Collaborative Filtering

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User-based Neighbor Algorithms

Given is the following user-item matrix:

User / Item	A	B	C	D
X		3	5	
Y	4	2		1
Z		1	5	5

1. Please create recommendations for user X based on the similar users Y and Z. The similarity between the users shall be calculate using the cosine metric. Use the following recommendation algorithms:
   1. Simple average r_1: $r_1(U, i) = \frac{1}{|S_i|} \sum_{s \in S_i} s[i]$
   2. Weighted average r_2: $r_2(U, i) = \frac{1}{\sum_{s \in S_i} sim(U,s)} \sum_{s \in S_i} sim(U, s) * s[i]$

Solution: User-based Neighbor Algorithms

$$
\begin{aligned}
S &= \{Y, Z\} \\
S_A &= \{Y\} \\
S_D &= \{Y, Z\} \\
r_1(X, A) &= \frac{1}{|S_A|} * \sum_{s \in S_A} s[i] \\
&= \frac{1}{1} * 4 = 4 \\
r_1(X, D) &= \frac{1}{|S_D|} * \sum_{s \in S_D} s[i] \\
&= \frac{1}{2} * (1 + 5) = 3
\end{aligned}
$$

Solution: User-based Neighbor Algorithms

$$
\begin{aligned}
cosim(X, Y) &= \frac{(3) \cdot (2)}{\sqrt{9} \cdot \sqrt{4}} = 1 \\
cosim(X, Z) &= \frac{\begin{pmatrix}3\\5\end{pmatrix} \cdot \begin{pmatrix}1\\5\end{pmatrix}}{\sqrt{34} \cdot \sqrt{26}} = 0,942 \\
r_2(X, A) &= \frac{1}{\sum_{s \in S_A} sim(X, s)} \sum_{s \in S_A} sim(X, s) * s[i] = \\
&= \frac{1}{sim(X, Y)} * sim(X, Y) * Y[A] = \\
&= \frac{1}{1} * 1 * 4 = 4 \\
r_2(X, D) &= \frac{1}{\sum_{s \in S_D} sim(X, s)} \sum_{s \in S_D} sim(X, s) * s[i] = \\
&= \frac{1}{sim(X, Y) + sim(X, Z)} * (sim(X, Y) * Y[D] + sim(X, Z) * Z[D]) = \\
&= \frac{1}{1,942} * (1 * 1 + 0,942 * 5) = 2,94
\end{aligned}
$$

Item-based Neighbor Algorithm

Given is the following user-item matrix:

User / Item	A	B	C	D
X		3	1	5
Y	4	2		1
Z		3	5	5

1. Based on the user-item matrix create the corresponding item-item matrix.
2. Please generate recommendations for the user X.

Remark: Use the cosine similarity to calculate the similarities:
$cosim(i,j) = \frac{\overrightarrow{r_i} * \overrightarrow{r_j}}{|\overrightarrow{r_i}| * |\overrightarrow{r_j}|}$

Solution: Item-based Neighbor Algorithm

$$
\begin{aligned}
cosim(A, B) &= \frac{(4) \cdot (2)}{\sqrt{16} \cdot \sqrt{4}} = 1 \\
cosim(A, C) &= n.a. \\
cosim(A, D) &= 1 \\
cosim(B, C) &= \frac{\begin{pmatrix}3\\3\end{pmatrix} \cdot \begin{pmatrix}1\\5\end{pmatrix}}{\sqrt{18} \cdot \sqrt{26}} \approx 0.83 \\
cosim(B, D) &= \frac{\begin{pmatrix}3\\2\\3\end{pmatrix} \cdot \begin{pmatrix}5\\1\\5\end{pmatrix}}{\sqrt{22} \cdot \sqrt{51}} \approx 0.96 \\
cosim(C, D) &= \frac{\begin{pmatrix}1\\5\end{pmatrix} \cdot \begin{pmatrix}5\\5\end{pmatrix}}{\sqrt{26} \cdot \sqrt{50}} \approx 0.83
\end{aligned}
$$

Item / Item	A	B	C	D
A	-	1	n.a.	1
B	**1**	-	**0.83**	**0.96**
C	n.a.	**0.83**	-	**0.83**
D	**1**	**0.96**	**0.83**	-
Similarity vector:	**1**	0.895	0.83	0.895

X is recommended A with score 1.