Lösung 1 **(Kombinatorik, Wahrscheinlichkeit)**

1. $N = 4$

   a)
$$k = 0: \quad \binom{4}{0} = 1$$
$$k = 1: \quad \binom{4}{1} = 4$$
$$k = 2: \quad \binom{4}{2} = 6$$
$$k = 3: \quad \binom{4}{3} = 4$$
$$k = 4: \quad \binom{4}{4} = 1$$

   b)
$$k = 0: \quad \sum_{i=0}^{0} \binom{4}{i} = 1$$
$$k = 1: \quad \sum_{i=0}^{1} \binom{4}{i} = 5$$
$$k = 2: \quad \sum_{i=0}^{2} \binom{4}{i} = 11$$
$$k = 3: \quad \sum_{i=0}^{3} \binom{4}{i} = 15$$
$$k = 4: \quad \sum_{i=0}^{4} \binom{4}{i} = 16$$

2. a) 1 Fehler in 1 Bit
   $P_e = p = 10^{-3}$

   b) ≥ 2 Fehler in 3 Bits
   $P_e = \binom{3}{2}p^2(1-p) + p^3 = 2,998 \cdot 10^{-6}$

   c) ≥ 3 Fehler in 5 Bits
   $P_e = \binom{5}{3}p^3(1-p)^2 + \binom{5}{4}p^4(1-p) + p^5 = 9,985006 \cdot 10^{-9}$

   d) $\geq k$ Fehler in $2k+1$ Bits
   $P_e = \sum_{i=k+1}^{2k+1} \binom{2k+1}{i} p^i (1-p)^{2k+1-i}$