Tarea 6:

$$G(s) = \frac{20(5+s)}{s(s+1)(s+4)} \quad \Big\} \quad \begin{array}{l} 0,5\ \% \quad 9,5\ \% \\ t_s = 0,74\ seg \end{array}$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \longrightarrow \boxed{20s+100} \longrightarrow Y(s)$$

$$\therefore \frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s} \longrightarrow X_1(s)(s^3+5s^2+4s) = U(s)$$

$$\xrightarrow{\mathcal{L}^{-1}} \dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

Asignando variables de estado:

$$q_1 = x_1 \ ; \ q_2 = \dot{q}_1 = \dot{x}_1 \ ; \ q_3 = \dot{q}_2 = \ddot{x}_1 \ ; \ \dot{q}_3 = \dddot{x}_1$$

$$\rightarrow \dot{q}_3 + 5q_3 + 4q_2 = u \longrightarrow \dot{q}_3 = -5q_3 - 4q_2 + u$$

$$\bullet\ Y(s) = (20s + 100)X_1(s) \xrightarrow{\mathcal{L}^{-1}} (20\dot{x}_1 + 100x_1)$$

$$\rightarrow y = 20q_2 + 100q_1$$

Espacio de estados:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \ ; \quad y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$\bullet\ 0,5\% = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 \ ; \ 0,95\% = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100$$

$$\rightarrow \ln(0,095) = \ln(e^{-(\zeta\pi/\sqrt{1-\zeta^2})}) \rightarrow 2,35(\sqrt{1-\zeta^2}) = \zeta\pi$$

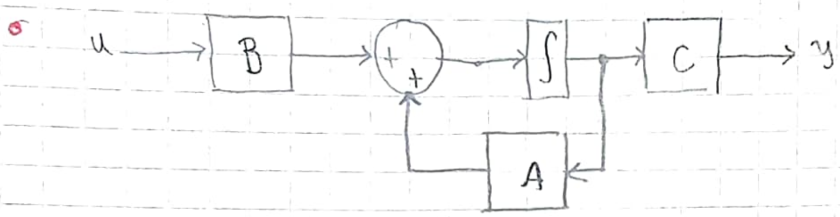
$$\therefore 5,54(1-\zeta^2) = \zeta^2\pi^2 \rightarrow \text{Despejando } \zeta: \sqrt{\frac{5,54}{5,54+\pi^2}} = 0,599$$

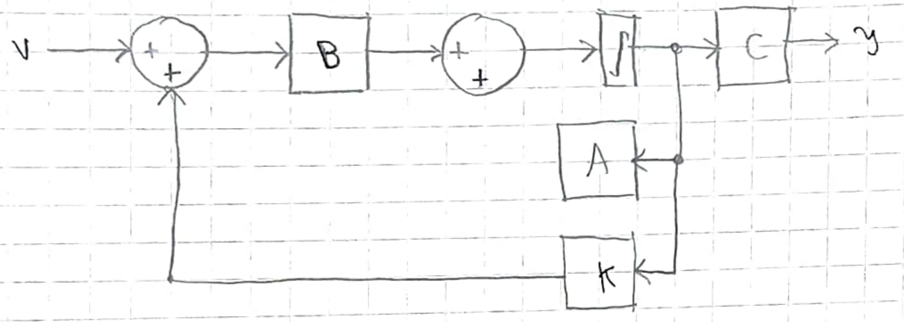
$$\bullet\ s = \sigma + j\omega d \ ; \ \sigma = \zeta\omega n \rightarrow \omega n = \frac{\sigma}{\zeta}$$

$$- \ \phi = \arccos(0,599) = 53,16^\circ$$

$$- \ t_s = \frac{4}{\sigma} \rightarrow 0,74\ seg = \frac{4}{\sigma} \rightarrow \sigma = 5,4 \rightarrow \omega n = 9,02$$

$$- \ \tan\phi = \frac{\omega d}{\sigma} \rightarrow \omega d = \text{arcotan}(53,16)(5,41) = 7,27$$



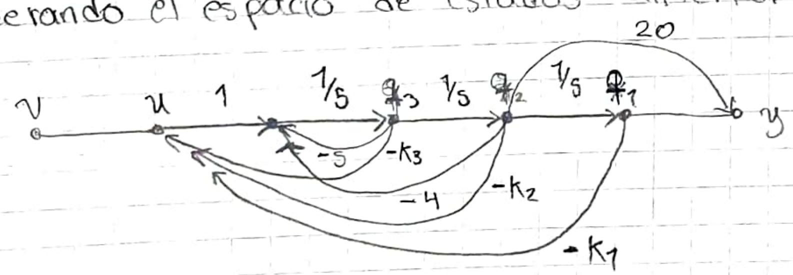


$- \dot{x} = Ax + Bx$; $y = Cx$

$\rightarrow \dot{x} = Ax + Bu = Ax + B(-Kx + v) = Ax - BKx + Bv$

$\rightarrow \dot{x} = (A - B)x + v$

Considerando el espacio de estados anterior:



$\dot{x}_3 = -4q_2 - 5q_3 + u = -4q_2 - 5q_3 + v[-k_3q_3 - k_2q_2 - k_1q_1] + v$

$= -k_1q_1 - (4+k_2)q_2 - (5+k_3)q_3 + v$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -(4+k_2) & -(5+k_3) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v \rightarrow s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = 0$$