, to be 1000 ($1 * 1000$). Therefore from the original signal a snapshot of length 0.25 seconds is taken, however the signal is padded with zeros to form a signal of length $1\,$s (padded with 750 zeros).

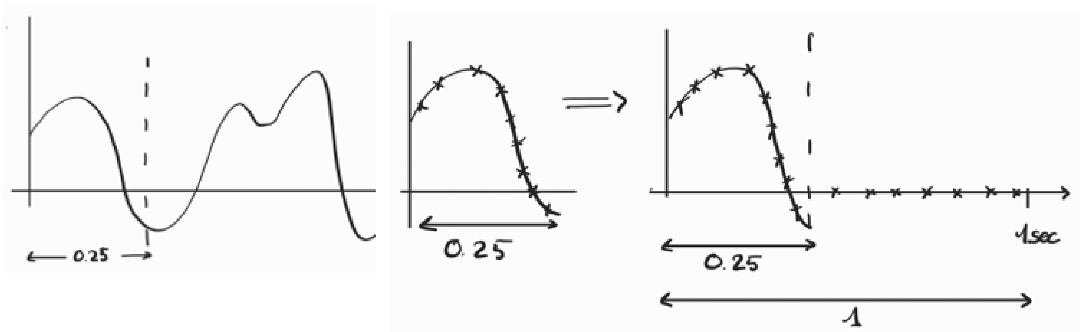


Figure 1.1: A sketch for the above situation, where the padding is applied to the signal

1.2. 1b

Problem statement: Next, we want to have twice as many frequencies, i.e. a DFT with a frequency step of 0.5 Hz. Find the required length N of your DFT. Using the approach from above it can be found that here N should be 2000, since $T = \frac{1}{0.5} = 2, N = 1000 * 2$. As a result the signal of length $0.25\,$s should be padded with 1750 zeros.

2
Discrete Fourier Transform

2.1. Part I

Fourier transforming Equation 2.1 and plotting the absolute value , $|X_r|$ results in Figure 2.1.

$$x_k = \sin(2\pi f_1 k\Delta) + 0.1\sin(2\pi f_2 k\Delta), \quad k = 0, \ldots, N-1 \tag{2.1}$$

Where f1 = 100 Hz, f2 = 125 Hz and $\Delta = 1$ ms. Within this section a value for N of 128 is utilised.

Within Figure 2.1, the frequencies on the x-axis where found applying the following relation [2].

$$f = \frac{k}{N\Delta} \tag{2.2}$$

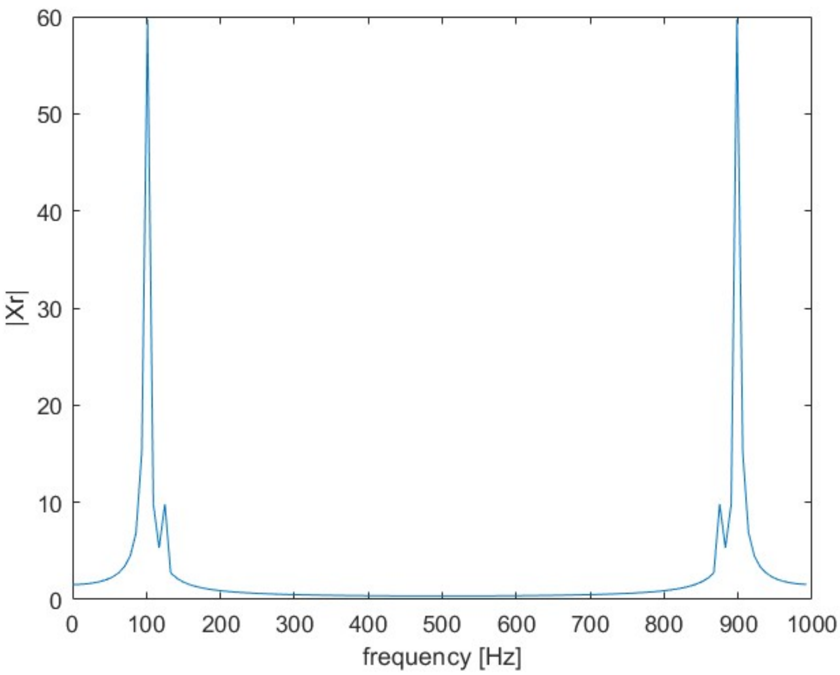


Figure 2.1: LinBlock16 with a_n included

Two peaks can be observed, one at 101.56 Hz and the second at 125 Hz. To obtain the relative amplitude, the peak amplitude values must be divided by the amplitudes of the sin's in Equation 2.1. This corresponds to 1 for the first sin and 0.1 for the second one. Consequently, the relative amplitude for the second sine is 98.1740.

2.2. Part II

After adding zero padding the signal to ensure N is 2048 (By padding the original sample with N=128 with 1920 zeros), the Fourier transform, X_r, is recomputed. The result is plotted vs frequency in Figure 2.2. Comparing Figure 2.1 and Figure 2.2, the frequency resolution does not change but there is better sampling of the DFT (steps in frequency are smaller). Zero padding allows for better estimation of the amplitude. However it can also be noticed that is does become more difficult to clearly distinct the frequency content of the signal. Many small side lobes can be observed, therefore these could mistakenly be interpreted as other various frequency content of the signal.

The second peak occurs at a frequency of 126.465 Hz with a relative amplitude of 112.2636.

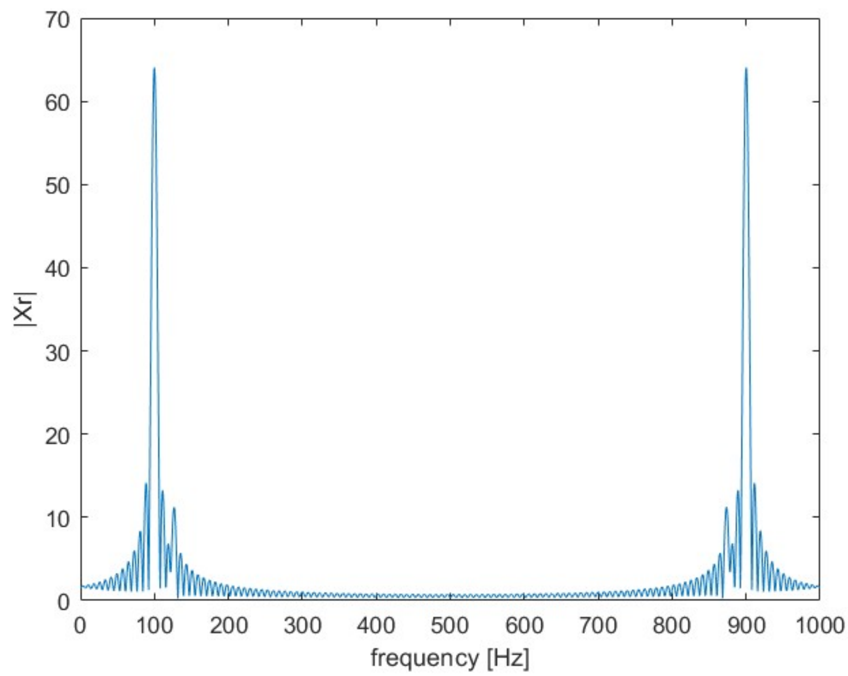
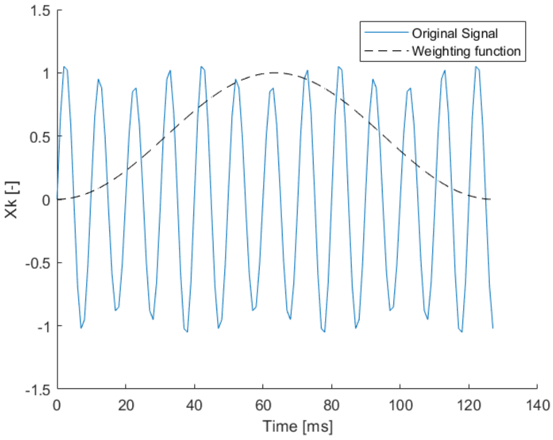


Figure 2.2: X_r with zero padding vs frequency [Hz]

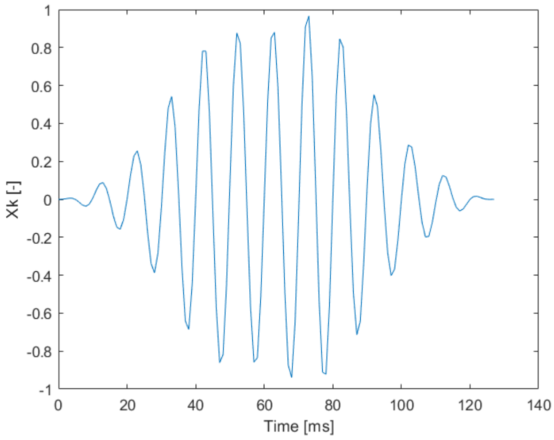
2.3. Part III

Previously it could be seen that by adding zeros, a better sampling could be achieved of the signal. However it could also be observed that many small peaks still exist besides the main two peaks of interest (100 Hz and 125 Hz). Making it harder to observe these two distinct frequencies.

First the Hanning weighting is applied to the original signal. Both the original signal and the weighting functions can be observed in Figure 2.3a. The Hanning function multiplies the values of the original signal by values between 0 and 1 (1 at the middle of the sampled time, 0 at the end). As a result of this multiplication the following signal can be found with the weighting applied Figure 2.3b.



(a) Original simulated signal x_k together with the hanning weighting function



(b) Hanning weighting applied to x_k

Similar to the procedure explained in section 2.2, here to the signal is padded to ensure N is 2048. As a result 1920 zeros are added to hanning weighted signal. Similar to section 2.2 this was done to decrease the spacing between the frequency nodes (better sampling) within the the plot containing the absolute value of the Fourier coefficients. Below in Figure 2.4

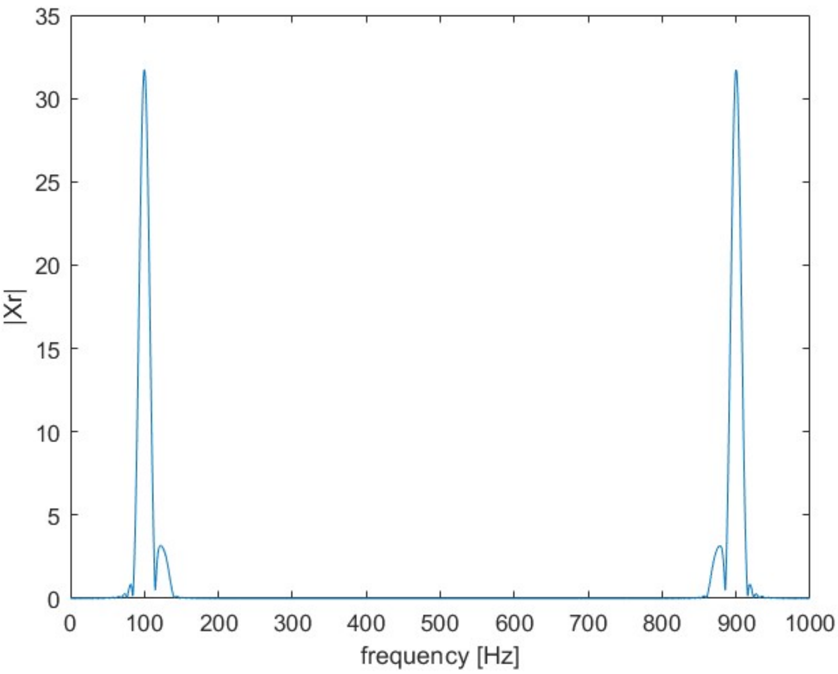


Figure 2.4: Magnitude of the Fourier coefficients of the weighted and padded signal x_k

Firstly it can be observed that the amount of small side peaks has significantly reduced. There are now two distinct peaks which can be observed. The first peak can be observed at 100.098 Hz (within the array containing the frequency values, this is denoted by index 206), secondly a second peak can be observed at 121.582 Hz (denoted by index 250). Therefore the frequency of the first sine is estimated to be 100.098 Hz and the second sine is estimated to be 121.582 Hz. The relative amplitude of the first sine was determined to be 31.7322 and for the second since the relative amplitude was found the be 31.6567 (peak on the plot located at 121.582 Hz divided by 0.1 (amplitude second sine in xk)).

When comparing these results to the ones obtained in the previous question, a range of items can be observed. Firstly, it can be observed that the peaks have become wider in comparison to both the padded and non-padded signal. Secondly, the second peak (corresponding to the second sine) has now moved to the left meaning the frequency of this sine is now less accurately determined. However the frequency accuracy of the first sine was maintained. Lastly, it can be seen that the relative magnitude has significantly reduced, due to the Hanning weighting. Furthermore, the relative magnitude for both peaks is nearly identical (≈ 31.7).

2.4. Part IV

Firstly, the frequencies at which large peaks existed, were the frequencies of the original signal. Secondly, the magnitude of the peak, described how significant the frequency was in the overall signal. When this magnitude was then divided by the magnitude of the original sine component, the relative magnitude could be obtained, ideally this value should be equal for both sines.

Within this section it could be observed that through the addition of padding and/or weighting a signal could be altered, to then analyse its frequencies using the Fourier transform. Firstly, it could be observed that the addition of padding, provided better sampling of the signal (as also described in [1]). It must be noted that the frequency resolution doesn't change, however due to the reduction in distance between the sampled frequencies, it becomes more feasible to evaluate certain frequencies within the signal (Possibly a higher accuracy when determining the magnitude of the signal). However it could be observed that due to the frequency resolution staying the same, many small peaks/ lobes can now be observed within the the figure. Due to this effect, it can be complicated to distinct the critical frequencies from the non critical frequencies within the signal.

Through the introduction of the Hanning weighting the small side lobes, previously observable due to the padding could be significantly reduced. As a result making it more feasible to identify the frequencies which are observable within the signal. However, the magnitude of both peaks significantly decreased and the width of the lobes increased, resulting in a less accurate determination of the frequency content.

To conclude, both zero padding and weighting should be utilised in combination when analysing a signal. Through the introduction of padding the signal could be better sampled resulting in more observable frequencies. Secondly through the introduction of the Hanning weighting the results from the Fourier transform were observed to be smoother, allowing for better determination of the frequency content of the signal. It must be said that without the two methods applied it could still be possible to effectively analyse the signal, however for the case where the frequency of the signal is not sampled (due to large steps between frequencies) the analysis will result in poor results.

References

[1] Prof. dr. ir. M. Snellen. *Further background on filtering in the frequency domain-spectral analysis*. 2023.

[2] Prof. dr. ir. M. Snellen Prof. dr. D.G. Simons. *Digital Signal Processing an introduction*. Delft, Netherlands: TU Delft, Dec. 2023.

3 Matlab code

```
%% Assignment 2: Advanced Aircraft Noise
% By: Elisabeth and Joshua

clear;

%% Constants

delta = 0.001;
N = 128;
f1 = 100;
f2 = 125;

k = 0:1:N-1;

xk = sin(2*pi*f1*k*delta) + 0.1*sin(2*pi*f2*k*delta);

plot(k, xk)

%% Part I

fr = k / (N*delta);

Xr = fft(xk);

peak1 = abs(Xr(14)) / 1;
peak2 = abs(Xr(17)) / 0.1;

plot(fr, abs(Xr))

%% Part II

N = 128;
k = 0:1:N-1;

xk = sin(2*pi*f1*k*delta) + 0.1*sin(2*pi*f2*k*delta);
xk_padded = resize(xk, 2048);

N_padded = 2048;
k_padded = 0:1:N_padded-1;

Xr_padded = fft(xk_padded);
fr_padded = k_padded / (N_padded*delta);

plot(fr_padded, abs(Xr_padded))

peak1_padded = abs(Xr_padded(206))/1;
peak2_padded = abs(Xr_padded(260))/0.1;

%% Part III

N = 128;
k = 0:1:N-1;
xk = sin(2*pi*f1*k*delta) + 0.1*sin(2*pi*f2*k*delta);

hanning_weighting = hann(N).';
xk_weighted = xk.*hanning_weighting;
```

```
fr = k / (N*delta);

figure();
hold on
plot(k, xk)
plot(k, hanning_weighting, "k--")
xlabel("Time [ms]")
ylabel("Xk [-]")
legend("Original Signal", "Weighting function")

figure();
plot(k, xk_weighted)
xlabel("Time [ms]")
ylabel("Xk [-]")

xk_weighted_padded = resize(xk_weighted, 2048);

N_padded = 2048;
k_padded = 0:1:N_padded-1;

Xr_weighted_padded = fft(xk_weighted_padded);
fr_weighted_padded = k_padded / (N_padded*delta);

figure();
plot(fr_weighted_padded, abs(Xr_weighted_padded))

peak1_weighted = abs(Xr_weighted_padded(206));
peak2_weighted = abs(Xr_weighted_padded(250)) / 0.1;

%% END
```