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Principles of Software

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Homework 1

1. Condition Strength

1)

{ x % 2 = 0 ∧ y is even } ← { x is divisible by 6 ∧ y = x + 2 } ← { x = 18 ∧ y = 20 } ← { x = 2k + 1 ∧ x = x + 1 ∧ y = 10 }

{ x = 18 ∧ y = 20 } ← { x = 2k + 1 ∧ x = x + 1 ∧ y = 10 }

Is true because the stronger statement because it is false, but the first statement is not.

{ x is divisible by 6 ∧ y = x + 2 } ← { x = 18 ∧ y = 20 }

X = 18 and y = 20 is stringer because it implies the other statement, plugging in x and y outputs the same answer, but { x = 18 ∧ y = 20 } only is true in this specific case.

{ x % 2 = 0 ∧ y is even } ← { x is divisible by 6 ∧ y = x + 2 }

This is true because x in the stronger statement is divisible by 6 which implies x is even and if x is even, y being 2 more than x makes y even, but x cannot be every even number in { x is divisible by 6 ∧ y = x + 2 }.

2)

{ k ≥ -10 } ← { -10 < k ≤ 1 } ← { 10 ≤ k ≤ -10 }

{ 5 ≤ k < 5 }

is not in the chain since it is equally strong to { 10 ≤ k ≤ -10 }, both are always false.

{ 5 ≤ k < 5 } → { 10 ≤ k ≤ -10 }, k = 5:

{ 5 ≤ 5 < 5 } → { 10 ≤ 5 ≤ -10 } = false → false

{ 5 ≤ k < 5 } ← { 10 ≤ k ≤ -10 }, k = 5:

{ 5 ≤ 5 < 5 } ← { 10 ≤ 5 ≤ -10 } = false → false

{ -10 < k ≤ 1 } ← { 10 ≤ k ≤ -10 }

This is true because { 10 ≤ k ≤ -10 } is always false, while { -10 < k ≤ 1 } is sometimes true.

{ k ≥ -10 } ← { -10 < k ≤ 1 }

This is true because k = 8 would be false for { -10 < k ≤ 1 }, but true for { k ≥ -10 }. K = -9 would work for both of these conditions.

3)

$\boxed{\{ x > 1 \lor y > 1 \} \leftarrow \{ x \geq 0 \land y > x \} \leftarrow \{ x = 3 \land y > 10 \}}$

$\{ x \geq 0 \land y > x \} \leftarrow \{ x = 3 \land y > 10 \}$

This is true, since x = 3 and y has to be bigger than 10. 10 > 3 so this implication be implied to x being greater than 0 (since it was 3) and y being larger than x (10 > 3).

$\{ x > 1 \lor y > 1 \} \leftarrow \{ x \geq 0 \land y > x \}$

This is true because if x is greater than or equal to zero, then y is always bigger 0 since y is larger than x.

$\{x = y \ \% \ 10\}$

$\{x = y \ \% \ 10\} \leftarrow \{ x = 3 \land y > 10 \}$

Y = 11

$\{x = y \ \% \ 10\} \leftarrow \{ x \geq 0 \land y > x \}$

Y = 1

$\{x = y \ \% \ 10\} \leftarrow \{ x > 1 \lor y > 1 \}$

Y = 0

$\{x = y \ \% \ 10\} \rightarrow \{ x = 3 \land y > 10 \}$

Y = 11

$\{x = y \ \% \ 10\} \rightarrow \{ x \geq 0 \land y > x \}$

Y = 1

$\{x = y \ \% \ 10\} \rightarrow \{ x > 1 \lor y > 1 \}$

Y = 0

$\{xy = 0\}$

$\{xy = 0\} \leftarrow \{ x = 3 \land y > 10 \}$

X =3, Y = 11

$\{xy = 0\} \leftarrow \{ x \geq 0 \land y > x \}$

X = 2, Y = 3

$\{xy = 0\} \leftarrow \{ x > 1 \lor y > 1 \}$

X = 2, Y = 3

$\{xy = 0\} \rightarrow \{ x = 3 \land y > 10 \}$

X = 0, Y = 11

$\{xy = 0\} \rightarrow \{ x \geq 0 \land y > x \}$

X = 1, Y = 0

$\{xy = 0\} \rightarrow \{ x > 1 \lor y > 1 \}$

X = 0, Y = 0

4)

$\{\boldsymbol{z} \in \mathbb{R}\} \leftarrow \{\boldsymbol{z} \in \mathbb{Q}\} \leftarrow \{\boldsymbol{z} \in \mathbb{Z}\} \leftarrow \{\boldsymbol{z} \in \mathbb{N}\} \leftarrow$ **{z = y % 2;}**

$\{z \in \mathbb{N}\} \leftarrow$ {z = y % 2;}
This is true because z can only be 0 or 1 which are natural numbers.
$\{z \in \mathbb{Z}\} \leftarrow \{z \in \mathbb{N}\}$
This is true because all natural numbers are integers.
$\{z \in \mathbb{Q}\} \leftarrow \{z \in \mathbb{Z}\}$
This is true because all integers are rational numbers.
$\{z \in \mathbb{R}\} \leftarrow \{z \in \mathbb{Q}\}$
This is true because all rational numbers are real numbers.

$z = \sqrt{-1}$

$\{z = \sqrt{-1}\} \leftarrow \{z \in \mathbb{N}\}$
No, z is an imaginary number and cannot be weaker than a natural number.
$\{z = \sqrt{-1}\} \leftarrow \{z \in \mathbb{Z}\}$
No, z is an imaginary number and cannot be weaker than an integer.
$\{z = \sqrt{-1}\} \leftarrow \{z \in \mathbb{Q}\}$
No, z is an imaginary number and cannot be weaker than a rational number.
$\{z = \sqrt{-1}\} \leftarrow \{z \in \mathbb{R}\}$
No, z is an imaginary number and cannot be weaker than a real number.

$\{z = \sqrt{-1}\} \rightarrow \{z \in \mathbb{N}\}$
No, natural numbers are not implied by an imaginary number.
$\{z = \sqrt{-1}\} \rightarrow \{z \in \mathbb{Z}\}$
No, natural numbers are not implied by an integer.
$\{z = \sqrt{-1}\} \rightarrow \{z \in \mathbb{Q}\}$
No, natural numbers are not implied by a rational number.
$\{z = \sqrt{-1}\} \rightarrow \{z \in \mathbb{R}\}$
No, natural numbers are not implied by a real number.

$z = \frac{m}{n};$

$\{z = \frac{m}{n}\} \leftarrow \{z \in \mathbb{N}\}$ $\qquad$ $\{z = \frac{m}{n}\} \rightarrow \{z \in \mathbb{N}\}$

Z = 0/4 , which is not a natural number

$\{z = \frac{m}{n}\} \leftarrow \{z \in \mathbb{Z}\}$ $\qquad$ $\{z = \frac{m}{n}\} \rightarrow \{z \in \mathbb{Z}\}$

Z = 1/4 which is not an integer

Any rational number can be written as a fraction so $z = \frac{m}{n}$; is neither stronger nor weaker than $\{z \in \mathbb{Q}\}$.

$z = y / 2$

$\{z = \frac{y}{2}\} \leftarrow \{z \in \mathbb{N}\}$ $\qquad$ $\{z = \frac{y}{2}\} \rightarrow \{z \in \mathbb{N}\}$

Z = 0/2 , which is not a natural number

$\{z = \frac{y}{2}\} \leftarrow \{z \in \mathbb{Z}\}$ $\qquad$ $\{z = \frac{y}{2}\} \rightarrow \{z \in \mathbb{Z}\}$

Z = 1/2 which is not an integer

A rational number can be written as a fraction so $z = \frac{y}{2}$; is neither stronger nor weaker than $\{z \in \mathbb{Q}\}$.

5)

{ -5 < x ≤ 10 } ← { -1 ≤ x ≤ 1 }

{ -5 < x ≤ 10 } ← { -1 ≤ x ≤ 1 }
This is true because all of { -1 ≤ x ≤ 1 } is within { -5 < x ≤ 10 }

{ y = log(x) }

{ y = log(x) } ← **{ -1 ≤ x ≤ 1 }**
X = -1
{ y = log(x) } ← **{ -5 < x ≤ 10 }**
X = -1

{ y = log(x) } → **{ -1 ≤ x ≤ 1 }**
X = 10, y = 1
{ y = log(x) } ← **{ -5 < x ≤ 10 }**
X = 100, y = 2

{ -7 < x < 0 }

{ -7 < x < 0 }← **{ -1 ≤ x ≤ 1 }**
X = 1
{ -7 < x < 0 }← **{ -5 < x ≤ 10 }**
X = 1

{ -7 < x < 0 }→ **{ -1 ≤ x ≤ 1 }**
X = -2
{ -7 < x < 0 }← **{ -5 < x ≤ 10 }**
X = -6

{ x = 1 ∧ y ≥ 1 }

{ x = 1 ∧ y ≥ 1 }← **{ -1 ≤ x ≤ 1 }**
Y = 0
{ x = 1 ∧ y ≥ 1 }← **{ -5 < x ≤ 10 }**
Y = 0

{ x = 1 ∧ y ≥ 1 }→ **{ -1 ≤ x ≤ 1 }**
Y = 0
{ x = 1 ∧ y ≥ 1 }← **{ -5 < x ≤ 10 }**
Y = 0

6)

{ |result - sin(x)| ≥ x^2 } ← { |result - sin(x)| ≤ 1 } ← { |result - sin(x)| ≤ 0.01 } ←
{ |result - sin(x)| ≤ 10^{-10}} ← { |result - sin(x)| ≤ -0.01 }

{ |result - sin(x)| ≤ 10^{-10}} ← { |result - sin(x)| ≤ -0.01 }

This is true because |result - sin(x)| ≤ -0.01 is always false.

{ |result - sin(x)| ≤ 0.01 } ← { |result - sin(x)| ≤ 10^{-10}}

This is true because 10^-10 is a smaller area than 0.01.

{ |result - sin(x)| ≤ 1 } ← { |result - sin(x)| ≤ 0.01 }

This is true because 0.01 is a smaller area than 1.

{ |result - sin(x)| ≥ x^2 } ← { |result - sin(x)| ≤ 1 }

This is true because x^2 can be small relative to the value of result.

2. Hoare Triples

1) $\{ x = 5\}$
   $x = x * 2;$
   $\{ x = 10 \vee \mathrm{x} \neq 0 \}$

Valid, x always starts out at 5, so the output should always be 10.

2) $\{ \sqrt{x-1} > k \}$
   $x = x + 1;$
   $\{ k \geq 0 \}$

Invalid, this only works for cases where k is greater than or equal to zero, but what happens in cases where k is a negative number like k = -1? The case fails. The post condition should be changed to $\{ \sqrt{x} \geq k \}$. This will always be the case, so it would be valid.

3) $\{ i + j \neq 0 \wedge i * j = 0 \}$
   $i = j - 1;$
   $j = i + 1;$
   $\{ (i = 0 \vee i \neq -j) \wedge k \in \mathbb{Q} \}$

Valid

4) $\{ n < 0 \wedge n = \sqrt{m} \}$

```
if (n < m)
    x = n;
else
    y = m;
```

   $\{ x \neq y \}$

Invalid, we do not know the values of x or y before the precondition, this means that x and y could be any values (or not even initialized). Thus, there could be the case that n = 2, so m = 4. This would cause x = 2, but if y was initialized with a value of 2 the post condition would fail. To fix this, change the post condition to: $\{ (n < m) \vee (n \geq m) \}$

3. *General Hoare Triples*

A, B, C, D, E, and F are logical conditions (logical formulas). The following are true:

- $A \to B$ (A implies B, i.e., A is stronger than B)
- $B \to C$
- $C \to B$
- $D \to E$
- $E \to F$
- { B } code { E }

1) {C} code {D}:

Invalid:
because C is stronger than E and D is stronger than E, but C and D have no set relation.
B = x > 4, E = x > 3, code x = x + 2, C = x > 4, D = x > 5

2) {B} code {C}:

valid

3) {A} code {D}:

Invalid:
we know A -> B -> E and D -> E. There is no evidence to show A -> D.
A = x > 3, B = x > 2, code = x = x + 1, E = x > 1, D = x > 7

4) {A} code {F}:

Valid

4. Forward Reasoning

1) $\{ z \neq 0 \}$

$y = 0;$

$\{\boldsymbol{y = 0 \ \wedge \ z \neq 0}\}$

$x = y + 2;$

$\{\boldsymbol{x = 2 \ \wedge \ y = 0 \ \wedge \ z \neq 0}\}$

$z = x + y;$

$\boxed{\{\boldsymbol{x = 2 \ \wedge \ y = 0 \ \wedge \ z = 2}\}}$

2) $\{ |x| > 5 \}$

$x = x \ \% \ 10;$

$\{\boldsymbol{0 \ \leq x \ \wedge \ x \ \leq 9}\}$

$x = x * x;$

$\{\boldsymbol{0 \leq x \leq 81 \ \ \wedge \ \ x \div \sqrt{x} == \sqrt{x}}\}$

$x = \ -x;$

$\boxed{\left\{\boldsymbol{-81 \leq x \leq 0 \ \wedge \ -x \div \sqrt{x} == -\sqrt{x}}\right\}}$

3) $\{ z < \ 5 \}$

If $(z > 0)$ {

$\{\boldsymbol{0 < z < 5}\}$

$z = \ -z;$

$\{ \boldsymbol{-5 < z < 0} \}$

}

$\boxed{\{ \boldsymbol{z \leq 0}\}}$

5. Backward Reasoning

1)

$\{\boldsymbol{wp}(\text{"x =1;"},\ \boldsymbol{y} > -\mathbf{3}\boldsymbol{x} \wedge \boldsymbol{y} < \mathbf{10} - \mathbf{3}\boldsymbol{x}) = \boldsymbol{y} > \mathbf{3} \wedge \boldsymbol{y} < \mathbf{13}\}$

$x = -1;$

$\{ wp(\text{"z = 3*x+y;"}, 0 < z < 10) = y > -3x \wedge y < 10 - 3x\}$

$z = 3 * x + y;$

$\{ 0 < z < 10\}$

2)

$\{ \boldsymbol{wp}(\text{"if (y > 0) x = x/y; else y = 4*x"}, \boldsymbol{x} \geq \boldsymbol{y} \wedge \boldsymbol{y} \in \mathbb{Q}) = (\boldsymbol{x} \geq \boldsymbol{y} \wedge \boldsymbol{y} > \mathbf{0})\}$

If (y > 0){

$\{ wp(\text{"x = x/y;"}, x > 1 \wedge y = 0) = x/y > 1 \wedge y \in \mathbb{Q} = x \geq y \wedge y \in \mathbb{Q}\}$

x = x / y;

$\{ wp(\text{"y = 0;"}, x > 1 \wedge y = 0) = x > 1 \wedge y \in \mathbb{Q} \}$

y = 0;

$\{ x > 1 \wedge y = 0 \}$

} else{

$\{ wp(\text{"y=4*x;"}, x > 1 \wedge y = 0) = \frac{y}{4} > 1 \wedge y = 0 = false \}$

y = 4 * x;

$\{ x > 1 \wedge y = 0 \}$

}

$\{ x > 1 \wedge y = 0 \}$

3)

{wp(“if($x \geq 0$) z = Math.min(z,x); ”,$\boldsymbol{Math}.\mathbf{min}(\boldsymbol{z},\boldsymbol{x}) \neq \mathbf{0} \wedge \boldsymbol{y} \geq \mathbf{0} \vee \boldsymbol{Math}.\mathbf{min}(\boldsymbol{z},\boldsymbol{x}) + \boldsymbol{y} \geq \mathbf{0}$) = $\boldsymbol{Math}.\mathbf{min}(\boldsymbol{z},\boldsymbol{x}) \neq \mathbf{0} \wedge \boldsymbol{y} \geq \mathbf{0} \vee \boldsymbol{Math}.\mathbf{min}(\boldsymbol{z},\boldsymbol{x}) + \boldsymbol{y} \geq \mathbf{0} \wedge \boldsymbol{x} \geq \mathbf{0}$}

If ($x \geq 0$){

{wp(“z = Math.min(z,x);”,$\mathrm{z} \neq 0 \wedge \mathrm{y} \geq 0 \vee \mathrm{z} + \mathrm{y} \geq 0$)

= $Math.\min(z,x) \neq 0 \wedge y \geq 0 \vee Math.\min(z,x) + y \geq 0$}

z = Math.min(z, x);

{wp(“$x = z + y;$”,$z \neq 0 \wedge y \geq 0$)= $\mathrm{z} \neq 0 \wedge \mathrm{y} \geq 0 \vee \mathrm{z} + \mathrm{y} \geq 0$}

x = z + y

$\{ \mathrm{z} \neq 0 \wedge \mathrm{y} \geq 0 \vee \mathrm{x} \geq 0\}$

}

$\{ \mathrm{z} \neq 0 \wedge \mathrm{y} \geq 0 \vee \mathrm{x} \geq 0\}$

4)

{wp(“If(math.abs(x))<=5 z = x-2; else if (x<=-5) ”,
(math.abs(x) <= 5 $\wedge$ $-1 \le x \le 5$) $\vee$ ($(\text{math.abs}(x) > 6 \wedge -9 \le x \le -5 \vee -1 \le x \le 1)$)}

If (math.abs(x) <= 5){

{wp(“z=x-2;” $-3 \le z \le 3$) = $-1 \le x \le 5$}

Z = x – 2;

{$-3 \le z \le 3$}

} else {

{wp(“if (x<=-5) z = x+6; else z = 3*x;”, $-9 \le x \le -3 \vee -1 \le x \le 1$) = $-9 \le x \le -5 \vee -1 \le x \le 1$}

If (x <= -5){

{wp(“z=x+6;”, {$-3 \le z \le 3$) = {$-9 \le x \le -3$}

Z = x + 6;

{$-3 \le z \le 3$}

}else{

{wp(“z=3*x;”, {$-3 \le z \le 3$) = {$-1 \le x \le 1$}

Z = 3 * x;

{$-3 \le z \le 3$}

}

{$-3 \le z \le 3$}

}

{$-3 \le z \le 3$}

5)

$\{\boldsymbol{wp}(\text{"}\boldsymbol{x = y/2};\text{"}, \boldsymbol{y \neq 1 \wedge y > -2})\}$

$x = y/2;$

$\{wp(\text{"}z = x + 1;\text{"}, x \neq 0.5 \wedge z > 0) = x \neq 0.5 \wedge x > -1\}$

$z = x + 1;$

$\{ x \neq 0.5 \wedge z > 0 \}$

6. Sufficient/Insufficient Correctness

1) { x < 2}

    $\{wp(\text{"z=x-1;"}, x > 3) = x > 3 \land z > 4\}$

    z = x – 1;

    $\{wp(\text{"w=x-1"}, w > 2) = x > 3\}$

    w = x – 1;

    $\{wp(\text{"z = w - 1;"}, z > 1) = w > 2\}$

    z = w – 1;

    { z > 1}

Insufficient: {x < 2} is weaker than { x > 3 ∧ z > 4 }, thus doesn't suffice.

2) $\{x = y \land y > 0 \lor y \neq x \land x \geq 0\}$

    $\{(x \leq y \land x \geq 1) \lor (y/2 < y \land y/2 \geq 0)\}$

    If (x < y) {

    $\{wp(\text{"x--;"}, x < y \land x \geq 0) = x \leq y \land x \geq 1\}$

    x--;

    $\{x < y \land x \geq 0\}$

    } else{

    $\{wp(\text{"x=y/2;"}, x < y \land x \geq 0) = y/2 < y \land y/2 \geq 0\}$

    x = y / 2;

    $\{x < y \land x \geq 0\}$

    }

    { x < y ∧ x ≥ 0}

Sufficient: $\{(\boldsymbol{x} \leq \boldsymbol{y} \land \boldsymbol{x} \geq \mathbf{1}) \lor (\boldsymbol{y}/\mathbf{2} < \boldsymbol{y} \land \boldsymbol{y}/\mathbf{2} \geq \mathbf{0})\}$ is weaker than $\{\boldsymbol{x} = \boldsymbol{y} \land \boldsymbol{y} > \mathbf{0} \lor \boldsymbol{y} \neq \boldsymbol{x} \land \boldsymbol{x} \geq \mathbf{0}$