Assignment 4

Digital Signal Processing

AE4463P-23: Advanced Aircraft Noise

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by

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Cover: https://unsplash.com/photos/white-and-blue-airplane-under-white-clouds-during-daytime-bNVbyBl870A
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1D beamforming of real experimental data

1.1. Theoretical questions

What is the highest frequency at which you can do beamforming at a steering angle of 0 degrees without having grating lobes? A steering angle of 0 degrees means no steering angle is utilised. The highest frequencies describe the region in which no grating lobes exist. For the case where no steering is applied ($\theta_s = 0$), the first set of grating lobes are located at Equation 1.1

$$\frac{Kd}{2} = \pm\pi, \qquad K = \frac{2\pi}{\lambda} * \sin\theta \tag{1.1}$$

Filling in the mentioned K, the grating lobes are described as

$$\frac{\pi d}{\lambda} \sin\theta = \pm\pi \tag{1.2}$$

To successfully apply beamforming there should exist no grating lobes with the regions from -90 degrees to 90 degrees. To ensure this statement is true d (distances between microphones) should be less then λ (wave length). When utilising this condition of $d < \lambda$, Equation 1.2 will have no solution thus meaning no grating lobes exist [2].

Analysing this condition for the case of the assignment the following observation is made. The speed of sound (c) within water is 1500 m/s, and the distance between microphones (d) is 2 m. Given that the velocity of a wave is described as $c = f * \lambda$, and $d < \lambda$ the following condition needs to satisfied.

$$d < \lambda \Rightarrow d < \frac{c}{f} \Rightarrow f < \frac{c}{d} = \frac{1500}{2} = 750Hz \tag{1.3}$$

Filling in the values it can be seen that the maximum frequency for which beamforming can be applied with no steering is 750Hz. When this condition is met no grating lobes will exist, and the data can be utilised reliably.

What is the highest frequency at which you can do beamforming at a steering angle of 90 degrees without having grating lobes? Within this case there is a steering angle, and as a result the condition for beamforming is altered. Now the charateristic equation for no grating lobes is defined as:

$$d \leq \frac{\lambda}{2} \tag{1.4}$$

again reformulating this using the definition of wavelength (λ), the following condition is formed.

$$d \leq \frac{\lambda}{2} \Rightarrow d \leq \frac{c}{2f} \Rightarrow f \leq \frac{c}{2d} = \frac{1500}{2 * 2} = 375Hz \tag{1.5}$$

Filling in the values it can be observed that the maximum frequency for which beamforming can be applied with a steering angle of 90 degrees is 375Hz

Lastly utilising Equation 1.6 from the reader [1] it is possible to express the frequency limitations for all angles between 0 and 90 degrees. The results can be observed within Figure 1.1. Within the plot the previously mentioned values can also be retrieved, further conforming the obtained results.

$$\theta_{s,max} = \text{arcsin}\,\frac{\lambda}{d} - 1 \tag{1.6}$$

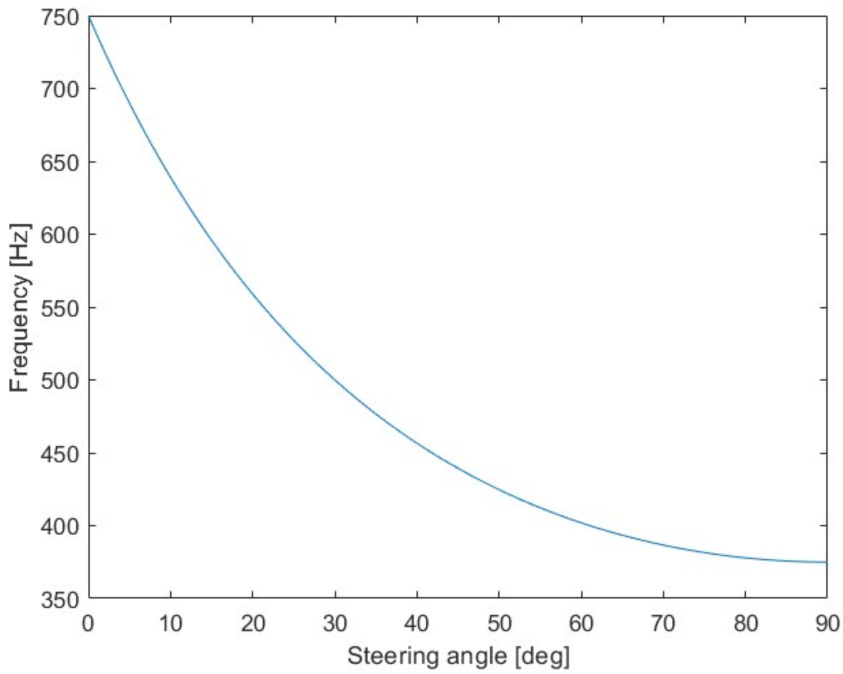

Figure 1.1: Maximum frequency for which beamforming can be applied as a function of steering angle

1.2. Beam forming applied on real data

1.2.1. 2D Beamforming Plot

The 2D beamforming plot shows the relationship between the frequency and steering angles. The logic behind the procedure is outlined below:

$$x_n\left(t_k\right) \xrightarrow{\text{Fourier Transform}} X_n\left(f_k\right) \xrightarrow{\text{Beamform}} \sum_n X_n\left(f_k\right) e^{2\pi i f_k \tau_n} \tag{1.7}$$

The provided microphone data for all 128 microphones is first fourier transformed, meaning the fourier coefficients are obtained for each individual microphone. Then, for each frequency, the fourier coefficients of each individual microphone are summed up while also multiplied with an exponential term. Note that since the sample frequency is 6000Hz only frequencies up to 3000Hz can be accurately measured. The exponential term represents a phase shift and incorporates the steering angle in $\tau_n = \frac{d}{c} n \sin\theta_s$. This results in a 1D array for each steering angle with the columns corresponding to the frequency values. Appending all rows, a final matrix of size is obtained. This matrix contained the power value in decibels for every pair of steering value and frequency. Note that the steering angles were take between -75° and 75°. The results can be observed in Figure 1.2. The colourscale demonstrates the strength in dB. Strong dB peaks originating at the bottom and reaching towards the top can be observed these are called the real sources. The real sources will be utilised in the next sections to obtain compute the expected grating lobes and beam width. They real sources fade in strength, however the beamformed data for a certain steering angle is only relevant up to a certain frequency as mentioned previously. Furthermore, within the plot sweeps can be observed both on the left and right side (symmetry). The shape of the sweep has a strong resemblance with the results observed in Equation 1.6, and there meaning is similar. The sweep represent the grating lobes and are not real sound sources. They are an created due to applying beamforming, and the frequencies which are effected are based on the steering angle utilised.

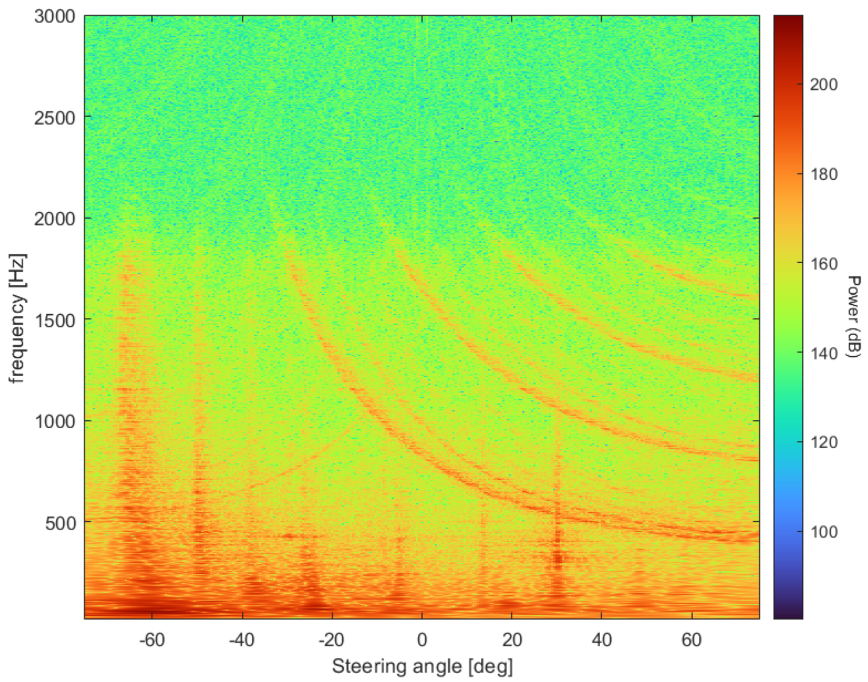

Figure 1.2: 2D beamformed data: frequency vs steering angle plot

1.2.2. Grating Lobes

When steering the array in the directions different from the desired one, directions may exist for which the response adds maximally. This is known as grating lobes. Grating lobes occur at angles obtained from Equation 1.8. One shall note these lobes are not 'real', meaning although there is an indication on the plot, there is no physical entity.

$$sin(\theta) = sin(\theta_s) \pm m \cdot \frac{\lambda}{d} \Leftrightarrow \theta = arcsin(sin(\theta_s) \pm m \cdot \frac{\lambda}{d} \tag{1.8}$$

Looking at Figure 1.2, real sources appear very strongly at steering angles -62°, -25°, and 30°. Hence,

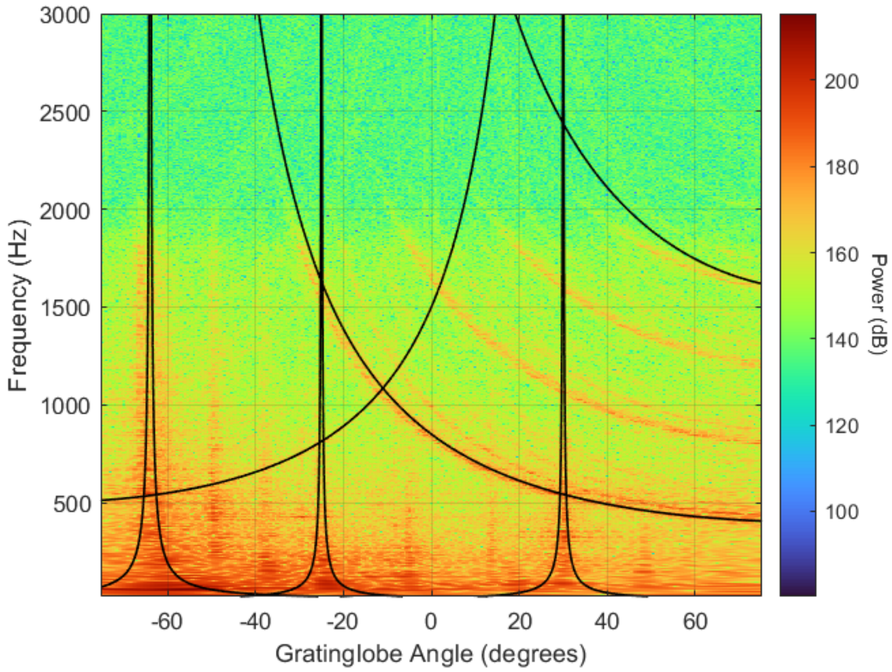

Figure 1.3: Grating lobe pattern

for each of these angles, the Equation 1.8 was implemented for each frequency.The resulting plot is shown by Figure 1.3. One can observe that the lines of the calculated grating lobes follow the beamformed plot data very well. Still, some tuning of the 'm' parameter, which makes sure all integer multiples are considered, was necessary. Both negative and positive values have been tried. Finally, 'm' was chosen to be 1 for -62°, 3 for -25°, and -1 for 30°. The -1 for 30° results in the grating lobe entering the plot from the left side. The lowest grating lobe entering from the right side of the plot is linked to the -62 degrees.

1.2.3. Expected beam width

Similar to the procedure utilised previously, here two it is possible to estimate the beam with of the various real sources. To ensure consistency the same real sources will be utilised as subsection 1.2.2, meaning that the sources at steering angles -62°, -25°, and 30° are considered.

To estimate the beam with for a given steering angle the following formulation is utilised, extracted from [2] (equation 8.24 and 8.40).

$$\theta_{B,s} = \frac{\theta_B}{\sin \theta_s} \qquad \text{With} \quad \theta_B = \frac{\lambda}{L} \tag{1.9}$$

Here L is the length of the array, which is computed by multiplying the amount of microphones with the distance between the microphones. Lambda is the wave length, meaning the beam with in radians is frequency dependent. Lastly θ_s is the steering angle. Note all angles are expressed as radians but converted to degrees for visualisations.

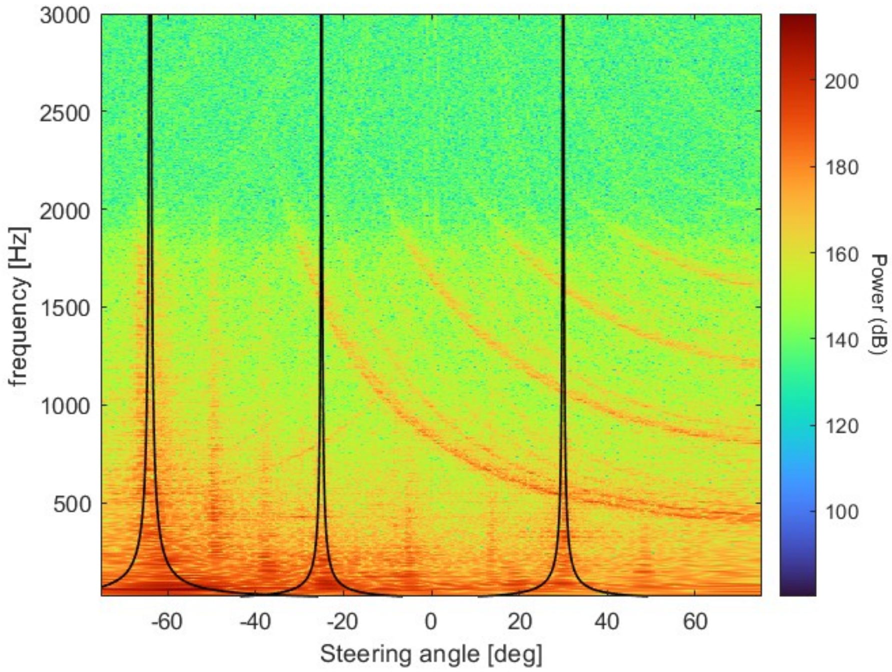

Figure 1.4: The expected beam with, plotted for a select amount of real sources, located at three distinct steering angles

It can be observed that it is possible to estimate the beam width of the main lobe for a certain steering angle with good accuracy. Firstly it can observed that, with increasing steering angle the beam width increases meaning the radial/ angular resolution becomes worse. As a result it will become more and more difficult to distinguish sources from each other at those steering angles. However it is seen that the expected beam width formulation is able to match this decrease in resolution. However only for the large steering angles, such as the one located at -62° the measured beam width is larger then estimated.

1.2.4. Usable Plot Area

The usable area was determined to be under the lowest observable grating lobes. Within Figure 1.5 the usable area was highlighted. It was determined that everything above this range cannot be utilised since within these regions fictitious sources will start forming, due to the applied beam forming. At these location there are no sources however when applying beamforming the frequencies which are observable are dependent on the steering angle.

Looking again at Figure 1.1, the frequency range goes up to 750 Hz for an angle of 0 degrees. This overlaps nicely with the lowest observable grating lobe and thus the usable regions previously indicated. When observing the positive steering angle it can be seen that values obtained using the theoretical formulation are lower, however a general strong match is observable in the general outline. Although the negative angles cannot be directly compared with a theoretical formulation, there it can be seen that the negative side is reasonably symmetrical to the positive steering angles.

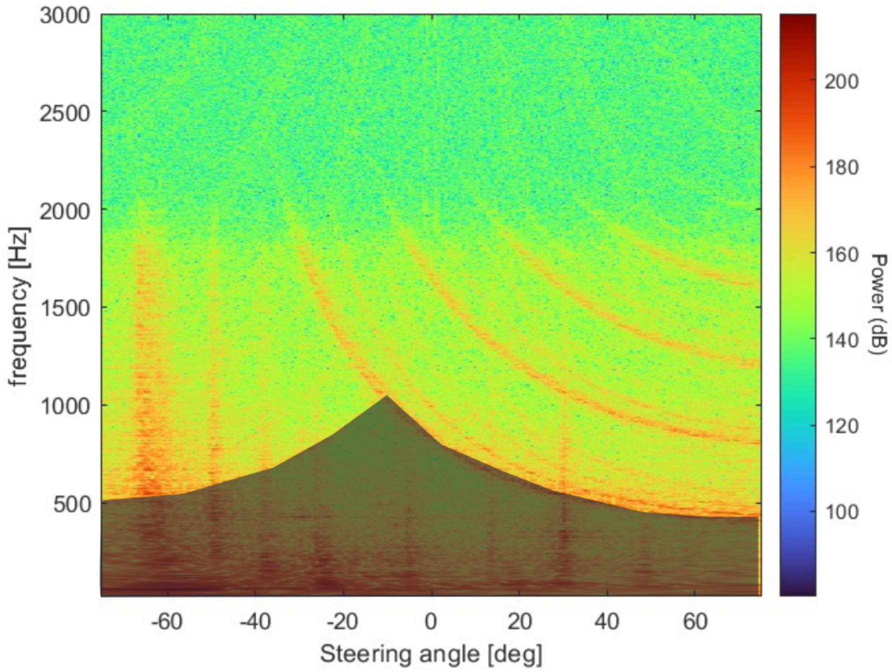

Figure 1.5: usable area of the beamform plot for the general cases

Note that the very low frequencies (0-20Hz) have been omitted as they are not accurate and thus unusable (the very low frequencies cannot be accurately measured by a microphone). The area corresponding to the usable frequencies is shown in Figure 1.5. Furthermore it may also be noticed that the high frequency region near 3000Hz is anyway unusable due the anti-aliasing filter, which is utilised to sample the signal reliably. Furthermore it was noticed that one defect microphone exist namely with id 113, its value remains constant throughout the experiment.

References

[1] Prof. dr. ir. M. Snellen. *Principles of beamforming (1D)*. 2023.

[2] Prof. dr. ir. M. Snellen Prof. dr. D.G. Simons. *An introduction to general acoustics and aircraft noise*. 1th ed. Delft, Netherlands: TU Delft, 2023.

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Matlab code

```
%% Assignment 4: Advanced Aircraft Noise
% By: Elisabeth and Joshua

clear;
close all;

load('hydrophonedata_AE4463P.mat')

%% Start Script
global c
global d
global n_mic

c = 1500;        % Speed of sound in water [m/s]
fs = 6000;       % Smample frequency [Hz]
d = 2;           % Distance between microphones [m]
n_mic = 128;     % Number of microphones [-]
p_ref = 10^(-6);   % Refrence ressure for water [Pa]

% Visualise the data

figure();
%[X,Y] = meshgrid(1:0.5:10,1:20);
[X,Y] = meshgrid(1/fs:1/fs:1,1:128);
surf(X,Y,y1,'EdgeColor' ,'none')
view([0 90])
colorbar

figure(6);
imagesc(1/fs:1/fs:1,1:128,y1);
colormap turbo;
axis xy;
colorbar

%% Part for Q1 and Q2 (Theoretical questions)

angle = 90:-1:0;
f = ((d/c)*(sin(angle*pi/180)+1)).^(-1);

figure();
plot(angle, f)
xlabel("Steering angle [deg]")
ylabel("Frequency [Hz]")


%% Part 3 Beamforming

% Step 1: fourier transform data from one microphone
% Step 2: add phase delay from stearing
% Step 3: repeat for all microphones
% Step 4: repeat for all stearing angles
% Step 5: Convert to dB presentation

% Microphone 113 seems to be defect

steering_angles = -75:0.5:75;
```

```
fk = 1:fs;  % Frequencies

final = zeros(size(steering_angles,2),fs);
row = 0;

for steering_angle = steering_angles

    inter = zeros(1, fs); %Storage of results for a single stearing angle but all frequencies
    row = row + 1;

    for n = 1:n_mic

        d_mic = y1(n,:);     % Microphone data of microphone n

        fc_mic = fft(d_mic); % Fourier coefficient a signular microphone n
        % (Still to be converted to One sided PSD)

        %plot(fk,fc_mic) % View the fourier coef over frequency

        tau_n = (d/c) * n * sin(deg2rad(steering_angle));   % Phase shift parameter
        inter = inter + fc_mic.*exp(2*pi*1i*fk*tau_n);      % Sum over all microphones
    end

    inter = 10*log10((abs(inter).^2)/(p_ref^2));    % Convert to dB
    final(row,:) = inter;                           % append to solution...
    % array based on steering angle in evaluation
    % (note all frequencies are added for the sum of all microphones)
end

figure();
imagesc(steering_angles, 20:fs/2, final(:,20:fs/2).');
colormap turbo;
axis xy;
colorbar;
xlabel('Steering␣angle␣[deg]');
ylabel('frequency␣[Hz]');
cb = colorbar();
ylabel(cb,'Power␣(dB)','Rotation',270)
%clim(); % <--- set bounds on colorbar

%% Steering angle

f = 20:1:3000;
angle = 30;

Thetabs = (c) ./ (cos(deg2rad(angle)) * n_mic * d * f);

figure(16);
ax = imagesc(steering_angles, 20:fs/2, final(:,20:fs/2).');
hold on;
colormap turbo;
axis xy;
colorbar;
xlabel('Steering␣angle␣[deg]');
ylabel('frequency␣[Hz]');
cb = colorbar();
ylabel(cb,'Power␣(dB)','Rotation',270)

plot(rad2deg(Thetabs) + angle, f, LineWidth=1,Color="k")
plot(-rad2deg(Thetabs) + angle, f, LineWidth=1,Color="k")

addBeamWidth(-64, f)

addBeamWidth(-25, f)

% Above is due to anti aliassing filter
% Peaks are real sound sources, they really exist
% Curves are fales data (steering), they are not real sound sources
% Below also not really good data to use (low freq)

plotGratinglobePattern([-25], 3, f);
```

```
plotGratinglobePattern([-62], 1, f);
plotGratinglobePattern([30], -1, f);


%% Functions
function addBeamWidth(steeringangle, f)
    %{
    Function to add the expected beam with to the data plot currently in
    use. The analysed steering angle of the signal is provided as input
    toghetter with the analysed frequencies. The result is then super
    imposed onto the beamformed data figure.
    %}
    global c
    global n_mic
    global d

    BeamWidth = c ./ (cos(deg2rad(steeringangle)) * n_mic * d * f);

    plot(rad2deg(BeamWidth) + steeringangle, f, LineWidth=1, Color="k")
    plot(-rad2deg(BeamWidth) + steeringangle, f, LineWidth=1, Color="k")

end

function plotGratinglobePattern(steering_angles, m, f)

    global c
    global d

    lambda1 = c ./ f;

    for angle = steering_angles
        grating_lobe_values = sin(deg2rad(angle)) + m*lambda1 / d;
        theta_grating_lobe = asin(grating_lobe_values);
        grating_lobe_angles_deg = rad2deg(theta_grating_lobe);

        plot(grating_lobe_angles_deg, f, 'LineWidth', 1, Color='k');
    end

    xlabel('Gratinglobe␣Angle␣(degrees)');
    ylabel('Frequency␣(Hz)');
    title('Gratinglobe␣Pattern␣for␣Different␣Steering␣Angles');
    legend(cellstr(num2str(steering_angles')), 'Location', 'Best');
    grid on;
end
```