Advanced Topics in Computational Complexity

Exercise Session 6

Due 23.11.2015.

Exercise 1

Give a $DQBF$-formula that is equivalent to the QBF-formula

$$\exists p_0 \forall p_1 \exists p_2 \exists p_3 \forall p_4 \big((p_2 \wedge \neg p_3) \vee p_0 \big) \wedge (\neg p_1 \vee p_2).$$

Give a QBF-formula that is equivalent to the $DQBF$-formula

$$\Big(\forall p_0 \forall p_3 \forall p_2 \exists p_5 \exists p_4 \exists p_1 \big((p_2 \wedge \neg p_3) \vee p_0 \big) \wedge (\neg p_1 \vee p_2),\ \big(\emptyset, \{p_2, p_3\}, \{p_0, p_2, p_3\} \big) \Big).$$

Exercise 2

Compute $\mathrm{nbSubf}\Big(\big(p \wedge \Diamond (q \vee \Box \neg q) \big) \Big)$ and $\mathrm{nbSubf}\Big(\Diamond \big(p \wedge \Diamond \Box (q \wedge \neg q) \big) \Big)$.

Exercise 3

Give a formula of form (3) of Proposition 22 that is equivalent to the EMDL formula $\Diamond \big(q \wedge \mathrm{dep}(\Box p, q \vee p) \big)$.

Exercise 4

Prove Lemma 4 from the handout. (You may assume that Proposition 24 holds.)

Exercise 5

A formula $\varphi \in \mathrm{ML}$ is *valid in small models* if $\mathrm{K}, w \models \varphi$ holds for every Kripke model $\mathrm{K} = (W, R, V)$ and $w \in W$ such that $|W| \leq |\varphi|$. (Here $|W|$ and $|\varphi|$ refers to the sizes of the set W and the formula φ, respectively.)

Show that the decision problem whether a given formula of ML is valid in small models is in *co*NP.