Advanced Topics in Computational Complexity

Exercise Session 4

Due 9.11.2015.

Exercise 1

Proof Proposition 6 in the lecture notes for existential quantifier.

Exercise 2

Let $K = (W, R, V)$ be a Kripke model such that $W = \{1, 2, 3, 4, 5\}$, $R = \{(i, j) \in W^2 \mid i + j \leq 5\}$, $V(p) = \{1, 2, 4\}$, and $V(q) = \{4, 5, 6\}$. Which of the following claims hold?

1. $K, 3 \models (\neg p \wedge \Diamond \Box q)$
2. $K, 2 \models \Diamond\Diamond(\Diamond p \wedge \Box q)$
3. $K, 5 \models \Box p$

   Which of the points in K satisfy the formula $\Box\Diamond p$?

Exercise 3

Write the standard translation ST_x of the formula $\Diamond(\Box p \vee q)$ and ST_y of the formula $\big((p \wedge \Box q) \vee \Box\Box q\big)$.

Exercise 4

Write a formula of modal logic that is true in a pointed model K, w if and only if there exists a dead end within the distance of 4 from w (that is, for some $n \leq 4$ is there exists points in $a_0, \dots, a_n \in W$ such that $a_0 = w$, $(a_i, a_{i+1}) \in R$, for each $i < n$, and for all $b \in W$ it holds that $(a_n, b) \notin R$.