

Advanced Topics in Computational Complexity

Exercise Session 2

Due 26.10.2015.

Exercise 1

Let \mathfrak{A} be a model with domain $A = \{0, 1, 2\}$. Let X be the team defined by the following table

	x_0	x_1	x_2
s_1	1	2	2
s_2	2	1	2
s_3	0	1	2

Does $\mathfrak{A} \models_X \varphi$ hold when

1. $\varphi := x_0 = x_2$ or $\varphi := \neg x_0 = x_2$
2. $\varphi := \exists x_0 (x_0 = x_2)$
3. $\varphi := \forall x_3 (= (x_2))$
4. $\varphi := (= (x_0, x_1) \vee = (x_1, x_2))$

Exercise 2

Show that $\models \forall x_0 \forall x_1 (x_0 = x_1 \rightarrow = (x_0, x_1))$. (Recall that $\varphi \rightarrow \psi$ is a shorthand notation for $(\neg \varphi \vee \psi)$.)

Exercise 3

Let \mathfrak{A} be a model with domain that has at least two elements. Describe teams X and Y of \mathfrak{A} of domain $\{x_0, x_1, x_2\}$ such that

- 1) $\mathfrak{A} \models_X \exists x_0 (= (x_0) \wedge x_0 = x_1)$
- 2) $\mathfrak{A} \models_Y \exists x_0 (= (x_0) \wedge \neg x_0 = x_1)$

Exercise 4

Write a formula of dependence logic that defines the class of infinite structures.

Exercise 5

Proof Proposition 3 of the handout.