OpenCL™ Programming Webinar Series

Smoothed Particle Hydrodynamics

Application Example

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AMD
The future is fusion

Fluids

Navier-Stokes equations
Smoothed Particle Hydrodynamics
OpenCL simulation

Fluids

- Liquids, e.g. water
- Gasses, e.g. air
- Plasmas

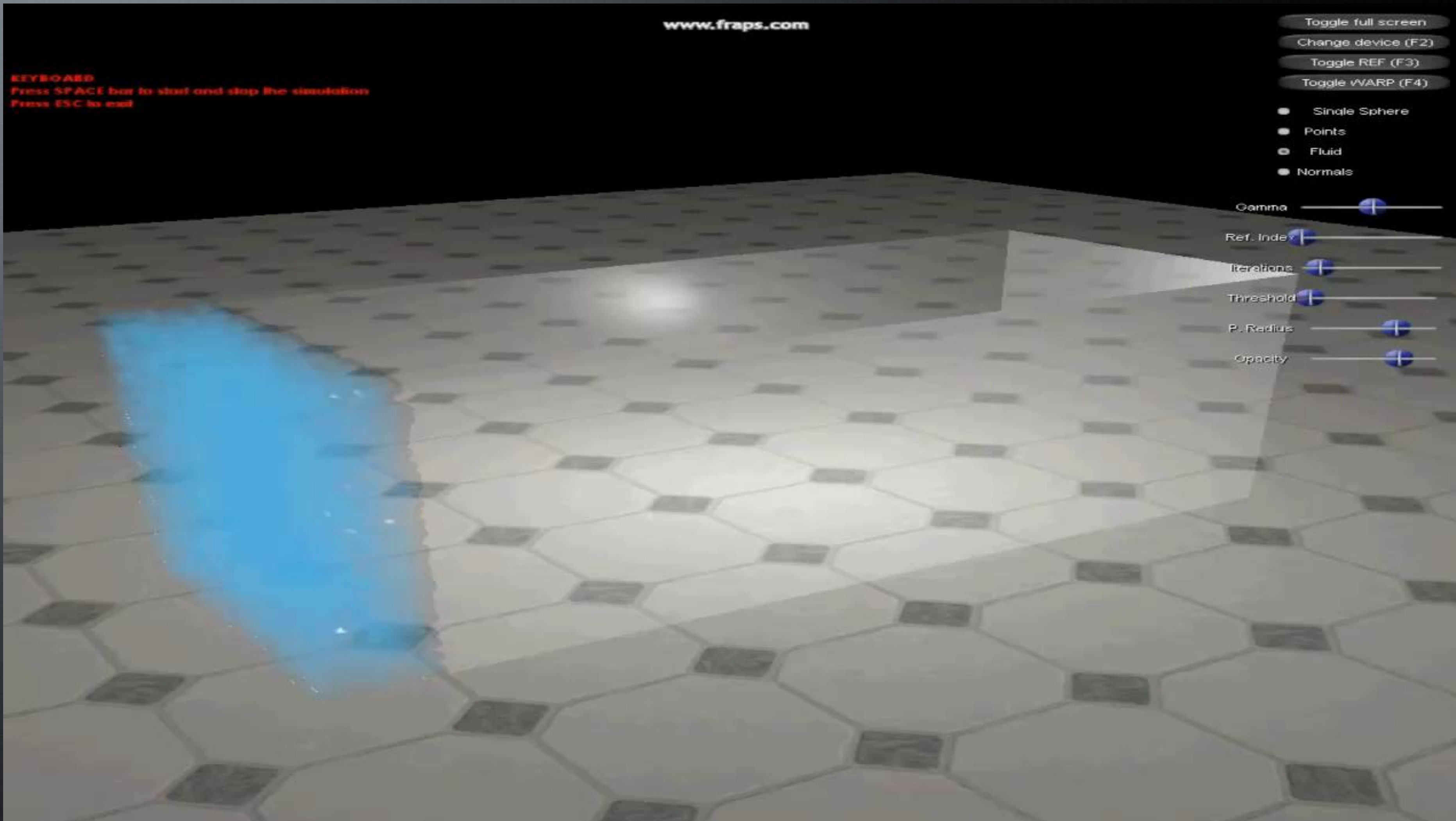
Fluids

- Described by (incompressible) Navier-Stokes equations

$$\rho\left[\frac{\partial v}{\partial t} + v \bullet \nabla v\right] = \rho g - \nabla p + \mu \nabla^2 v \qquad \rho(\nabla \bullet v) = 0$$

- Driven by gravity g, pressure ∇p and velocity $\mu \nabla^2 v$
  - Fluid flows from high pressure to low pressure
  - Viscosity μ determines fluid stickiness
    - Low viscosity: air, water
    - High viscosity: honey, mud

Fluids


www.fraps.com
KEYBOARD
Press SPACE bar to start and stop the simulation
Press ESC to exit
Toggle full screen
Change device (F2)
Toggle REF (F3)
Toggle WARP (F4)
Single Sphere
Points
Fluid
Normals
Gamma
Iterations
Threshold
P. Radius
Opacity

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Incompressible Navier-Stokes equations

$$\rho\left[\frac{\partial v}{\partial t} + v \bullet \nabla v\right] = \rho g - \nabla p + \mu \nabla^2 v$$

$$\rho(\nabla \bullet v) = 0$$ (mass continuity)

- ρ - density, p – pressure (scalars)
- g – gravity, v – velocity (vectors)
- $v \bullet \nabla v \equiv \left[v_x \frac{\partial v_x}{\partial x}, v_y \frac{\partial v_y}{\partial y}, v_z \frac{\partial v_z}{\partial z} \right]$ convective acceleration
- $\nabla p \equiv \left[\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z} \right]$ p = k(ρ - ρ0) resting density ρ0

$$\nabla^2 v \equiv \left[\nabla^2 v_x, \nabla^2 v_y, \nabla^2 v_z\right], \nabla^2 v_x \equiv \partial^2 v_x / \partial x^2 + \partial^2 v_x / \partial y^2 + \partial^2 v_x / \partial z^2$$

Incompressible Navier-Stokes equations

$$\rho\left[\frac{\partial v}{\partial t} + v \bullet \nabla v\right] = \rho g - \nabla p + \mu \nabla^2 v$$

– Mass continuity equation: $\rho(\nabla \bullet v) = 0$

$$\nabla \bullet v = \left(\partial v_x \big/ \partial x + \partial v_y \big/ \partial y + \partial v_z \big/ \partial z \right) = 0$$

– Mass continuity will be satisfied trivially by using a particle formulation, since each particle has constant mass and particles are neither created nor destroyed

Incompressible Navier-Stokes equations

$$\rho\left[\frac{\partial v}{\partial t} + v \bullet \nabla v\right] = \rho g - \nabla p + \mu \nabla^2 v$$

– The material derivative is the derivative along a path with velocity *v*. To simulate with particles take the material derivative

$$\rho \frac{Dv}{Dt} = \rho g - \nabla p + \mu \nabla^2 v$$

– For a single particle *i*

$$\frac{dv_i}{dt} = g - \frac{1}{\rho_i} \nabla p + \frac{\mu}{\rho_i} \nabla^2 v$$

Incompressible Navier-Stokes equations

- The Navier-Stokes equations are sensitive to scale, so we simulate them at 0.004x scale relative to the physical environment.

Fluids
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Smoothed Particle Hydrodynamics

- [Monaghan 1992] introduced smoothing kernels W

$$A_i(r) = \int A(r')W(r-r',h)dr' \approx \sum_b A(r_b)W(r-r_b,h)$$

- And approximations to terms of the N-S equations
- $\rho_i \approx \sum_j m_j W(r-r_j,h)$ m – mass, r – position, h - radius

$$\frac{\nabla p_i}{\rho_i} \approx \sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla W(r-r_j,h)$$

$$\frac{\mu}{\rho_i} \nabla^2 v_i \approx \frac{\mu}{\rho_i} \sum_j m_j \left(\frac{v_j - v_i}{\rho_j} \right) \nabla^2 W(r-r_j,h)$$

Smoothed Particle Hydrodynamics

- Over time the literature has converged on these W:
  - w = 0 at distance *h*
  - w sums to 1 over sphere of radius *h*

$$W(r - r_b, h) \equiv \frac{315}{64\pi h^9}\left(h^2 - \left\|r - r_b\right\|^2\right)^3$$

$$\nabla W(r - r_b, h) \equiv \frac{-45}{\pi h^6}\left(h - \left\|r - r_b\right\|\right)^2 \frac{r - r_b}{\left\|r - r_b\right\|}$$

$$\nabla^2 W(r - r_b, h) \equiv \frac{45}{\pi h^6}\left(h - \left\|r - r_b\right\|\right)$$

Smoothed Particle Hydrodynamics

- From Navier-Stokes to SPH:

$$\frac{dv_i}{dt} = g - \frac{1}{\rho_i}\nabla p + \frac{\mu}{\rho_i}\nabla^2 v \qquad (1)$$

$$\rho_i \approx \sum_j m_j \frac{315}{64\pi h^9}\left(h^2 - \|r - r_b\|^2\right)^3 \qquad (2)$$

$$\frac{\nabla p_i}{\rho_i} \approx \sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right)\frac{-45}{\pi h^6}\left(h - \|r - r_b\|\right)^2 \frac{r - r_b}{\|r - r_b\|} \qquad (3)$$

$$\frac{\mu}{\rho_i}\nabla^2 v_i \approx \frac{\mu}{\rho_i}\sum_j m_j \left(\frac{v_j - v_i}{\rho_j}\right)\frac{45}{\pi h^6}\left(h - \|r - r_b\|\right) \qquad (4)$$

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OpenCL simulation

- Numerical algorithm:
  - density ρ = equation (2).
  - pressure p = k(ρ - ρ0)
  - pressure gradient $\frac{\nabla p_i}{\rho_i}$ = equation (3).
  - viscous term $\frac{\mu}{\rho_i}\nabla^2 v_i$ = equation (4).
  - acceleration = equation (1).
  - numerically integrate velocity, position.

OpenCL simulation

- A naïve algorithm computes interactions among all particles
  - Gives correct result because W= 0 for particles beyond the interaction radius
  - But this has complexity O(n^2)
    - Need an algorithm that only computes interactions among particles that are within the interaction radius

OpenCL simulation

- A better algorithm partitions space into local regions
  - Divide into voxels of size 2*h* on a side
  - Each particle can only interact with particles in the same voxel, and in immediately adjacent voxels
    - Total search volume = 2x2x2 voxels
  - Further refinement: compute interactions with a limited number *m* of particles
    - *m* = 32 works well

OpenCL simulation

- The final algorithm:
  - Organize particles into voxels
  - Compute spatial index from voxel to particles
  - For every particle
    - Examine local region of 2x2x2 voxels
    - Compute interactions with 32 particles

OpenCL simulation

- Interop allows a buffer to be shared between OpenCL and a graphics subsystem.
  - This avoids an expensive round trip to host memory
  - This is crucial for high performance applications
- Due to limits of time we did not implement interop in the graphics code, however we will show you the OpenCL initialization for interop for reference.

OpenCL simulation

- To interop with dx10 include “cl_d3d10.h” and define USE_DX_INTEROP

```
#define USE_DX_INTEROP
#if defined(__APPLE__) || defined(__MACOSX)
#include <OpenCL/cl.hpp>
#include <OpenCL/cl_d3d10.h>
#else
#include <CL/cl.hpp>
#include <CL/cl_d3d10.h>
#endif
```

OpenCL simulation

- To interop with dx10 initialize the OpenCL context:

```
cl_context_properties *cprops;
cprops = new cl_context_properties[ 6 ];
cprops[ 0 ] = CL_CONTEXT_D3D10_DEVICE_KHR;
cprops[ 1 ] = (intptr_t) DXUTGetD3D10Device();
cprops[ 2 ] = CL_CONTEXT_PLATFORM;
cprops[ 3 ] = (cl_context_properties)(platformList[0])();
cprops[ 4 ] = cprops[ 5 ] = 0;
context = cl::Context( CL_DEVICE_TYPE_GPU, cprops, NULL, NULL,
&err);
```

OpenCL simulation

- Buffers
  - position, velocity, acceleration – float4
  - particleIndex – uint2
  - sortedPosition, sortedVelocity – float4
  - gridCellIndex, gridCellIndexFixedUp - uint
- Kernels
  - hashParticles, sort, sortPostPass
  - indexx, indexPostPass
  - findNeighbors
  - computeDensityPressure, computeAcceleration, integrate

OpenCL simulation

- The final algorithm:
  - Organize particles into voxels
    - hashParticles, sort, sortPostPass
  - Compute spatial index from voxel to particles
    - indexx, indexPostPass
  - For every particle
    - Examine local region of 2x2x2 voxels
      - findNeighbors
    - Compute interactions with 32 particles
      - computeDensityPressure, computeAcceleration, integrate

OpenCL simulation

- Organize particles into voxels: global_id(0)=particle id
  - hashParticles:
    - computes a scalar voxel id from position
      - Voxel size 2*h*x2*h*
    - stores voxel id in position.w;
    - writes {voxel id,global_id(0)} to particleIndex
  - sort:
    - sorts particleIndex by voxel id
      - radixSort works only on GPU, use qsort on CPU
  - sortPostPass:
    - rewrite position,velocity into sortedPosition,sortedVelocity according to order of particleIndex

OpenCL simulation

- Compute spatial index from voxel to particles: global_id(0) = voxel id
  - indexx:
    - computes gridCellIndex(i), index into sortedPosition of first particle in voxel i
      - Binary search in sortedPosition for lowest particle id
        - Leave -1 for empty voxels
  - indexPostPass:
    - Fills in index for empty voxels
      - gridCellIndex(i) = gridCellIndex(i+1) for i empty, i+1 nonempty

OpenCL simulation

- Examine local region of 2x2x2 voxels:
  - findNeighbors:
    - Locates particle in one corner of 2x2x2 voxel set
    - Searches up to 8 voxels until 32 neighbors are found
      - Retains only neighbors within interaction radius
      - Within each voxel search is randomized
        - Necessary to eliminate biasing artifacts
        - Specifically, compute random offset within voxel, then proceed sequentially
        - Alternate sequential directions according to odd/evenness of particle

OpenCL simulation

- Compute interactions with 32 particles:
  - computeDensityPressure:
    - Equation (2) followed by $p = k(\rho - \rho 0)$
  - computeAcceleration:
    - Equations (3), (4), (1)
  - integrate:
    - Semi-implicit Euler integration
    - $v = v + dt\ a$, $position = position + dt\ v$
      - Boundary conditions prevent particle escape

Summary

- Fluids
  - Governed by pressure, velocity
- Navier-Stokes equations
  - Incompressible equations, material derivative
- Smoothed Particle Hydrodynamics
  - Smoothing kernel approximations
  - Approximate ρ, ∇p, $\nabla^2 v$
- OpenCL simulation
  - Organize into voxels, create voxel index, compute equations (2), (3), (4), (1), integrate

Questions and Answers

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