Model-Based Software Engineering

Lecture 09 – Transformation

Prof. Dr. Joel Greenyer

SOFTWARE ENGINEERING

June 21, 2016

Leibniz Universität Hannover

in the last lecture...

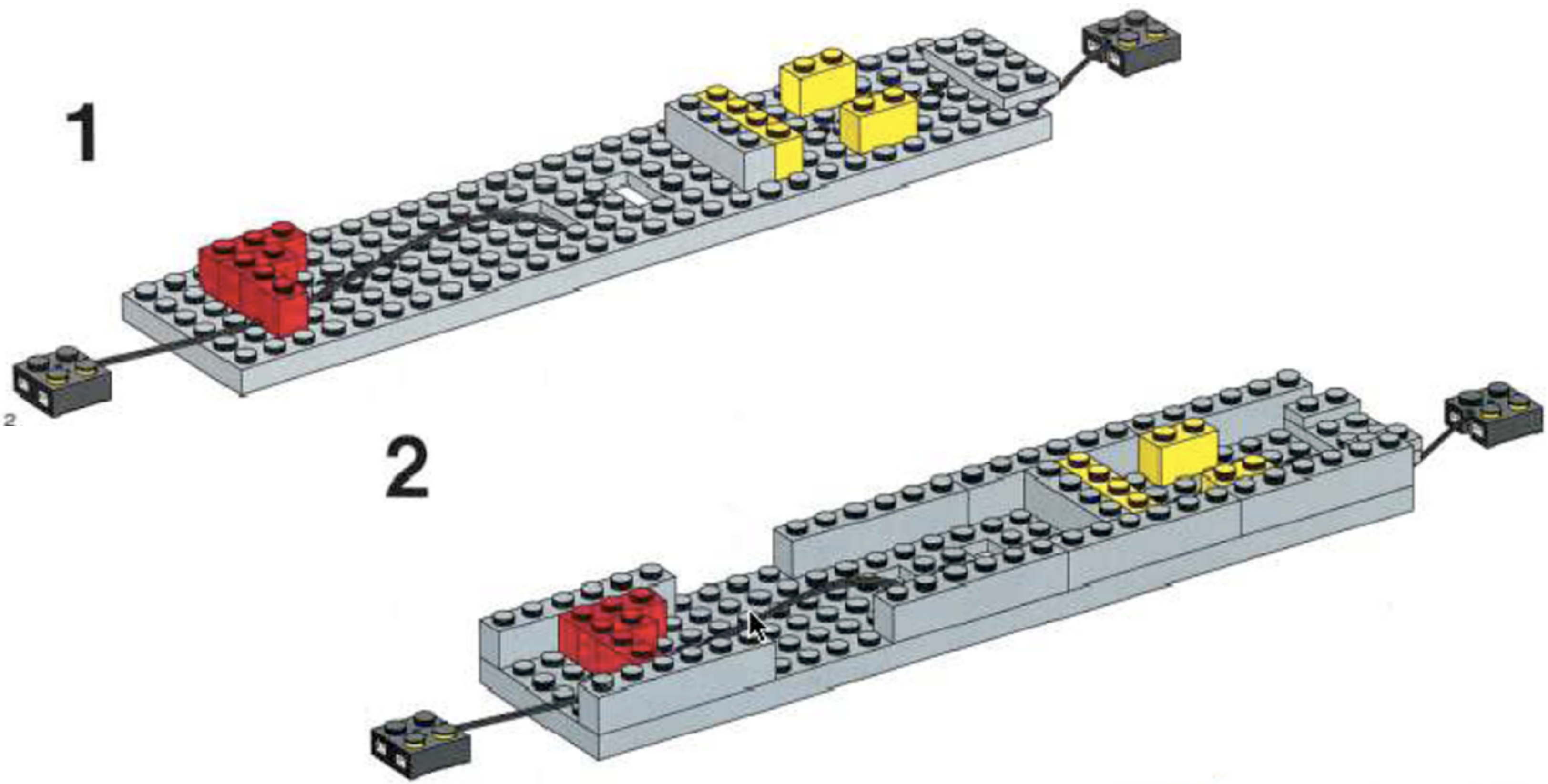
5.3. Model-to-model transformation – graph transformations

Leibniz Universität Hannover

Describe Structural Changes

in the last lecture...

- Most children understand this way of describing structural changes:

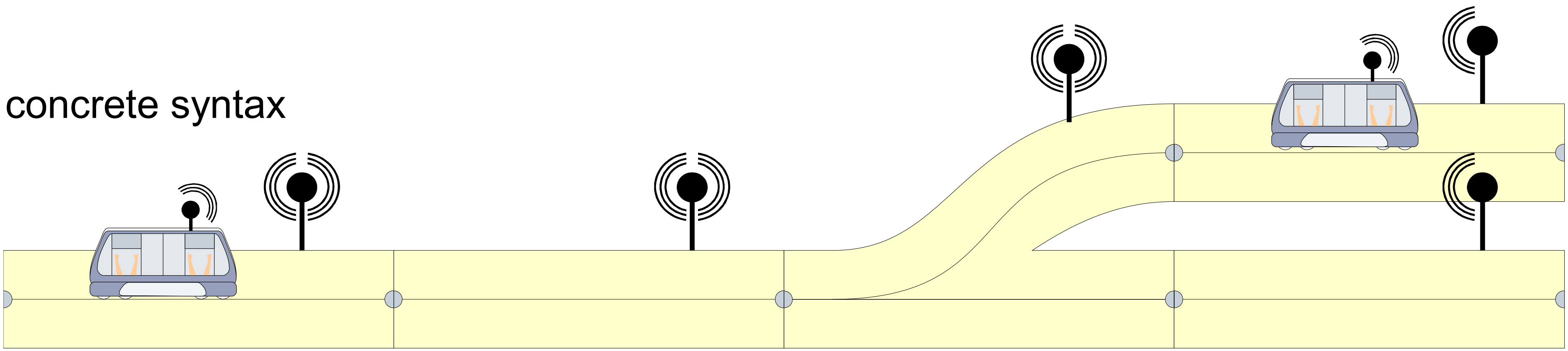


View the System as a Graph

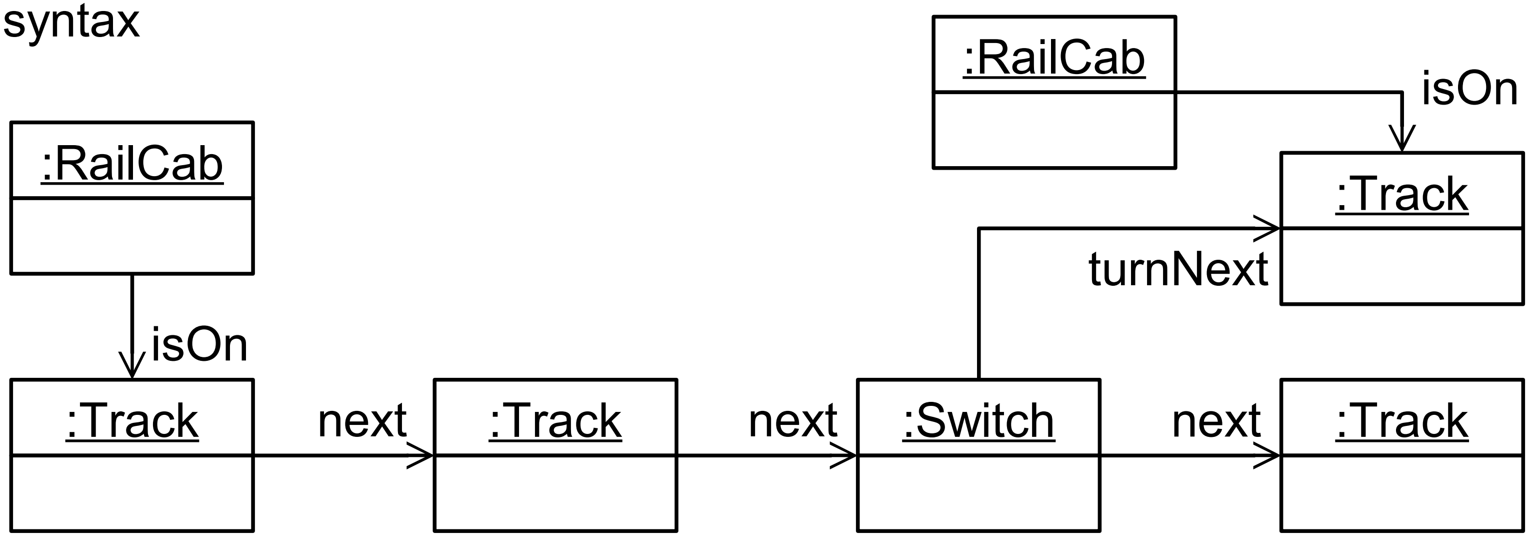
in the last lecture...

- Idea: View the model as a graph
- **Example**: train system “RailCab”

concrete syntax



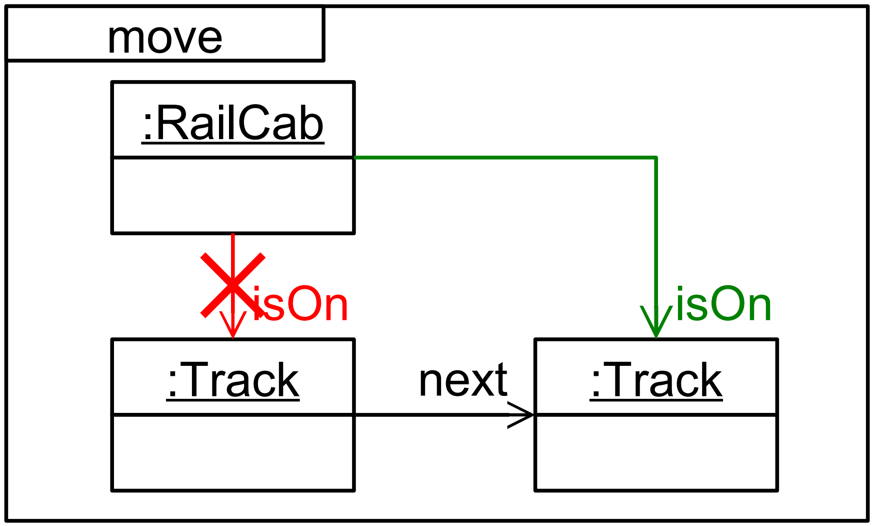
abstract syntax



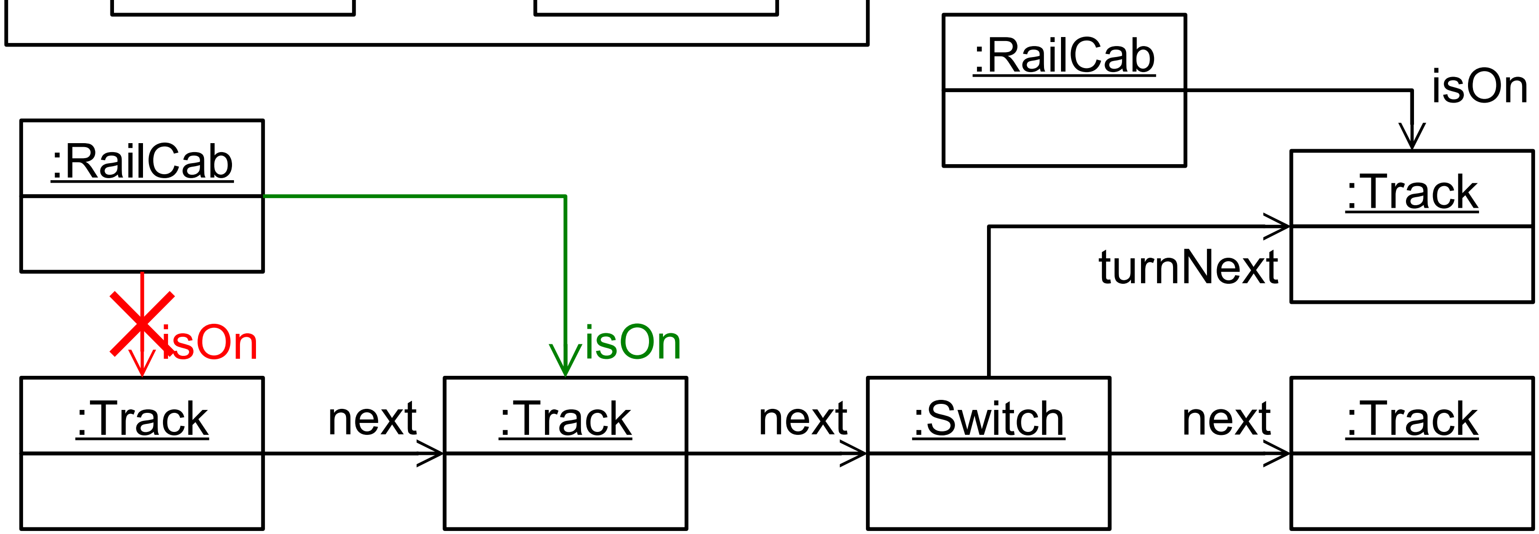
Graph Transformation Rule

in the last lecture...

- Describe the necessary **context of the change** and the **change itself** in a **graph transformation rule**



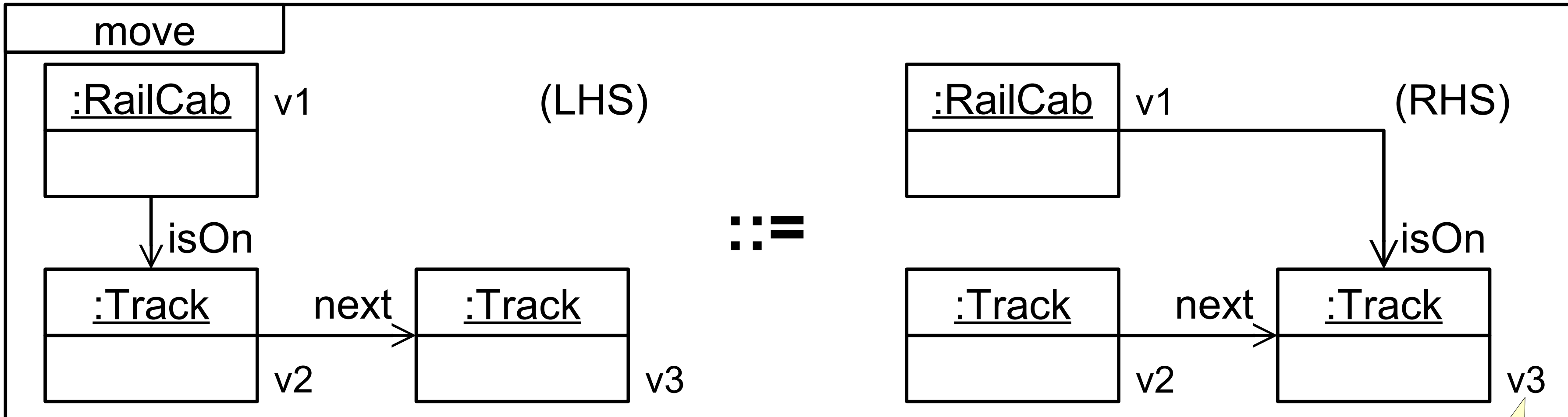
the rule's semantic is
clear intuitively,
but what does this
mean exactly?



Graph Grammar Rule

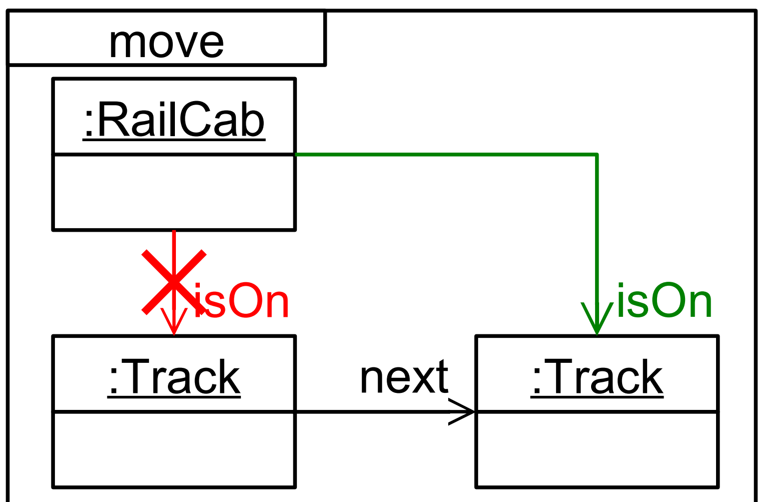
in the last lecture...

- A graph grammar rule consists of two typed graphs
  - called **left-hand side** (LHS) and **right-hand side** (RHS)



node identities

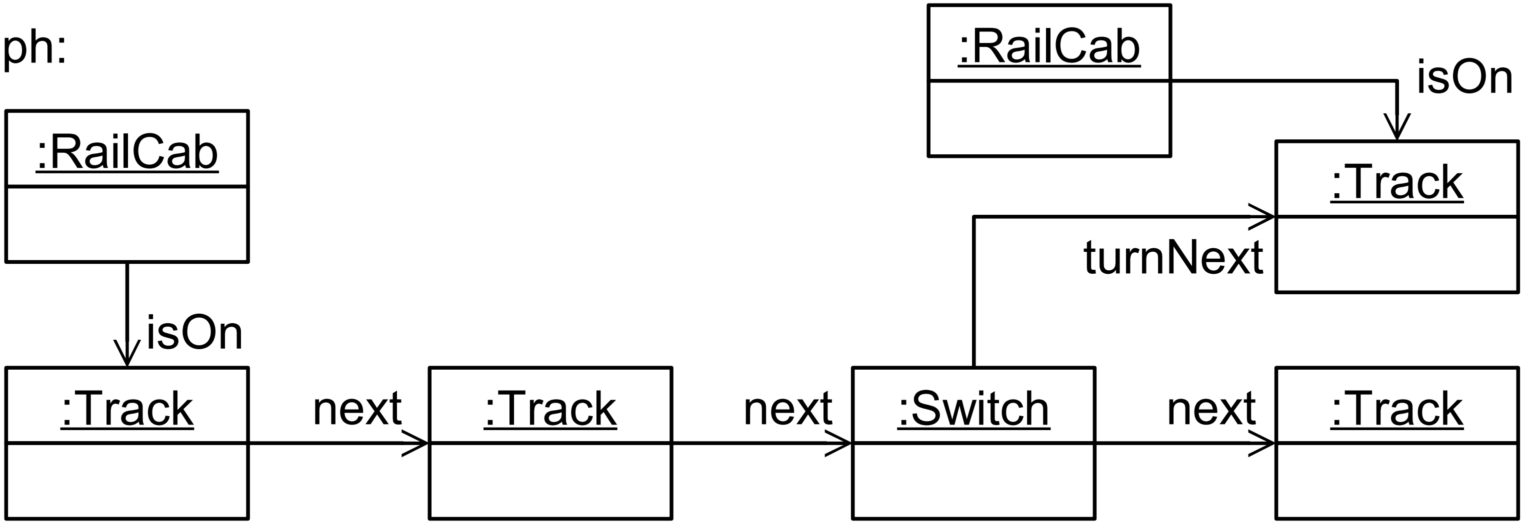
short-hand notation:

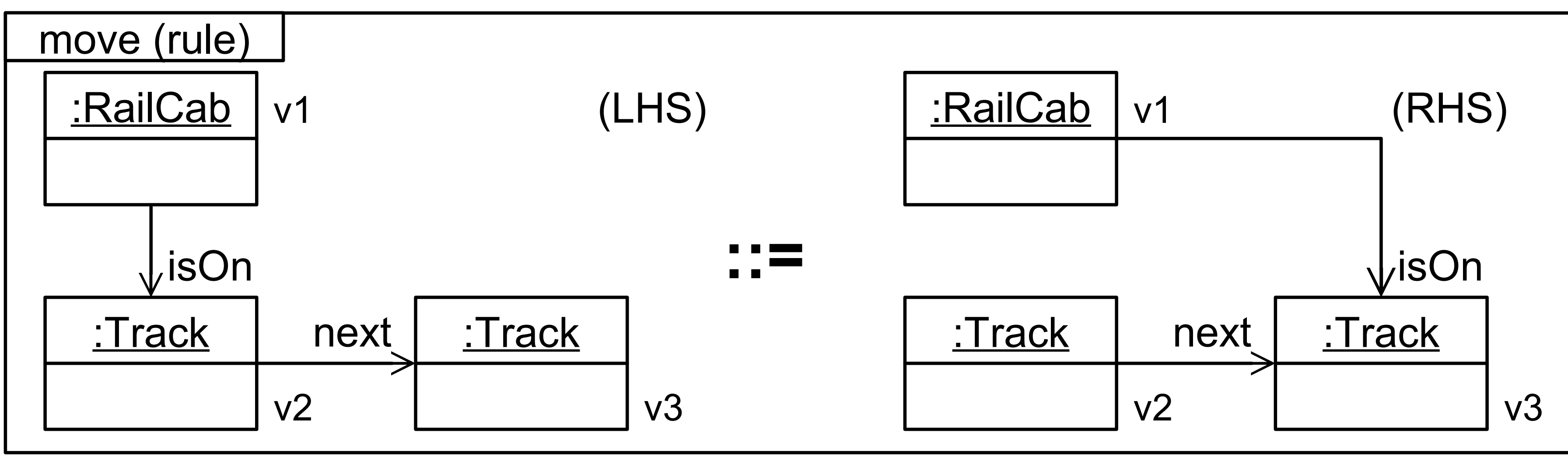


Graph Grammar Rule Application

in the last lecture...

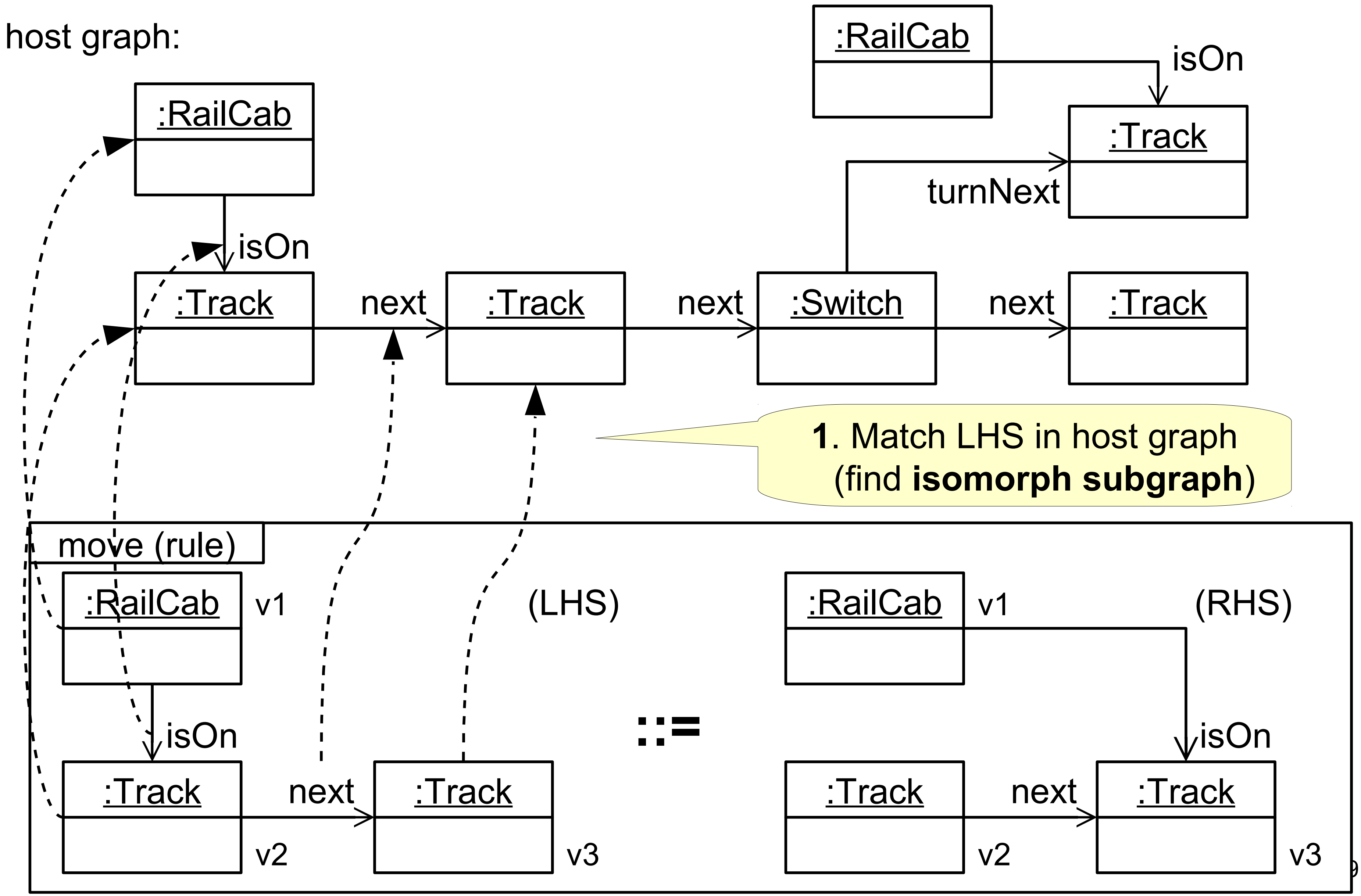
host graph:





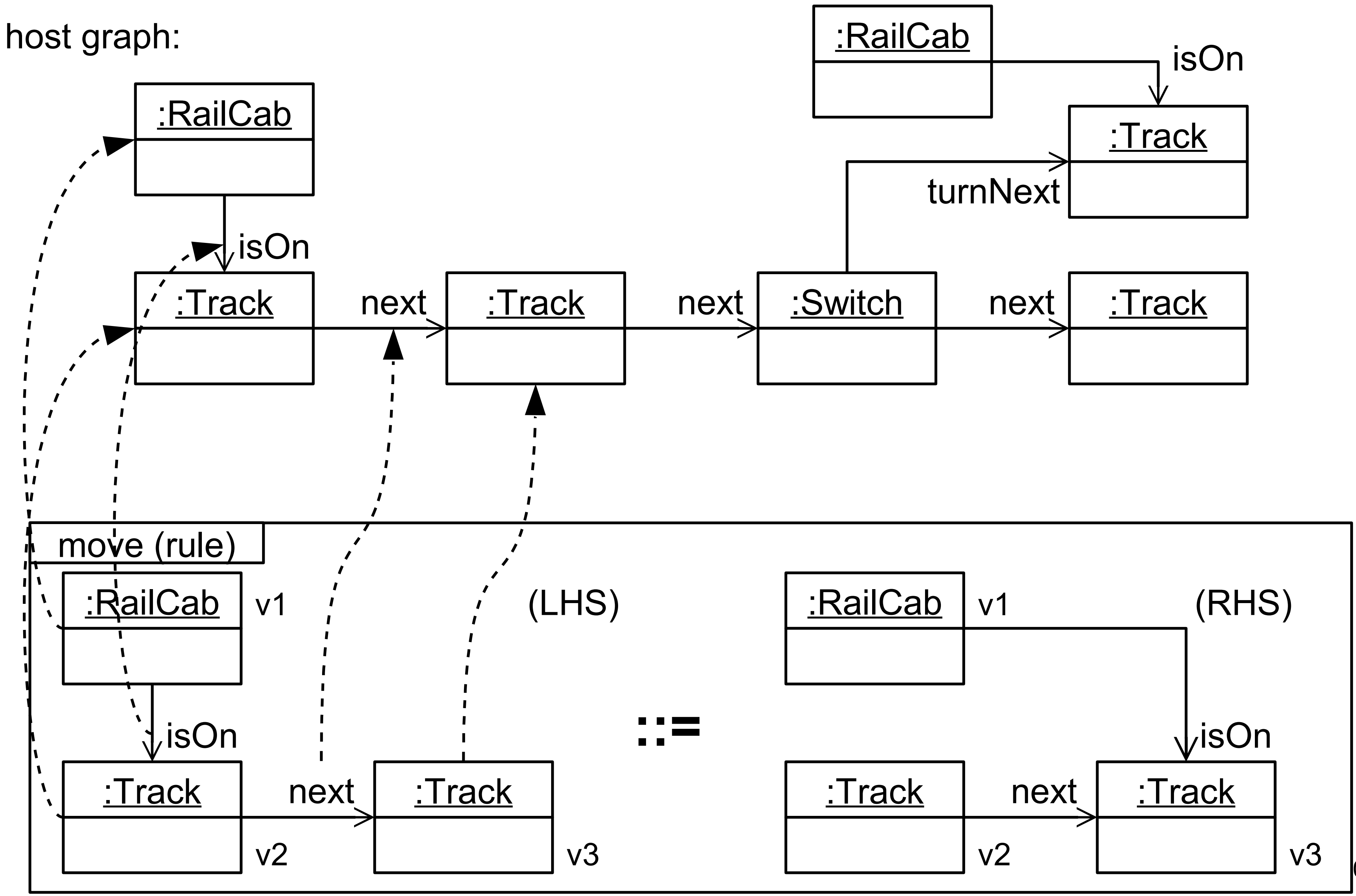
Graph Grammar Rule Application

in the last lecture...



Graph Grammar Rule Application

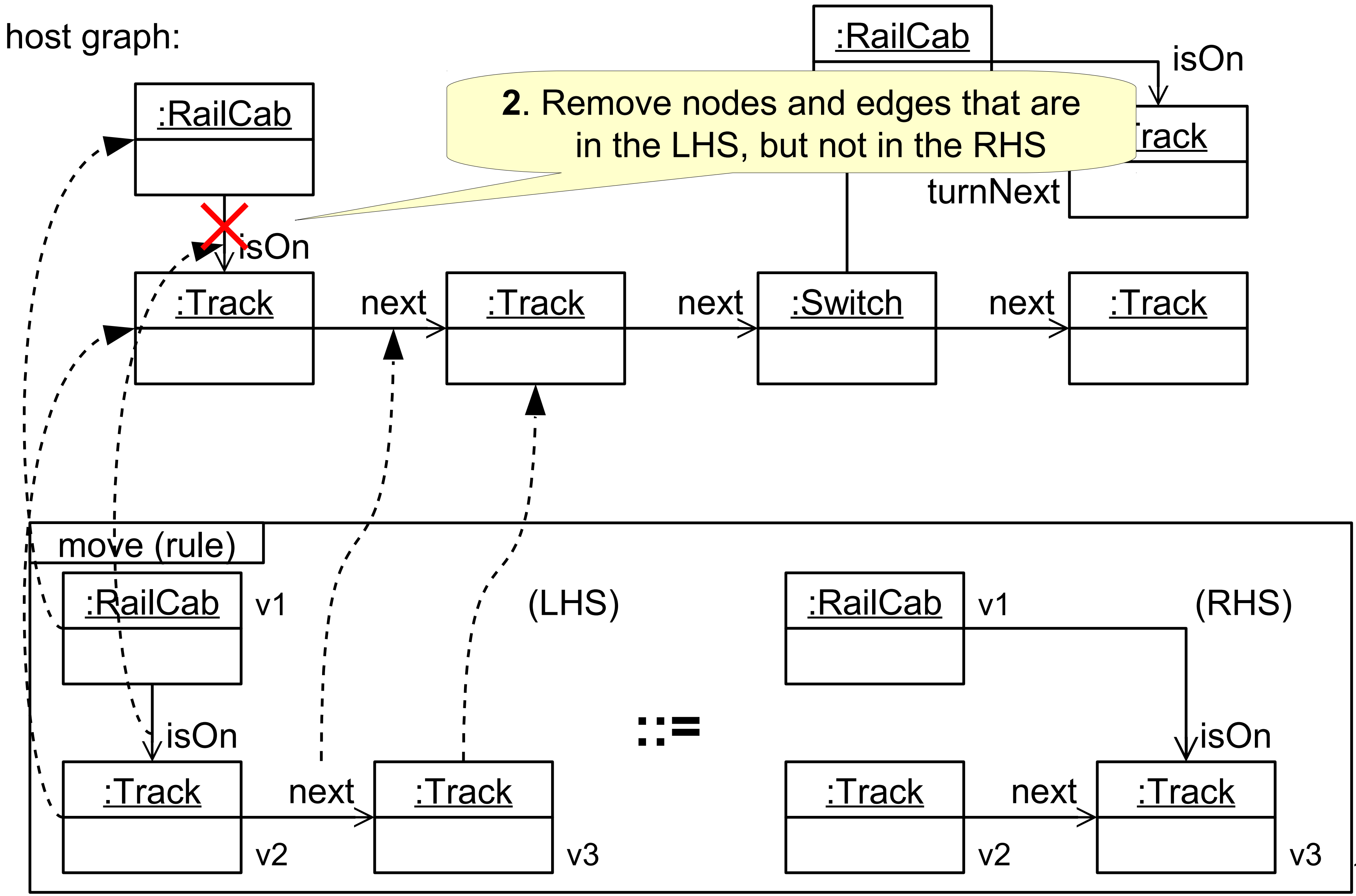
in the last lecture...



Graph Grammar Rule Application

in the last lecture...

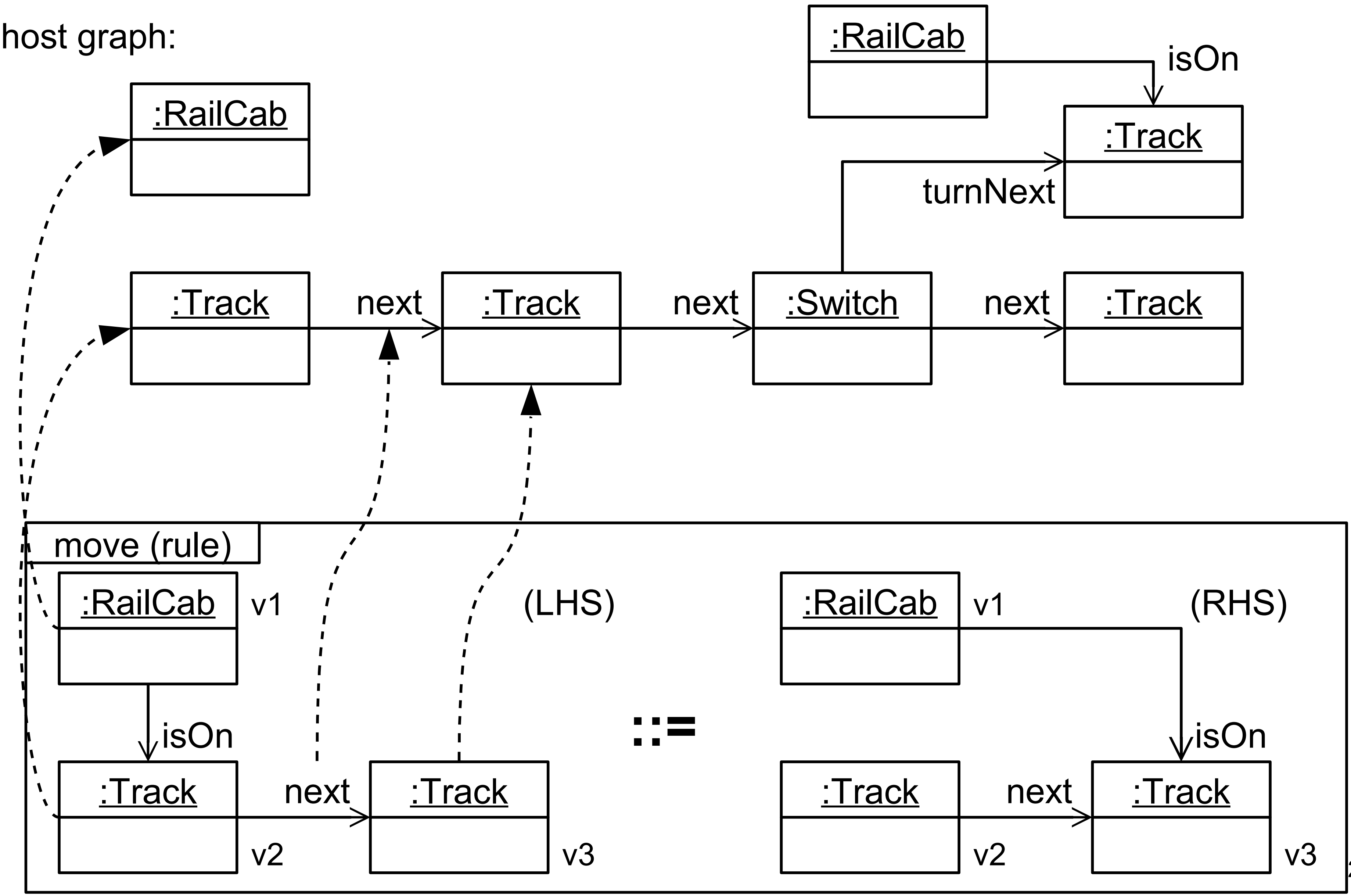
host graph:



Graph Grammar Rule Application

in the last lecture...

host graph:

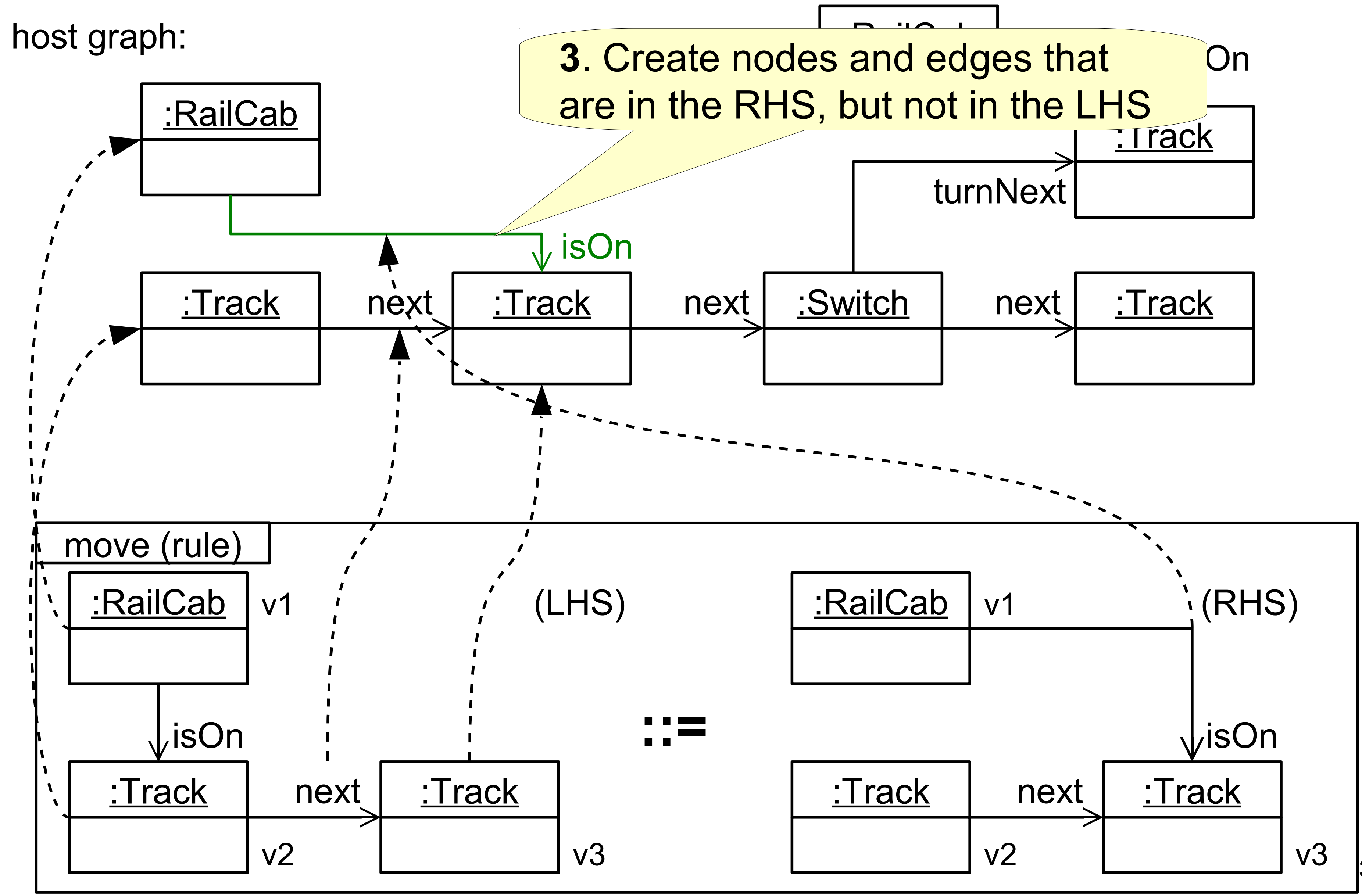


Graph Grammar Rule Application

in the last lecture...

host graph:

3. Create nodes and edges that are in the RHS, but not in the LHS



Eclipse Henshin

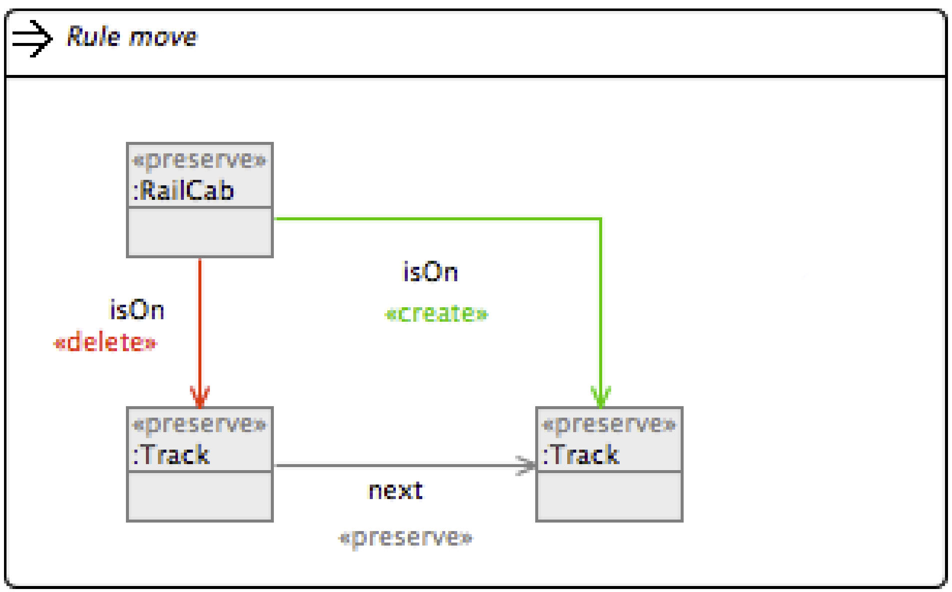
in the last lecture...

- An Eclipse project that supports the modeling, execution, and analysis of EMF-based graph transformation systems
  - https://www.eclipse.org/henshin/

Eclipse Henshin

in the last lecture...

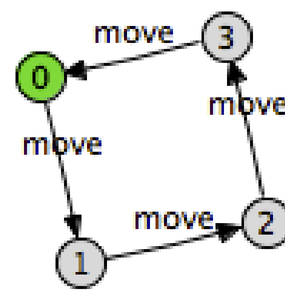
- An Eclipse project that supports the modeling, execution, and analysis of EMF-based graph transformation systems
  - https://www.eclipse.org/henshin/



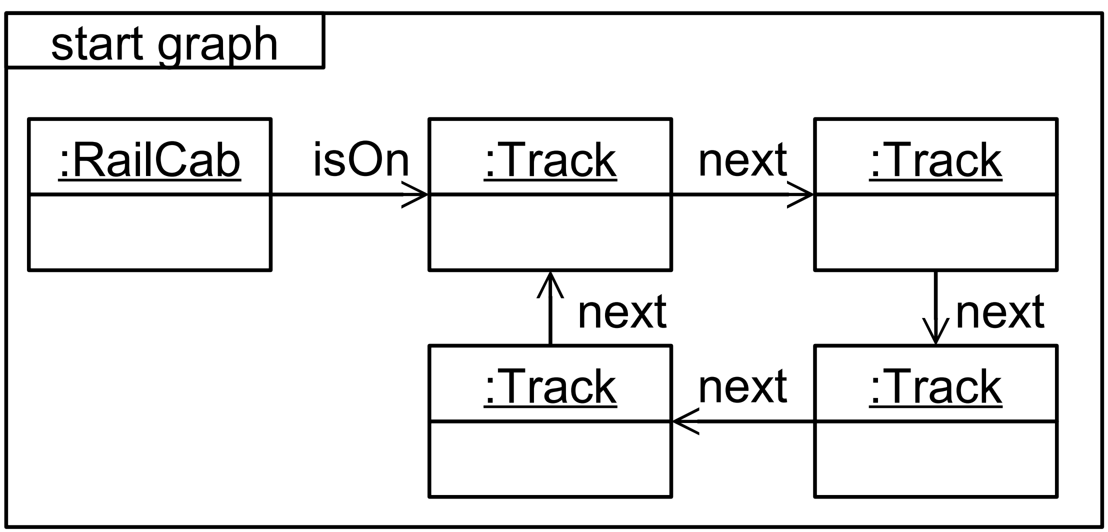
Exploring the State Space

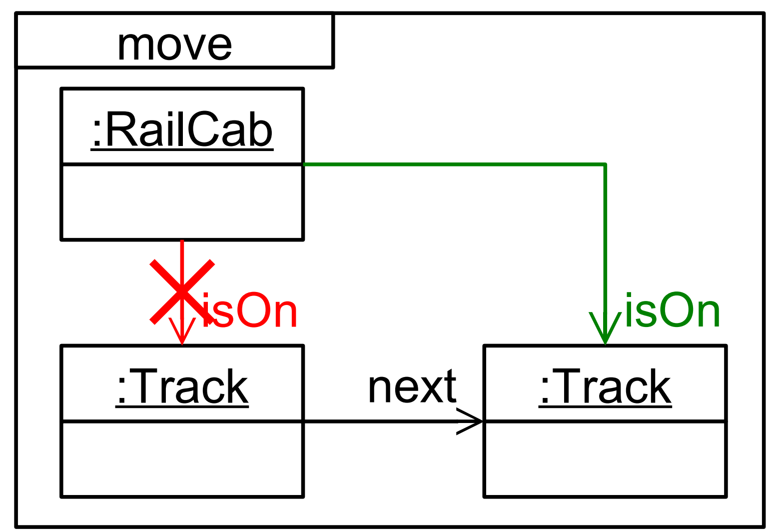
in the last lecture...

- A rule application can be considered a transition in a Labeled Transition System
  - source state: host graph before the rule application
  - transition: rule application
  - target state: host graph after the rule application



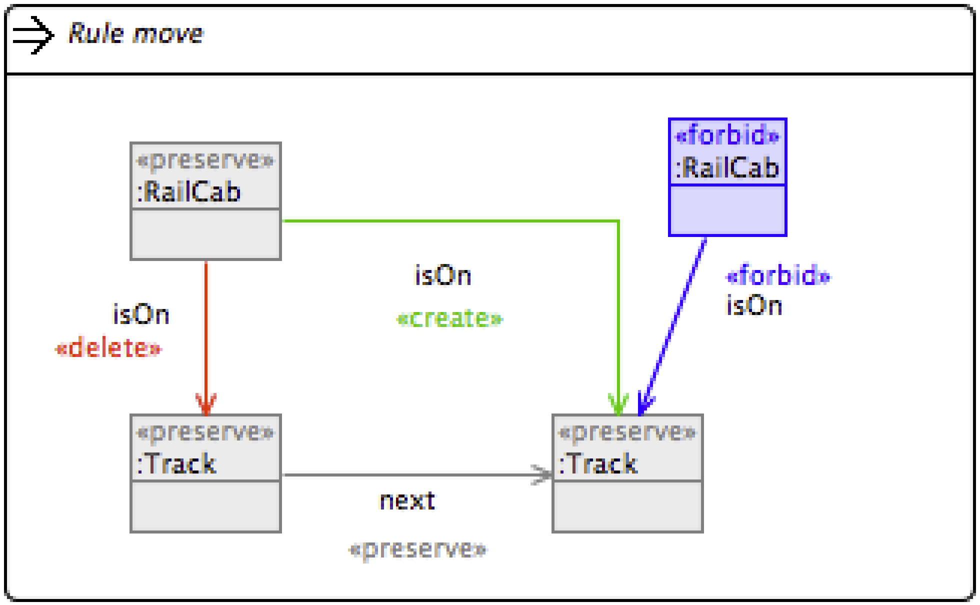
state space explored with Henshin: 4 different graphs; (graph after 4 applications of move rule is isomorphic⇒equal to the first)



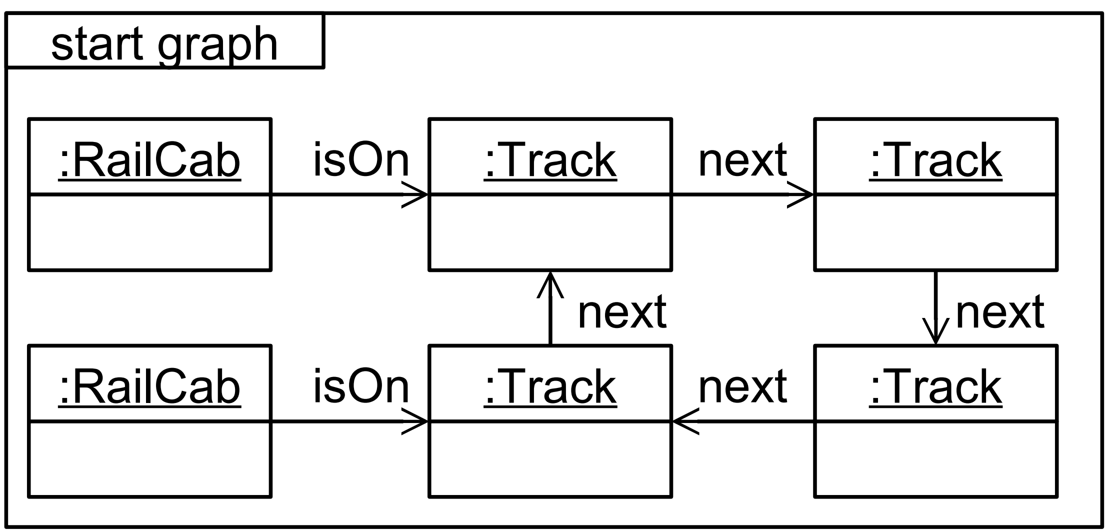


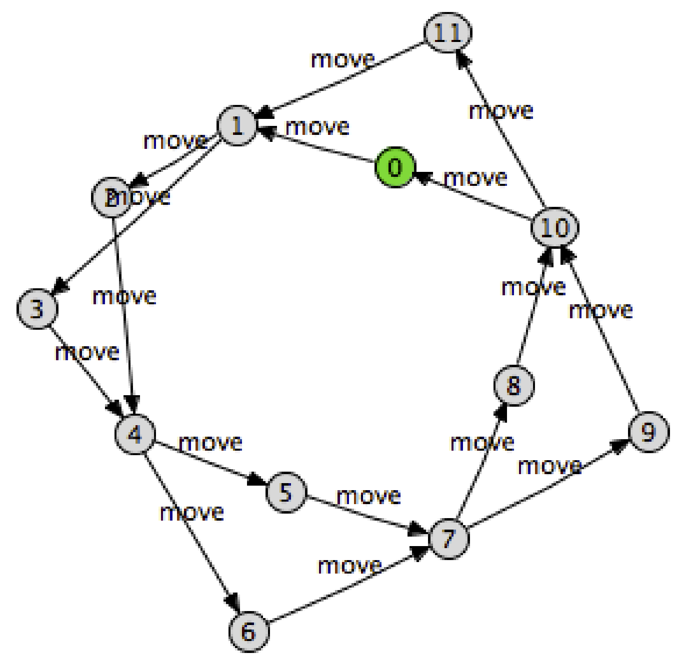
Exploring the State Space

in the last lecture...



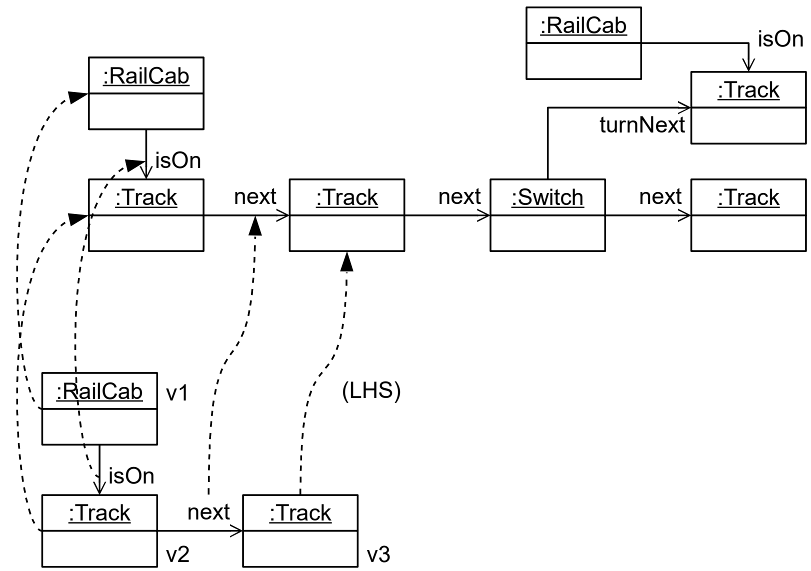
rule as specified in Henshin





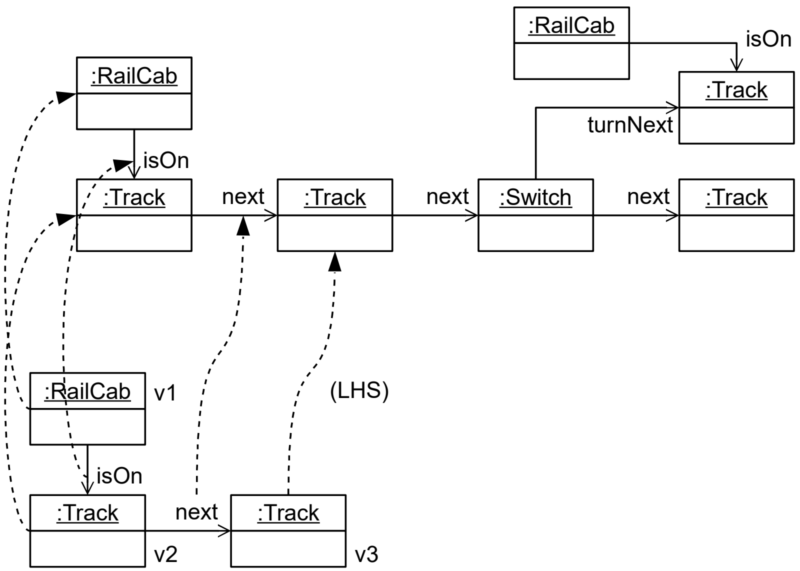
Graph Transformations More Formally

- A **match** of a rule graph in a host graph is a **typed graph isomorphism** between the rule graph and a subgraph of the host graph



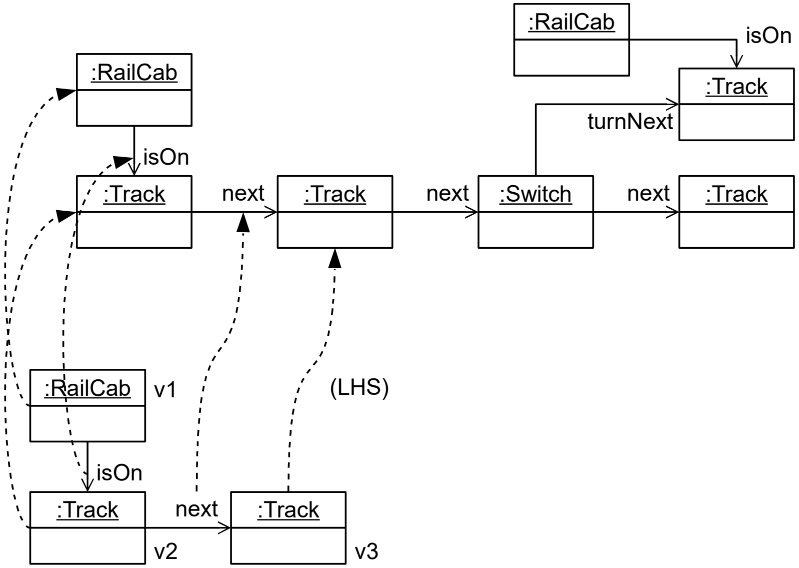
Graph Transformations More Formally

- A **match** of a rule graph in a host graph is a **typed graph isomorphism** between the rule graph and a subgraph of the host graph
  - What is a morphism?



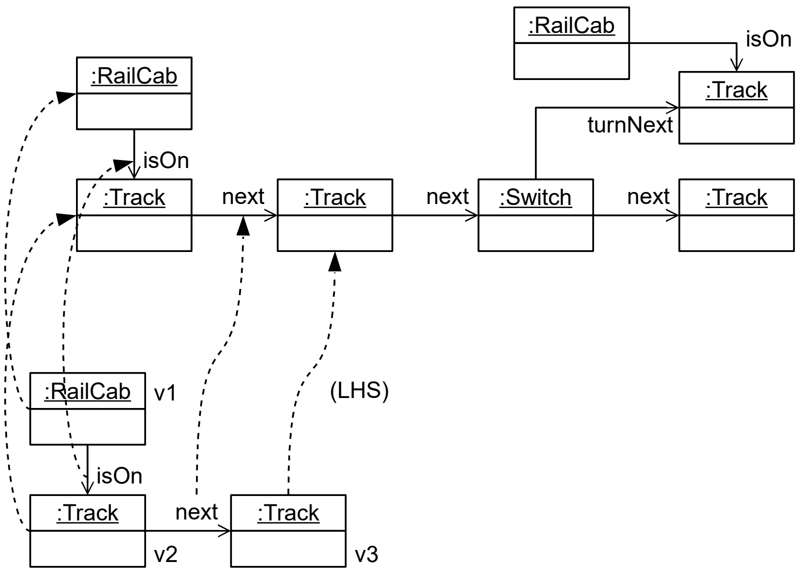
Graph Transformations More Formally

- A **match** of a rule graph in a host graph is a **typed graph isomorphism** between the rule graph and a subgraph of the host graph
  - What is a morphism?
  - What is a graph morphism?



Graph Transformations More Formally

- A **match** of a rule graph in a host graph is a **typed graph isomorphism** between the rule graph and a subgraph of the host graph
  - What is a morphism?
  - What is a graph morphism?
  - What is a graph isomorphism?



Graph Transformations More Formally

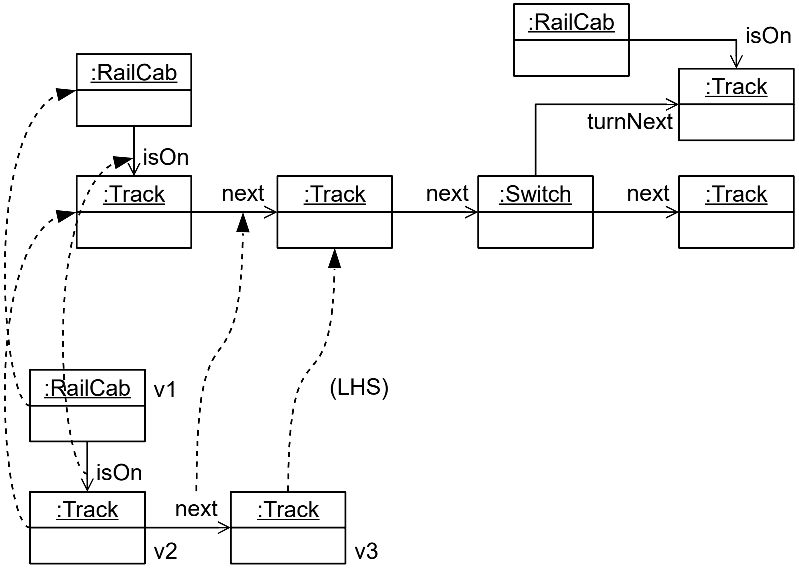
- A **match** of a rule graph in a host graph is a **typed graph isomorphism** between the rule graph and a subgraph of the host graph

  – What is a morphism?

  – What is a graph morphism?

  – What is a graph isomorphism?

  – What is a typed graph isomorphism?



Morphisms (Background)

- In mathematics, a **morphism** is a **structure-preserving mapping** from one mathematical structure to another

Morphisms (Background)

- In mathematics, a **morphism** is a **structure-preserving mapping** from one mathematical structure to another

- Example: A **group (homo)morphism** is a function that maps one group to another in an **operation-preserving** way

Morphisms (Background)

- In mathematics, a **morphism** is a **structure-preserving mapping** from one mathematical structure to another

- Example: A **group (homo)morphism** is a function that maps one group to another in an **operation-preserving** way
  - **group**: a **set** of elements and an **operation** that maps any two elements from that set to a third element from that set

Morphisms (Background)

- In mathematics, a **morphism** is a **structure-preserving mapping** from one mathematical structure to another

- Example: A **group (homo)morphism** is a function that maps one group to another in an **operation-preserving** way
  - **group**: a **set** of elements and an **operation** that maps any two elements from that set to a third element from that set
    - **example**: natural numbers with addition $(\mathbb{N}, +)$

Morphisms (Background)

- In mathematics, a **morphism** is a **structure-preserving mapping** from one mathematical structure to another

- Example: A **group (homo)morphism** is a function that maps one group to another in an **operation-preserving** way
  - **group**: a **set** of elements and an **operation** that maps any two elements from that set to a third element from that set
    - **example**: natural numbers with addition $(\mathbb{N}, +)$
  - given two groups $(G_1, *)$ and $(G_2, \#)$, a **homomorphism** $h: G_1 \to G_2$ is an operation-preserving function, i.e., for $a, b \in G_1$ it holds that $h(a * b) = h(a) \# h(b)$

Morphisms (Background)

- In mathematics, a **morphism** is a **structure-preserving mapping** from one mathematical structure to another

- Example: A **group (homo)morphism** is a function that maps one group to another in an **operation-preserving** way
  - **group**: a **set** of elements and an **operation** that maps any two elements from that set to a third element from that set
    - **example**: natural numbers with addition $(\mathbb{N}, +)$
  - given two groups $(G_1, *)$ and $(G_2, \#)$, a **homomorphism** $h: G_1 \rightarrow G_2$ is an operation-preserving function, i.e., for $a, b \in G_1$ it holds that $h(a * b) = h(a) \# h(b)$
    - **example**: given (STRING, ·) and $(\mathbb{N}_{\geq 0}, +)$, then $length: STRING \rightarrow \mathbb{N}$ is a homomorphism, since for two strings a and b, it holds that $length(a \cdot b) = length(a) + length(b)$ *(" · " means the concatenation of two strings)*

Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where

Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)

Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges

Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \to V$ is the source function, defining edges' source nodes

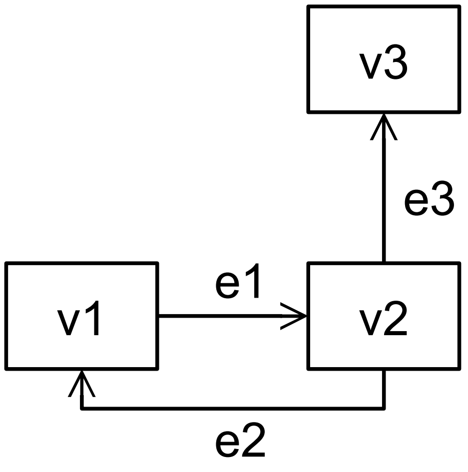
Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \rightarrow V$ is the source function, defining edges' source nodes
  - $t: E \rightarrow V$ is the target function, defining edges' target nodes

Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \rightarrow V$ is the source function, defining edges' source nodes
  - $t: E \rightarrow V$ is the target function, defining edges' target nodes

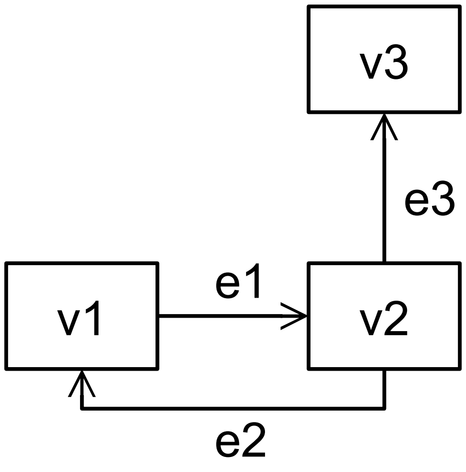
- Example: How to interpret this graph mathematically?



Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \rightarrow V$ is the source function, defining edges' source nodes
  - $t: E \rightarrow V$ is the target function, defining edges' target nodes

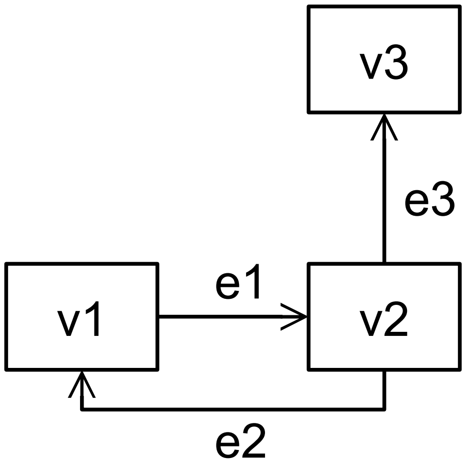
- Example: How to interpret this graph mathematically?
  - $V = \{v1, v2, v3\}$



Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \rightarrow V$ is the source function, defining edges' source nodes
  - $t: E \rightarrow V$ is the target function, defining edges' target nodes

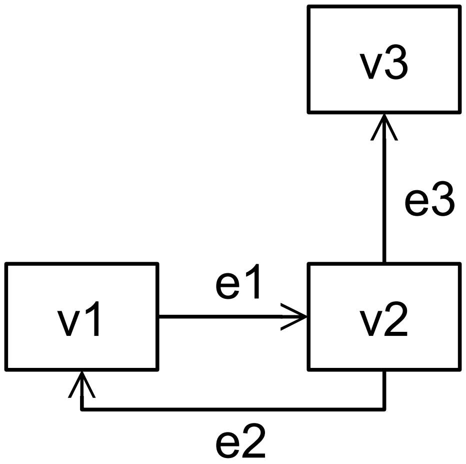
- Example: How to interpret this graph mathematically?
  - $V = \{v1, v2, v3\}$
  - $E = \{e1, e2, e3\}$



Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \rightarrow V$ is the source function, defining edges' source nodes
  - $t: E \rightarrow V$ is the target function, defining edges' target nodes

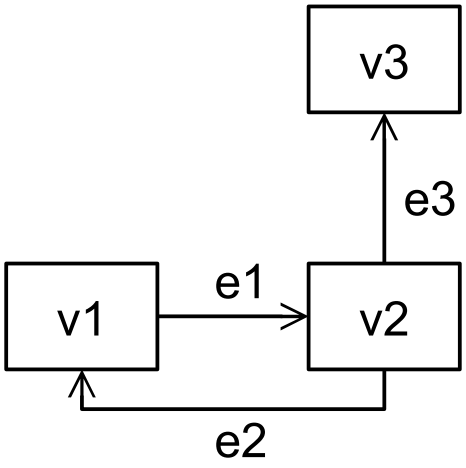
- Example: How to interpret this graph mathematically?
  - $V = \{v1, v2, v3\}$
  - $E = \{e1, e2, e3\}$
  - $s = \{(e1, v1), (e2, v2), (e3, v2)\}$



Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \rightarrow V$ is the source function, defining edges' source nodes
  - $t: E \rightarrow V$ is the target function, defining edges' target nodes

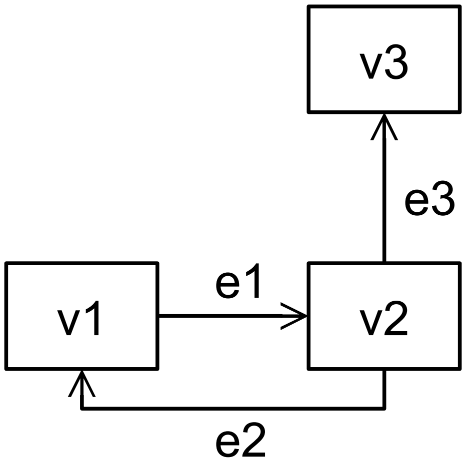
- Example: How to interpret this graph mathematically?
  - $V = \{v1, v2, v3\}$
  - $E = \{e1, e2, e3\}$
  - $s = \{(e1, v1), (e2, v2), (e3, v2)\}$
  - $t = \{(e1, v2), (e2, v1), (e3, v3)\}$



Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \rightarrow V$ is the source function, defining edges' source nodes
  - $t: E \rightarrow V$ is the target function, defining edges' target nodes

- Example: How to interpret this graph mathematically?
  - $V = \{v1, v2, v3\}$
  - $E = \{e1, e2, e3\}$
  - $s = \{(e1, v1), (e2, v2), (e3, v2)\}$
  - $t = \{(e1, v2), (e2, v1), (e3, v3)\}$

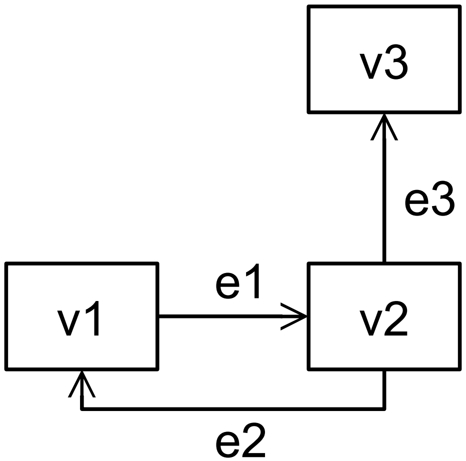


Definition: Graph

- We define a graph G as a tuple $G = (V, E, s, t)$ where
  - V is a finite set of nodes (vertices)
  - E is a finite set of edges
  - $s: E \to V$ is the source function, defining edges' source nodes
  - $t: E \to V$ is the target function, defining edges' target nodes

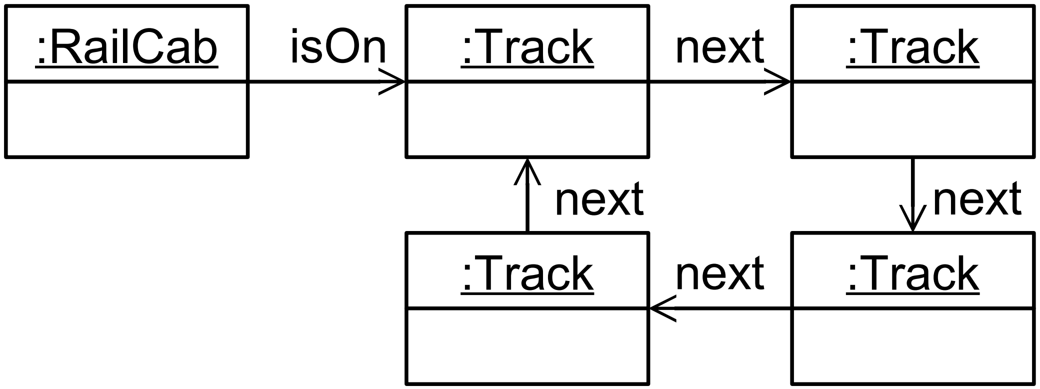
- Example: How to interpret this graph mathematically?
  - $V = \{v1, v2, v3\}$
  - $E = \{e1, e2, e3\}$
  - $s = \{(e1, v1), (e2, v2), (e3, v2)\}$
  - $t = \{(e1, v2), (e2, v1), (e3, v3)\}$

  We also write for example $s(e1) = v2$



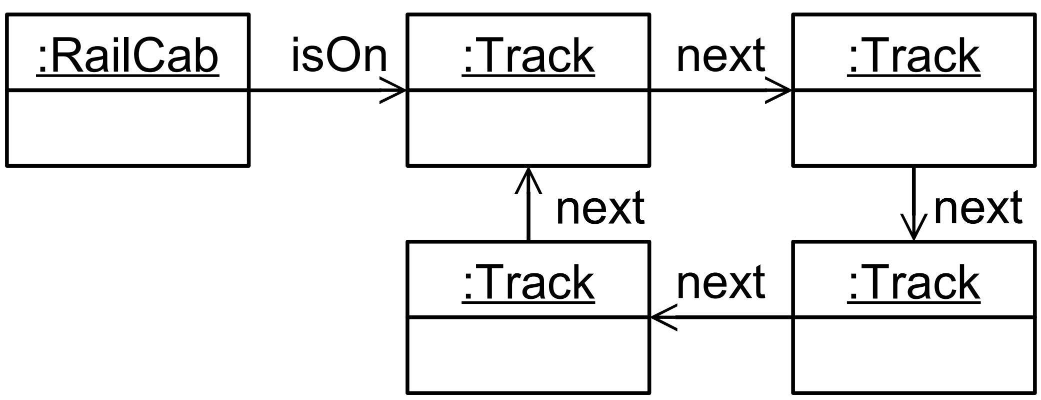
Labeled Graph

- Problem: How to formalize the following graph?



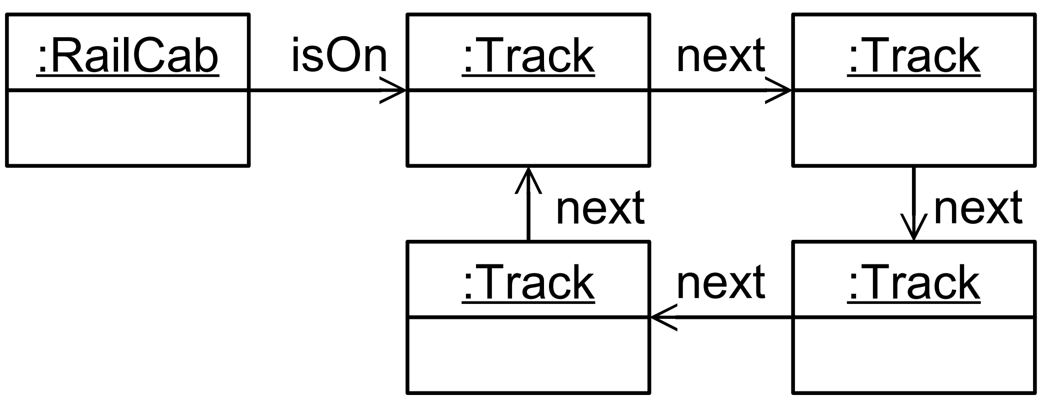
Labeled Graph

- Problem: How to formalize the following graph?
  - multiple nodes called “:Track”



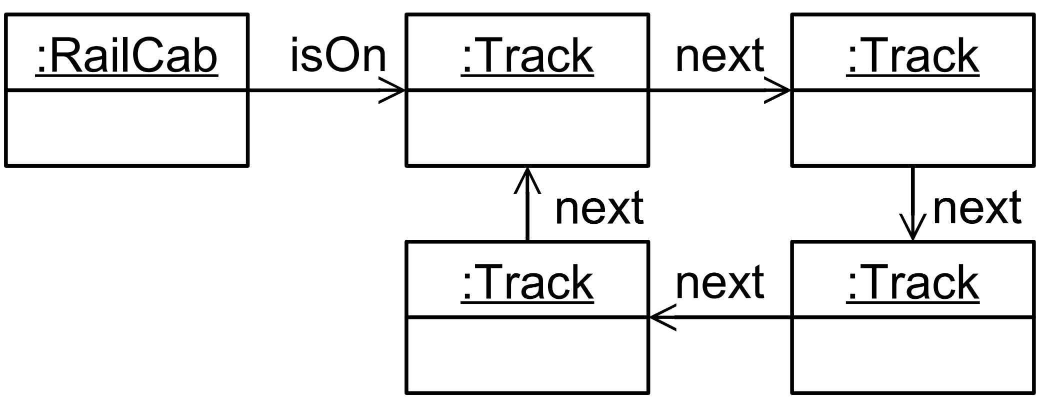
Labeled Graph

- Problem: How to formalize the following graph?
  - multiple nodes called “:Track”
  - multiple edges called “isOn”



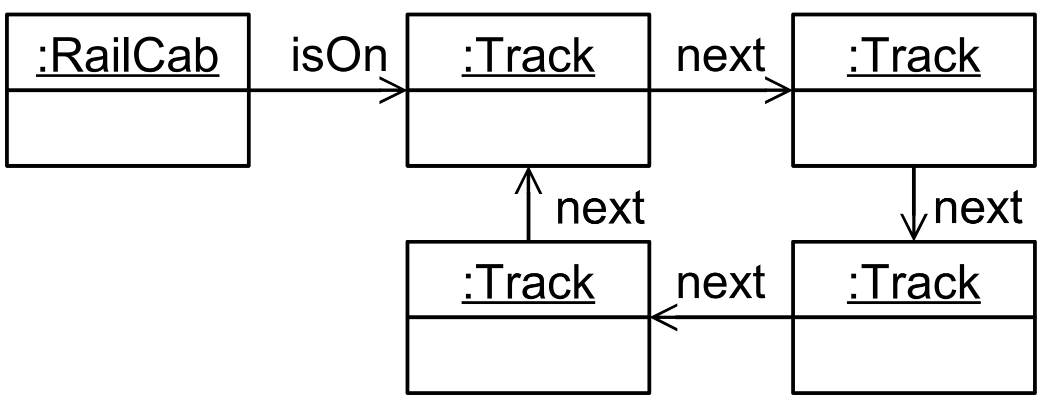
Labeled Graph

- Problem: How to formalize the following graph?
  - multiple nodes called “:Track”
  - multiple edges called “isOn”
- Element names are the same, but identities are not



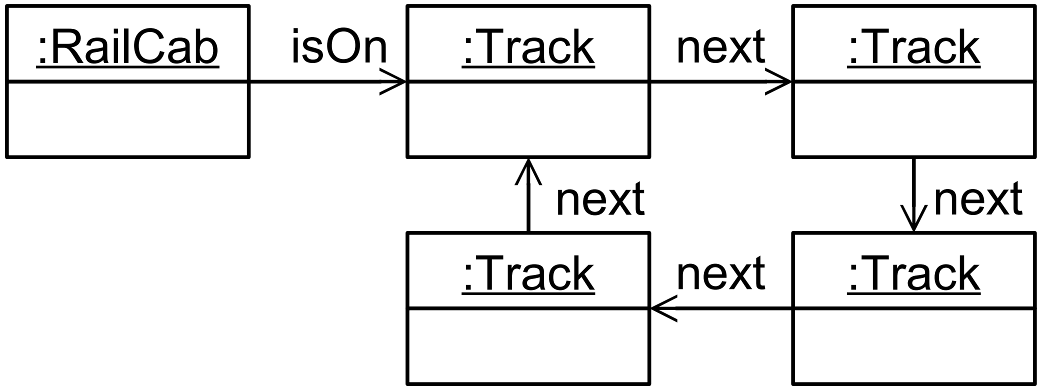
Labeled Graph

- Problem: How to formalize the following graph?
  - multiple nodes called “:Track”
  - multiple edges called “isOn”
- Element names are the same, but identities are not
- We cannot have a set with the same element occurring multiple times, e.g. $\cancel{E = \{isOn, next, next, next, next\}}$



Labeled Graph

- Problem: How to formalize the following graph?
  - multiple nodes called “:Track”
  - multiple edges called “isOn”
- Element names are the same, but identities are not
- We cannot have a set with the same element occurring multiple times, e.g. ~~$E = \{isOn, next, next, next, next\}$~~
- Solution: Model labels explicitly



Labeled Graph

- **Labels**: Giving names to nodes and edges

Labeled Graph

- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E

Labeled Graph

- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function

Labeled Graph

- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function
  - L_E: $V \rightarrow \Sigma$ is an edge labeling function

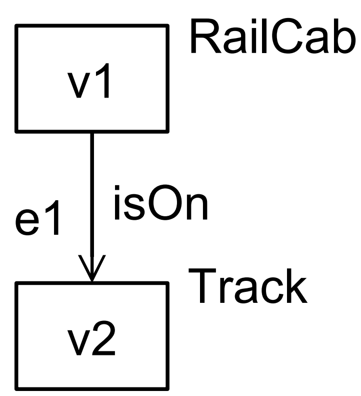
Labeled Graph

- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function
  - L_E: $V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels

Labeled Graph

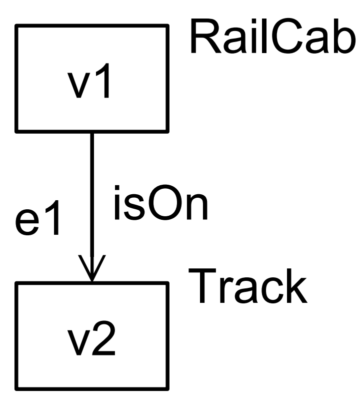
- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function
  - L_E: $V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels

- Example: How to interpret the given graph mathematically?



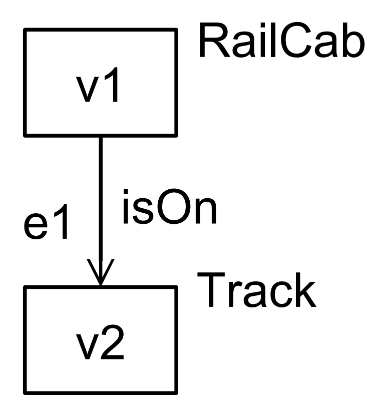
Labeled Graph

- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - $L_V: V \rightarrow \Sigma$ is a node labeling function
  - $L_E: V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels
- Example: How to interpret the given graph mathematically?
  - $V = \{v1, v2\}$



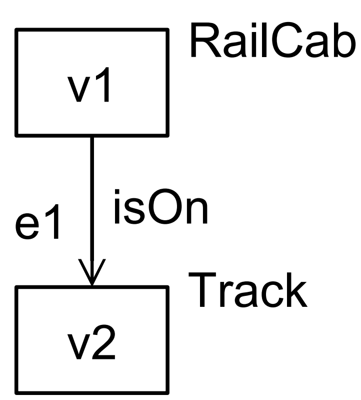
Labeled Graph

- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function
  - L_E: $V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels
- Example: How to interpret the given graph mathematically?
  - $V = \{v1, v2\}$
  - $E = \{e1\}$



Labeled Graph

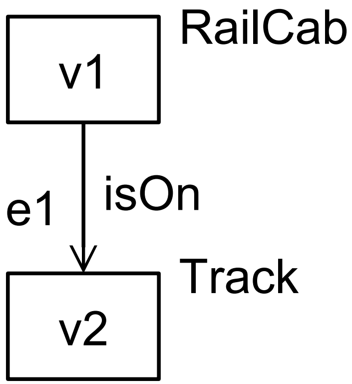
- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function
  - L_E: $V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels
- Example: How to interpret the given graph mathematically?
  - $V = \{v1, v2\}$
  - $E = \{e1\}$
  - $s = \{(e1, v1)\}$



Labeled Graph

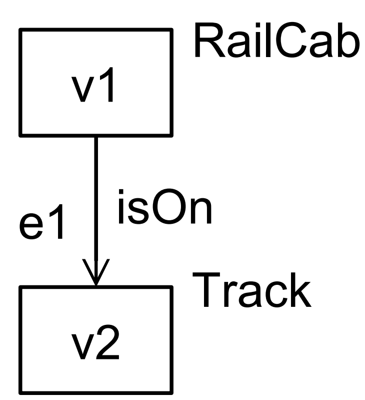
- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function
  - L_E: $V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels

- Example: How to interpret the given graph mathematically?
  - $V = \{v1, v2\}$
  - $E = \{e1\}$
  - $s = \{(e1, v1)\}$
  - $t = \{(e1, v2)\}$



Labeled Graph

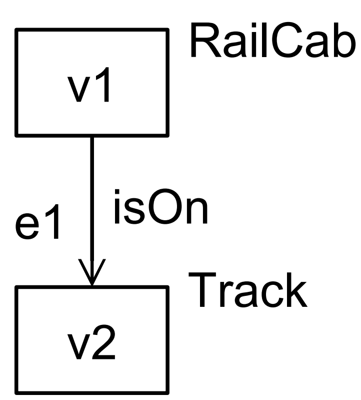
- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - $L_V: V \rightarrow \Sigma$ is a node labeling function
  - $L_E: V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels
- Example: How to interpret the given graph mathematically?
  - $V = \{v1, v2\}$
  - $E = \{e1\}$
  - $s = \{(e1, v1)\}$
  - $t = \{(e1, v2)\}$
  - $L_V = \{(v1, RailCab), (v2, Track)\}$



Labeled Graph

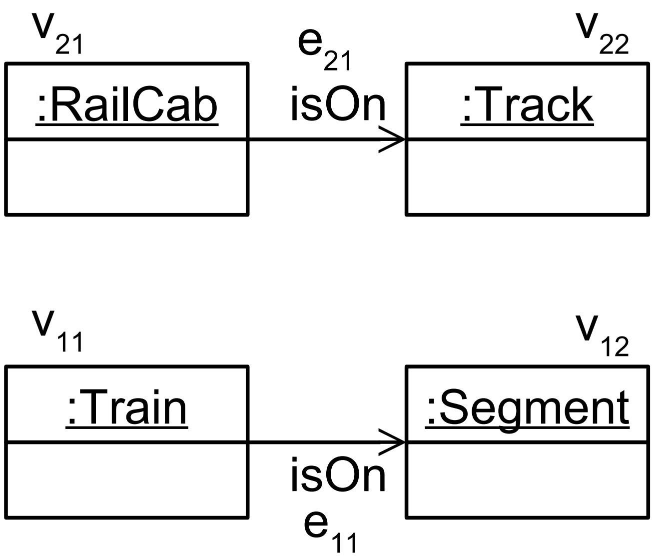
- **Labels**: Giving names to nodes and edges
- A graph G can be extended by labeling functions L_V and L_E
  - L_V: $V \rightarrow \Sigma$ is a node labeling function
  - L_E: $V \rightarrow \Sigma$ is an edge labeling function
  - Σ: is a set of labels

- Example: How to interpret the given graph mathematically?
  - $V = \{v1, v2\}$
  - $E = \{e1\}$
  - $s = \{(e1, v1)\}$
  - $t = \{(e1, v2)\}$
  - $L_V = \{(v1, RailCab), (v2, Track)\}$
  - $L_E = \{(e1, isOn)\}$



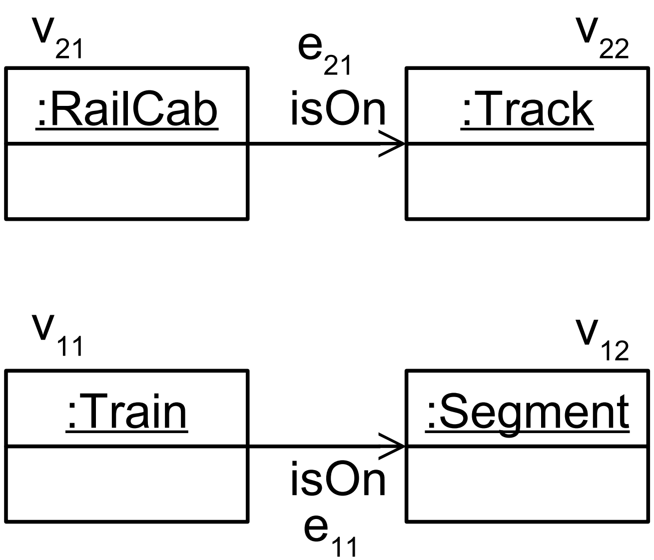
Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$



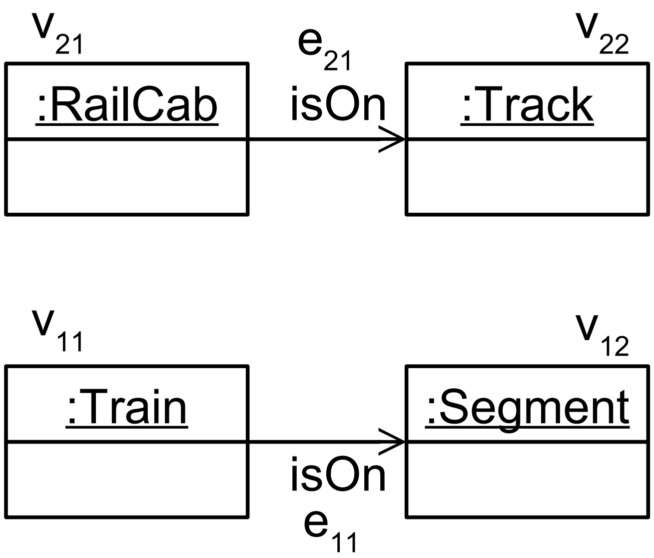
Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$



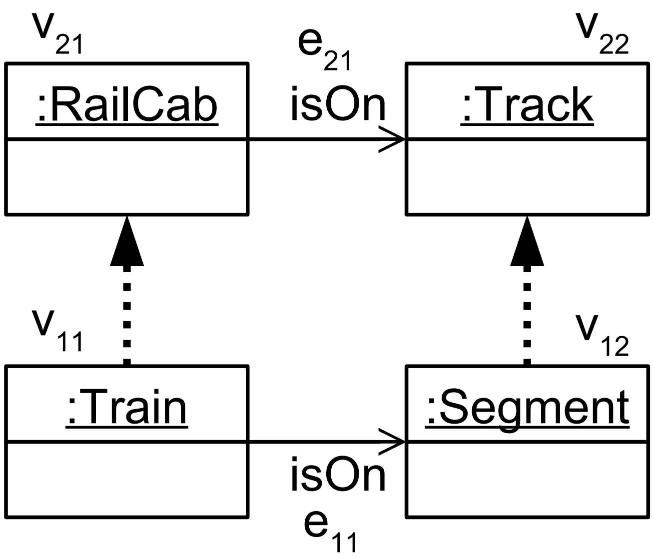
Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$



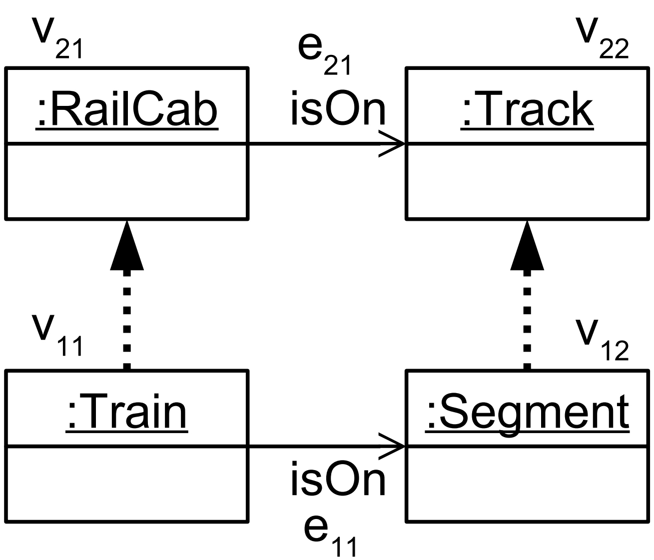
Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f\colon G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V\colon V_1 \rightarrow V_2$



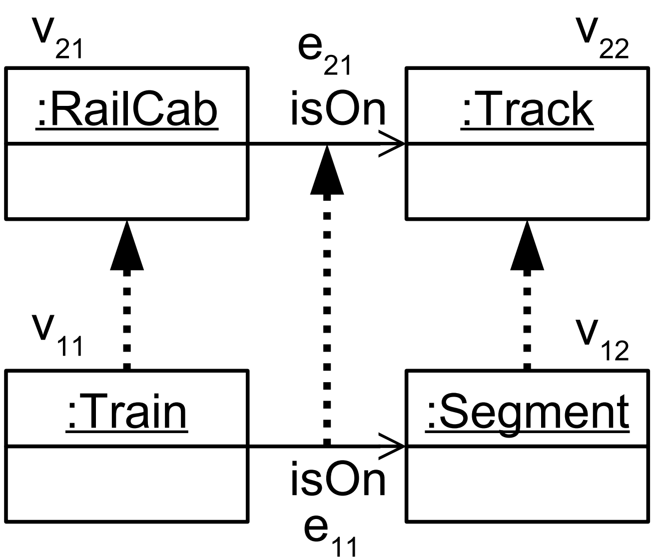
Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \to G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \to V_2$
  - $f_E: E_1 \to E_2$



Graph Morphism

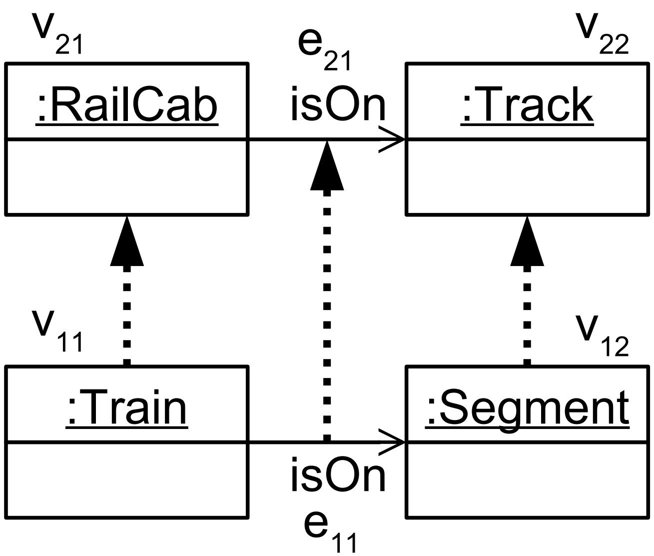
- Given two graphs $G_i = (V_i, E_i, s_i, t_i)$, $i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

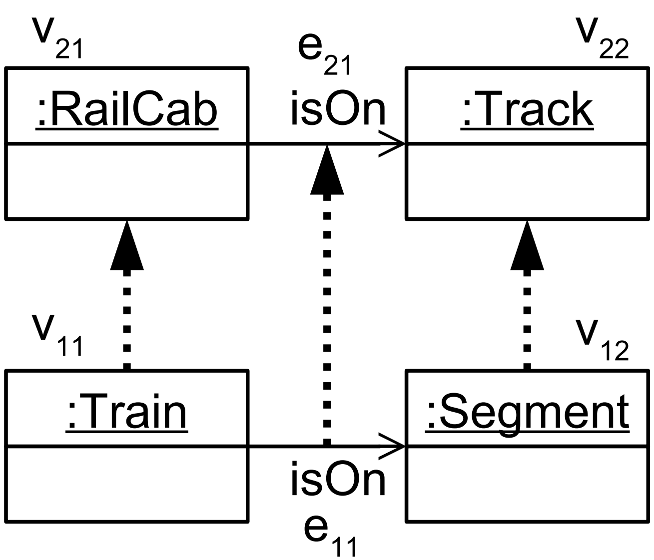
  that preserve the source and target functions, i.e.,



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \to G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \to V_2$
  - $f_E: E_1 \to E_2$

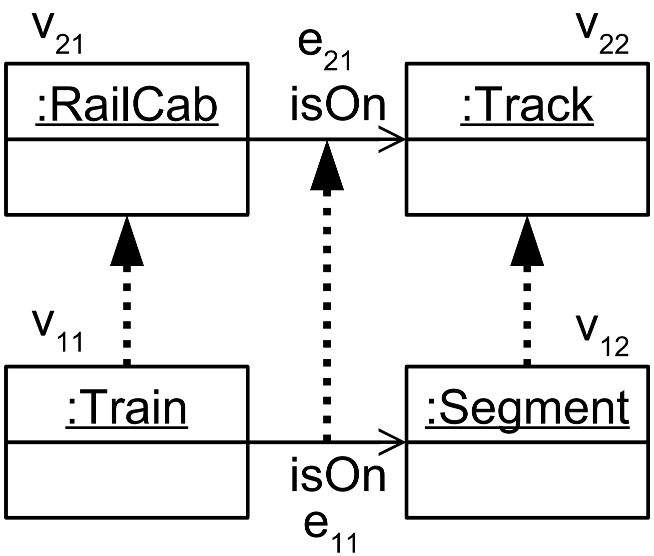
  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f\colon G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V\colon V_1 \rightarrow V_2$
  - $f_E\colon E_1 \rightarrow E_2$

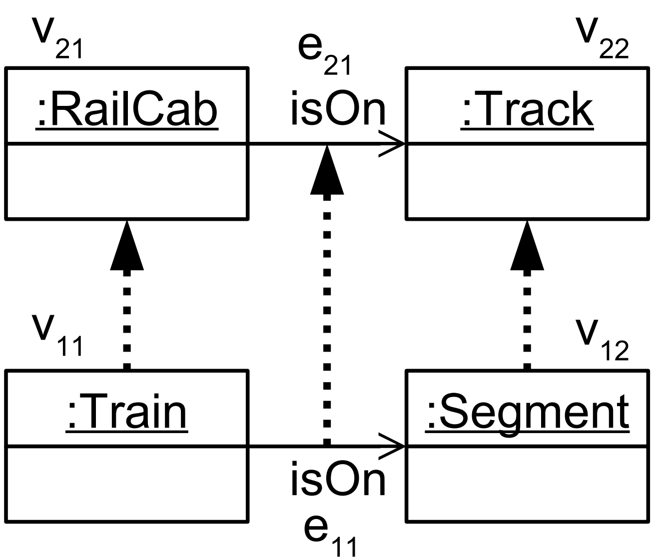
  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

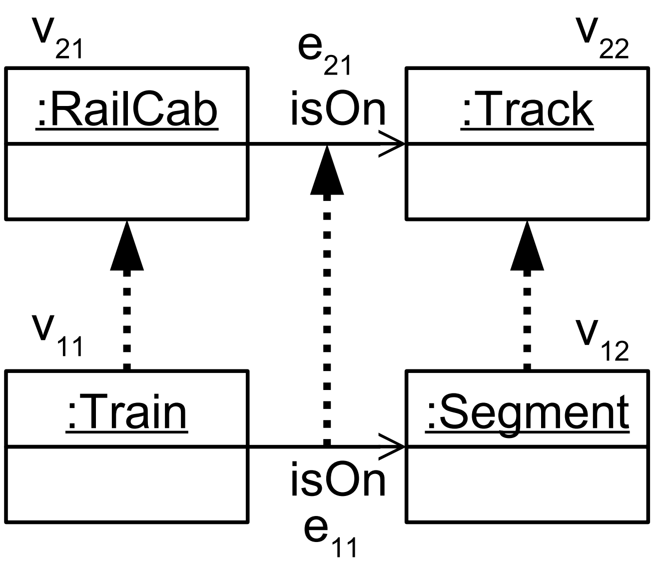
  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

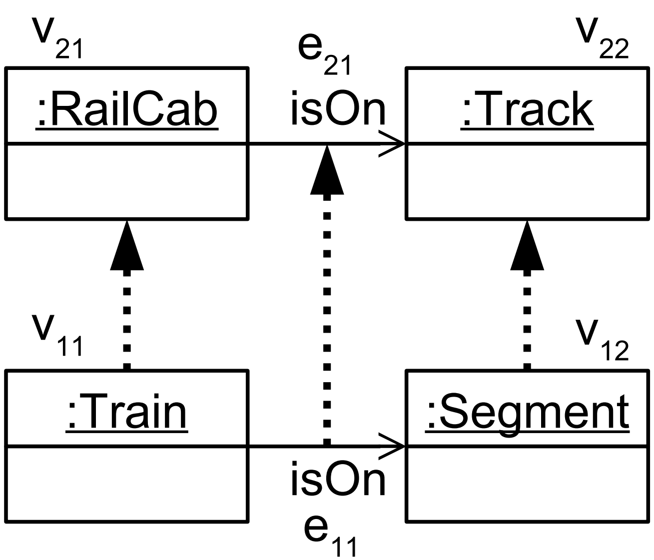
  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$
  consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

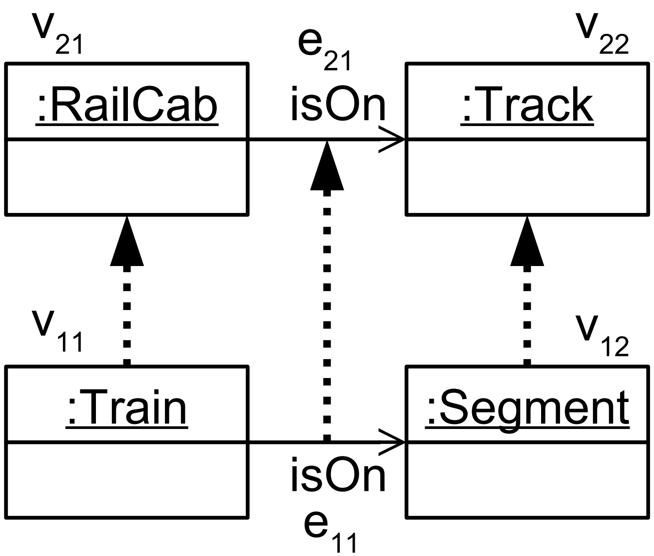


Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:



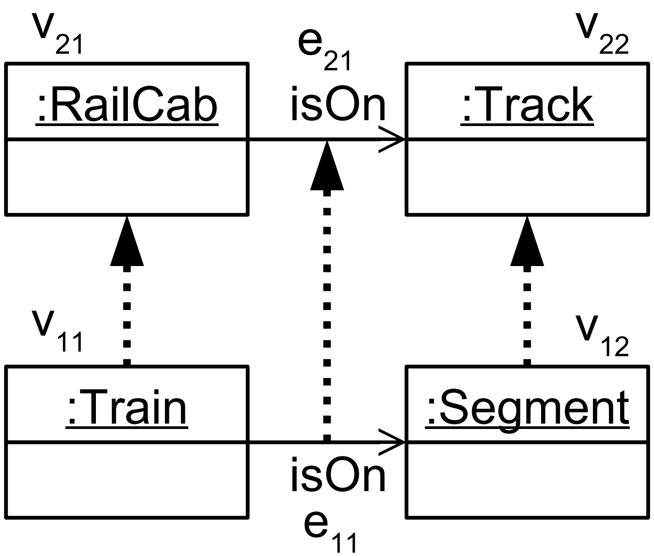
Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:

$f_V(s_1(e11))$



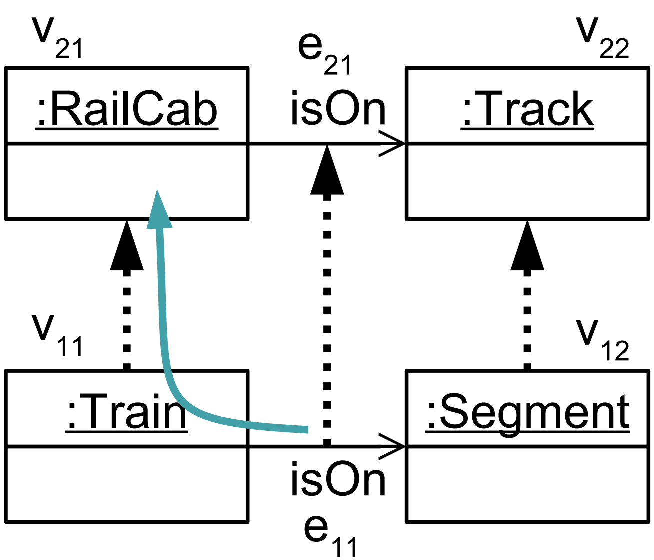
Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:

$f_V(s_1(e11))$



Graph Morphism

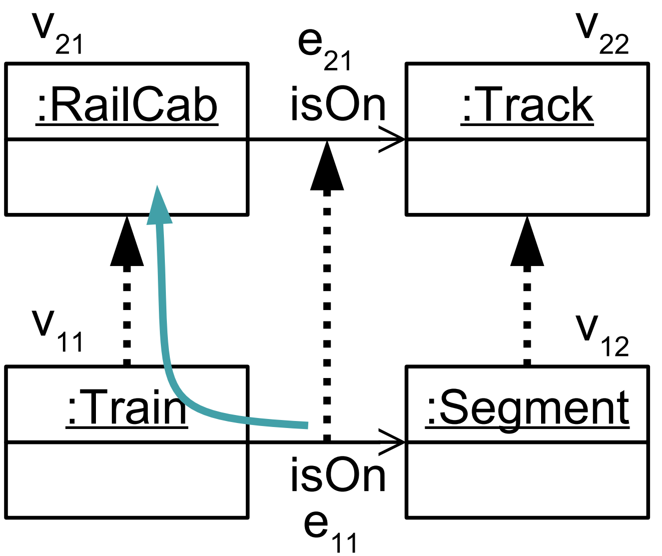
- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:

$f_V(s_1(e11))$

$= s_2(f_E(e11))$

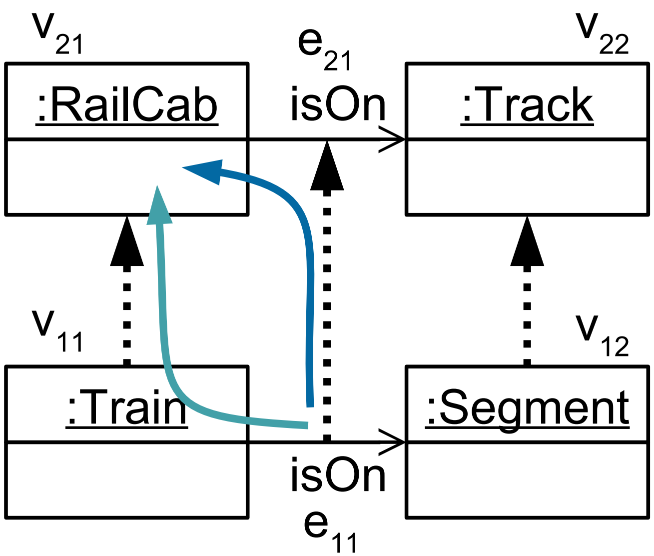


Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:
$f_V(s_1(e11))$
$= s_2(f_E(e11))$



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

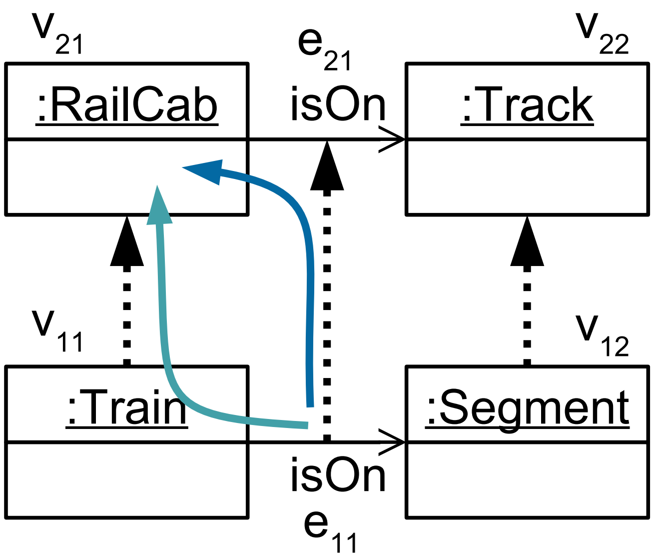
  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:

$f_V(s_1(e11))$

$= s_2(f_E(e11))$

$f_V(t_1(e11))$



Graph Morphism

- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f\colon G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V\colon V_1 \rightarrow V_2$
  - $f_E\colon E_1 \rightarrow E_2$

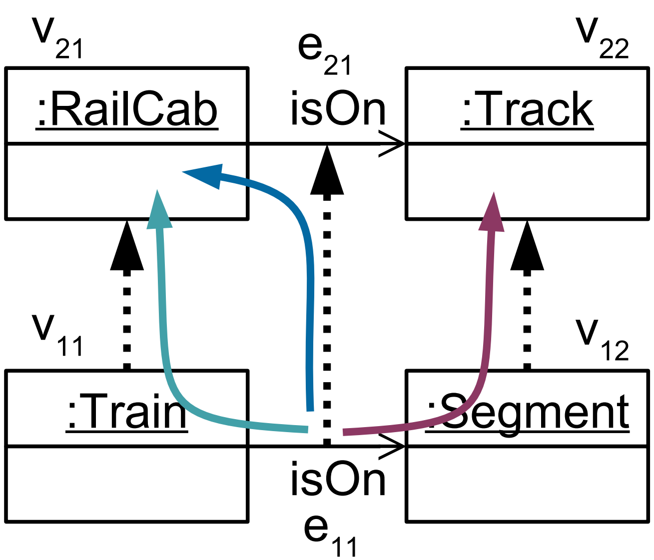
  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:

$f_V(s_1(e11))$

$= s_2(f_E(e11))$

$f_V(t_1(e11))$



Graph Morphism

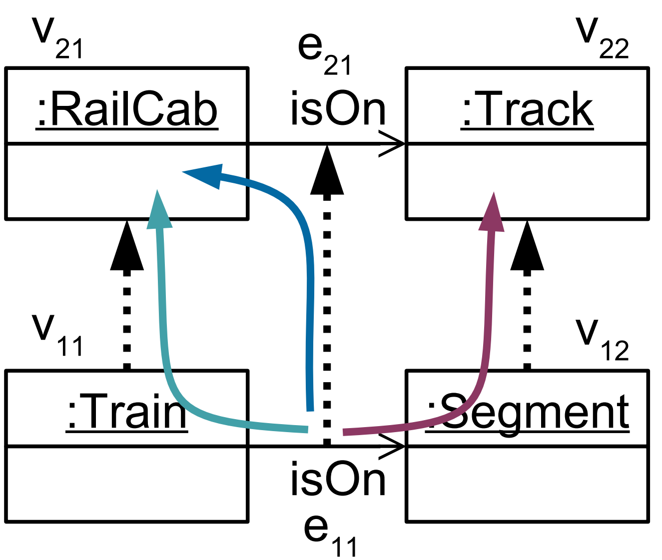
- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:

$f_V(s_1(e11))$
$= s_2(f_E(e11))$

$f_V(t_1(e11))$
$= t_2(f_E(e11))$



Graph Morphism

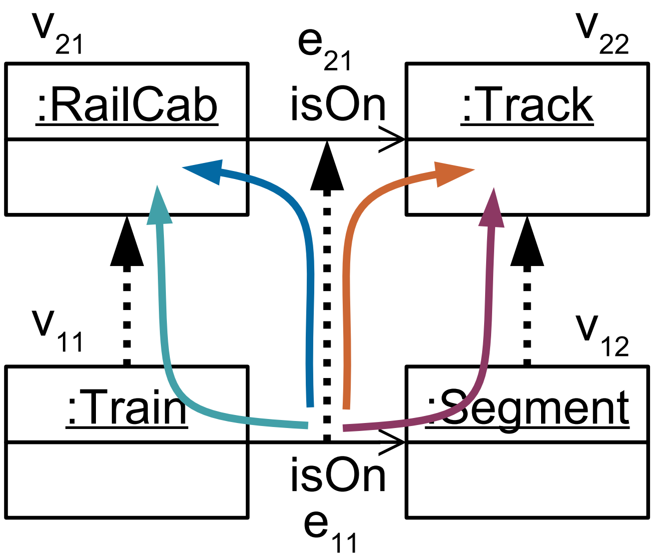
- Given two graphs $G_i = (V_i, E_i, s_i, t_i), i \in \{1, 2\}$
- A **graph morphism** $f: G_1 \rightarrow G_2$ consists of two functions $f = (f_V, f_E)$
  - $f_V: V_1 \rightarrow V_2$
  - $f_E: E_1 \rightarrow E_2$

  that preserve the source and target functions, i.e.,
  - $f_V \circ s_1 = s_2 \circ f_E$ and
  - $f_V \circ t_1 = t_2 \circ f_E$
- Example: for these two graphs a graph morphism would be
  - $f_V = \{(v_{11}, v_{21}), (v_{12}, v_{22})\}$
  - $f_E = \{(e_{11}, e_{21})\}$

Example:

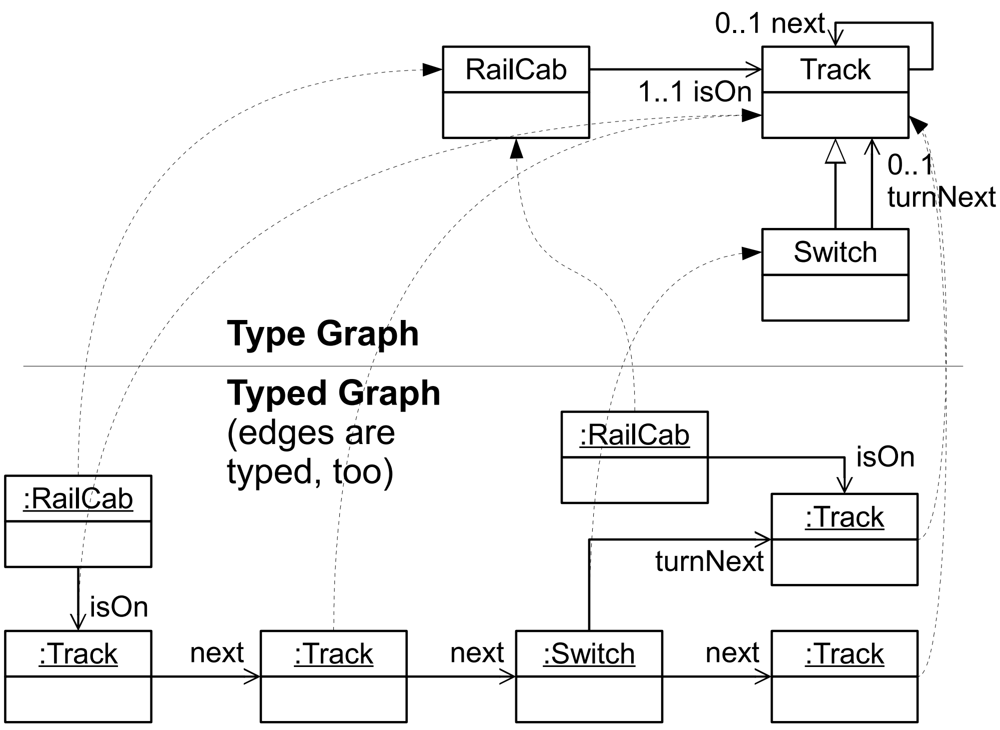
$f_V(s_1(e11))$
$= s_2(f_E(e11))$

$f_V(t_1(e11))$
$= t_2(f_E(e11))$



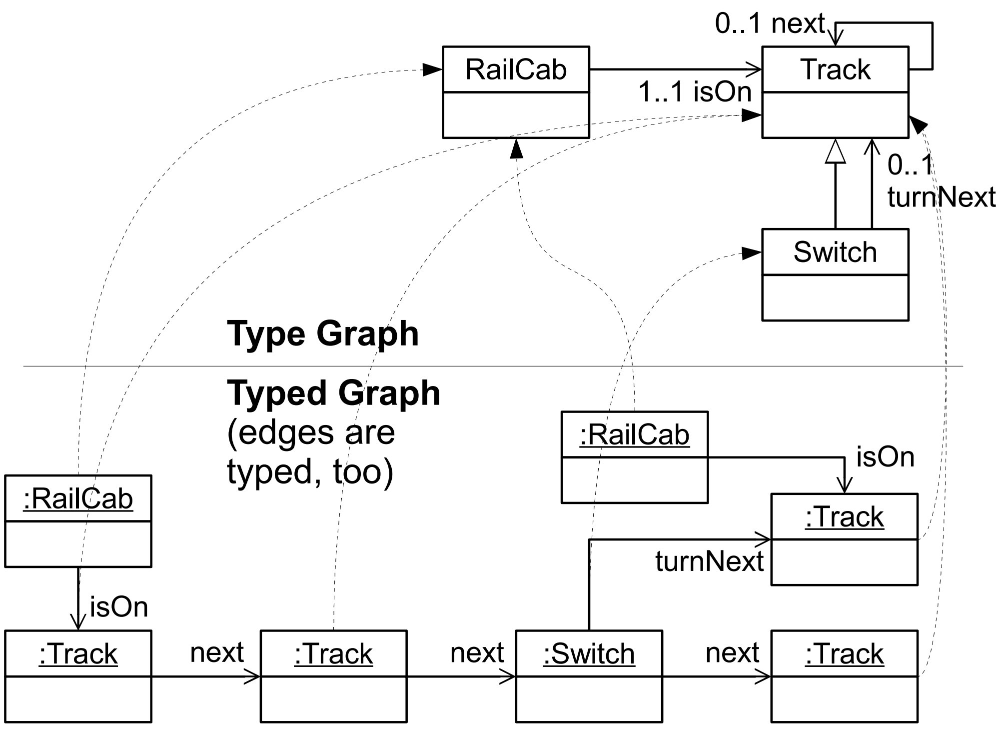
Typed Graph

- A graph G can be **typed** by giving a **graph morphism** $type: G \rightarrow G_{Type}$



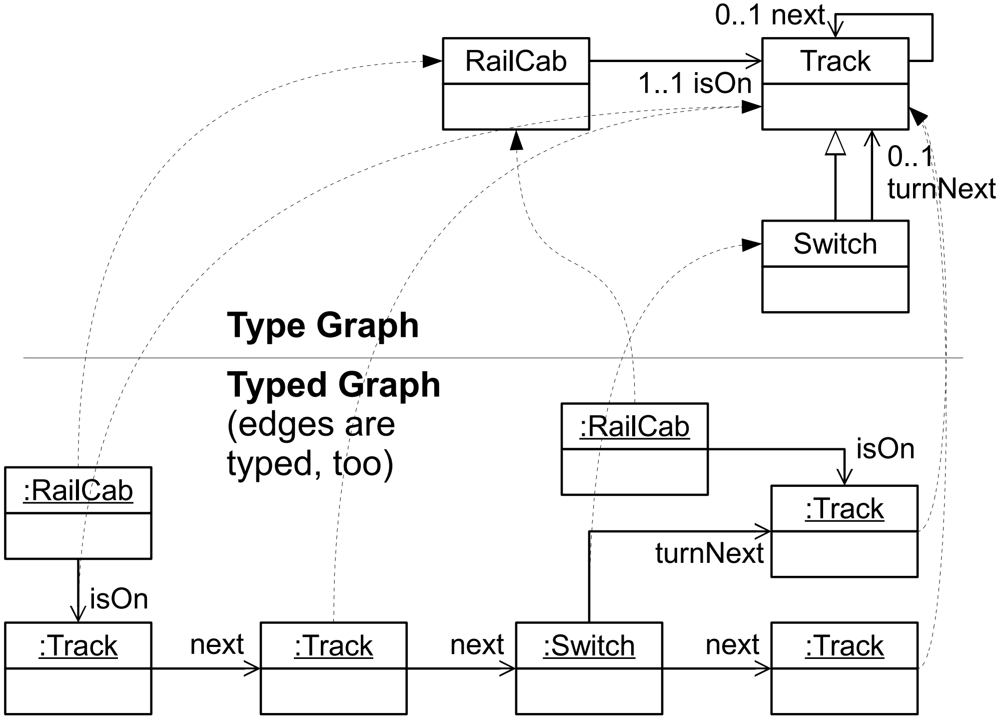
Typed Graph

- A graph G can be **typed** by giving a **graph morphism** $type: G \rightarrow G_{Type}$
  - G_{Type} is the **type graph,** the tuple $(G, type)$ is the **typed graph**



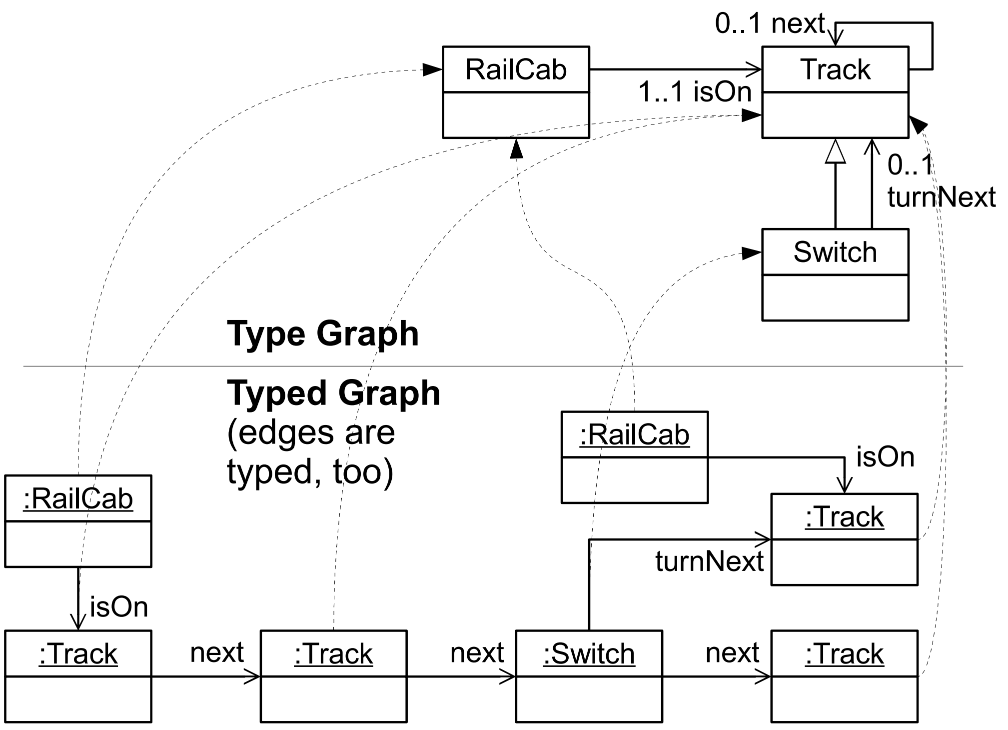
Typed Graph

- A graph G can be **typed** by giving a **graph morphism** $type: G \rightarrow G_{Type}$
  - G_{Type} is the **type graph,** the tuple $(G, type)$ is the **typed graph**
- A graph morphism (also **graph homomorphism**) is a **total function**



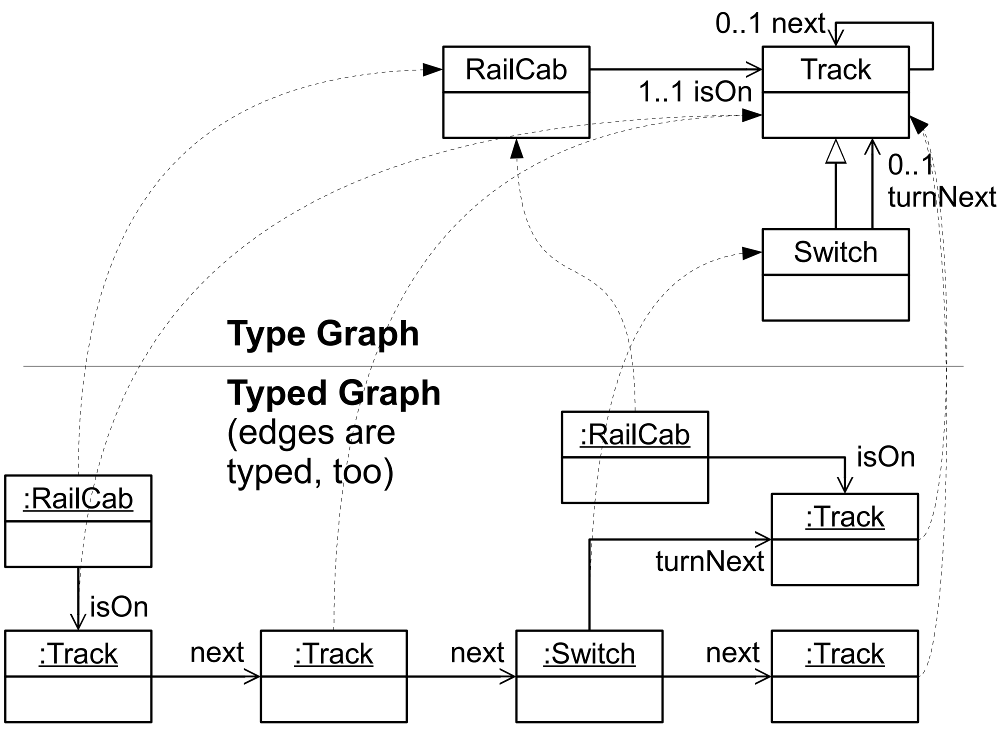
Typed Graph

- A graph G can be **typed** by giving a **graph morphism** $type: G \rightarrow G_{Type}$
  - G_{Type} is the **type graph,** the tuple $(G, type)$ is the **typed graph**
- A graph morphism (also **graph homomorphism**) is a **total function**
  - f_V and f_E are total



Typed Graph

- A graph G can be **typed** by giving a **graph morphism** $type\colon G \rightarrow G_{Type}$
  - G_{Type} is the **type graph,** the tuple $(G, type)$ is the **typed graph**
- A graph morphism (also **graph homomorphism**) is a **total function**
  - f_V and f_E are total
  - every element in the domain (typed graph) has to be related to exactly one element of the co-domain (type graph)



Graph Isomorphism

- A graph morphism $f = (f_V, f_E)$ is called a **graph isomorphism** if f_V and f_E are **bijective**
  - each element in the domain corresponds to exactly one element of the co-domain
  - the graph morphism is reversible

Graph Isomorphism

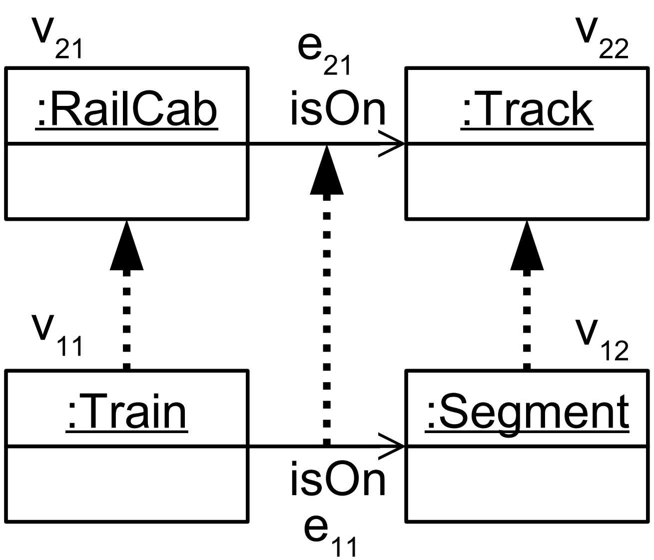
- A graph morphism $f = (f_V, f_E)$ is called a **graph isomorphism** if f_V and f_E are **bijective**
  - each element in the domain corresponds to exactly one element of the co-domain
  - the graph morphism is reversible

- Example:
  (from before)

Graph Isomorphism

- A graph morphism $f = (f_V, f_E)$ is called a **graph isomorphism** if f_V and f_E are **bijective**
  - each element in the domain corresponds to exactly one element of the co-domain
  - the graph morphism is reversible

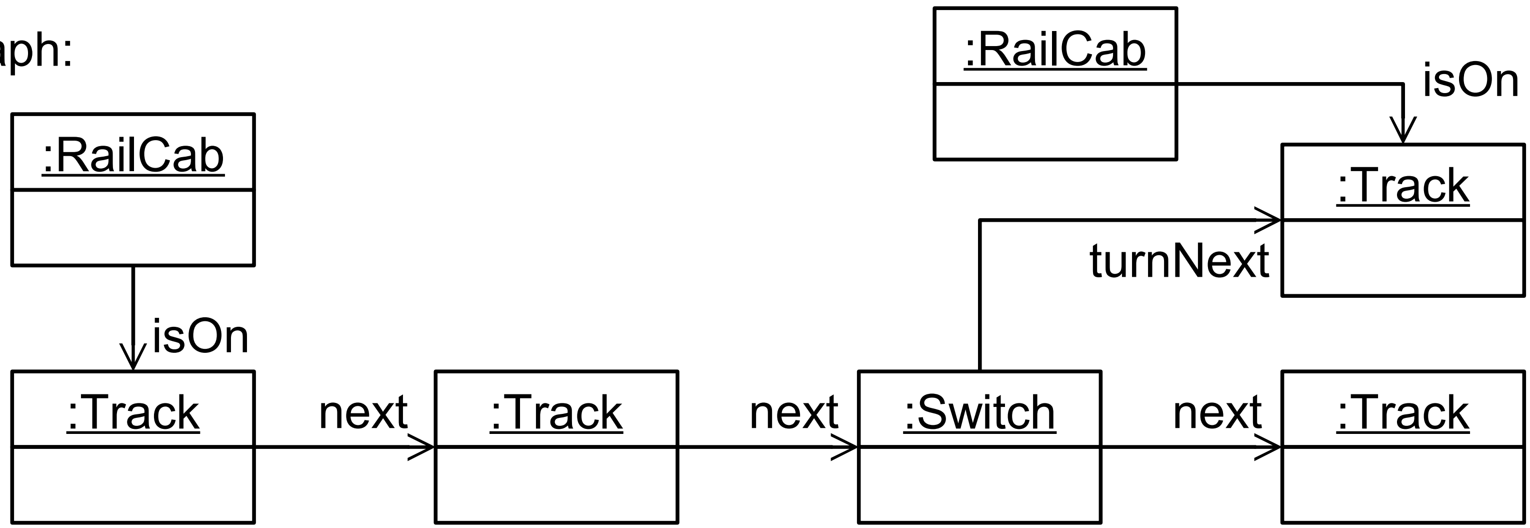
- Example:
(from before)

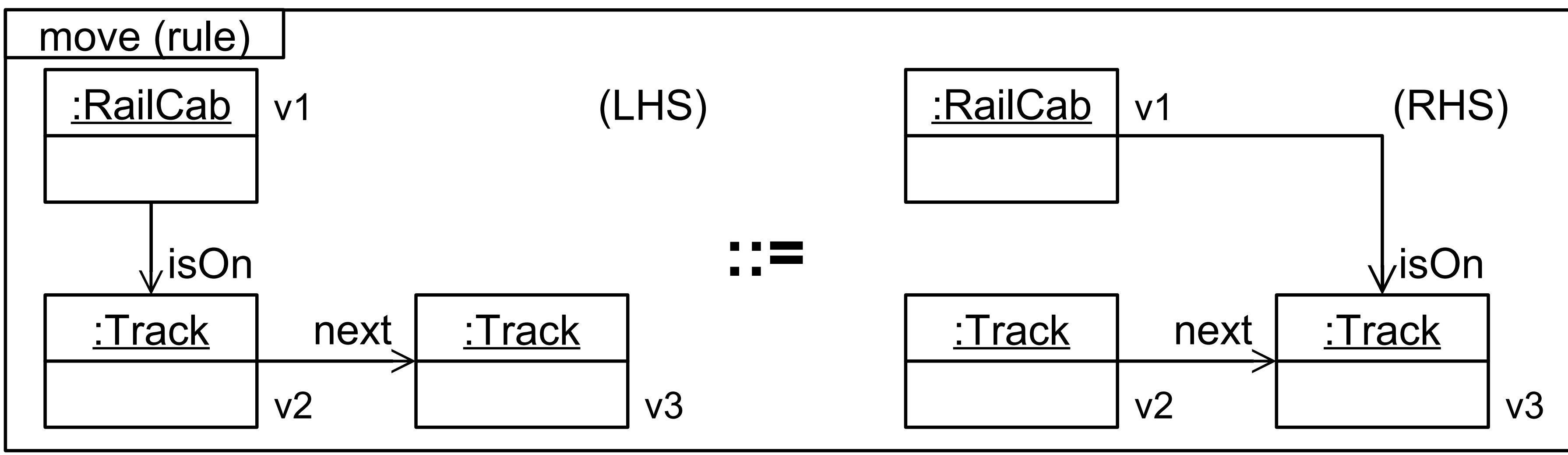


Graph Grammar Rule Application

in the last lecture...

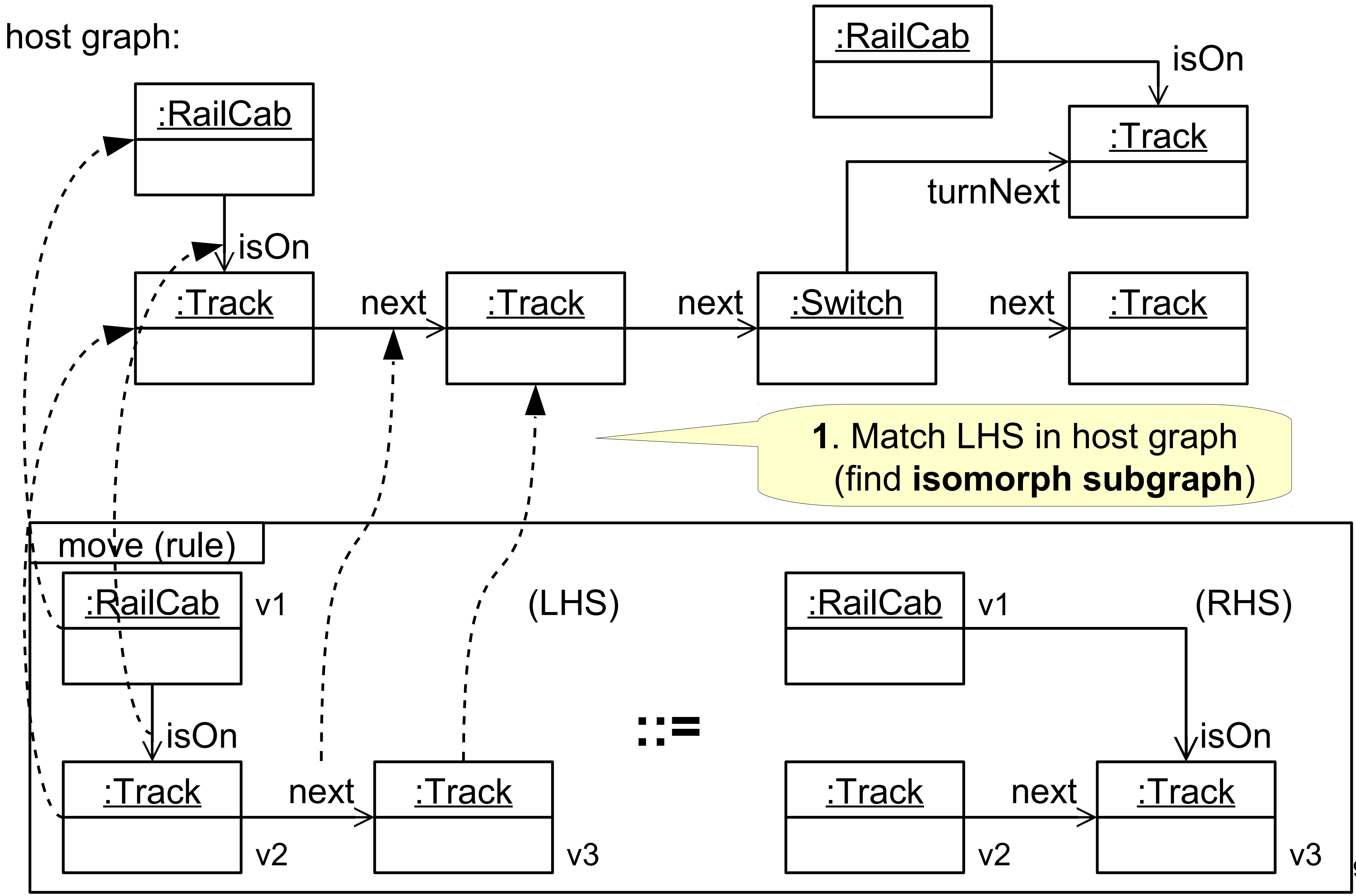
host graph:





Graph Grammar Rule Application

in the last lecture...



Subgraph

- A graph $G_{Sub} = (V_{Sub}, E_{Sub}, s_{Sub}, t_{Sub})$ is a **subgraph** of graph $G = (V, E, s, t)$ if

Subgraph

- A graph $G_{Sub} = (V_{Sub}, E_{Sub}, s_{Sub}, t_{Sub})$ is a **subgraph** of graph $G = (V, E, s, t)$ if
  - $V_{Sub} \subseteq V$
  - $E_{Sub} \subseteq E$
  - $s_{Sub} = s|E_{Sub}$
  - $t_{Sub} = t|E_{Sub}$

Subgraph

- A graph $G_{Sub} = (V_{Sub}, E_{Sub}, s_{Sub}, t_{Sub})$ is a **subgraph** of graph $G = (V, E, s, t)$ if
  - $V_{Sub} \subseteq V$
  - $E_{Sub} \subseteq E$
  - $s_{Sub} = s|E_{Sub}$
  - $t_{Sub} = t|E_{Sub}$

It means that the source and target functions for the subgraph are reduced to the edges which are in it.

Subgraph

- A graph $G_{Sub} = (V_{Sub}, E_{Sub}, s_{Sub}, t_{Sub})$ is a **subgraph** of graph $G = (V, E, s, t)$ if
  - $V_{Sub} \subseteq V$
  - $E_{Sub} \subseteq E$
  - $s_{Sub} = s|E_{Sub}$
  - $t_{Sub} = t|E_{Sub}$

It means that the source and target functions for the subgraph are reduced to the edges which are in it.

- If G_{Sub} is a subgraph of G, we also write $G_{Sub} \leq G$

Typed Isomophism

- When matching a rule graph to the host graph:

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**
  - there must be a ***typed subgraph isomorphism***

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**
  - there must be a ***typed subgraph isomorphism***

- Let $G_1 = (V_1, E_1, s_1, t_1)$ and $G_2 = (V_2, E_2, s_2, t_2)$ be two graphs

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**
  - there must be a ***typed subgraph isomorphism***
- Let $G_1 = (V_1, E_1, s_1, t_1)$ and $G_2 = (V_2, E_2, s_2, t_2)$ be two graphs
  - typed by graph morphisms
    $type_1 = (type_{1V}, type_{1E}): G_1 \rightarrow G_{Type}$ and
    $type_2 = (type_{2V}, type_{2E}): G_2 \rightarrow G_{Type}$

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**
  - there must be a ***typed subgraph isomorphism***
- Let $G_1 = (V_1, E_1, s_1, t_1)$ and $G_2 = (V_2, E_2, s_2, t_2)$ be two graphs
  - typed by graph morphisms
    $type_1 = (type_{1V}, type_{1E}): G_1 \rightarrow G_{Type}$ and
    $type_2 = (type_{2V}, type_{2E}): G_2 \rightarrow G_{Type}$
- An **(iso)morphism** $f = G_1 \rightarrow G_2, f = (f_V, f_E)$ is **typed** when

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**
  - there must be a ***typed subgraph isomorphism***

- Let $G_1 = (V_1, E_1, s_1, t_1)$ and $G_2 = (V_2, E_2, s_2, t_2)$ be two graphs
  - typed by graph morphisms
    $type_1 = (type_{1V}, type_{1E}): G_1 \to G_{Type}$ and
    $type_2 = (type_{2V}, type_{2E}): G_2 \to G_{Type}$

- An **(iso)morphism** $f = G_1 \to G_2, f = (f_V, f_E)$ is **typed** when
  - $type_1 = type_2 \circ f$, i.e.,

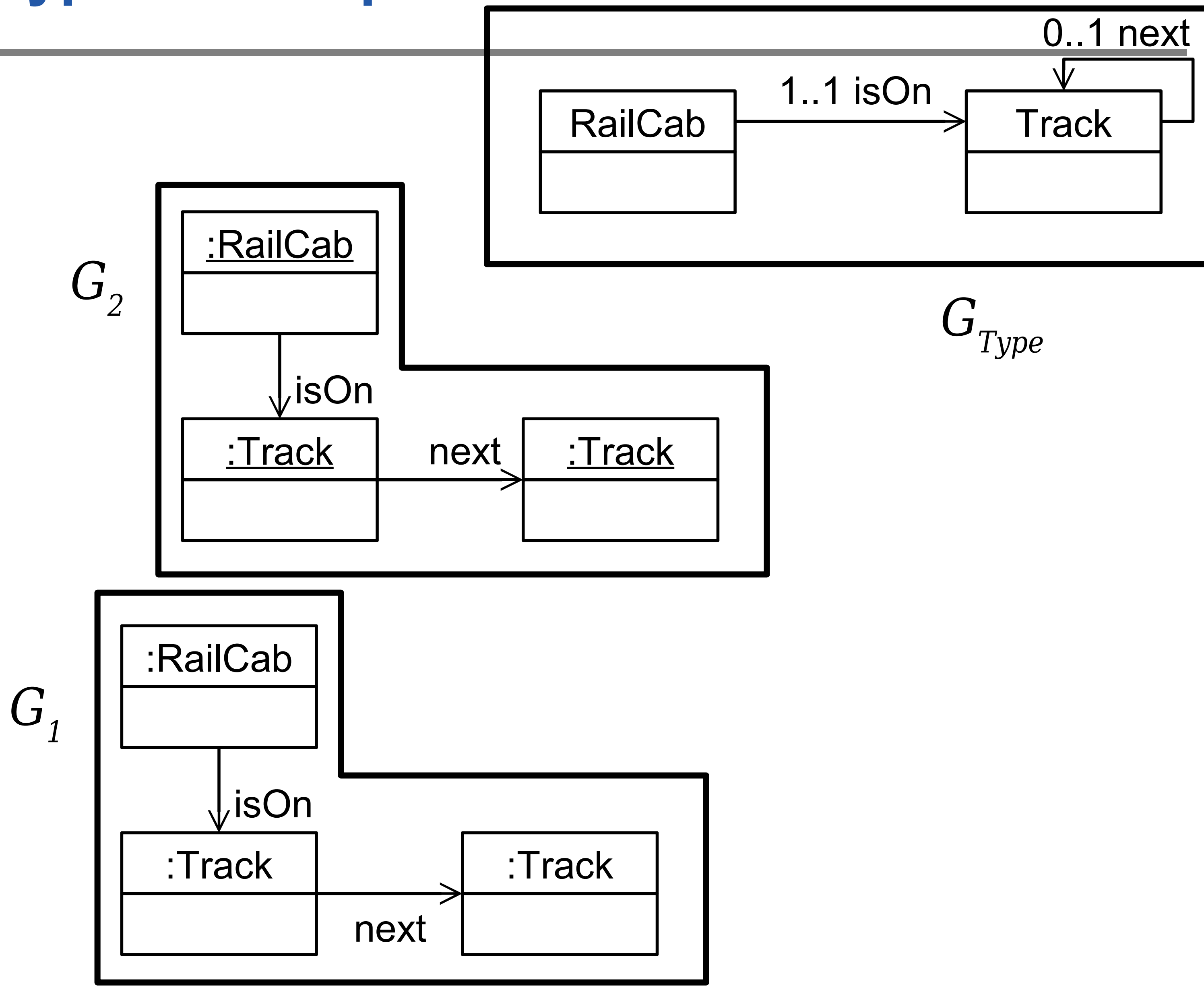
Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**
  - there must be a ***typed subgraph isomorphism***
- Let $G_1 = (V_1, E_1, s_1, t_1)$ and $G_2 = (V_2, E_2, s_2, t_2)$ be two graphs
  - typed by graph morphisms
    $type_1 = (type_{1V}, type_{1E}): G_1 \rightarrow G_{Type}$ and
    $type_2 = (type_{2V}, type_{2E}): G_2 \rightarrow G_{Type}$
- An **(iso)morphism** $f = G_1 \longrightarrow G_2, f = (f_V, f_E)$ is **typed** when
  - $type_1 = type_2 \circ f$, i.e.,
    - for all $v \in V_1$ it holds that $type_{1V}(v) = type_{2V}(f_V(v))$ and

Typed Isomophism

- When matching a rule graph to the host graph:
  - rule graph and host graph have the **same type graph**
  - the match must **respect the typing of the graphs**
  - there must be a ***typed subgraph isomorphism***
- Let $G_1 = (V_1, E_1, s_1, t_1)$ and $G_2 = (V_2, E_2, s_2, t_2)$ be two graphs
  - typed by graph morphisms
    $type_1 = (type_{1V}, type_{1E}): G_1 \rightarrow G_{Type}$ and
    $type_2 = (type_{2V}, type_{2E}): G_2 \rightarrow G_{Type}$
- An **(iso)morphism** $f = G_1 \rightarrow G_2, f = (f_V, f_E)$ is **typed** when
  - $type_1 = type_2 \circ f$, i.e.,
    - for all $v \in V_1$ it holds that $type_{1V}(v) = type_{2V}(f_V(v))$ and
    - for all $e \in E_1$ it holds that $type_{1E}(e) = type_{2E}(f_E(e))$

Typed Isomophism

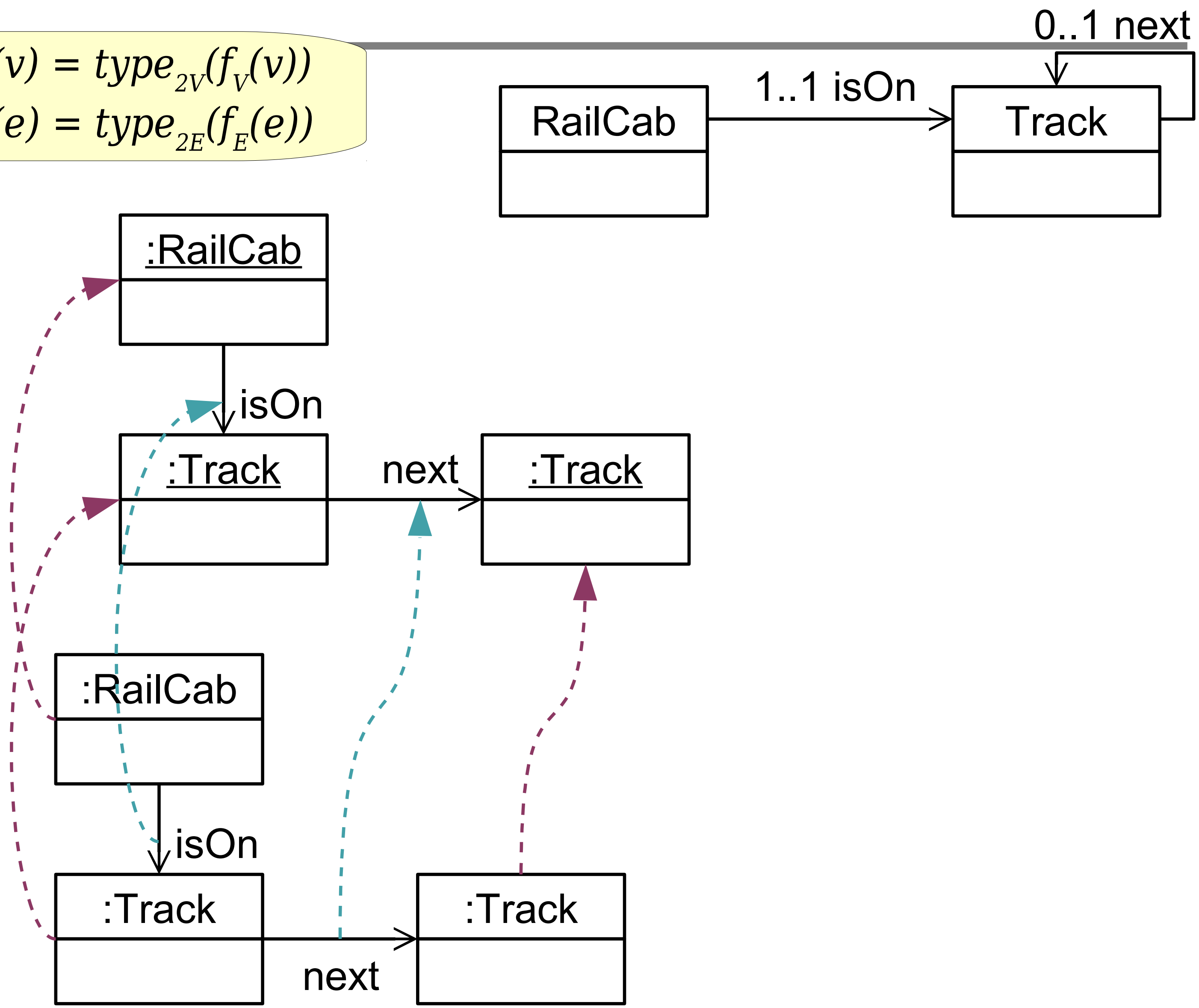


Typed Isomophism

for all $v \in V_1$ $type_{1V}(v) = type_{2V}(f_V(v))$

for all $e \in E_1$ $type_{1E}(e) = type_{2E}(f_E(e))$

f_V and f_E



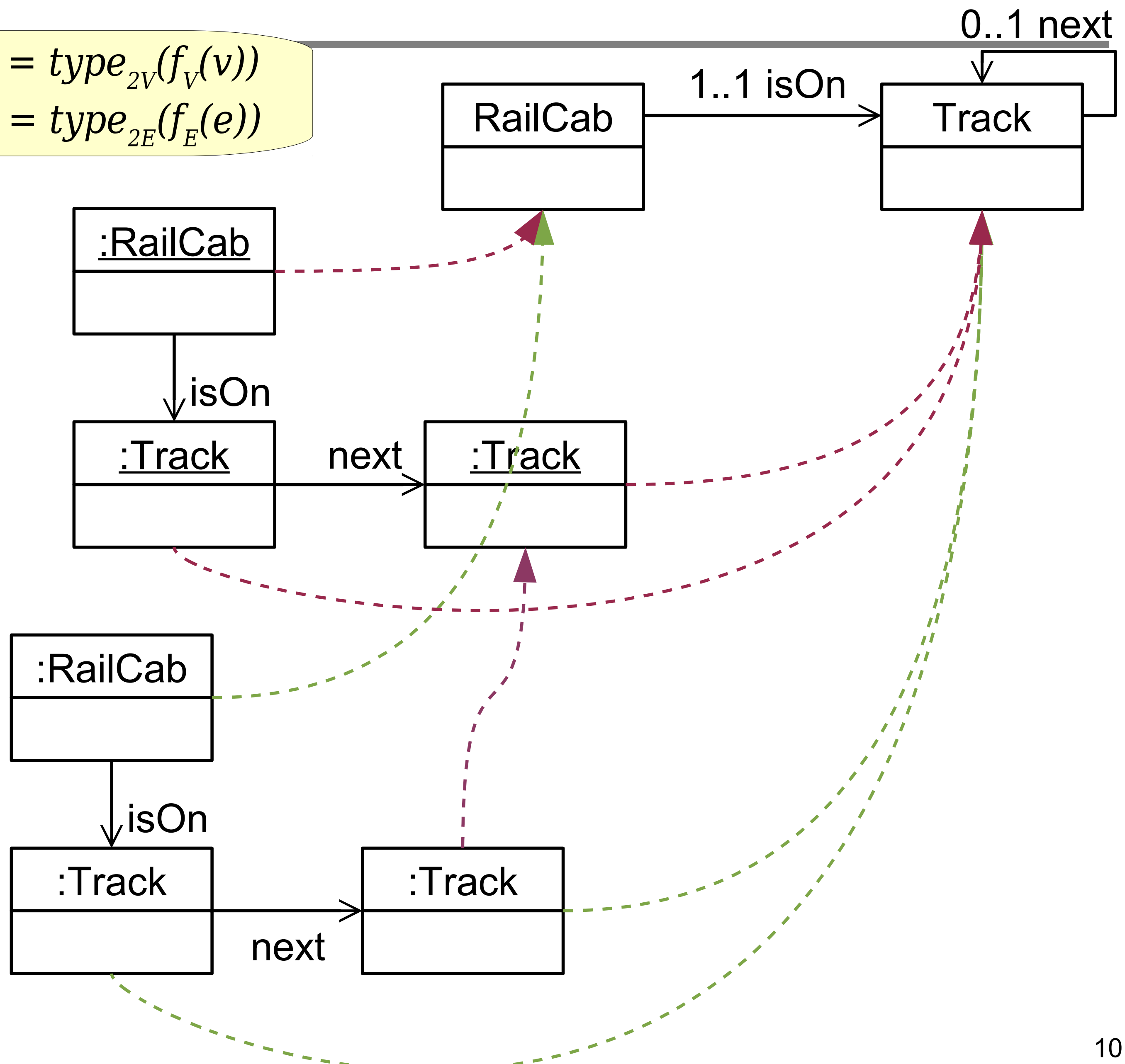
Typed Isomophism

for all $v \in V_1$ $type_{1V}(v) = type_{2V}(f_V(v))$

for all $e \in E_1$ $type_{1E}(e) = type_{2E}(f_E(e))$

$type_{1V}$ and

$type_{2V}$



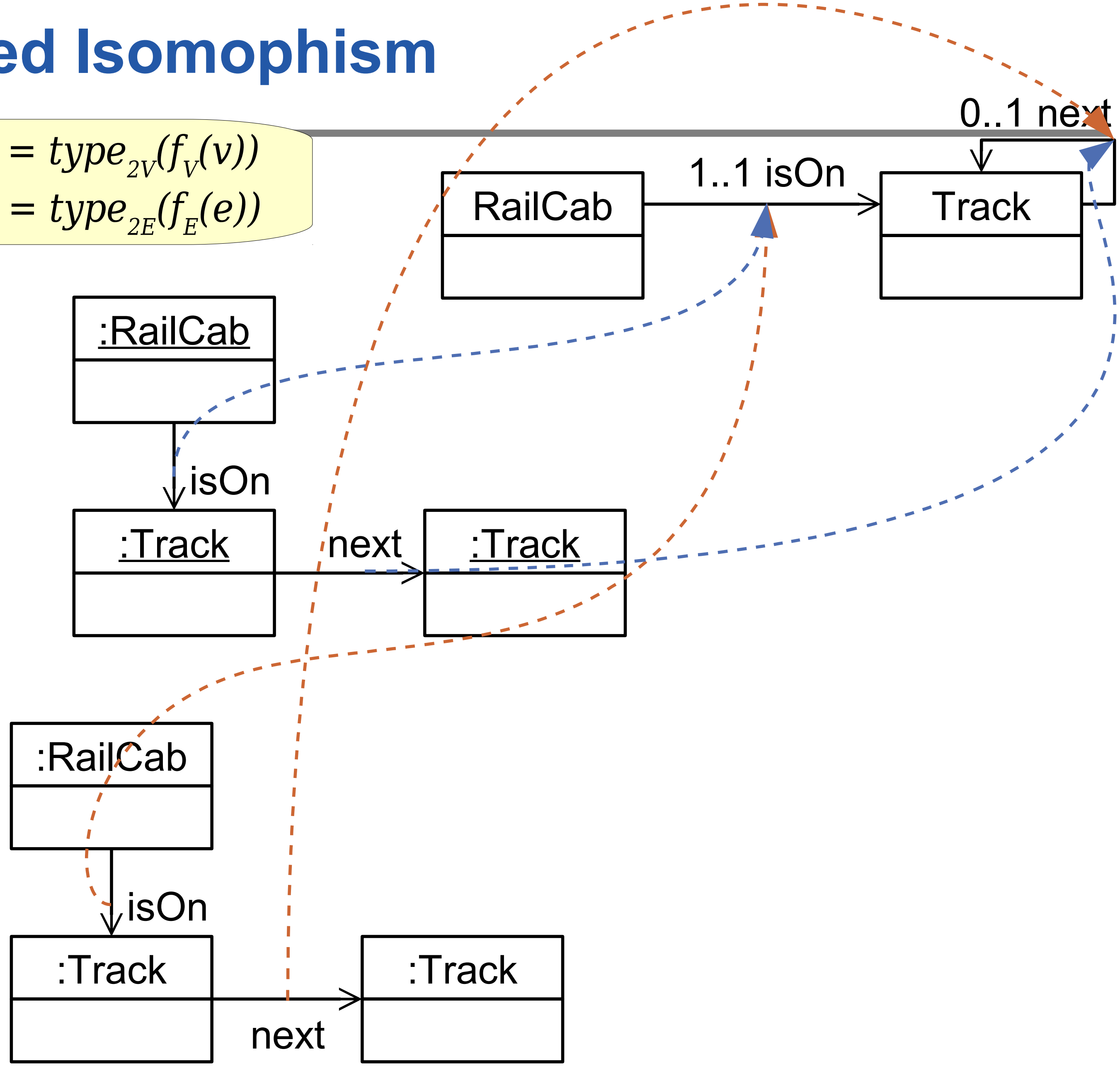
Typed Isomophism

for all $v \in V_1$ $type_{1V}(v) = type_{2V}(f_V(v))$

for all $e \in E_1$ $type_{1E}(e) = type_{2E}(f_E(e))$

$type_{1E}$ and

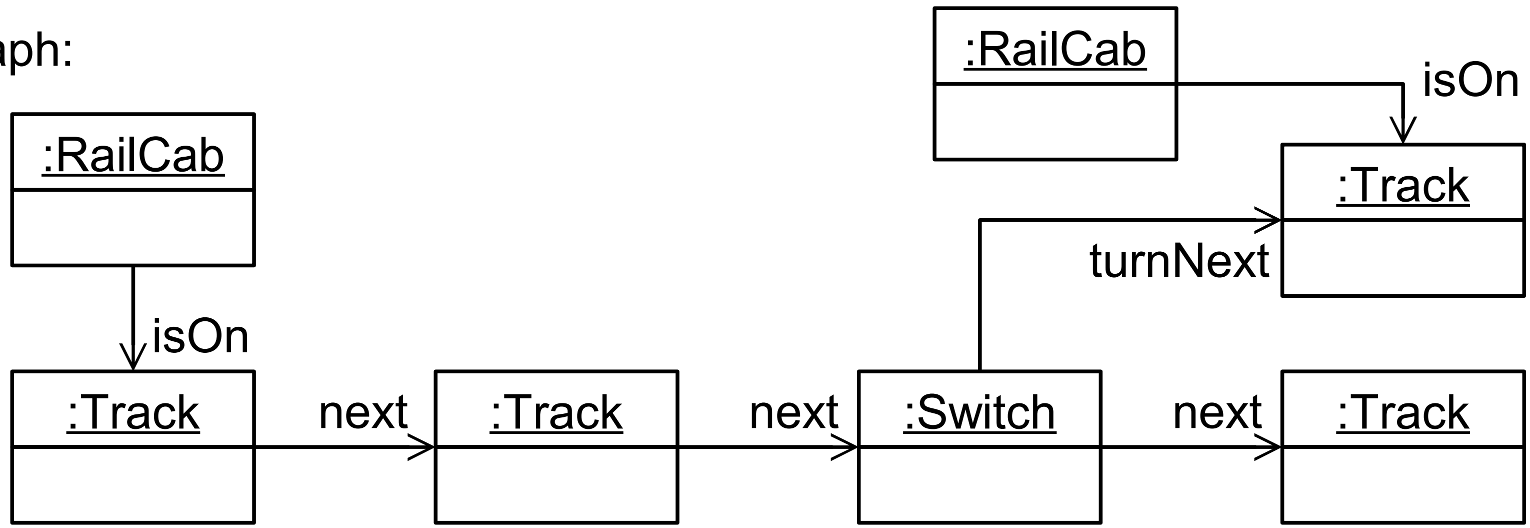
$type_{2E}$

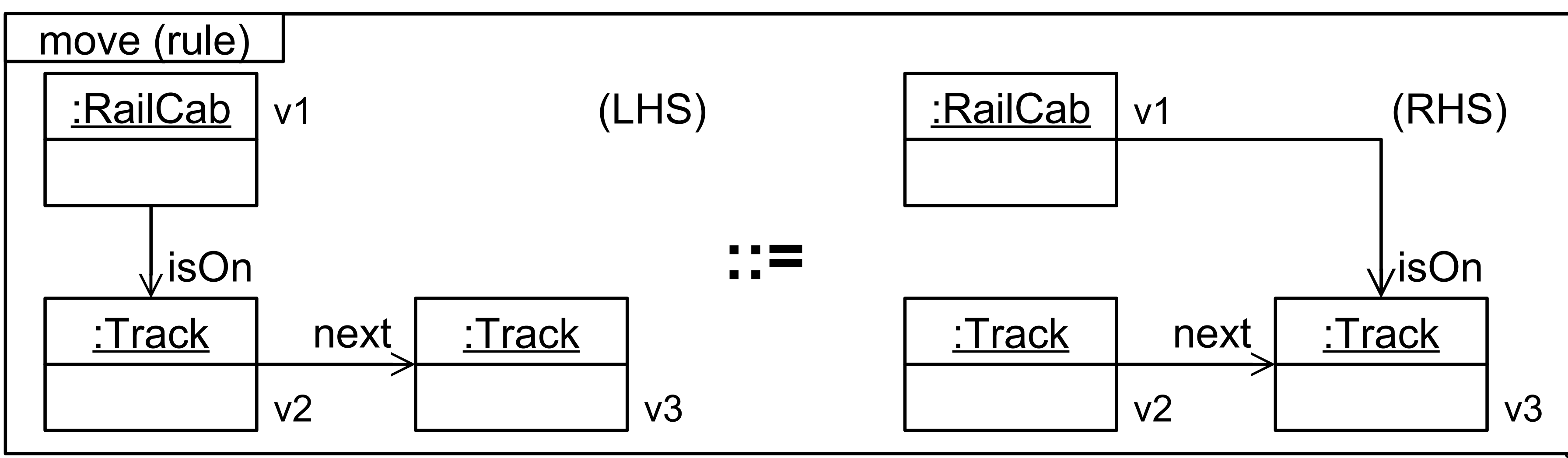


Graph Grammar Rule Application

in the last lecture...

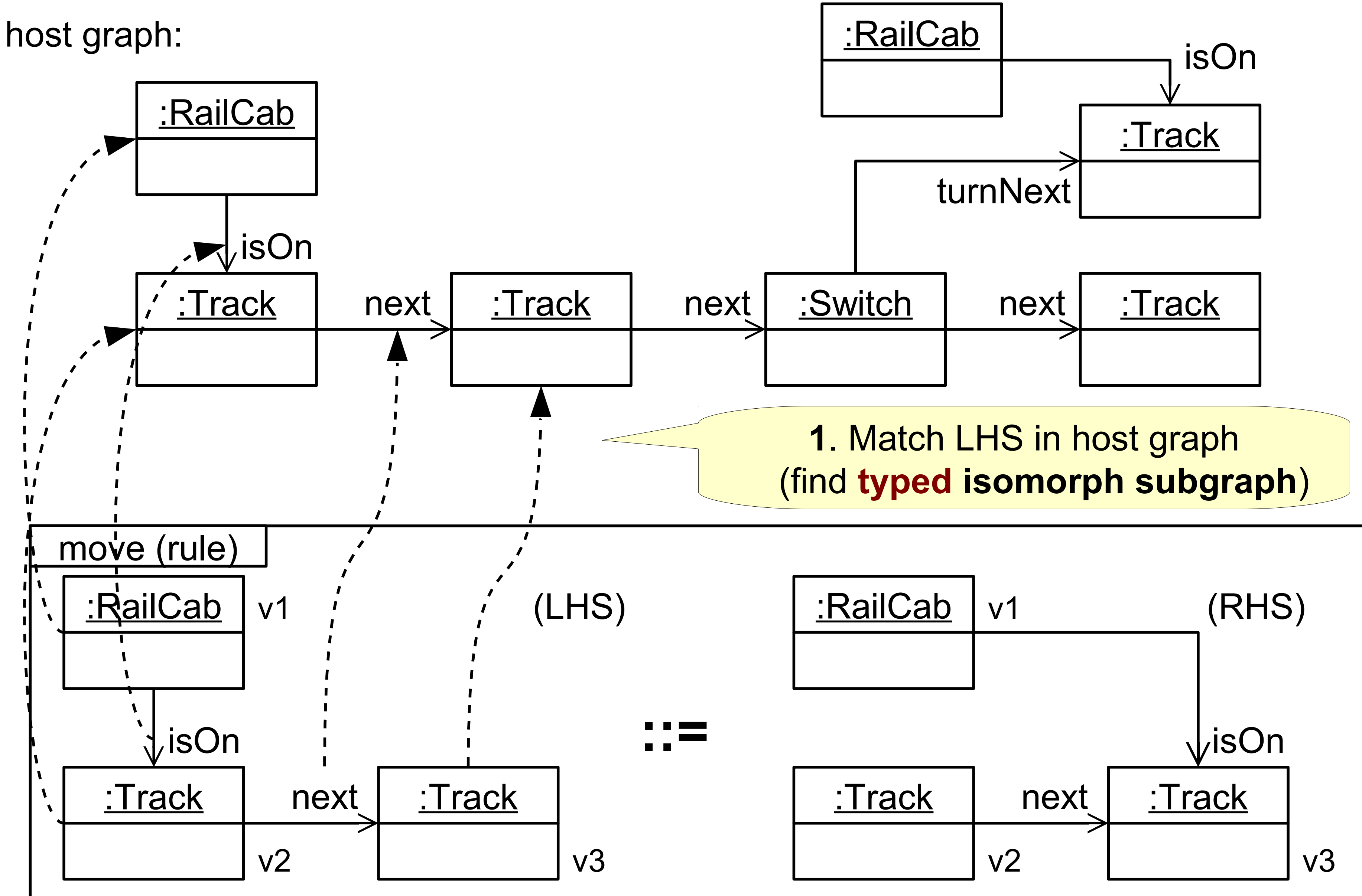
host graph:



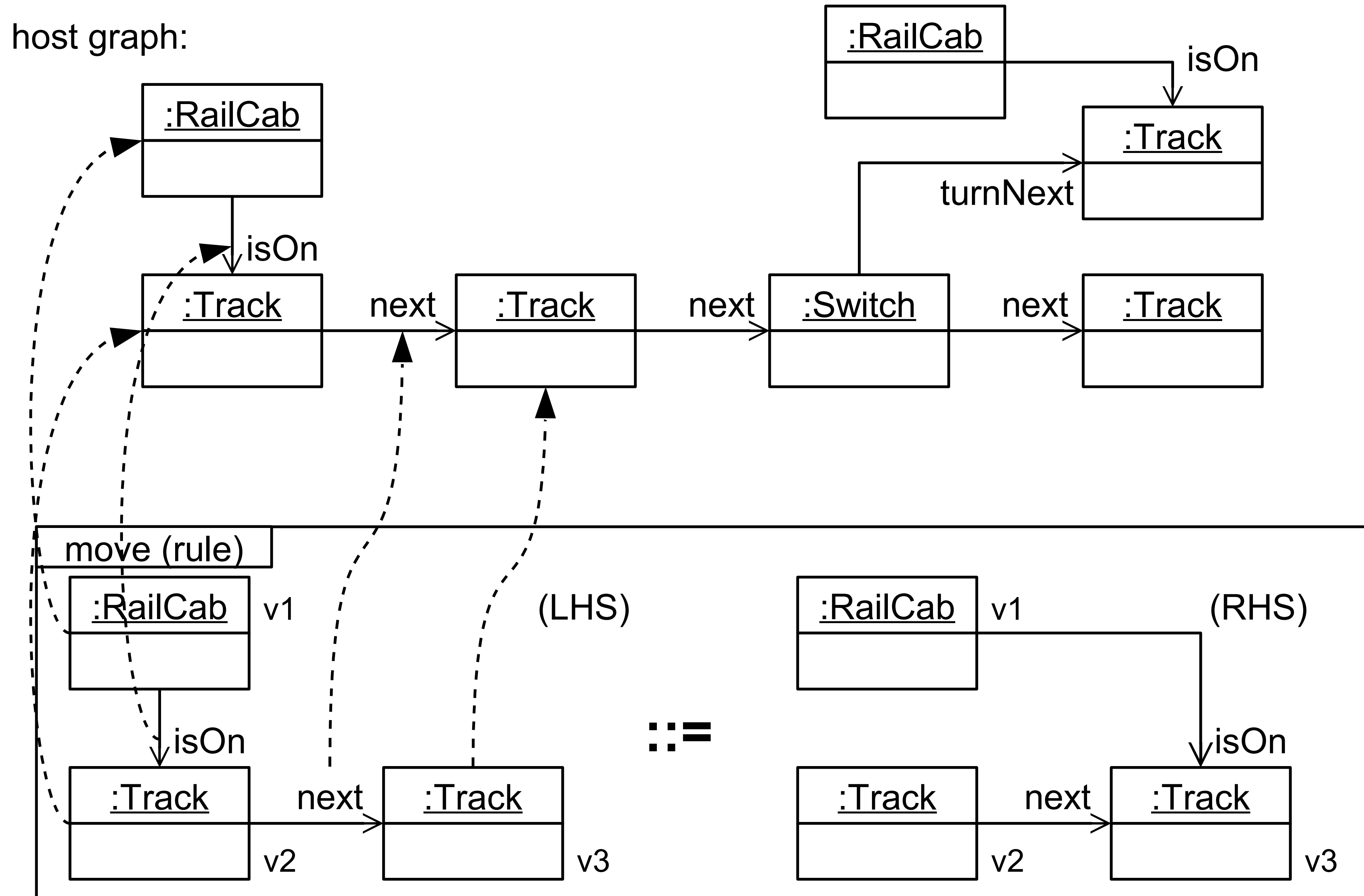


Graph Grammar Rule Application

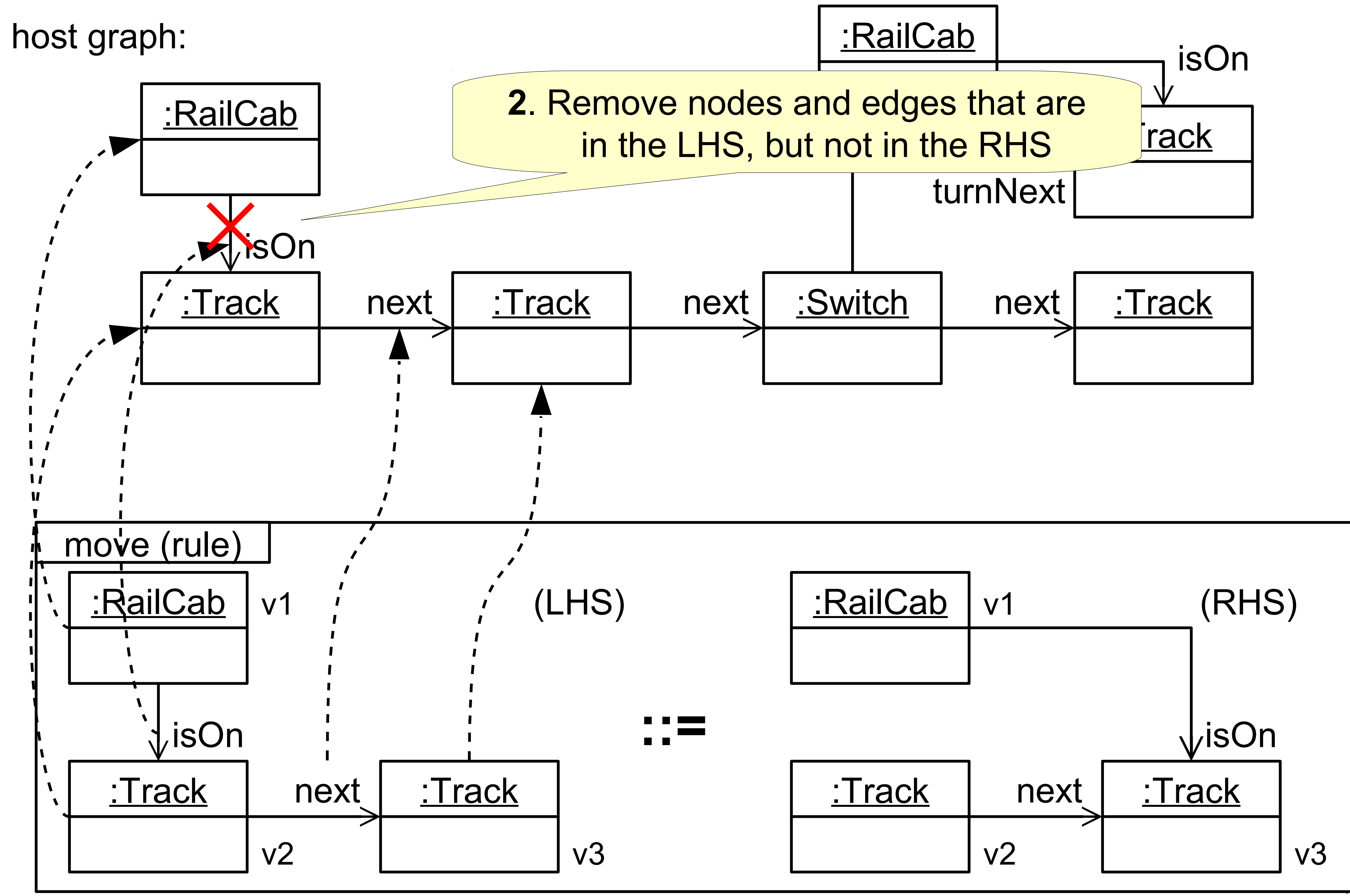
in the last lecture...



Graph Grammar Rule Application

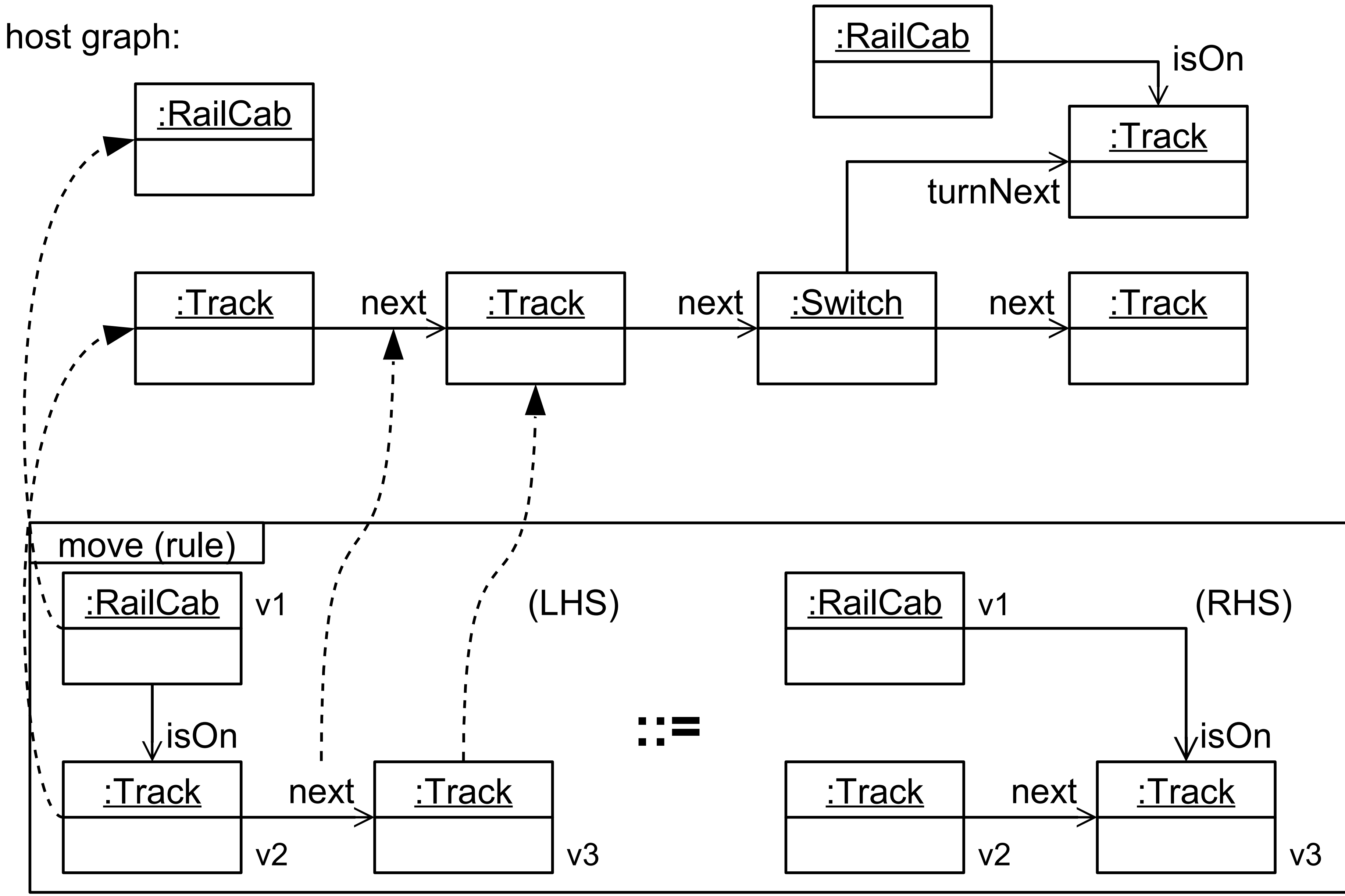


Graph Grammar Rule Application



Graph Grammar Rule Application

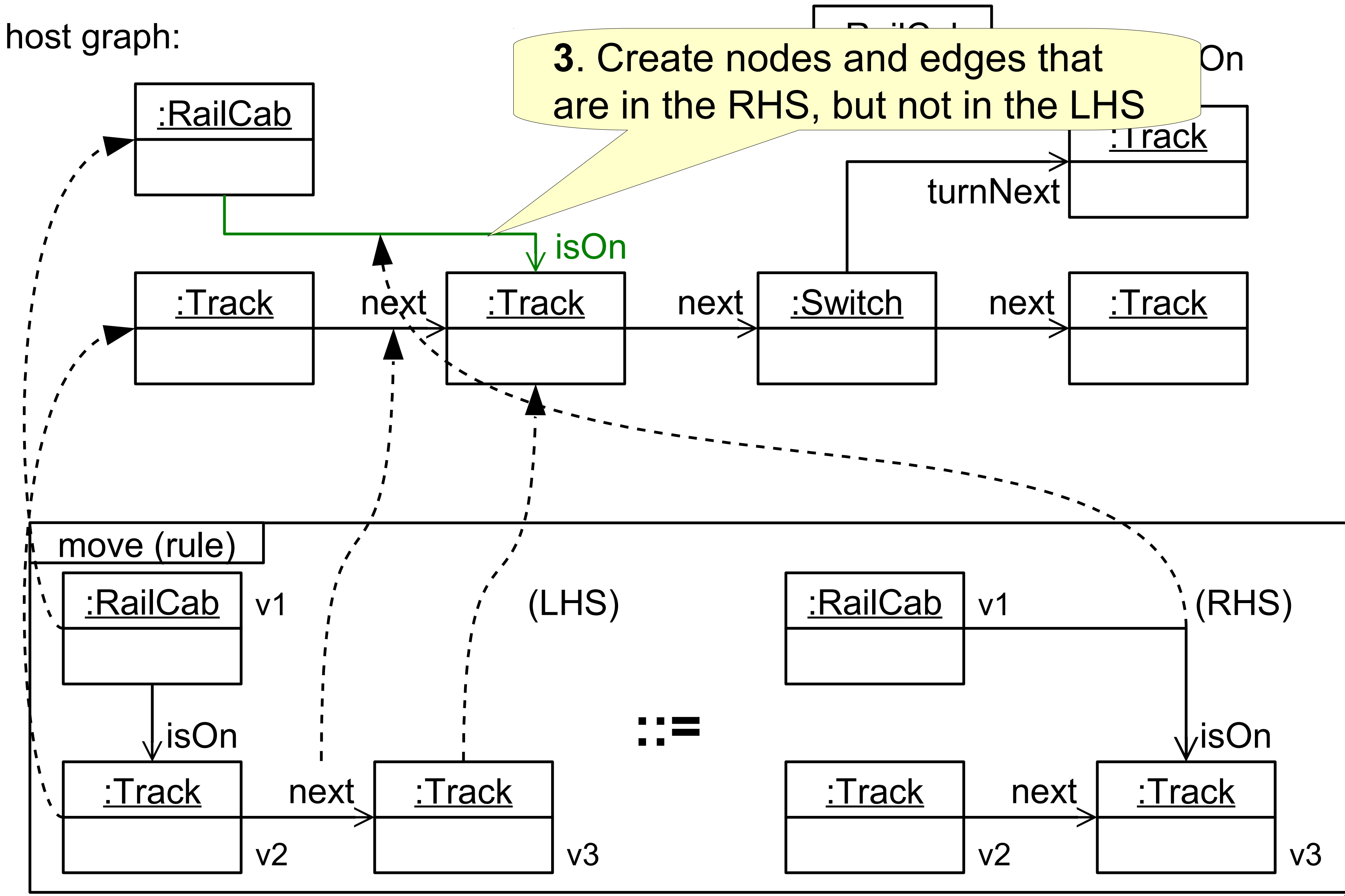
in the last lecture...



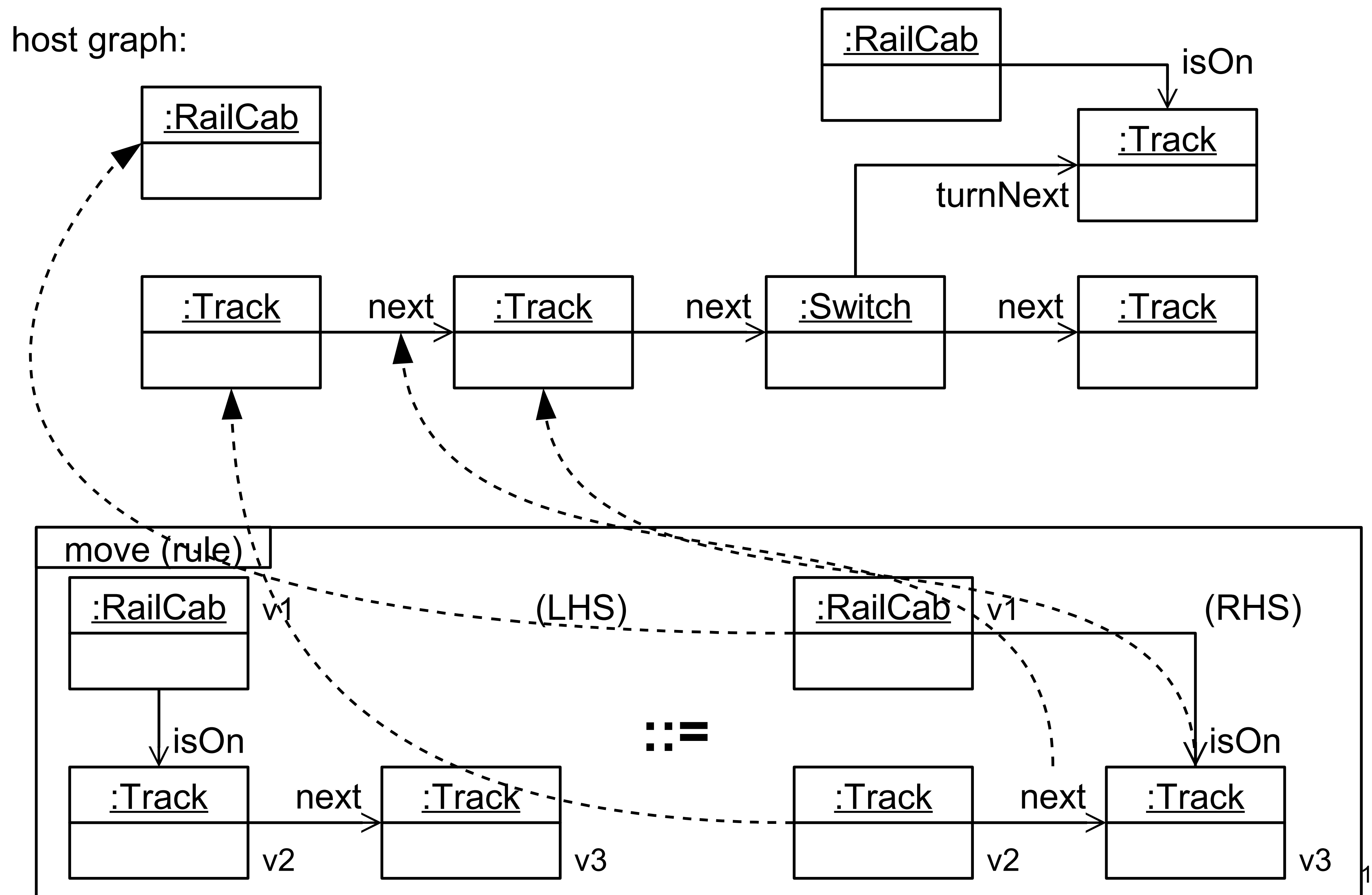
Graph Grammar Rule Application

in the last lecture...

host graph:



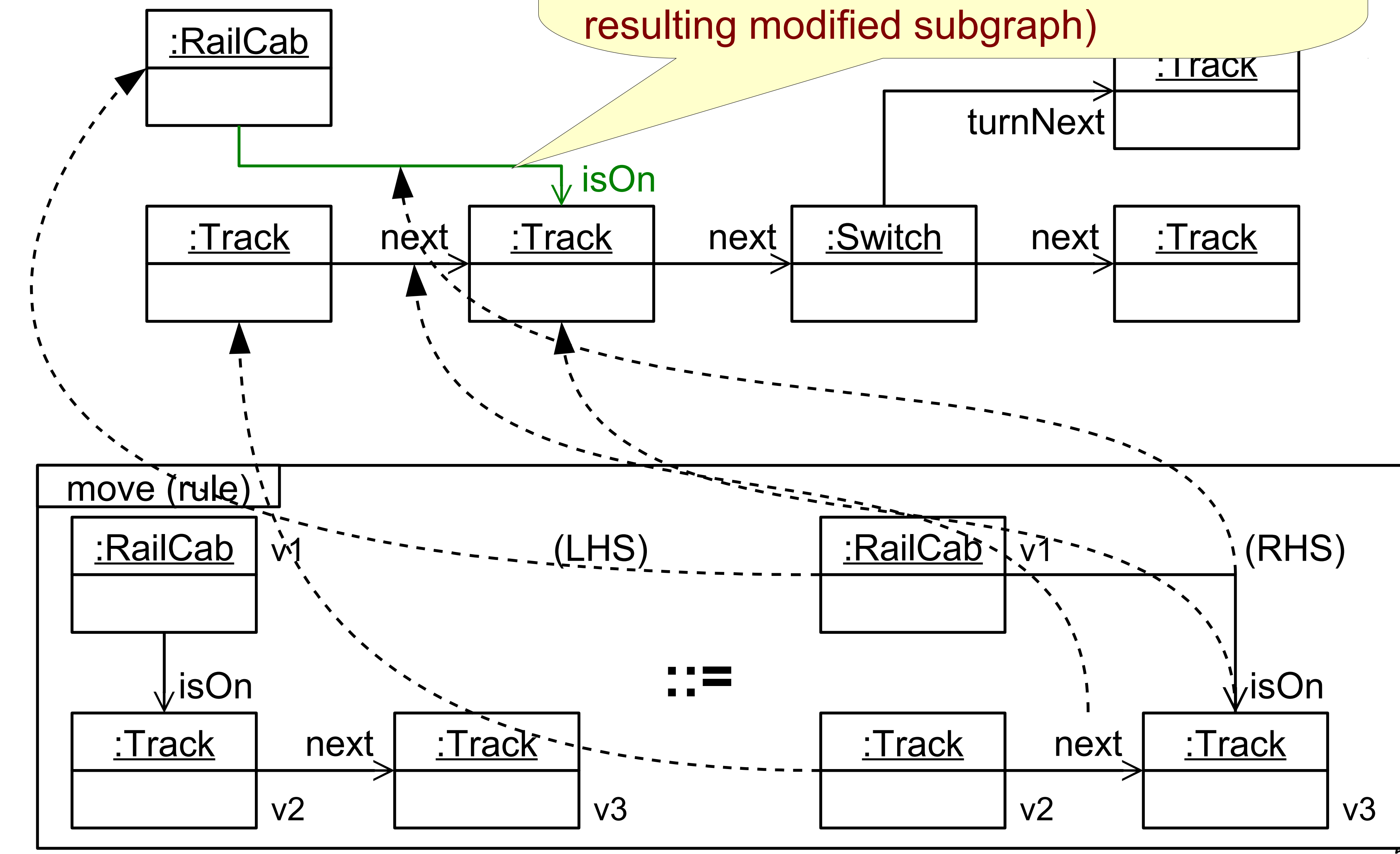
Graph Grammar Rule Application



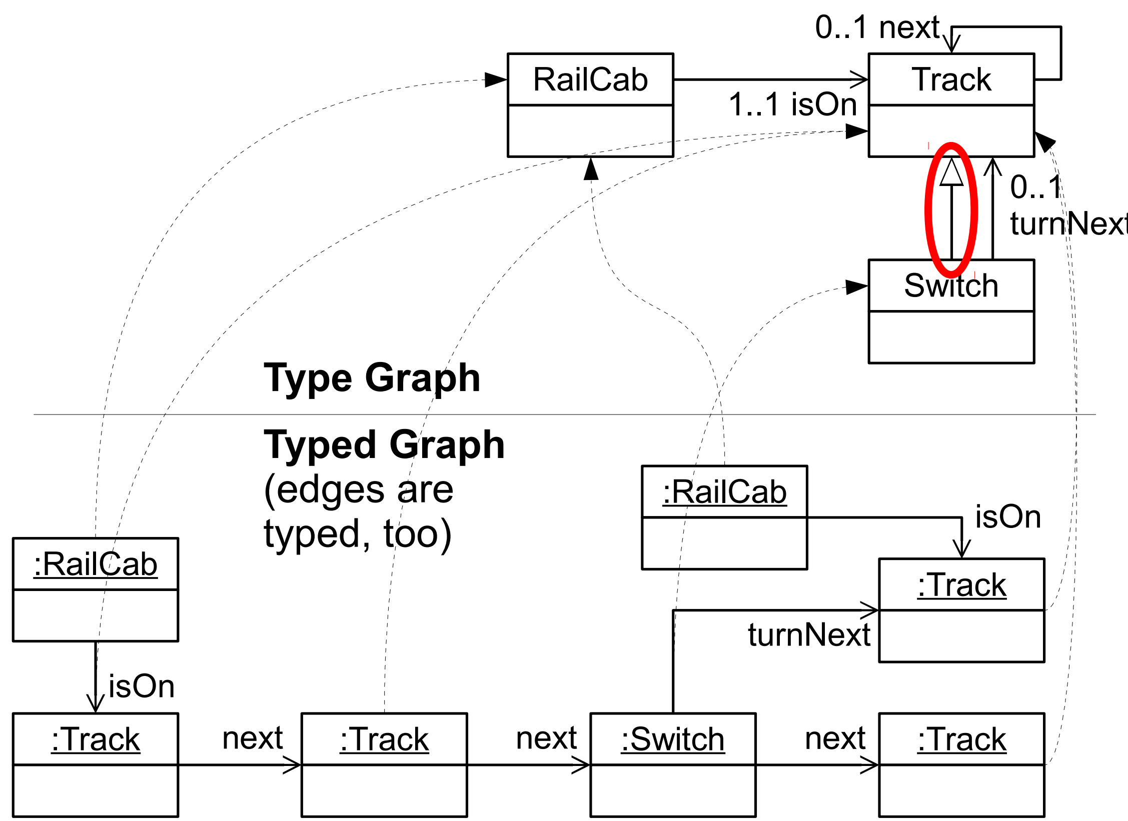
Graph G

3. Create nodes and edges that are in the RHS, but not in the LHS
(such that there is **typed isomorphism** between the rule's RHS graph and the resulting modified subgraph)

host graph:

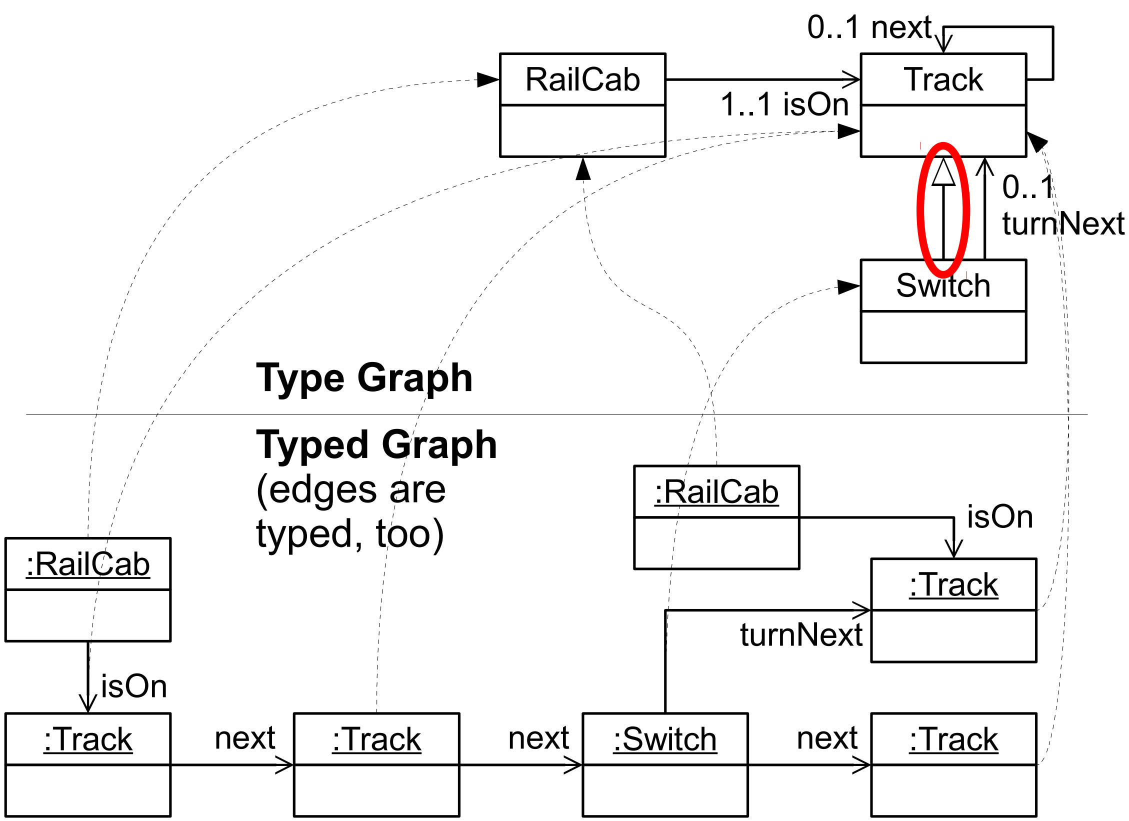


Graph Transformations More Formally



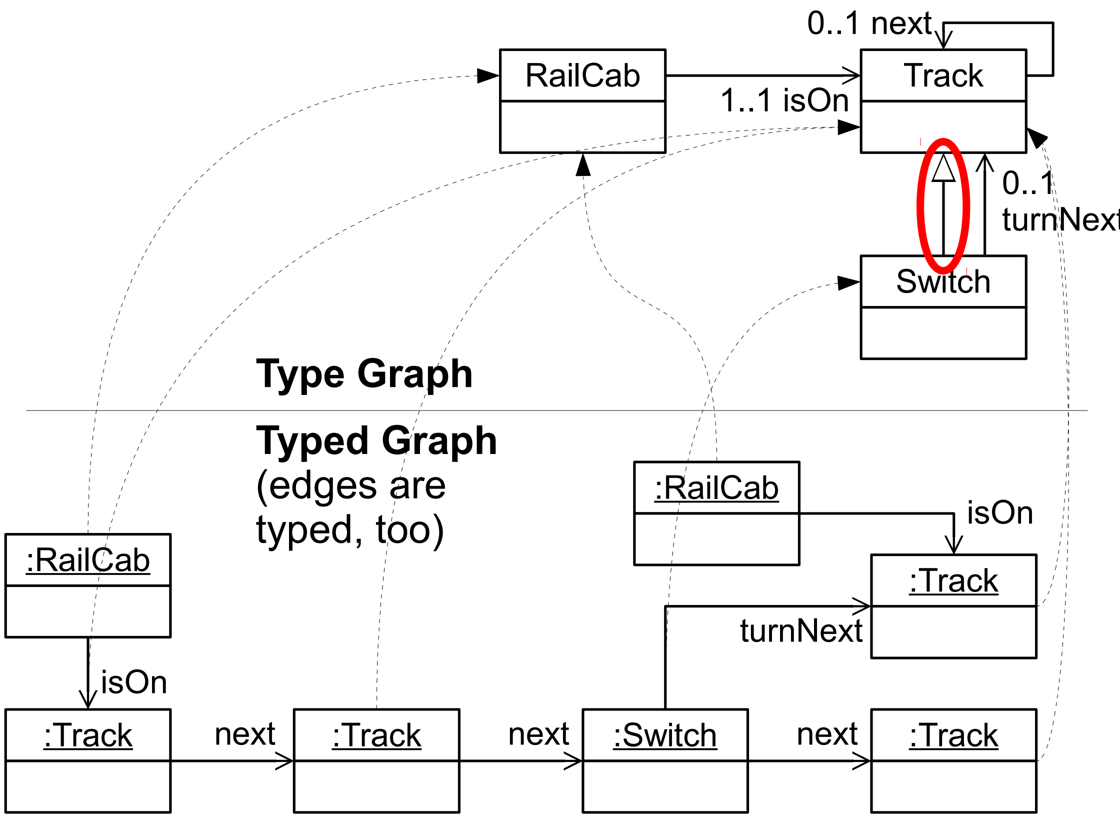
Graph Transformations More Formally

- How do we treat the concept of generalization (inheritance)?
  - this makes matters a bit more complicated...



Graph Transformations More Formally

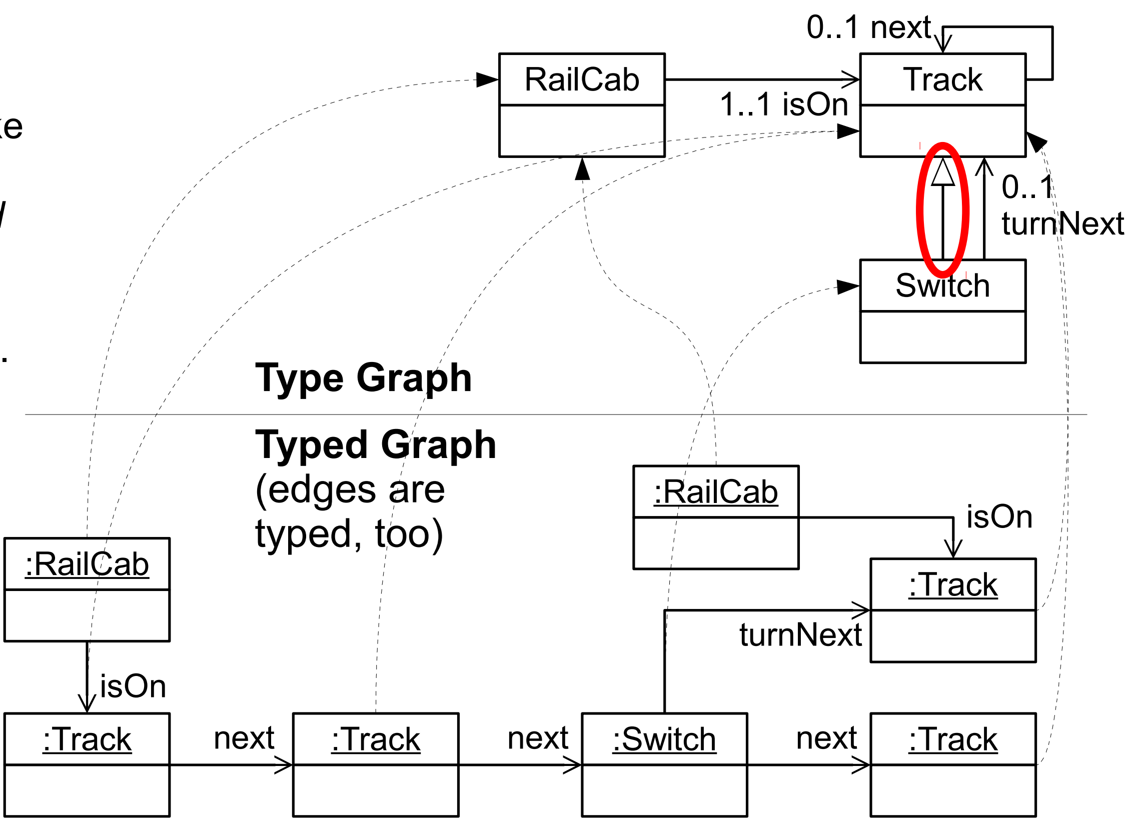
- How do we treat the concept of generalization (inheritance)?
  - this makes matters a bit more complicated...
- How do we treat attribute values?



Graph Transformations More Formally

- How do we treat the concept of generalization (inheritance)?
  - this makes matters a bit more complicated...
- How do we treat attribute values?

see for example: Juan de Lara, Roswitha Bardohl, Hartmut Ehrig, Karsten Ehrig, Ulrike Prange, Gabriele Taentzer, Fundamental Aspects of Software Engineering, *Attributed graph transformation with node type inheritance*, Theoretical Computer Science, Volume 376, Issue 3, 2007, Pages 139-163.



Graph Matching Algorithm

- Finding an isomorphic subgraph is an NP-complete problem

Graph Matching Algorithm

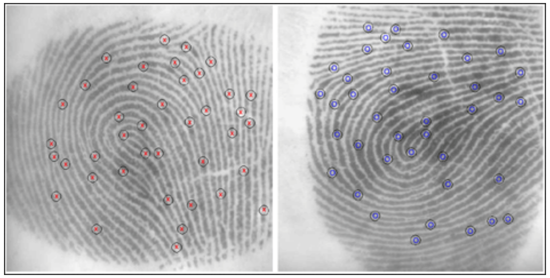
- Finding an isomorphic subgraph is an NP-complete problem
  - exponential in the size of the involved graphs
- In the MBSE context, the graphs are usually typed and often strongly structured
  - so matching graph transformation rule patterns can happen in practically acceptable time

- In some applications, graphs are not that structured, but then also heuristics can be employed to find close matches

Graph Matching Algorithm

- Finding an isomorphic subgraph is an NP-complete problem
  – exponential in the size of the involved graphs
- In the MBSE context, the graphs are usually typed and often strongly structured
  – so matching graph transformation rule patterns can happen in practically acceptable time

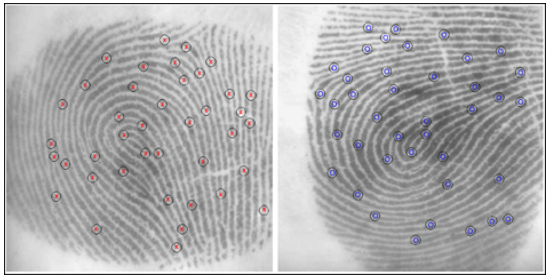
- In some applications, graphs are not that structured, but then also heuristics can be employed to find close matches

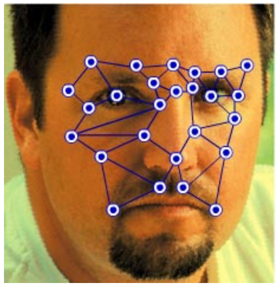


Graph Matching Algorithm

- Finding an isomorphic subgraph is an NP-complete problem
  - exponential in the size of the involved graphs
- In the MBSE context, the graphs are usually typed and often strongly structured
  - so matching graph transformation rule patterns can happen in practically acceptable time

- In some applications, graphs are not that structured, but then also heuristics can be employed to find close matches





Graph Transformations – Intermediate Summary

- Graph transformation rules allow us

Graph Transformations – Intermediate Summary

- Graph transformation rules allow us
  - to describe the behavior of systems (e.g. RailCab) formally

Graph Transformations – Intermediate Summary

- Graph transformation rules allow us
  - to describe the behavior of systems (e.g. RailCab) formally
  - to describe the behavior of object-oriented programs formally

Graph Transformations – Intermediate Summary

- Graph transformation rules allow us
  - to describe the behavior of systems (e.g. RailCab) formally
  - to describe the behavior of object-oriented programs formally

- The behavior can be analyzed formally

Graph Transformations – Intermediate Summary

- Graph transformation rules allow us
  - to describe the behavior of systems (e.g. RailCab) formally
  - to describe the behavior of object-oriented programs formally

- The behavior can be analyzed formally

- And the behavior can be execute

Graph Transformations – Intermediate Summary

- Graph transformation rules allow us
  - to describe the behavior of systems (e.g. RailCab) formally
  - to describe the behavior of object-oriented programs formally

- The behavior can be analyzed formally

- And the behavior can be execute
  - by and interpreter (like Henshin)

Graph Transformations – Intermediate Summary

- Graph transformation rules allow us
  - to describe the behavior of systems (e.g. RailCab) formally
  - to describe the behavior of object-oriented programs formally
- The behavior can be analyzed formally
- And the behavior can be execute
  - by and interpreter (like Henshin)
  - or by code generation (like SDMTools https://www.hpi.uni-potsdam.de/giese/public/mdelab/mdelab-projects/story-diagram-tools/)

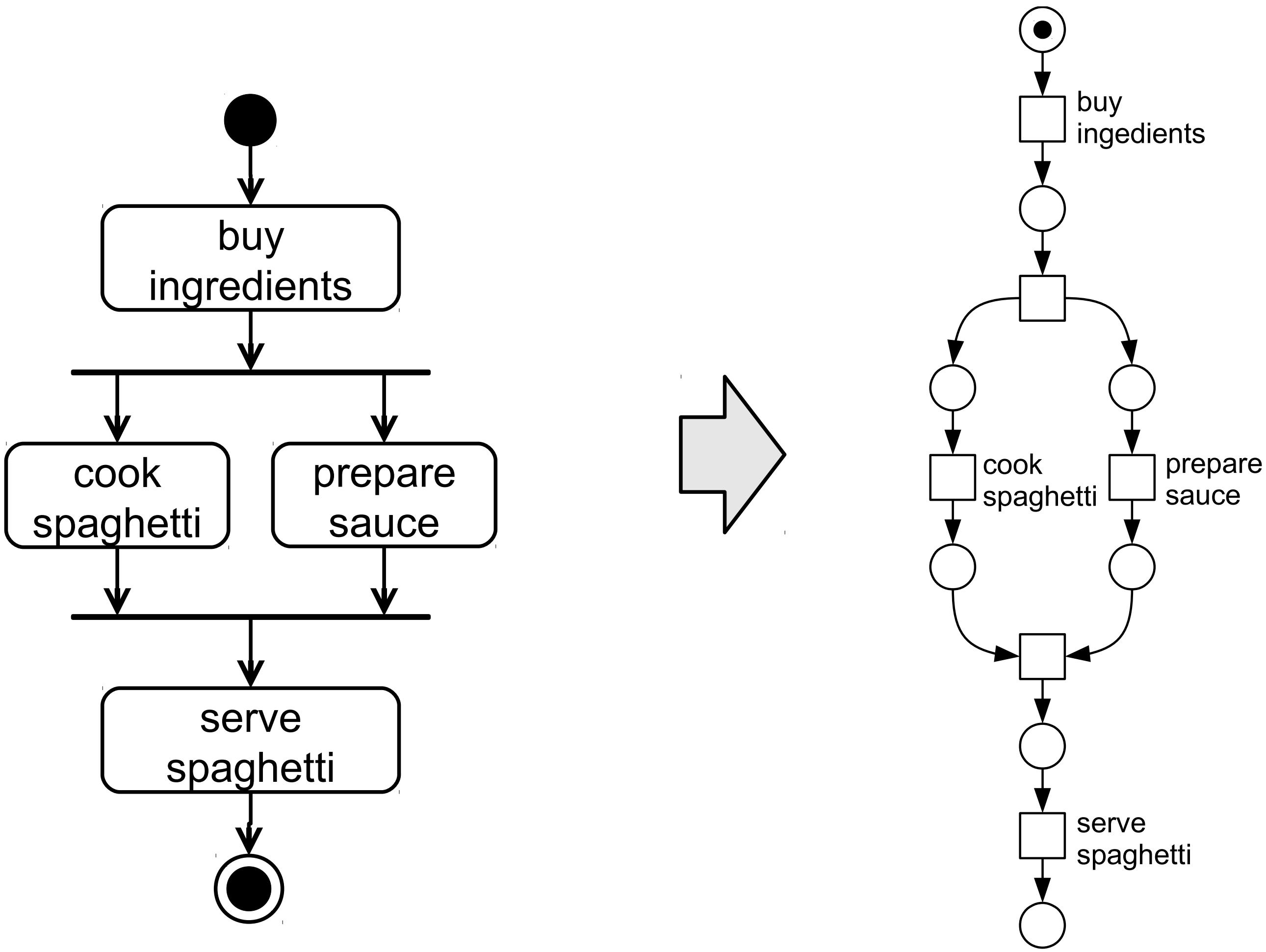
Graph Transformations – Intermediate Summary

- Graph transformation rules allow us
  - to describe the behavior of systems (e.g. RailCab) formally
  - to describe the behavior of object-oriented programs formally
- The behavior can be analyzed formally
- And the behavior can be execute
  - by and interpreter (like Henshin)
  - or by code generation (like SDMTools https://www.hpi.uni-potsdam.de/giese/public/mdelab/mdelab-projects/story-diagram-tools/)
- So far, we have mainly considered **endogenous** model transformations, how about **exogenous** ones?

5.4. Model-to-model transformation – Triple Graph Grammars

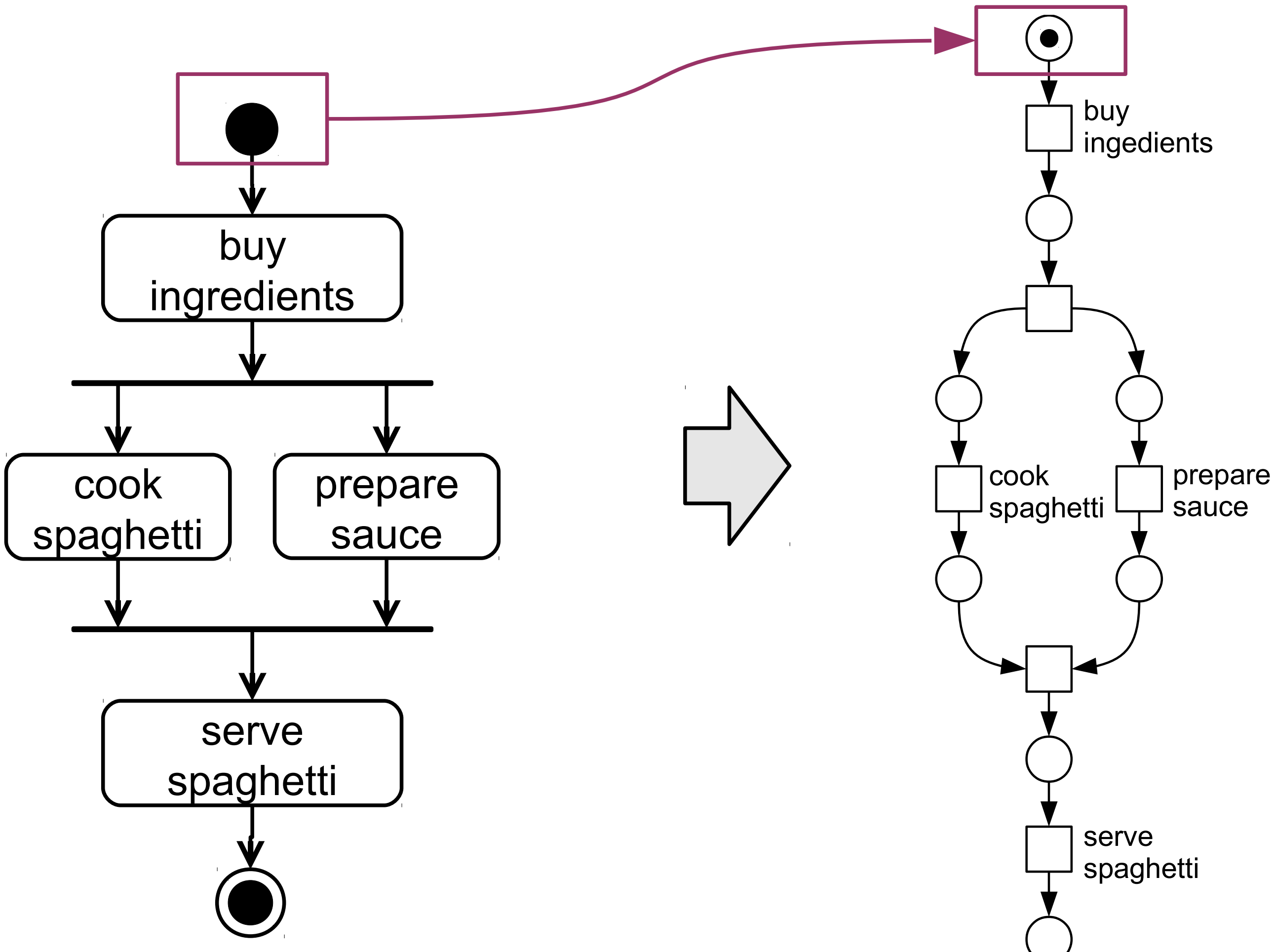
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



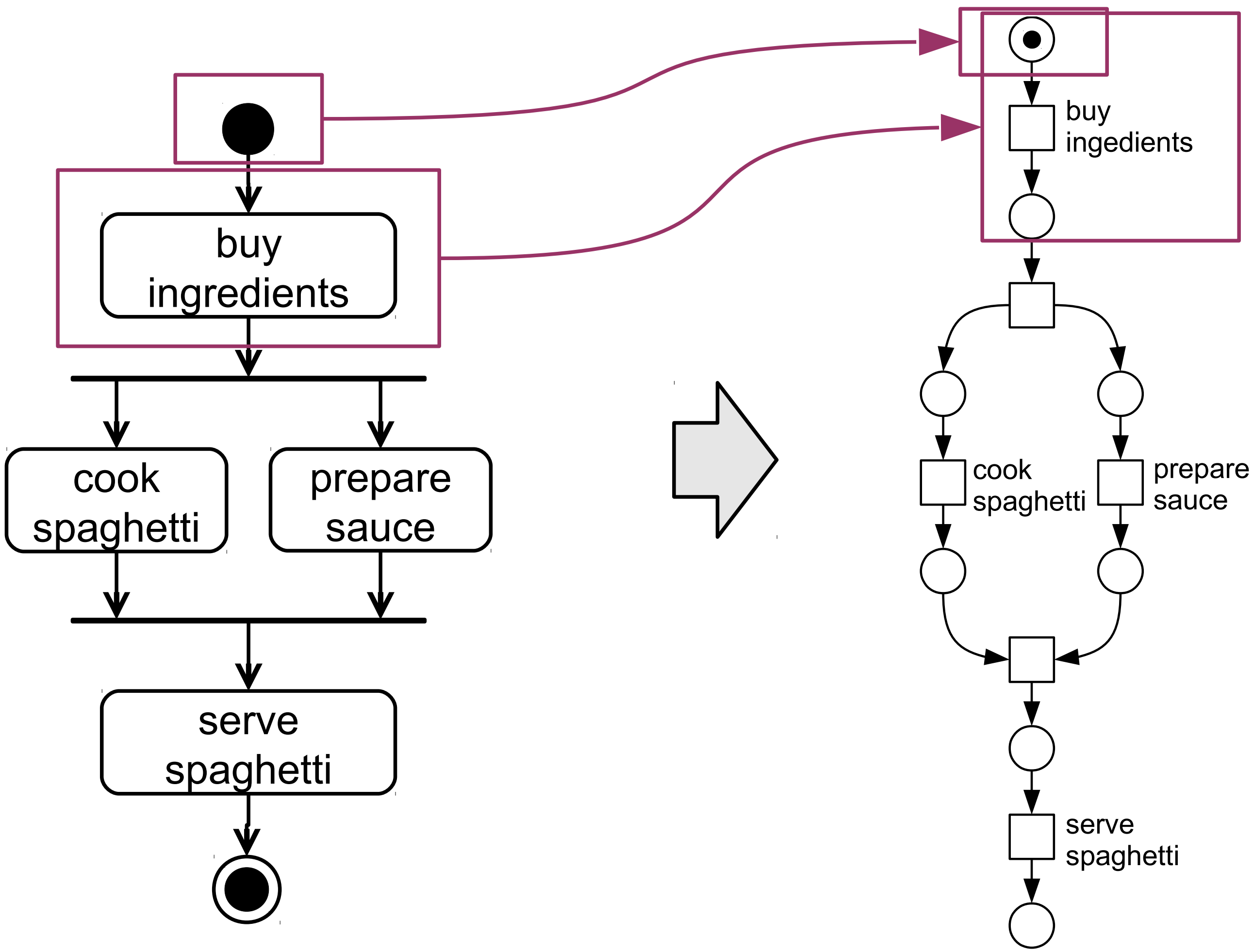
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



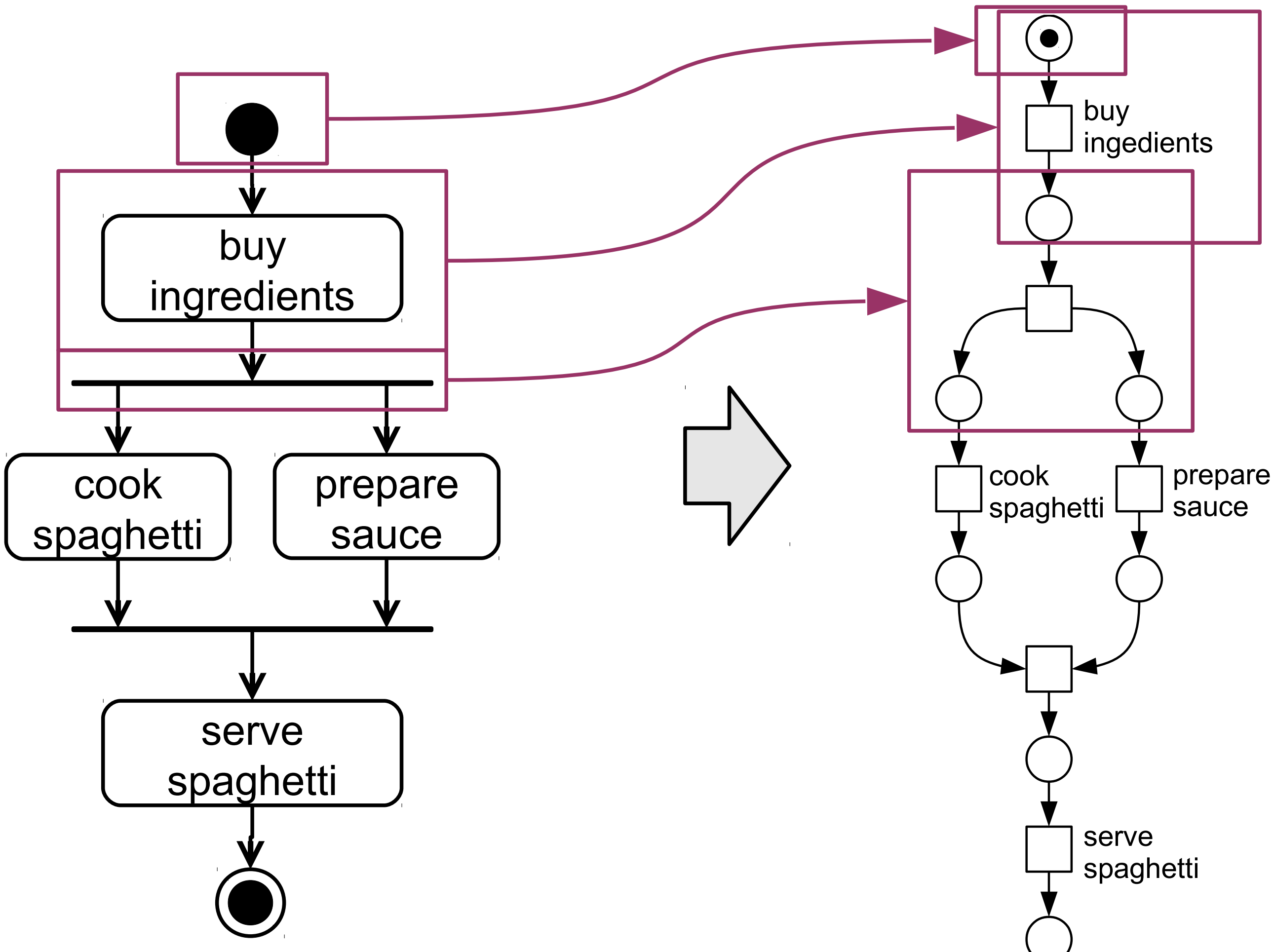
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



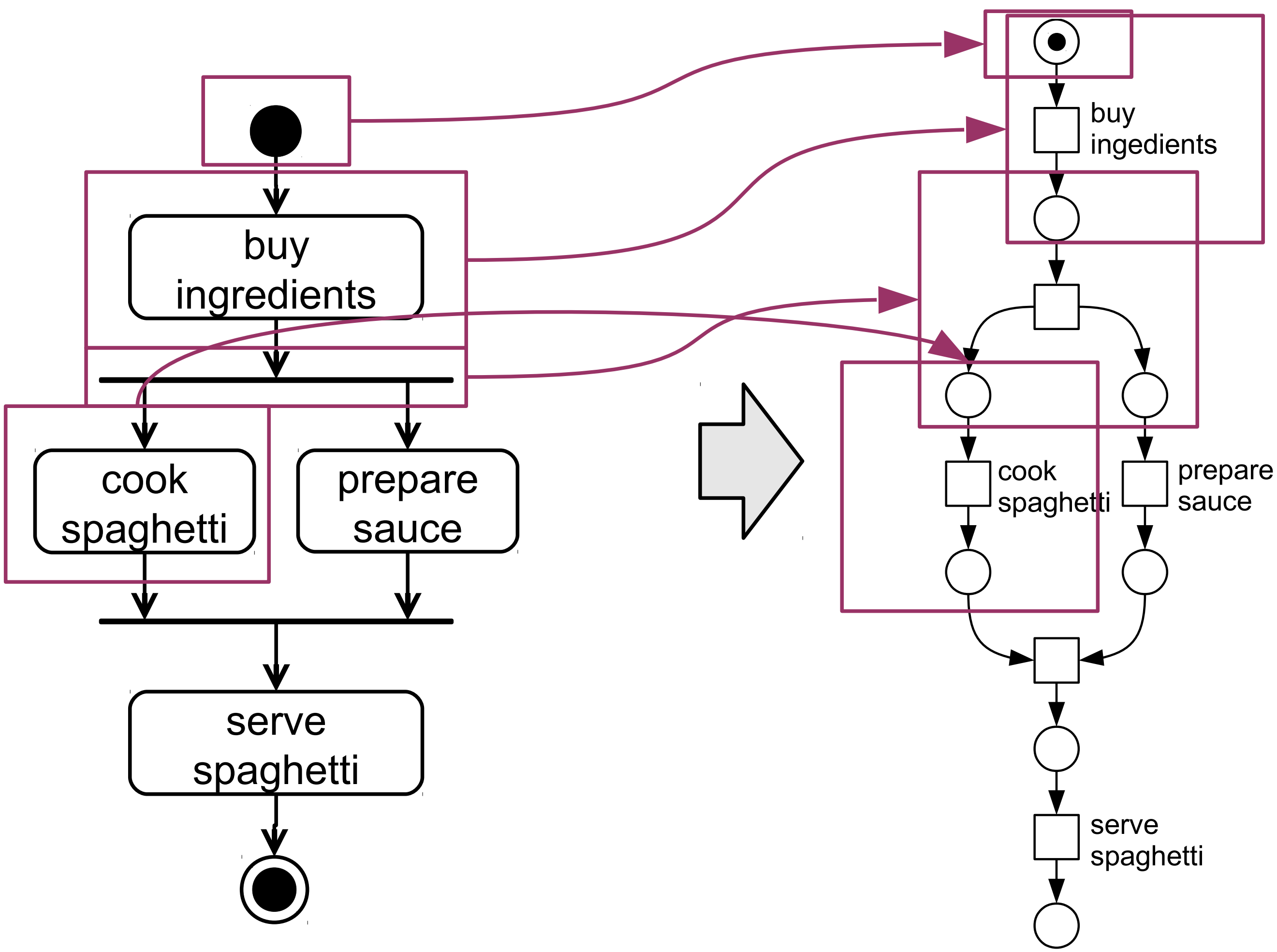
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



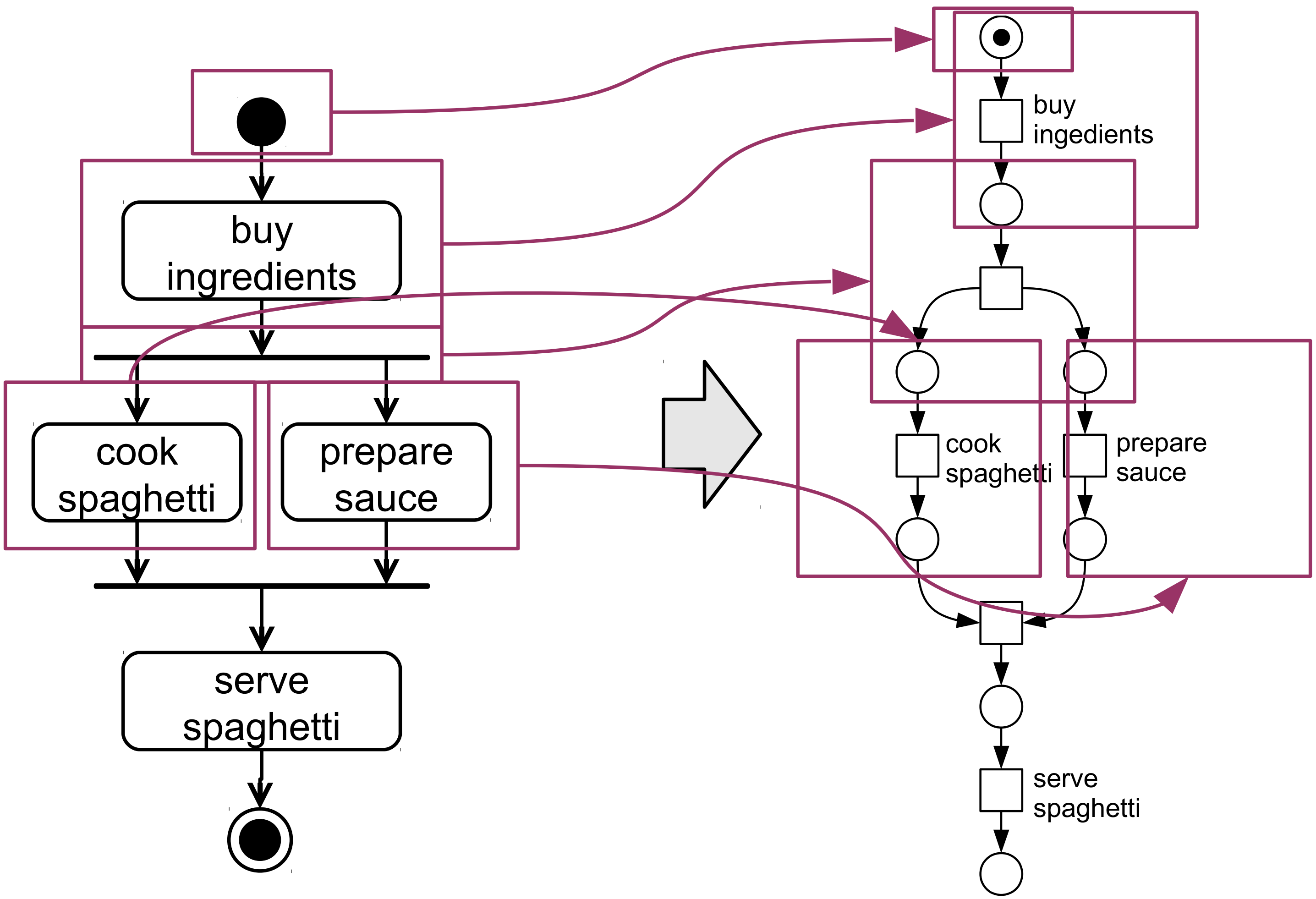
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



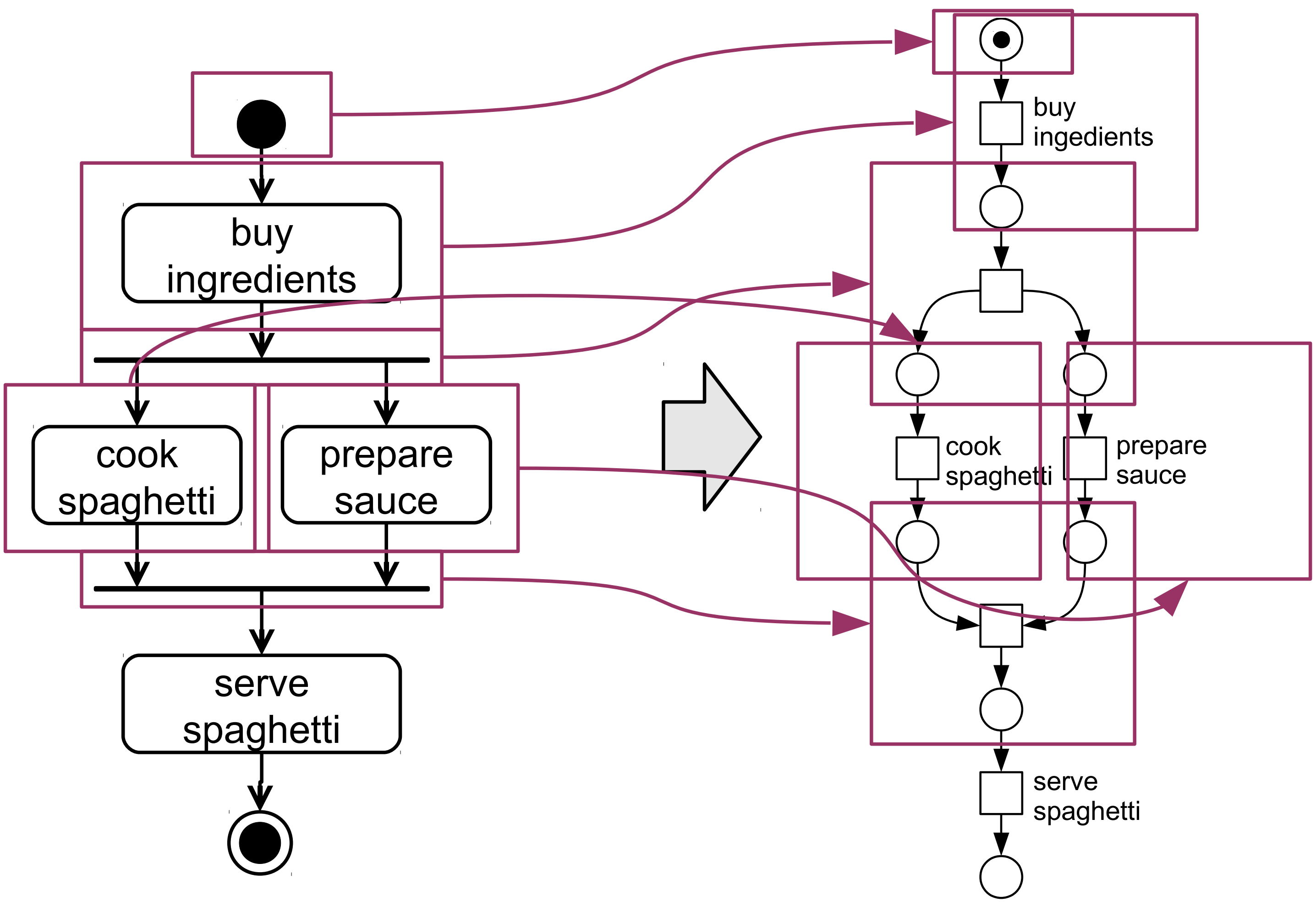
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



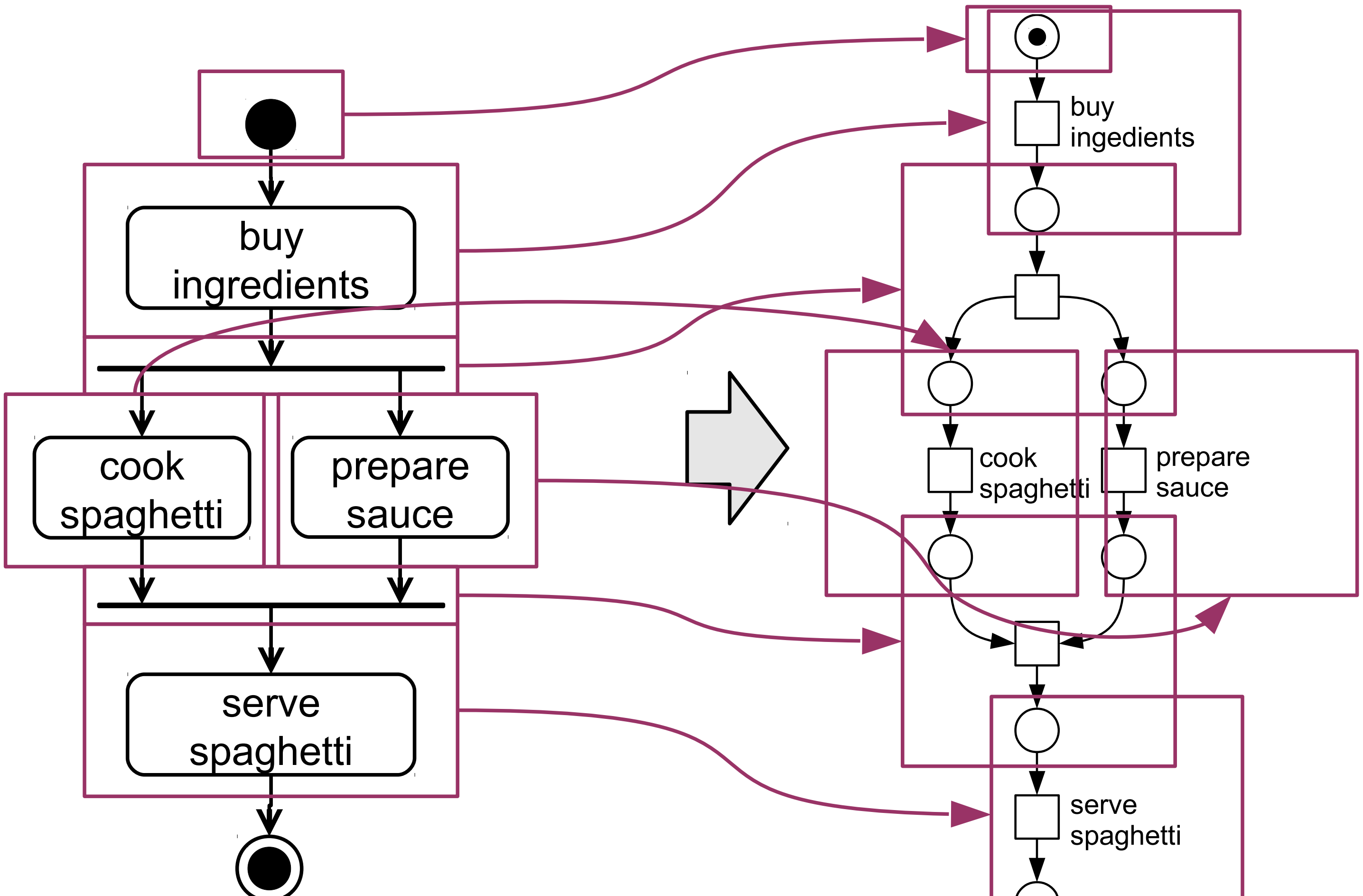
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



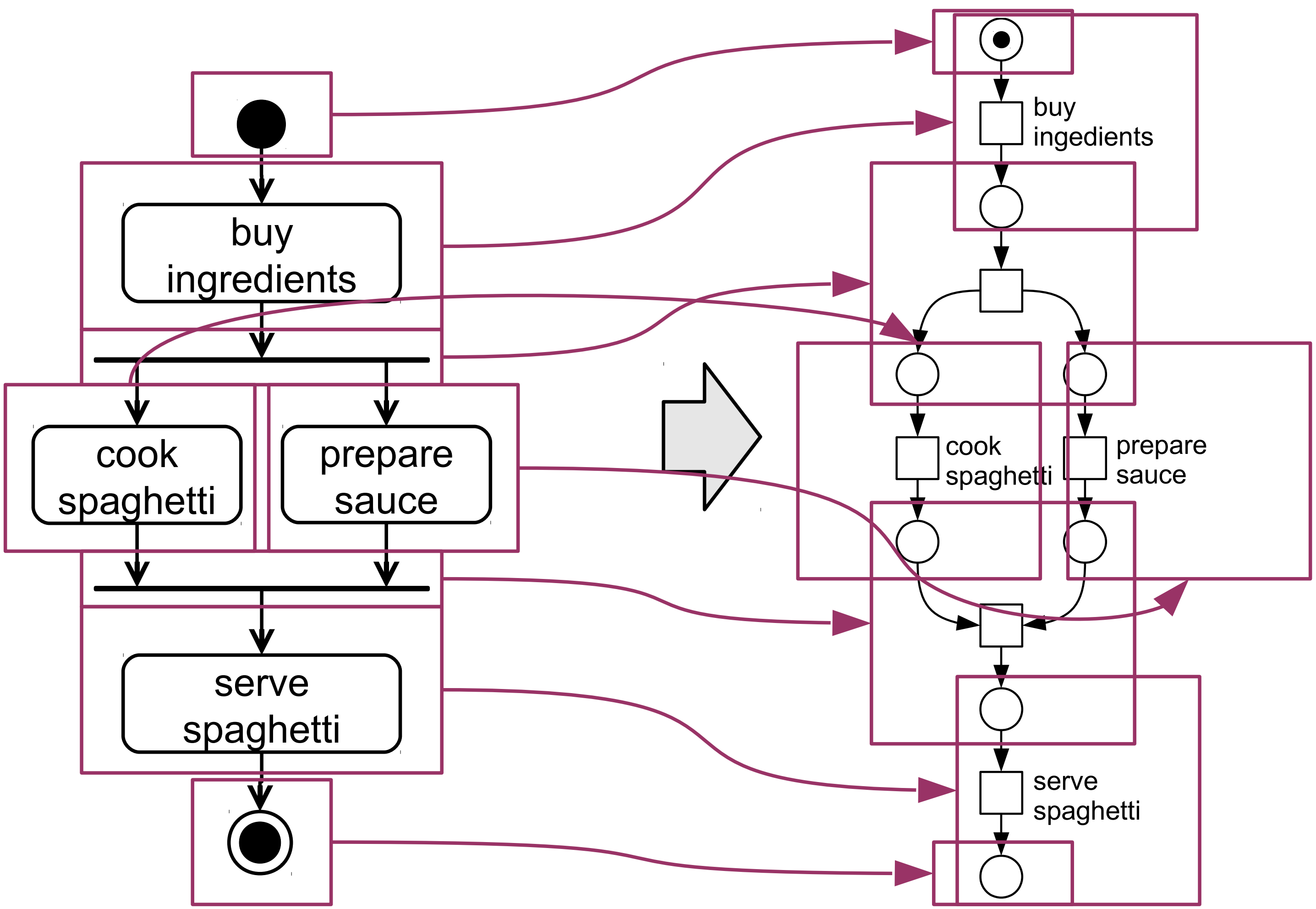
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



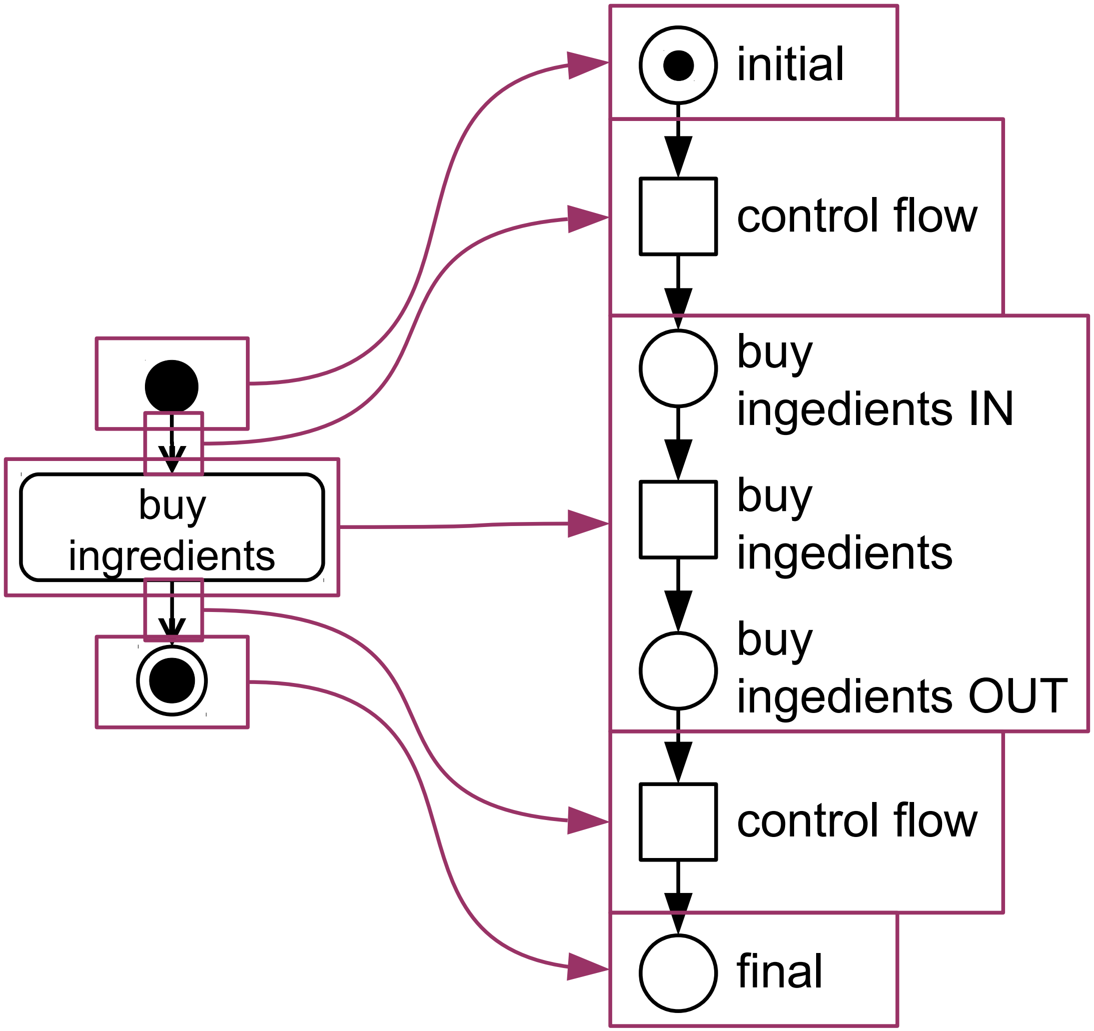
Exogenous Model Transformations

- Example: transform Activity Diagrams to Petri nets



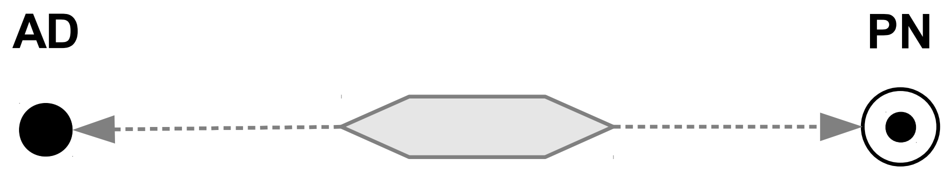
Example:Transform Activity Diagrams to Petri nets

- Let's start simple: How to transform
  - Initial nodes?
  - Final nodes?
  - Action nodes?
  - Control flow edges?



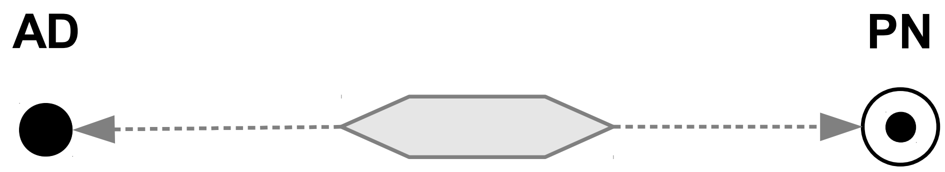
Relations Between Model Patterns
Example: Activity to Petri net

- Intial node ↔ Place with intial marking



Relations Between Model Patterns Example: Activity to Petri net

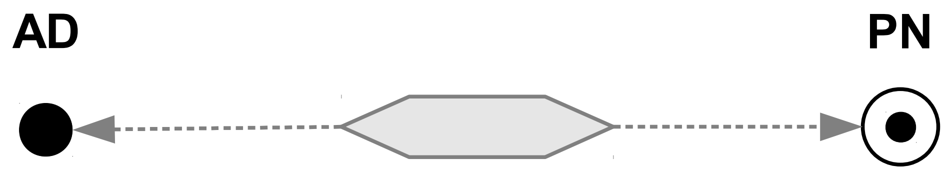
- Intial node ↔ Place with intial marking



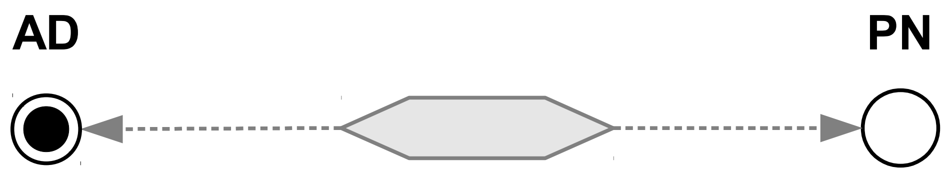
- Final node ↔ Empty Place

Relations Between Model Patterns Example: Activity to Petri net

- Intial node ↔ Place with intial marking

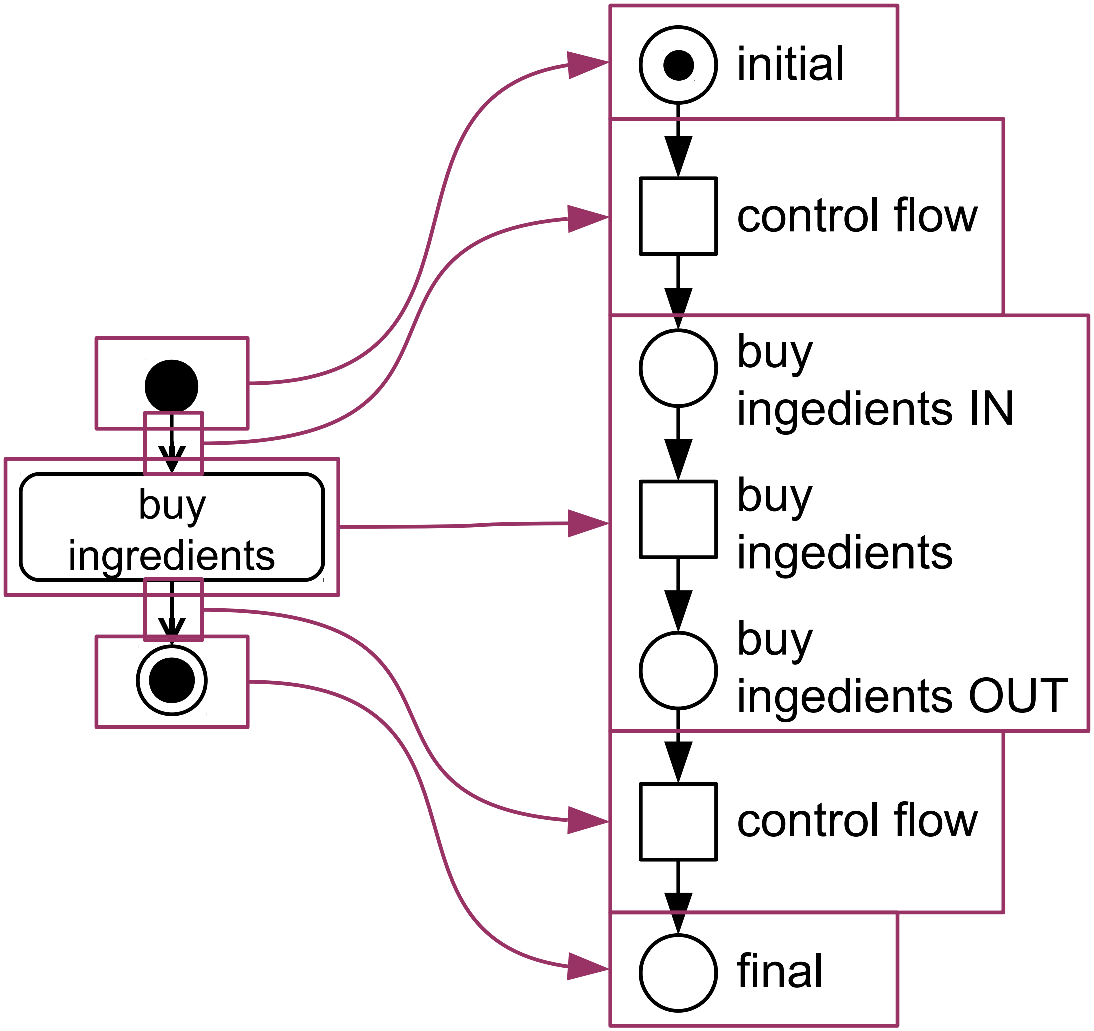


- Final node ↔ Empty Place



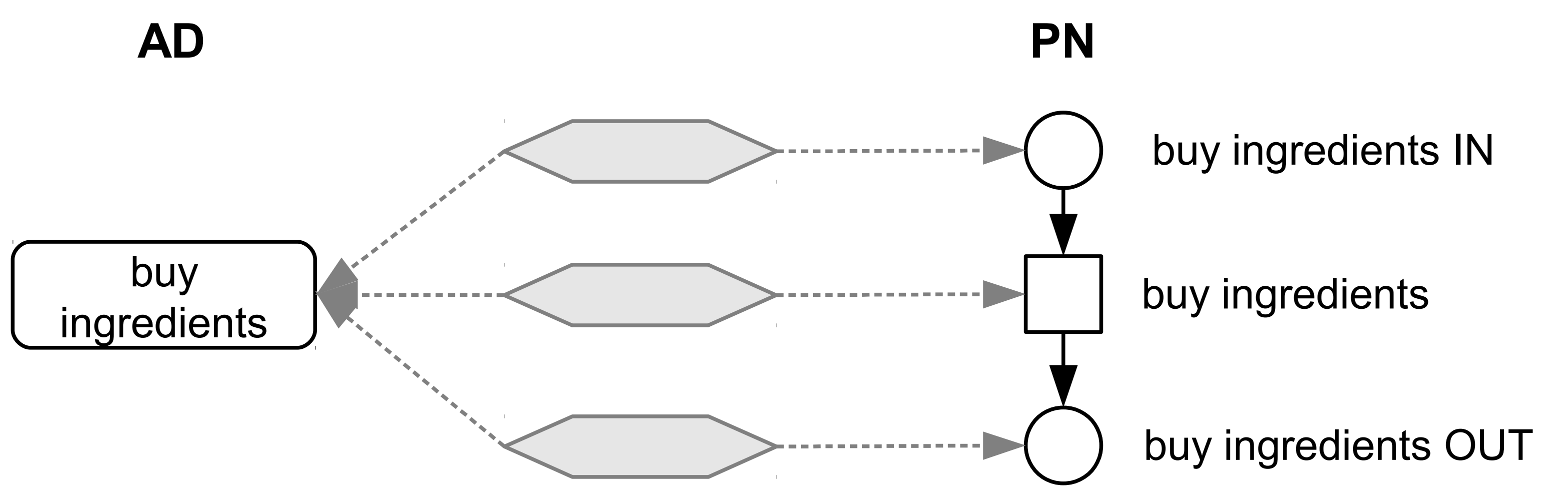
Example: Transform Activity Diagrams to Petri nets

- Let's start simple: How to transform
  - Initial nodes?
  - Final nodes?
  - Action nodes?
  - Control flow edges?



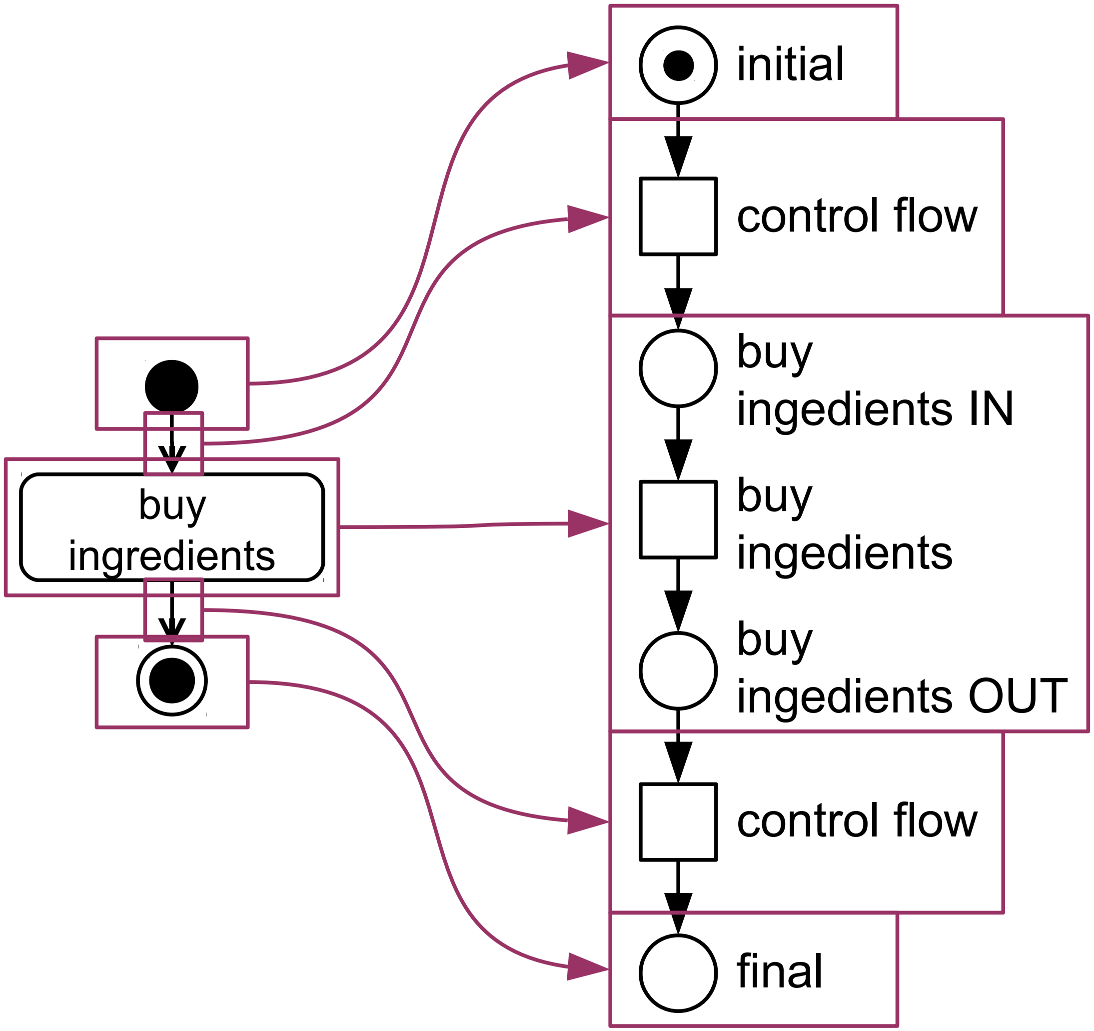
Relations Between Model Patterns Example: Activity to Petri net

- Action node ↔ Transition with input and output place



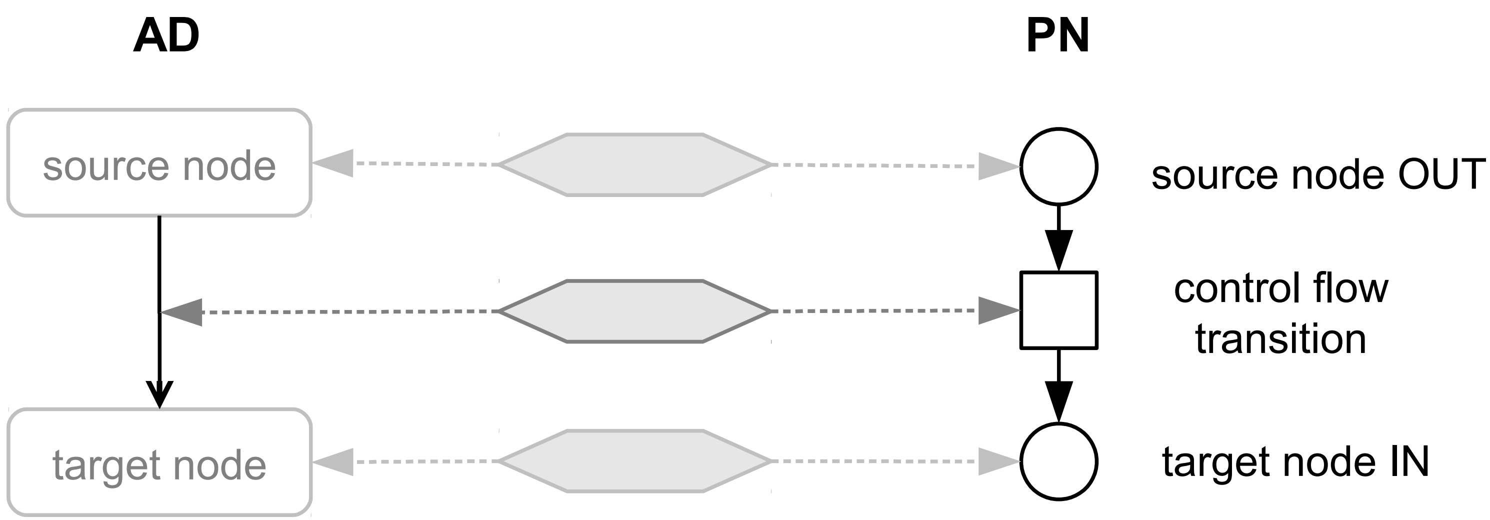
Example:Transform Activity Diagrams to Petri nets

- Let's start simple: How to transform
  - Initial nodes?
  - Final nodes?
  - Action nodes?
  - Control flow edges?



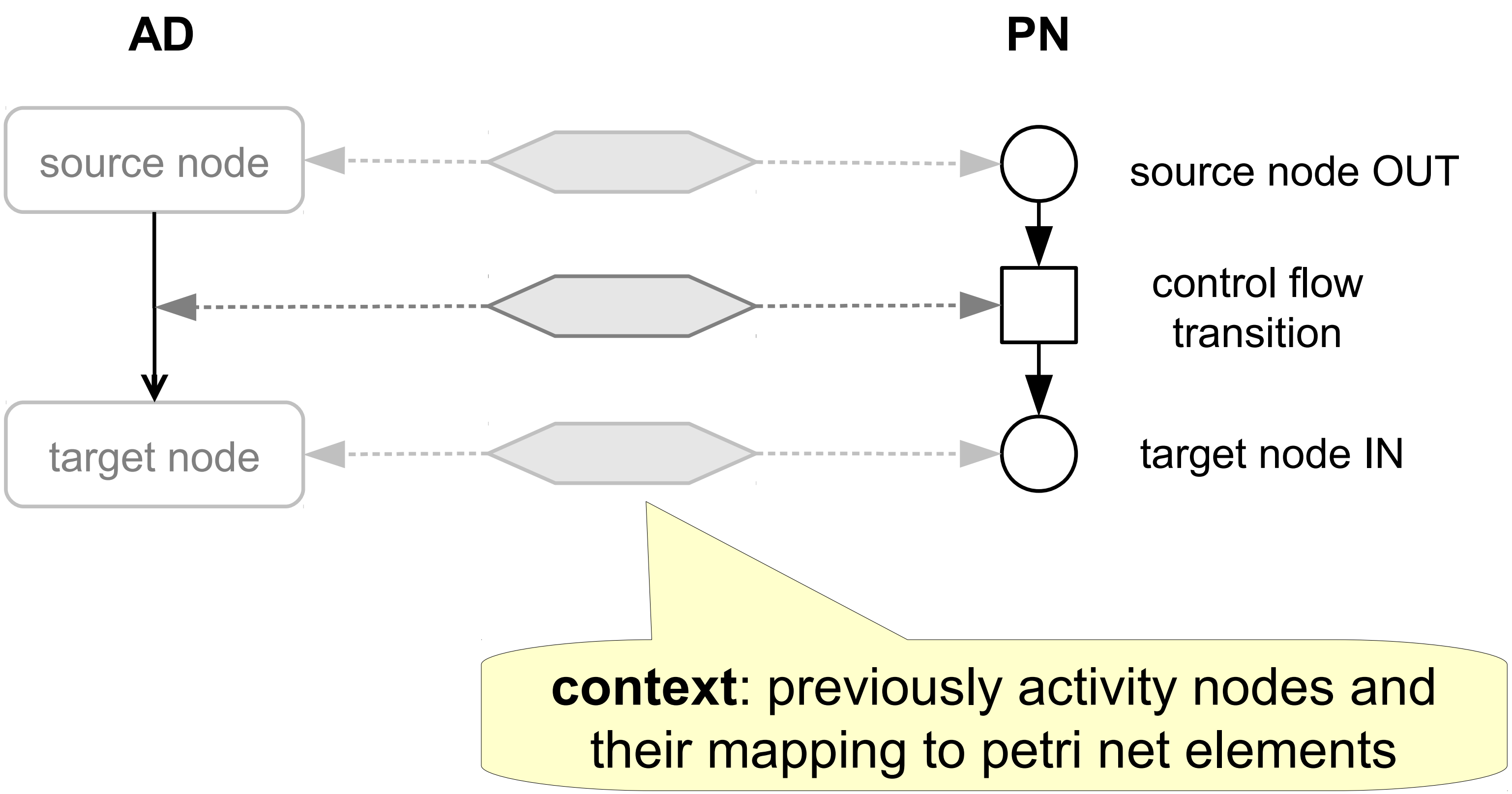
Relations Between Model Patterns Example: Activity to Petri net

- Control flow edge ↔ Transition connecting in- and out places that correspond to the edge's source and target nodes



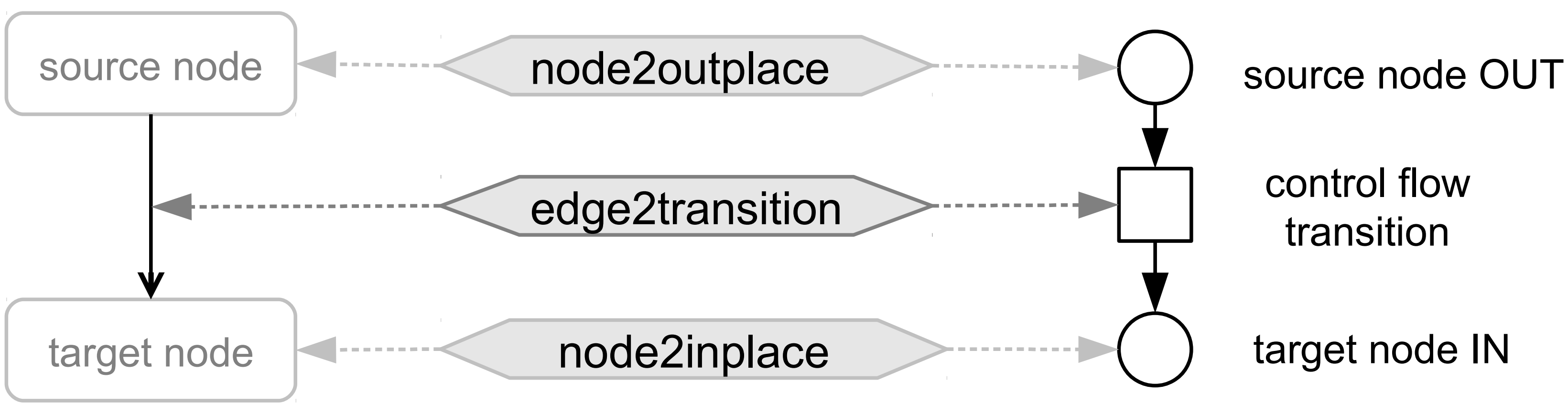
Relations Between Model Patterns Example: Activity to Petri net

- Control flow edge ↔ Transition connecting in- and out places that correspond to the edge's source and target nodes



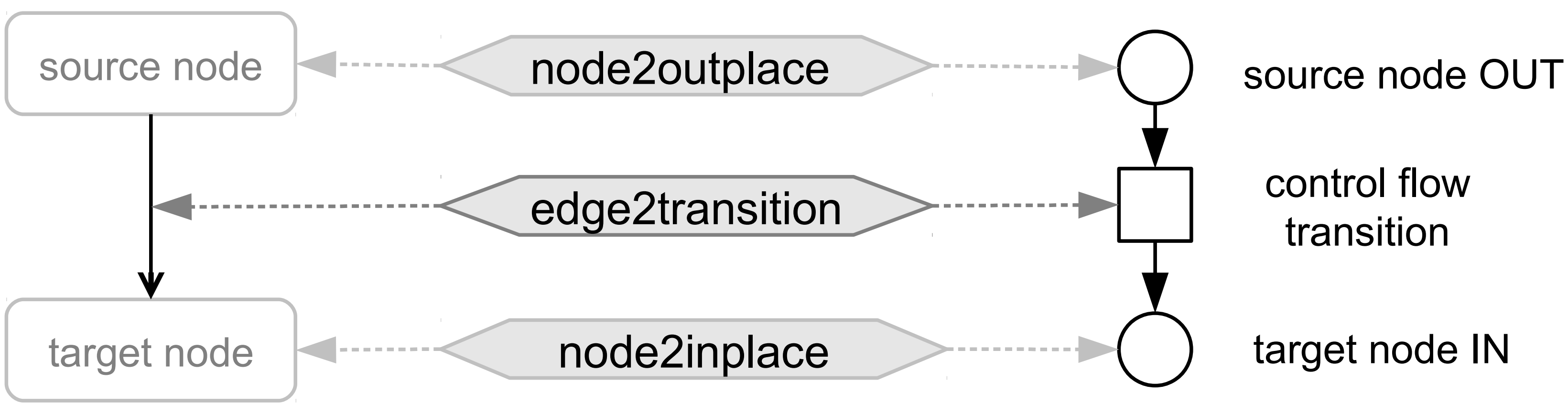
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**



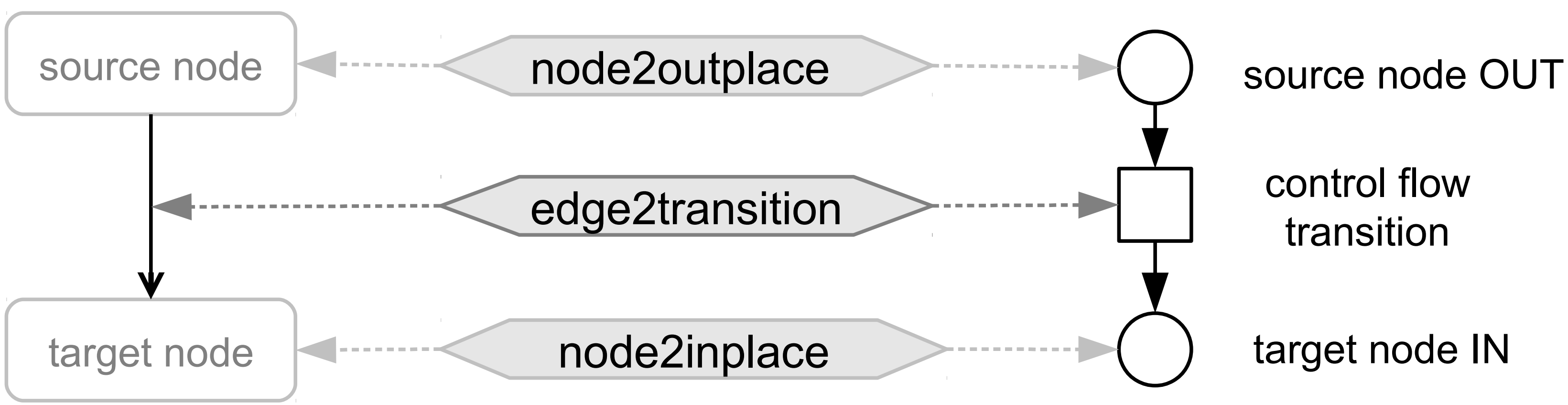
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**
- What does a **triple graph** consist of?



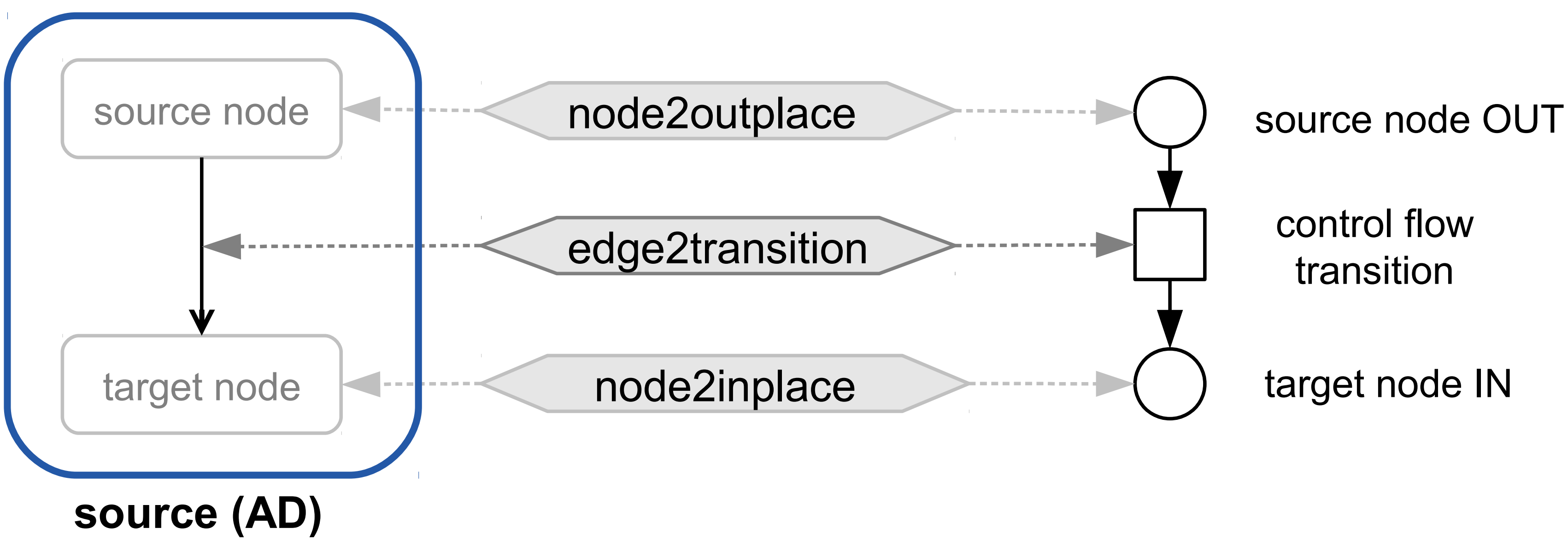
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**
- What does a **triple graph** consist of?
  - **source** graph (model)



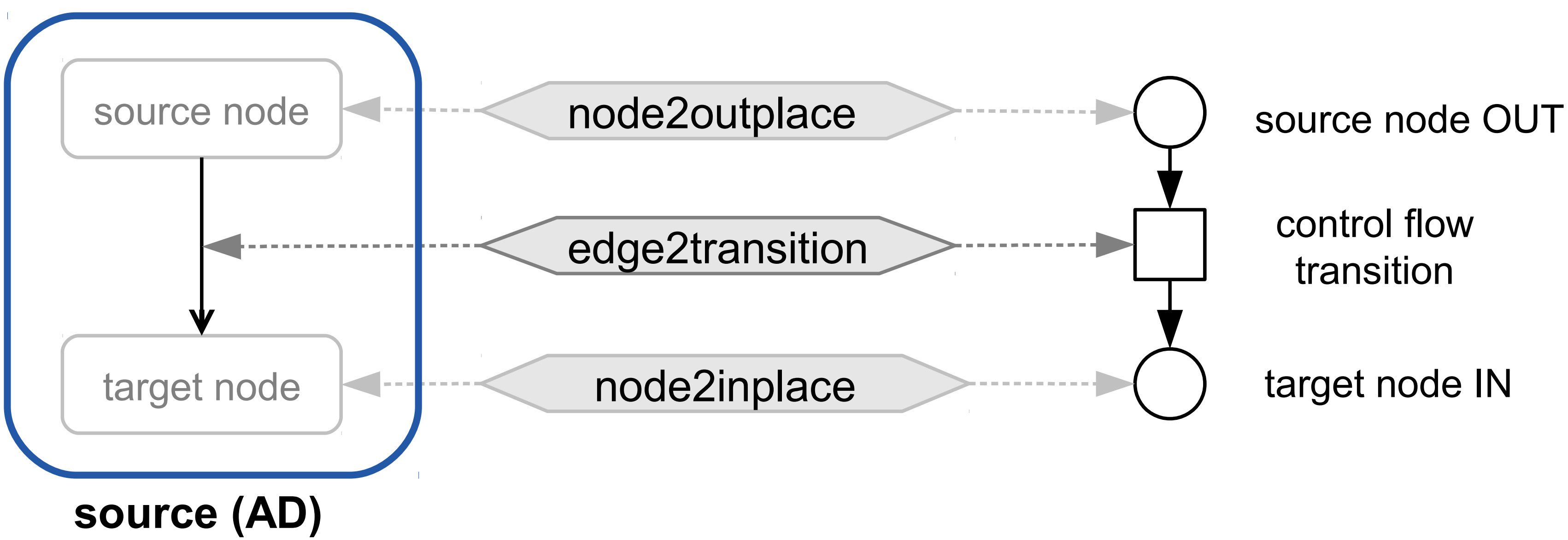
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**
- What does a **triple graph** consist of?
  - **source** graph (model)



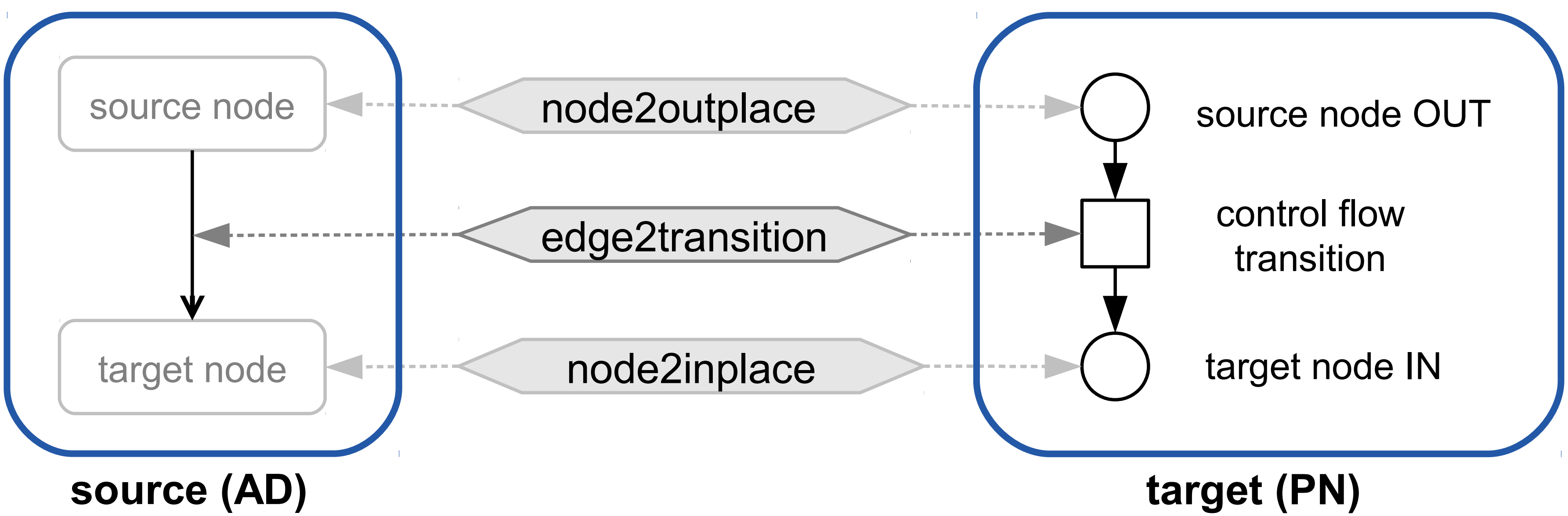
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**
- What does a **triple graph** consist of?
  - **source** graph (model)
  - **target** graph (model)



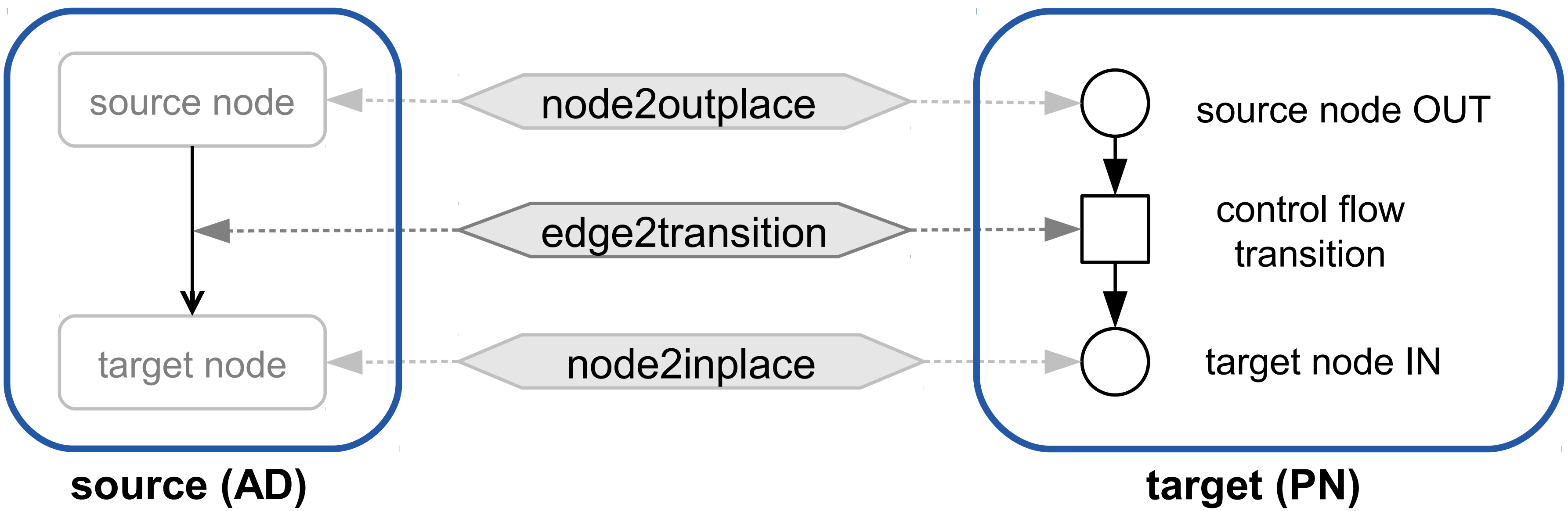
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**
- What does a **triple graph** consist of?
  - **source** graph (model)
  - **target** graph (model)



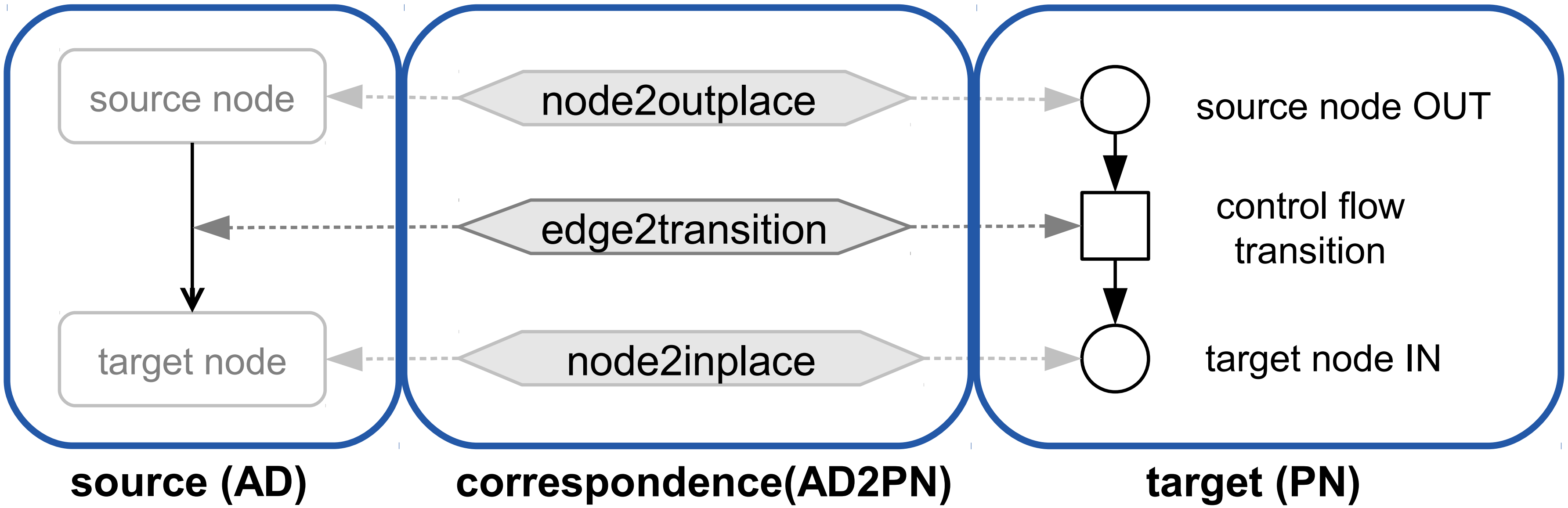
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**
- What does a **triple graph** consist of?
  - **source** graph (model)
  - **target** graph (model)
  - **correspondence** graph (model) that connects the source and target graphs (models)



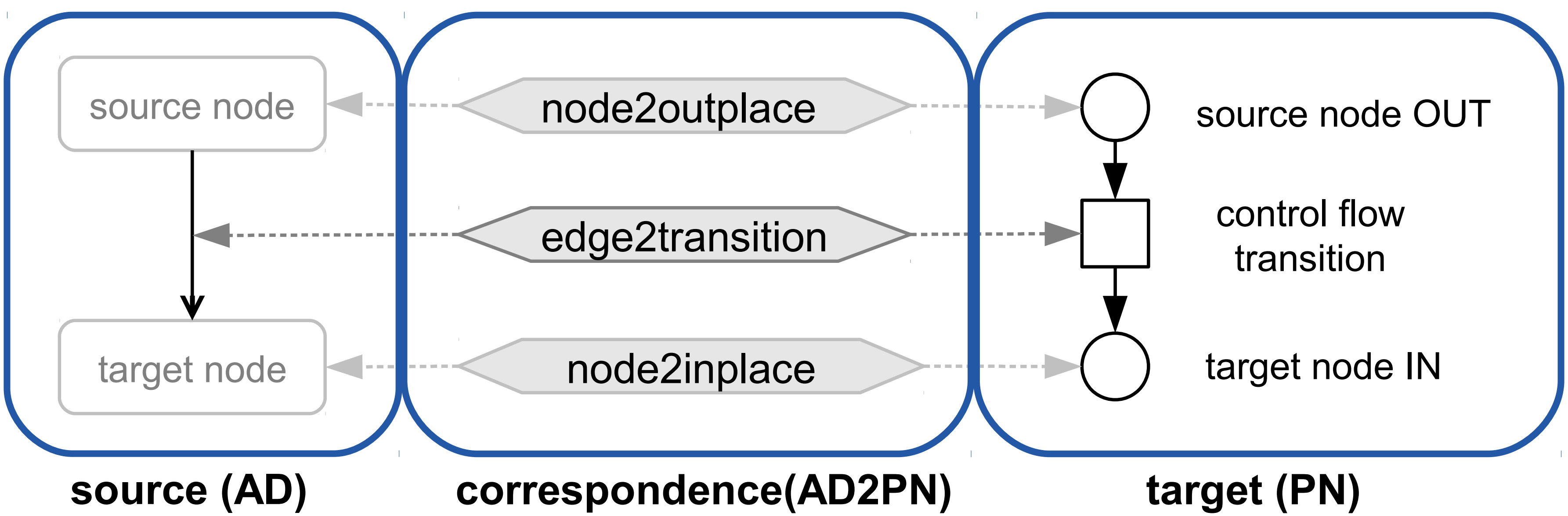
Triple Graphs

- **Idea 1**: describe the mapping of models as a **triple graph**
- What does a **triple graph** consist of?
  - **source** graph (model)
  - **target** graph (model)
  - **correspondence** graph (model) that connects the source and target graphs (models)



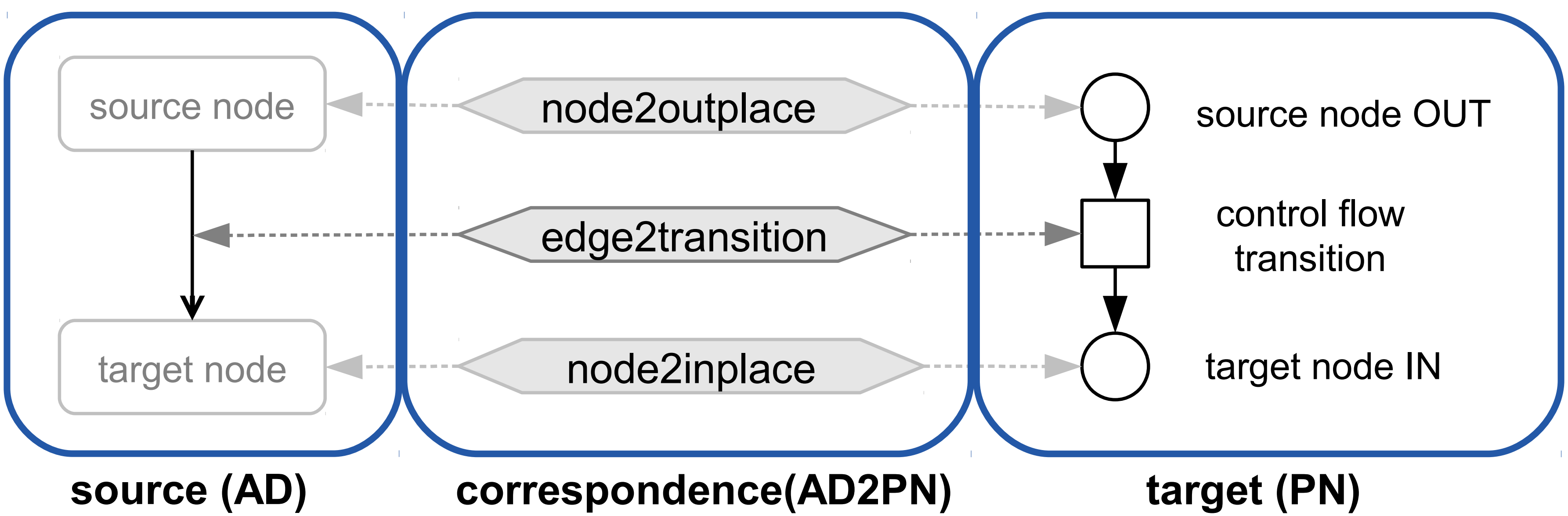
Triple Graphs

- The three different graphs (source, target, correspondence) are typed over (usually different) type graphs (metamodels)



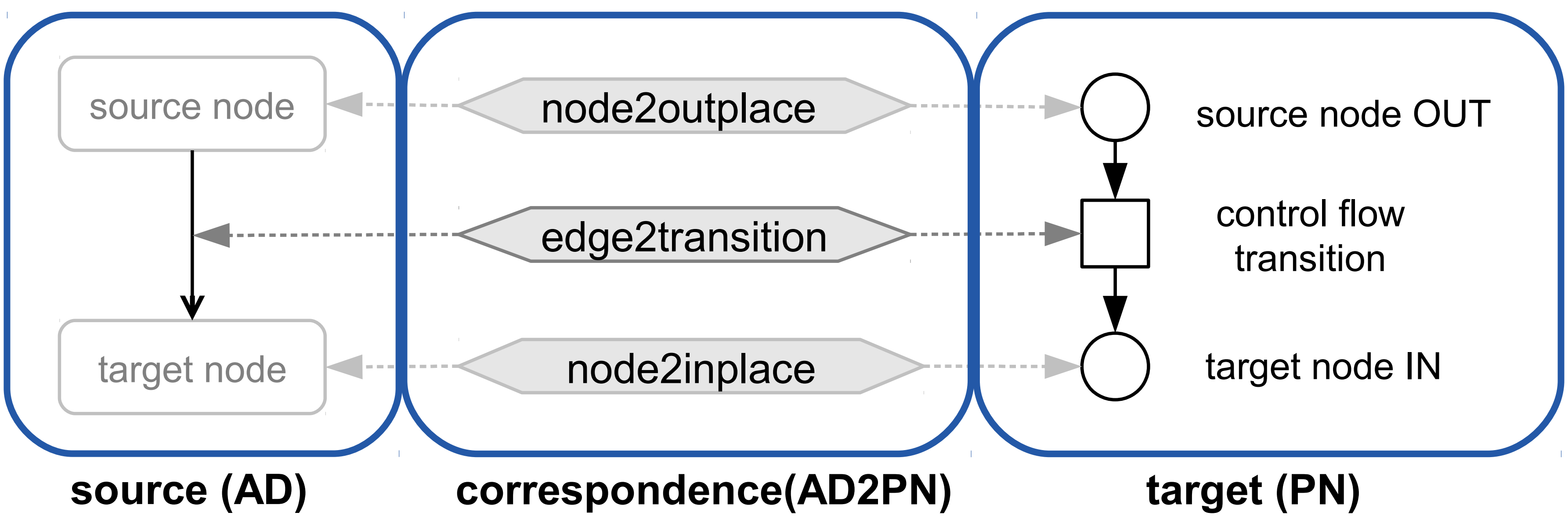
Triple Graphs

- The three different graphs (source, target, correspondence) are typed over (usually different) type graphs (metamodels)
- Also called source-, target-, and correspondence- **domain**



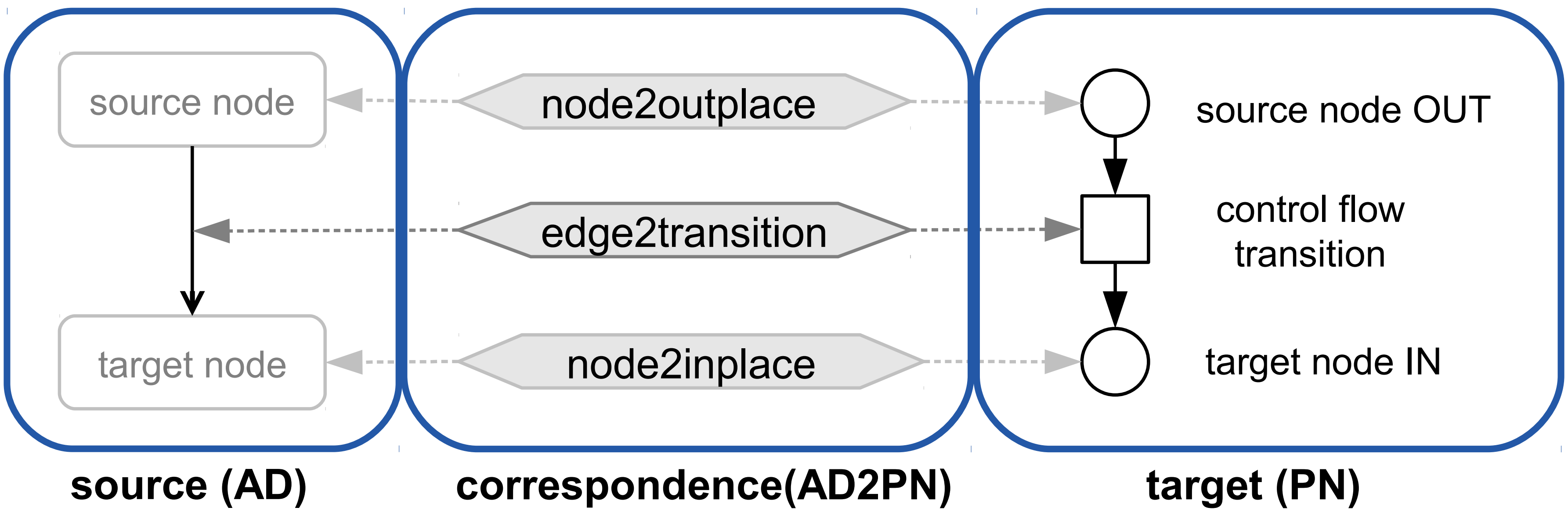
Triple Graphs

- The three different graphs (source, target, correspondence) are typed over (usually different) type graphs (metamodels)
- Also called source-, target-, and correspondence- **domain**
  - **source domain:** Activity Diagrams



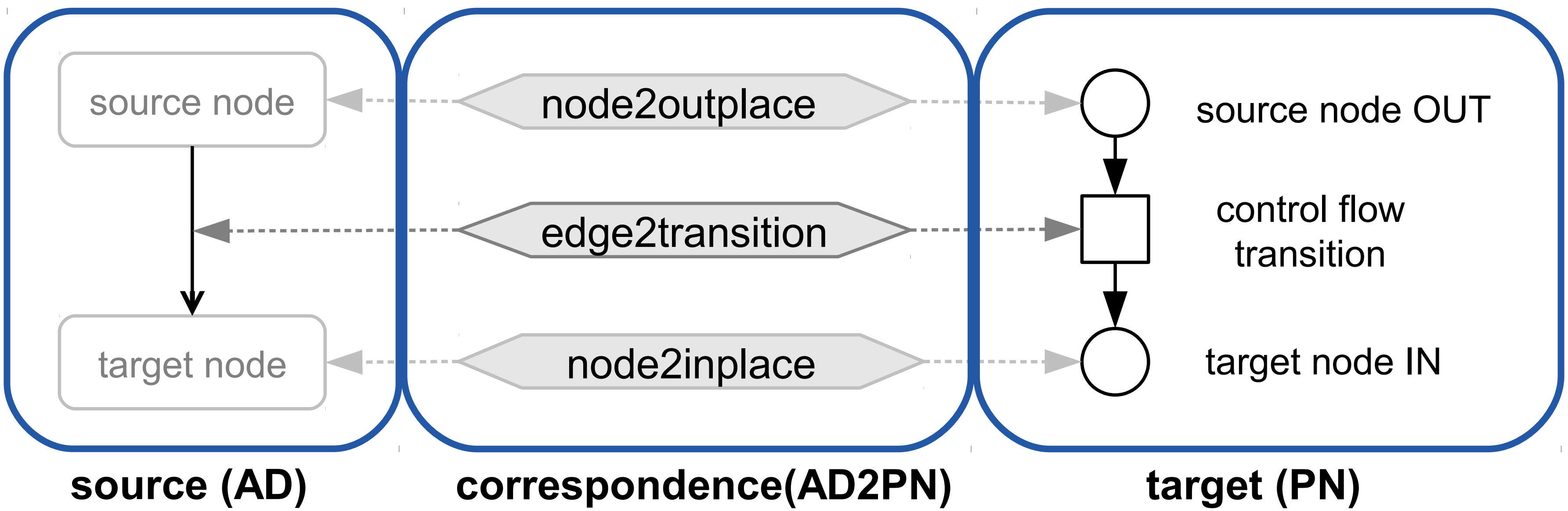
Triple Graphs

- The three different graphs (source, target, correspondence) are typed over (usually different) type graphs (metamodels)
- Also called source-, target-, and correspondence- **domain**
  - **source domain:** Activity Diagrams
  - **target domain:** Petri net



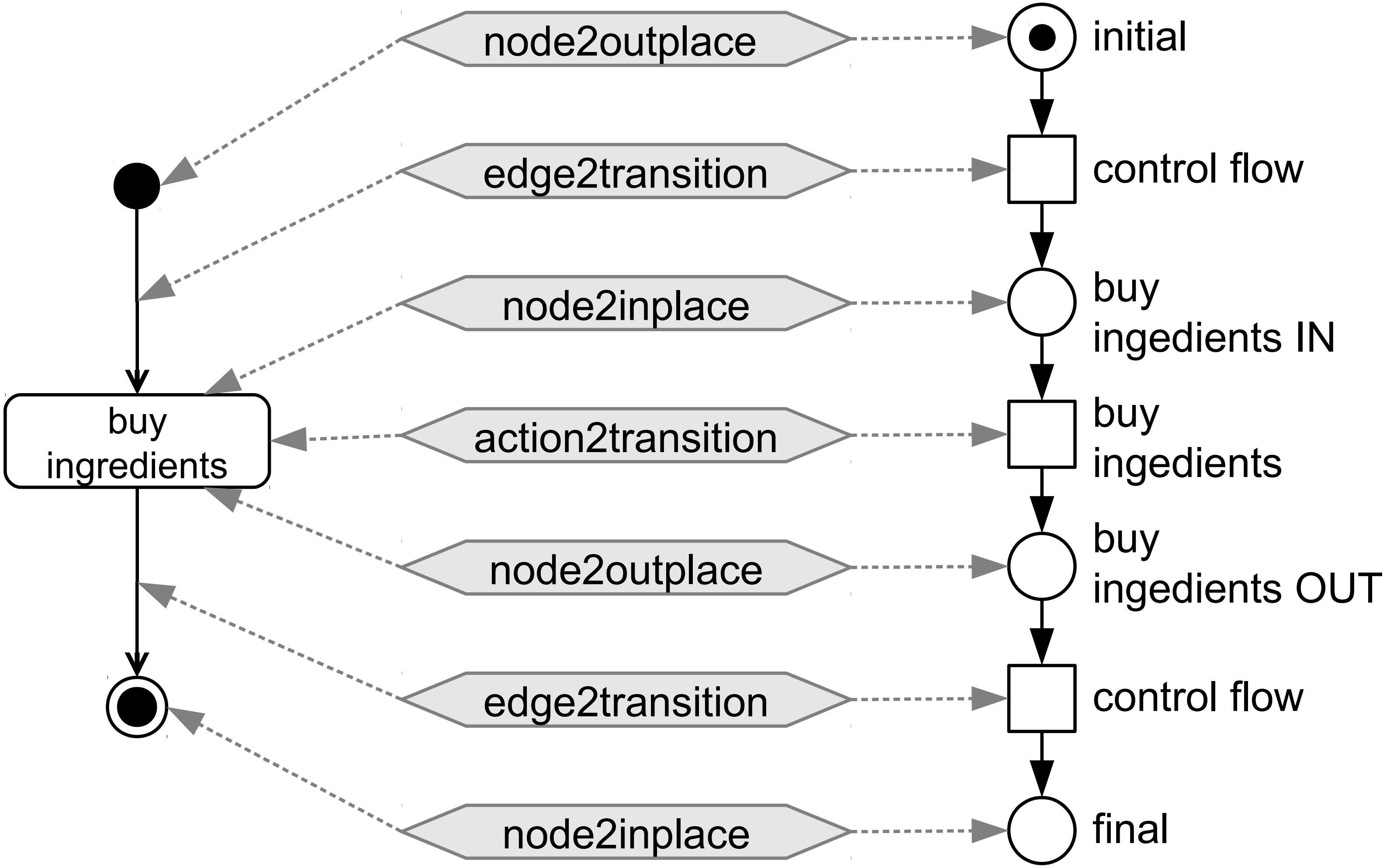
Triple Graphs

- The three different graphs (source, target, correspondence) are typed over (usually different) type graphs (metamodels)
- Also called source-, target-, and correspondence- **domain**
  - **source domain:** Activity Diagrams
  - **target domain:** Petri net
  - **correspondence domain:** AD2PN



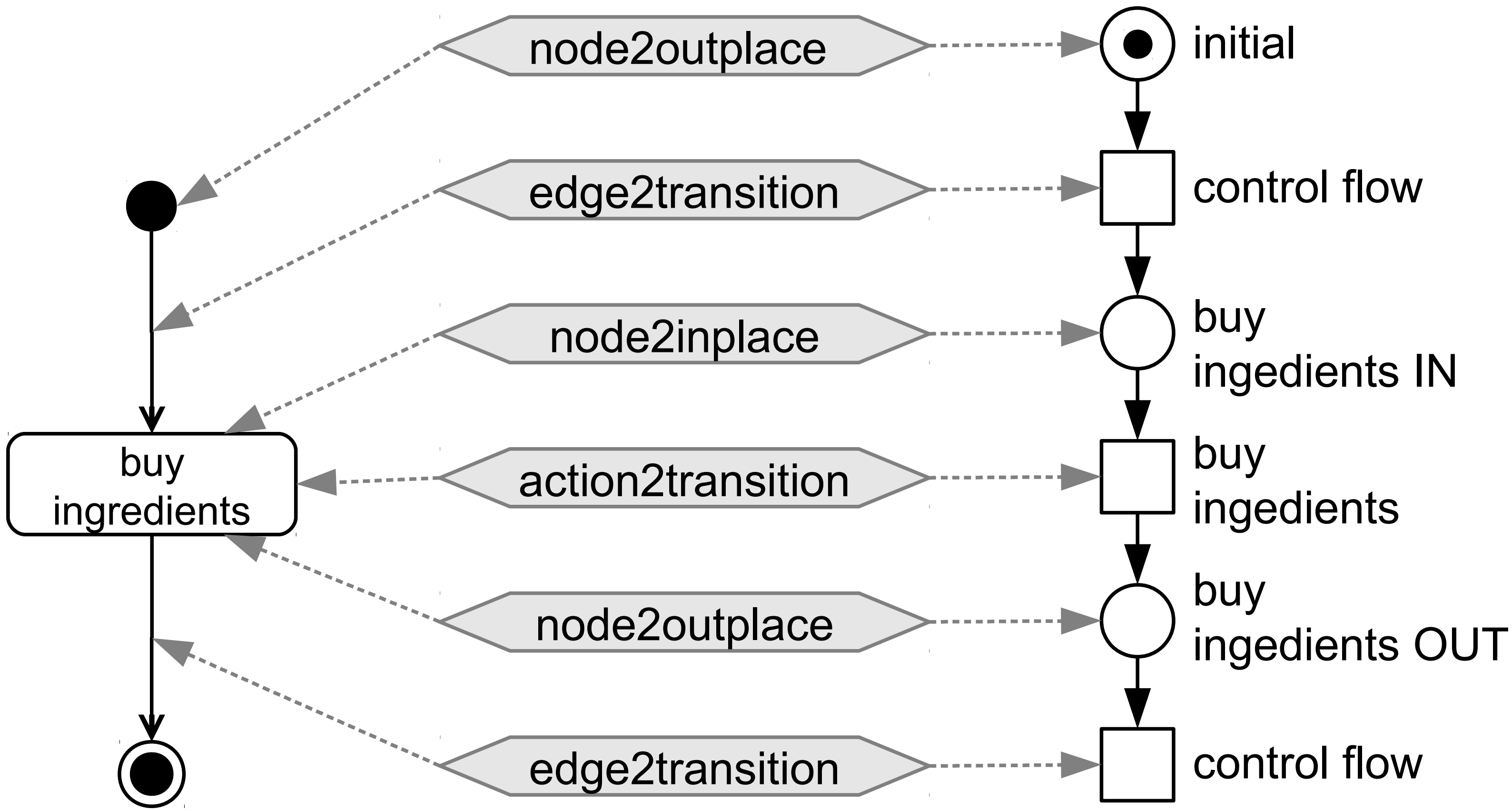
Triple Graphs

- Example of a bigger triple graph



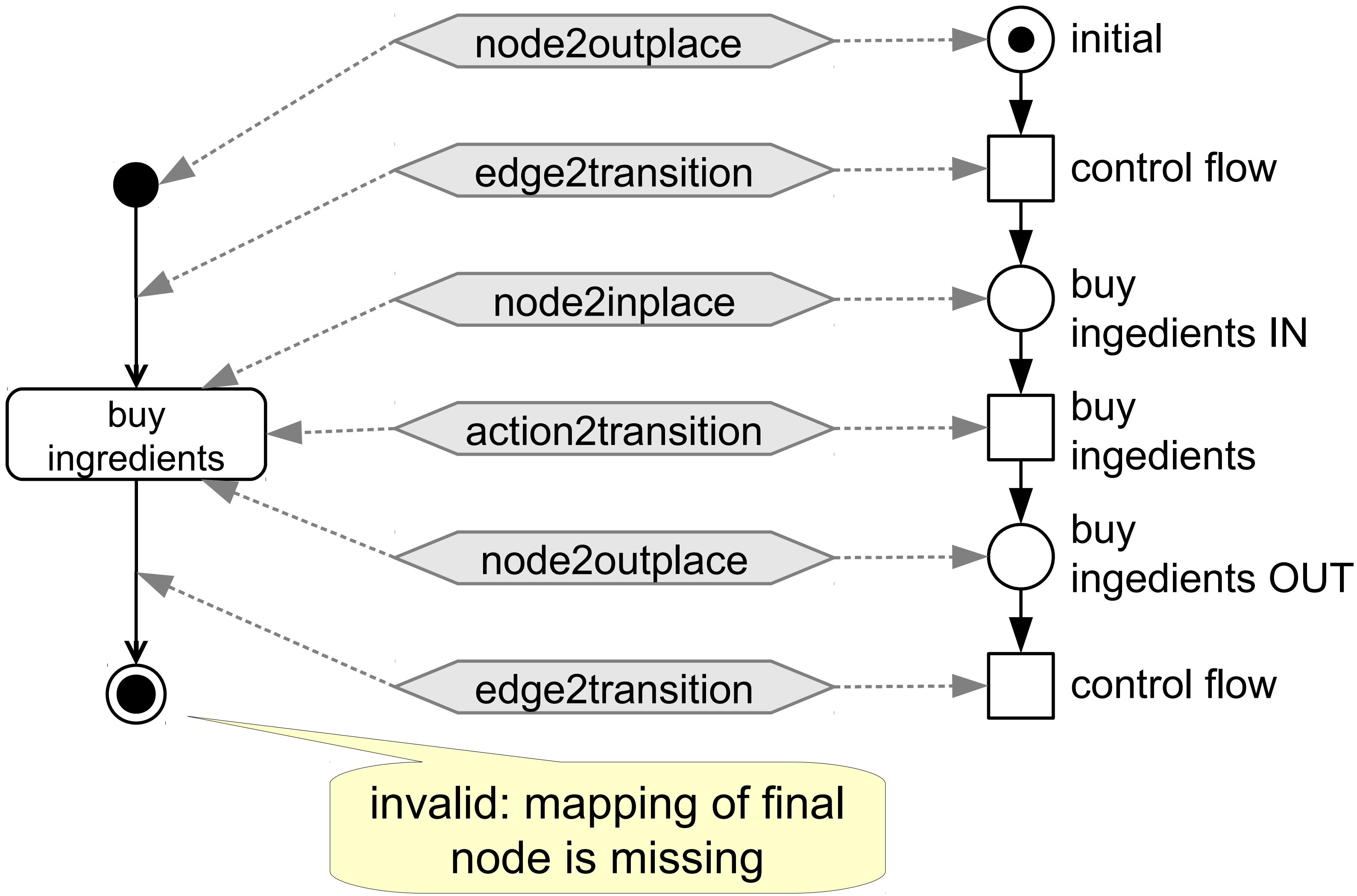
Triple Graphs

- An “invalid” triple graph



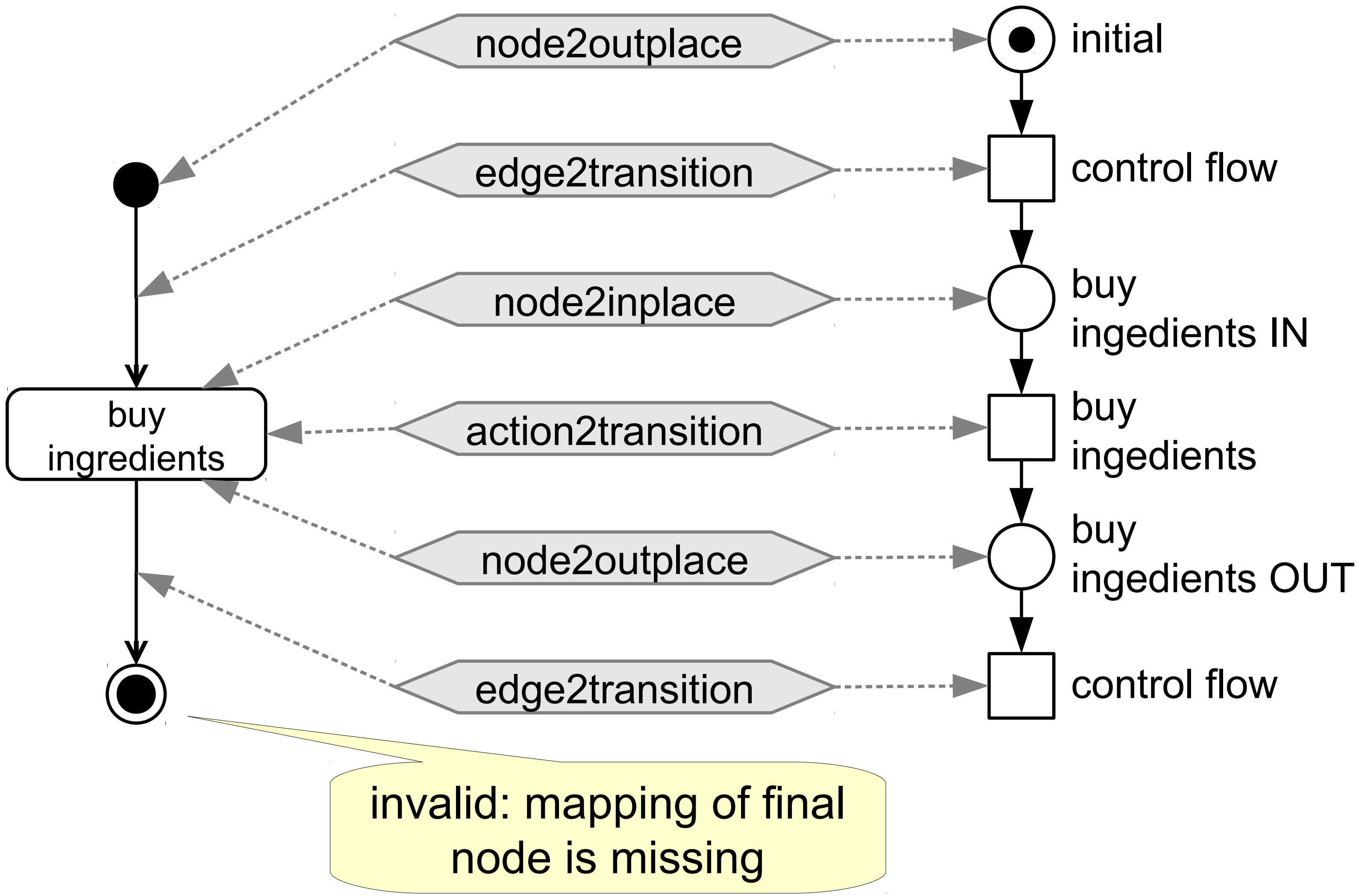
Triple Graphs

- An “invalid” triple graph



Triple Graphs

- An "invalid" triple graph



Triple Graph Grammar (TGG)

- How to describe which triple graphs are valid in which ones are not?
  - i.e., express which mappings are valid and which ones are not

Triple Graph Grammar (TGG)

- How to describe which triple graphs are valid in which ones are not?
  - i.e., express which mappings are valid and which ones are not

- **Idea 2**: Use a **graph grammar** that describes the production of valid triple graphs

Triple Graph Grammar (TGG)

- How to describe which triple graphs are valid in which ones are not?
  - i.e., express which mappings are valid and which ones are not

- **Idea 2**: Use a **graph grammar** that describes the production of valid triple graphs
  - → Triple Graph Grammar (TGG)