Mobilkommunikation - Mobile Communications

Lecture 5: Random Access

Prof. Dr.-Ing. Markus Fidler

Institute of Communications Technology
Leibniz Universität Hannover

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Previous lecture

So far multiple access is coordinated

- F/T/CDMA
- static allocation or dynamic assignment

However, wireless communication is often much more ad-hoc

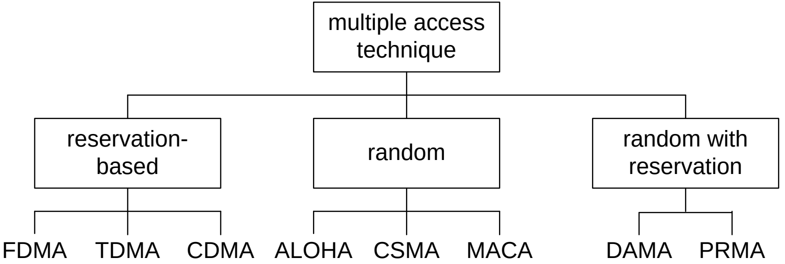
- new terminals have to register with the network
- terminals request access to the medium spontaneously
- in many cases there is no central control

Need other access methods

- distributed
- non-arbitrated

⇒ random access

Multiple access techniques



- reservation-based: fixed allocation of resources to terminals
- random access: no collision free allocation; terminals compete for the channel using randomized procedures

Multiple access to a shared medium

Many access networks, e.g., Local Area Networks (LANs), Wireless LANs, and radio access networks, use a shared medium, i.e., Ether.

Advantages:

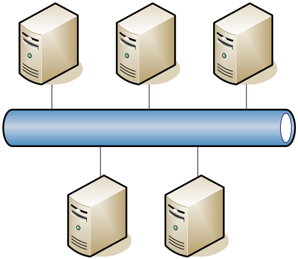
- access to the entire medium
- statistical multiplexing

Need: medium access control

- non-carrier sense
- vs. carrier sense

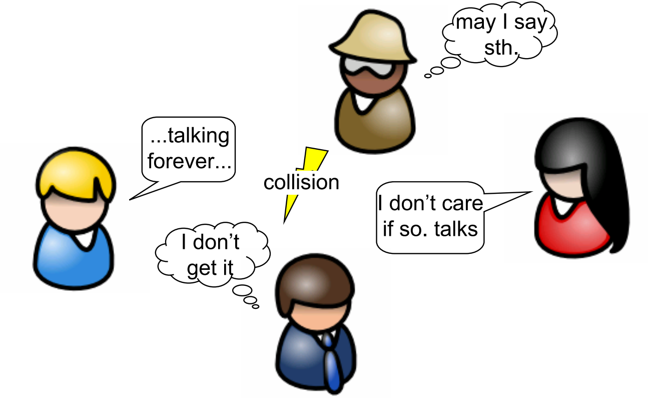
Choice depends on the quotient of

- propagation delay
- transmission delay



An analogy

Consider a number of participants at a typical meeting.



Think about it: What is the protocol for sharing the medium?

Outline

- ALOHA
  - Slotted ALOHA
  - Pure ALOHA
- Carrier sense multiple access
  - Renewal theory
  - CSMA throughput model
  - Collision avoidance
- Random access with reservation

ALOHA

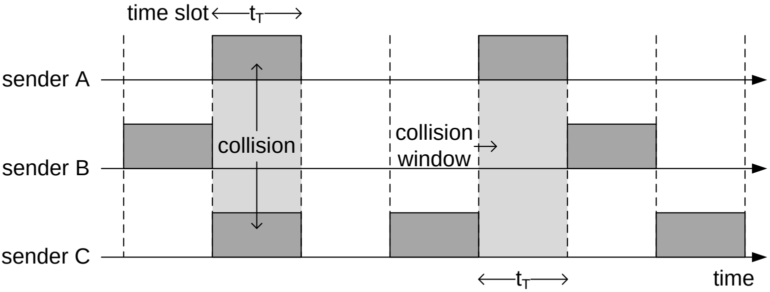
The ALOHA protocol was developed by Abramson for a wireless computer network between the Hawaii islands. It is used, e.g., for the GSM Random Access Channel.

A number of hosts share a wireless channel

- if a host has data for transmission it sends the data immediately
- the hosts do not consider other potentially sending hosts
- if two or more packets are transmitted at the same time they are destroyed
- these packets are retransmitted after a random time

What is the impact of packet collisions on the performance, i.e., what is the maximally achievable throughput?

Slotted ALOHA



- synchronous TDM scheme with slot time t_T
- constant sized packets with transmission time t_T
- collision window t_T

Slotted ALOHA throughput

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Assume N stations contend for the channel

- all stations are identical and statistically independent
- the probability that a station sends in a given time-slot is p
- the probability that a station transmits a packet without collision is $p(1-p)^{N-1}$
- the probability that any one out of N stations transmits a packet without collision is the throughput $S = Np(1-p)^{N-1}$

The throughput is maximized for

$$\frac{\partial}{\partial p} Np(1-p)^{N-1} = N(1-p)^{N-1} - Np(N-1)(1-p)^{N-2} = 0$$

and after simplification $1-p = (N-1)p$ yields $p = 1/N$ such that $S_{\max} = (1-1/N)^{N-1}$

Slotted ALOHA throughput continued

If the number of contending stations is large we have in the limit $N \to \infty$ that

$$\lim_{N\to\infty} S_{\max} = \lim_{N\to\infty}\left(1-\frac{1}{N}\right)^{N-1} = \frac{1}{e} \approx 0.368$$

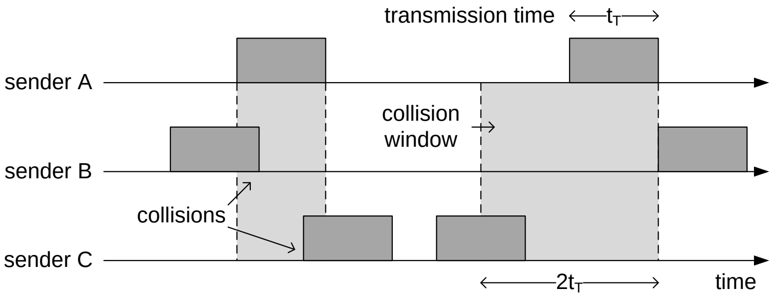
since

- $\lim_{N\to\infty}(1-1/N) = 1$
- $\lim_{N\to\infty}(1-1/N)^N = 1/e$

The fewer stations contend the better the throughput, e.g.,

- $S_{\max} = 1$ for $N = 1$
- $S_{\max} = 1/2$ for $N = 2$

Pure ALOHA



- asynchronous
- constant sized packets with transmission time t_T
- collision window $2t_T$

Pure ALOHA throughput

Assume all stations use packets with transmission duration t_T

- the probability that a station sends in $[t, t + t_T]$ is p
- the probability that no other station starts transmitting in $[t, t + t_T]$ is $(1 - p)^{N-1}$
- the probability that no other station started transmitting in $[t - t_T, t]$ is $(1 - p)^{N-1}$
- the probability that a station transmits a packet without collision is $p(1 - p)^{2(N-1)}$
- the probability that any one out of N stations transmits a packet without collision is the throughput $S = Np(1 - p)^{2(N-1)}$

Pure ALOHA throughput continued

The throughput is maximized for

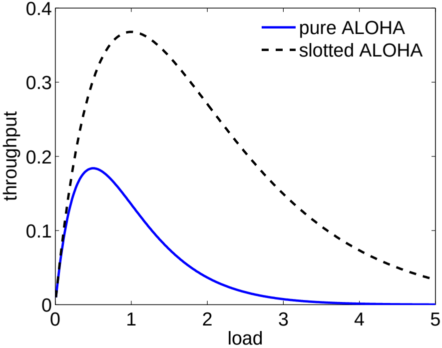
$$\frac{\partial}{\partial p} Np(1-p)^{2(N-1)} = N(1-p)^{2(N-1)} - Np2(N-1)(1-p)^{2(N-1)-1} = 0$$

and after simplification $1 - p = 2(N-1)p$ yields $p = 1/(2N-1)$ such that $S_{\max} = N/(2N-1)\ (1 - 1/(2N-1))^{2(N-1)}$

If the number of contending stations is large we have in the limit $N \to \infty$ that

$$\lim_{N\to\infty} S_{\max} = \lim_{N\to\infty} \frac{N}{(2N-1)} \left(1 - \frac{1}{2N-1}\right)^{2(N-1)} = \frac{1}{2e} \approx 0.184$$

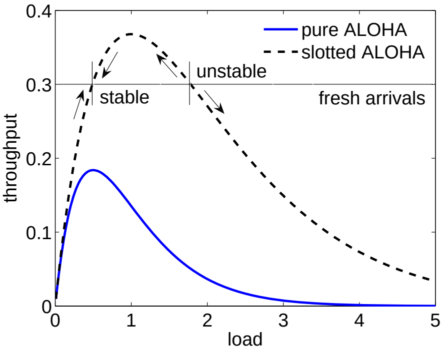
ALOHA throughput vs. load



The simple model neglects retransmissions respectively retrials.

- As an extension: load = fresh arrivals + retransmissions
- For stability: rate of fresh arrivals = throughput

ALOHA throughput vs. load



The simple model neglects retransmissions respectively retrials.

- As an extension: load = fresh arrivals + retransmissions
- For stability: rate of fresh arrivals = throughput

Pure and slotted ALOHA are unstable for loads larger than 0.5, respectively, 1. ALOHA requires a cautious retransmission strategy.

Outline

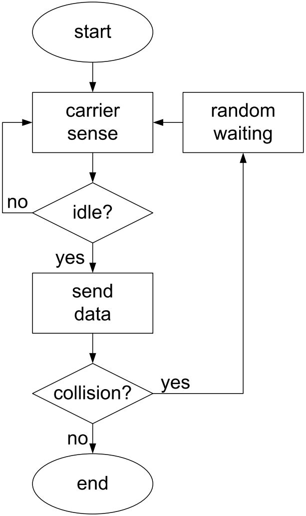
ALOHA
- Slotted ALOHA
- Pure ALOHA

Carrier sense multiple access
- Renewal theory
- CSMA throughput model
- Collision avoidance

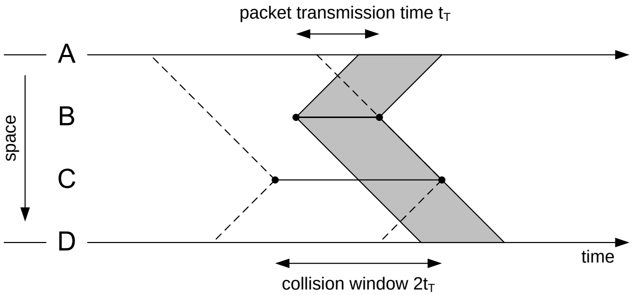
Random access with reservation

Carrier sense multiple access (CSMA)

- stations sense the channel before transmitting (listen before talk)
  - if the station finds the channel idle it starts sending
  - if the station finds the channel busy it defers sending
    - non-persistent: try again after random waiting time
    - 1-persistent: try again immediately
    - p-persistent: try again, if idle send with probability p, wait one slot with $1 - p$
- if no acknowledgement is received, a collision is assumed
- does not solve the hidden and exposed station problems

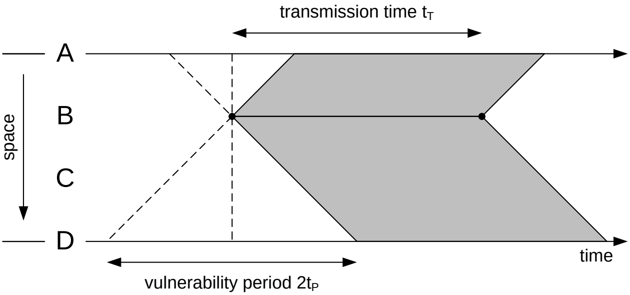


ALOHA space time diagram



- transmission time t_T
- propagation delay t_P
- collision window $2t_T$ resp. t_T

CSMA space time diagram



A packet transmission, e.g., from B is vulnerable

- until all stations sense the ongoing transmission
- once the medium is sensed busy the protocol forbids that other stations start sending
- vulnerability period $2t_P$

CSMA vs. ALOHA

Is CSMA generally better than ALOHA?

1. Case 1: local area network
   - vulnerability period at most $2t_P$
   - packet size 1500 Byte, 100 Mbps link: $t_T = 0.12$ ms
   - 100 meter distance: $t_P \approx 0.5$ μs
   - $t_T \gg t_P$ favors CSMA

2. Case 2: satellite link
   - collision window $2t_T$ respectively t_T
   - geosynchronous satellite (RFC 2488): $t_P \approx 250$ ms
   - $t_P \gg t_T$ favors ALOHA
   - carrier sense only provides old, outdated information

Probability density function

Given a random variable X. The integral of the probability density function (pdf) $f_X(x) \geq 0$ denotes the probability that X takes a value within an interval $[a, b]$

$$P(a \leq X \leq b) = \int_a^b f_X(x)dx.$$

Clearly, it also holds that

$$\int_{-\infty}^{\infty} f_X(x)dx = 1.$$

The cumulative distribution function (cdf) is defined as

$$F_X(a) = P(X \leq a) = \int_{-\infty}^{a} f_X(x)dx.$$

Conversely, the probability density function follows as

$$f_X(x) = \frac{dF_X(x)}{dx}.$$

Expected value

The expected value of a random variable X is defined as

$$\mathsf{E}[X] = \int_{-\infty}^{\infty} x f_X(x) dx.$$

For the expected value of the sum of two random variables $X + Y$ it holds that

$$\mathsf{E}[X + Y] = \mathsf{E}[X] + \mathsf{E}[Y].$$

Renewal processes

Counting process $K(t)$

- counts the number of arrivals (random events) $K(t)$ in $[0, t]$

Inter-arrival times X_i

- time between arrival i and $i - 1$

The counting process $K(t)$ is a renewal process if the inter-arrival times X_i are independent and identically distributed (iid).

Example: Consider light bulbs that fail after some iid random time. A single bulb is used at a time and replaced immediately if it fails. The process $K(t)$ denotes the number renewals by time t.

Poisson process

A well-known counting process is the Poisson process where

$$\mathsf{P}[K(t) = x] = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$$

The expected value of $K(t)$ is $\mathsf{E}[K(t)] = \lambda t$ where λ is the mean arrival rate.

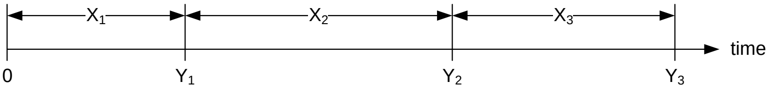
For the time X until the next arrival takes place it follows that

$$\mathsf{P}[X > t] = \mathsf{P}[K(t) = 0] = e^{-\lambda t},$$

i.e., the time between two arrivals is exponentially distributed. The expected value of X is $\mathsf{E}[X] = 1/\lambda$.

Mean time between renewals

Inter-arrival times X_i and renewal times Y_i



The time of the k-th renewal can be expressed as

$$Y_k = \sum_{i=1}^{k} X_i$$

Under the assumptions of the strong law of large numbers the sample average converges to the expected value

$$\frac{Y_k}{k} = \frac{1}{k} \sum_{i=1}^{k} X_i \to \mathsf{E}[X] \qquad \text{as } k \to \infty$$

Renewal theory

Consider

- $Y_{K(t)}$ the time of the last renewal before or at time t
- $Y_{K(t)+1}$ the time of the first renewal after t

such that

$$\frac{Y_{K(t)}}{K(t)} \leq \frac{t}{K(t)} < \frac{Y_{K(t)+1}}{K(t)}$$

Renewal theory

Consider

- $Y_{K(t)}$ the time of the last renewal before or at time t
- $Y_{K(t)+1}$ the time of the first renewal after t

such that

$$\frac{Y_{K(t)}}{K(t)} \leq \frac{t}{K(t)} < \frac{Y_{K(t)+1}}{K(t)}$$

For $t \to \infty$ we have $K(t) \to \infty$ and both sides converge to $\mathsf{E}[X]$ such that

$$\frac{K(t)}{t} \to \frac{1}{\mathsf{E}[X]} \qquad \text{as } t \to \infty$$

Example: from the mean inter-arrival time $\mathsf{E}[X]$ it follows that $K(t)/t = 1/\mathsf{E}[X]$ for $t \to \infty$ is the average rate of renewals.

Renewal reward processes

Each renewal k may come with a reward (or cost) denoted R_k. The total reward earned by time t is

$$R(t) = \sum_{k=1}^{K(t)} R_k$$

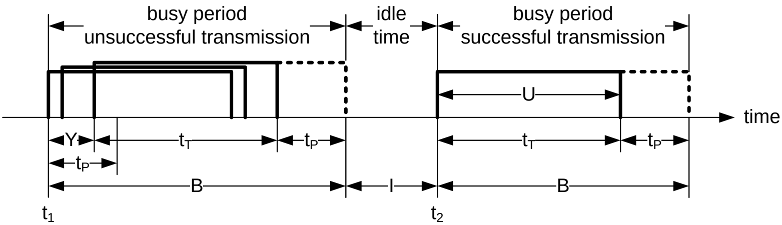
The average reward becomes

$$\frac{R(t)}{t} = \left(\frac{R(t)}{K(t)}\right)\left(\frac{K(t)}{t}\right) = \left(\frac{\sum_{k=1}^{K(t)} R_k}{K(t)}\right)\left(\frac{K(t)}{t}\right)$$

and for $t \to \infty$ the two factors converge (owing to the strong law of large numbers) to $\mathsf{E}[R]$ and $1/\mathsf{E}[X]$ respectively. The long term average reward becomes

$$\lim_{t\to\infty} \frac{R(t)}{t} = \frac{\mathsf{E}[R]}{\mathsf{E}[X]},$$

i.e., mean reward per renewal divided by mean length of a renewal.



Required notation: Denote

- B the duration of a busy period
- I the duration of an idle period
- U the time during which the channel is used without conflicts
- Y the time of the last arrival that causes a conflict if any

We normalize the model with respect to t_T such that $t_T = 1$.

CSMA throughput model continued

We want to compute the average throughput for the simplest case, i.e., non-persistent CSMA.

Phrasing the problem using renewal theory we have

- length of a renewal $B + I$
- reward per renewal U

The reward per renewal U is the time during which the channel is used without conflicts. Hence, the long term average reward is the average utilization such that

$$S = \frac{\mathsf{E}[U]}{\mathsf{E}[B+I]} = \frac{\mathsf{E}[U]}{\mathsf{E}[B] + \mathsf{E}[I]}.$$

The reward is

- t_T if there is no collision (we assume $t_T = 1$)
- 0 if there is a collision

CSMA throughput model continued

The number of packets that are transmitted during a time interval of length t is a random variable K.

Under relatively general assumptions (large number of independent stations) it can be modeled as a Poisson process with mean rate λ

$$\mathsf{P}[K = x] = p(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}.$$

The probability that there is no collision, i.e., no further arrival during t_P is

$$p(0) = e^{-\lambda t_P}.$$

The mean reward becomes (with $t_T = 1$)

$$\mathsf{E}[U] = t_T e^{-\lambda t_P} = e^{-\lambda t_P}.$$

CSMA throughput model continued

The mean duration of an idle period is simply the expected inter-arrival time $\mathsf{E}[I] = 1/\lambda$ of the Poisson arrival process.

The mean duration of a busy period is $\mathsf{E}[B] = \mathsf{E}[Y] + t_T + t_P$. The distribution of Y is for $0 \leq y \leq t_P$

$$F_Y(y) = \mathsf{P}[Y \leq y] = \mathsf{P}[\text{no arrival in } t_P - y] = e^{-\lambda(t_P - y)}$$

The density function is derived by differentiation

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \lambda e^{-\lambda t_P} e^{\lambda y}$$

and the expected value follows (using partial integration $\int u'v = uv - \int uv'$) as

$$\mathsf{E}[Y] = \int_0^{t_P} y f_Y(y) dy = t_P - \frac{1}{\lambda}(1 - e^{-\lambda t_P})$$

CSMA throughput model continued

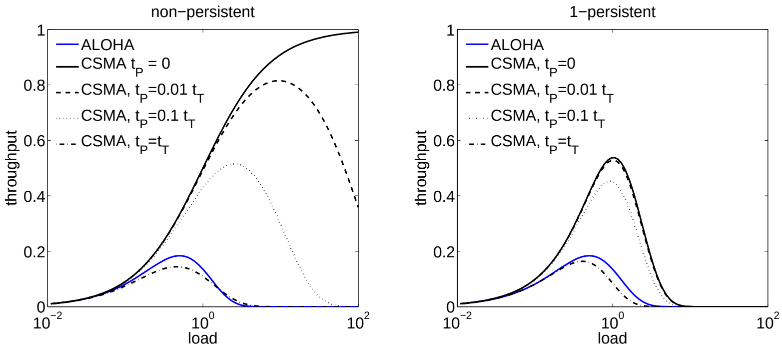
Putting all pieces together, the throughput of non-persistent CSMA is

$$S = \frac{\lambda e^{-\lambda t_P}}{\lambda(1+2t_P)+e^{-\lambda t_P}}$$

More involved are models for 1- and p-persistent CSMA. For 1-persistent CSMA the throughput can be computed as

$$S = \frac{\lambda(1+\lambda+t_P\lambda(1+\lambda+t_P\lambda/2))e^{-\lambda(1+2t_P)}}{\lambda(1+2t_P)-(1-e^{-\lambda t_P})+(1+t_P\lambda)e^{-\lambda(1+t_P)}}$$

CSMA vs. ALOHA



For small propagation delays t_P, e.g., in local area networks CSMA outperforms ALOHA significantly.

Non-persistent CSMA achieves higher throughput (why?) than 1-persistent CSMA, however, at the cost of additional latencies.

Collision avoidance

MACA uses a two step signalling procedure to address the hidden and exposed terminal problems

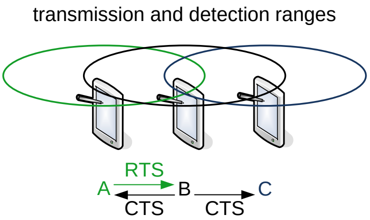
- request to send (RTS): sender broadcast a request to send
- clear to send (CTS): receiver broadcasts a clear to send

Signalling packets contain

- sender address
- receiver address
- packet size
  - network allocation vector (NAV)
  - duration during which other stations have to keep quiet to avoid a collision

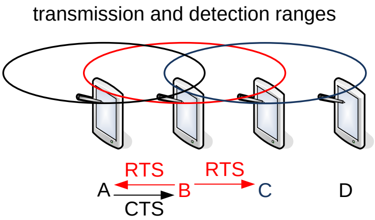
Hidden terminal C

- C does not hear A
- but C hears the CTS
- C keeps silent



Exposed terminal C

- A does not hear C
- but C hears B
- C does not hear the CTS
- C may send, e.g., to D



Outline

Demand assigned multiple access (DAMA)

Motivation

- the efficiency of ALOHA is very poor (18 %, 36 %)
- reservation can significantly increase efficiency

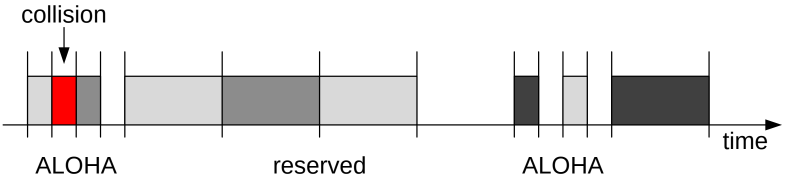
DAMA, also called reservation ALOHA, allows a sender to reserve timeslots. Two phase approach

- reservation phase: contention using slotted ALOHA and short reservation packets
- transmission phase: collision-free transmission using reserved timeslots

Assessment

- advantage: only short reservation messages collide
- disadvantage: adds additional delays

Demand assigned multiple access (DAMA)



Alternating (in TDM fashion)

- reservation phase
- transmission phase

$\Rightarrow$ explicit reservation

Packet reservation multiple access (PRMA)

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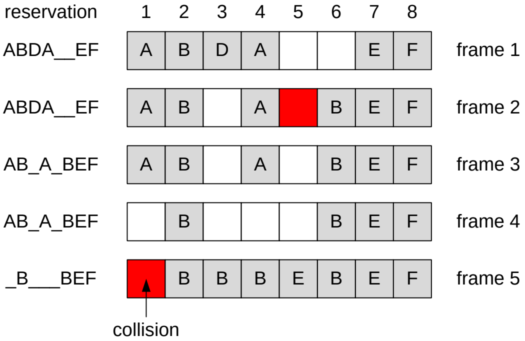
PRMA uses a repeating frame structure of slots

- slotted ALOHA is used to compete for free slots
- if a station wins, the slot is reserved in subsequent frames
- slots become free if stations stop sending

$\Rightarrow$ implicit reservation



Literature

- J. Schiller, Mobile Communications, Second Edition, Addison-Wesley, 2003.
- N. Abramson: *The ALOHA System - Another alternative for computer communications*, AFIPS Conference Proceedings, Vol. 36, 1970, pp. 295-298.
- L. Kleinrock, and F. A. Tobagi: *Packet Switching in Radio Channels: Part 1 - Carrier Sense Multiple-Access Modes and Their Throughput-Delay Characteristics*, IEEE Transactions on Communications, 23(12):1400-1416, 1975.