Teams

- A team is just a *set* of assignments for a model.
- Special cases:
  - Empty team $\emptyset$.
    - Database with no rows.
  - The team $\{\emptyset\}$ with the empty assignment.
    - Database with no columns, and hence with at most one row.

Dependence logic *D*

$$t = t', \quad Rt_1 ... t_n$$

$$=\!(t_1, ..., t_n)$$

$$\varphi \vee \psi, \neg\varphi, \exists x_n \varphi$$

$$\mathfrak{M} \models_X t_1 = t_2 \quad \textit{iff} \quad \forall s \in X(t_1^{\mathfrak{M}}\langle s\rangle = t_2^{\mathfrak{M}}\langle s\rangle)$$
$$\mathfrak{M} \models_X t_1 \neq t_2 \quad \textit{iff} \quad \forall s \in X(t_1^{\mathfrak{M}}\langle s\rangle \neq t_2^{\mathfrak{M}}\langle s\rangle)$$

A team satisfies a relation $Rt_1...t_n$ if every team member does.

A team satisfies a relation $\neg Rt_1...t_n$ if every team member does.

	x_0	x_1	x_2
s_0	0	0	0
s_1	0	1	1
s_2	2	5	5

$$\mathfrak{M} \models_X =(t_1, \ldots, t_n)$$

$\forall s, s' \in X(t_1^{\mathfrak{M}}\langle s\rangle \neq t_1^{\mathfrak{M}}\langle s'\rangle$ *or*
$\ldots$ *or* $t_{n-1}^{\mathfrak{M}}\langle s\rangle \neq t_{n-1}^{\mathfrak{M}}\langle s'\rangle$ *or* $t_n^{\mathfrak{M}}\langle s\rangle = t_n^{\mathfrak{M}}\langle s'\rangle)$

$\mathfrak{M} \models_X \phi \vee \psi$

there are X_0 *and* X_1 *such that*
$\mathfrak{M} \models_{X_0} \phi$, $\mathfrak{M} \models_{X_1} \psi$, *and* $X \subseteq X_0 \cup X_1$

$\mathfrak{M} \models_X \phi \wedge \psi$

both $\mathfrak{M} \models_X \phi$ *and* $\mathfrak{M} \models_X \psi$

$$\mathfrak{M} \models_X \exists x\phi$$

there is Y *such that* $\mathfrak{M} \models_Y \phi$ *and for every*
$s \in X$ *we have* $s[a/x] \in Y$ *for some* $a \in M$

$$\mathfrak{M} \models_X \forall x \phi$$

there is Y such that $\mathfrak{M} \models_Y \phi$ and for every $s \in X$ we have $s[a/x] \in Y$ for every $a \in M$

Logical consequence and equivalence

ψ follows logically from φ

$\phi \Rightarrow \psi$ $\quad\quad$ $\mathcal{M} \models_X \phi$ implies $\mathcal{M} \models_X \psi$

ψ is logically equivalent with φ

$$\phi \equiv \psi, \textit{ if } \phi \Rightarrow \psi \textit{ and } \psi \Rightarrow \phi$$

Armstrong's rules

Always =(x,x)

If =(x,y,z), then =(y,x,z).

If =(x,x,y), then =(x,y).

If =(x,z), then =(x,y,z).

If =(x,y) and =(y,z), then =(x,z).

Propositional rules

- From $\varphi\wedge\psi$ follows $\psi\wedge\varphi$.
- From $\varphi\vee\psi$ follows $\psi\vee\varphi$.

Commutative

- From $\varphi\wedge(\psi\wedge\theta)$ follows $(\varphi\wedge\psi)\wedge\theta$.
- From $\varphi\vee(\psi\vee\theta)$ follows $(\varphi\vee\psi)\vee\theta$.

Associative

- From$(\varphi\vee\eta)\wedge(\psi\vee\theta)$ follows $(\varphi\wedge\psi)\vee(\varphi\wedge\theta)\vee(\eta\wedge\psi)\vee(\eta\wedge\theta)$.
- From$(\varphi\wedge\eta)\vee(\psi\wedge\theta)$ follows $(\varphi\vee\psi)\wedge(\varphi\vee\theta)\wedge(\eta\vee\psi)\wedge(\eta\vee\theta)$.

"Almost" distributive

- From φ and ψ follows $\varphi\wedge\psi$.
- From $\varphi\wedge\psi$ follows φ.
- From φ follows $\varphi\vee\psi$.

Incorrect rules

No absortion

- From φ∨φ follows φ. Wrong!
- From(φ∧ψ)∨(φ∧θ) follows φ∧(ψ∨θ). Wrong!
- From(φ∨ψ)∧(φ∨θ) follows φ∨(ψ∧θ). Wrong!

Non-distributive

Quantifier rules

- From ∀xφ∧∀xψ follows ∀x(φ∧ψ), and vice versa.
- From ∃xφ∨∃xψ follows ∃x(φ∨ψ), and vice versa.
- From φ∨∀xψ follows ∀x(φ∨ψ), and vice versa, provided that x is not free in φ.
- From φ∧∃xψ follows ∃x(φ∧ψ), and vice versa, provided that x is not free in φ.
- From ∀x∀yφ follows ∀y∀xφ.
- From ∃x∃yφ follows ∃y∃xφ.
- From φ follows ∃xφ.
- From ∀xφ follows φ.

Conservative over FO

Corollary 22 *Let ϕ be a first order L-formula of dependence logic. Then:*

1. $\mathcal{M} \models_{\{s\}} \phi$ *if and only if* $\mathcal{M} \models_s \phi$.
2. $\mathcal{M} \models_X \phi$ *if and only if* $\mathcal{M} \models_s \phi$ *for all* $s \in X$.

Example: even cardinality

$$\begin{aligned}\forall x_0 \exists x_1 \forall x_2 \exists x_3 (=(x_2, x_3) \ \wedge\ & \neg(x_0 = x_1) \\ \wedge\ & (x_0 = x_2 \rightarrow x_1 = x_3) \\ \wedge\ & (x_1 = x_2 \rightarrow x_3 = x_0))\end{aligned}$$

Example: infinity

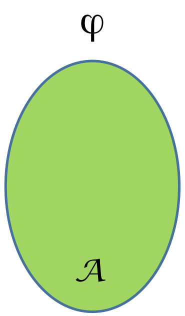
$$\exists x_4 \forall x_0 \exists x_1 \forall x_2 \exists x_3 (=\!(x_2, x_3) \ \wedge\ \neg(x_1 = x_4) \ \wedge\ (x_0 = x_2 \leftrightarrow x_1 = x_3))$$

"There is a bijection to a proper subset."

Uniform strategy

- A strategy of II is uniform if whenever the game ends in $(=(t_1,\ldots,t_n),s)$ with the same $=(t_1,\ldots,t_n)$ and the same values of $t_1,\ldots,t_{n-1}$, then also the value of t_n is the same.
- Imperfect information game!

Game theoretical semantics of *D*



φ is true in $\mathcal{A}$ if and only if II has a uniform winning strategy

Model theory of dependence logic

- Basic reduction:

New predicate

$\phi(x_{i_1}, \ldots, x_{i_n})$

$d \in \{0,1\}$

Σ^1_1-*sentence* $\tau_{d,\phi}(S)$

Equivalent

1. $(\phi, X, d) \in \mathcal{T}$
2. $(\mathcal{M}, X) \models \tau_{d,\phi}(S)$

Team becomes predicate

Application

Theorem 58 (Compactness Theorem of $\mathcal{D}$) *Suppose Γ is an arbitrary set of sentences of dependence logic such that every finite subset of Γ has a model. Then Γ itself has a model.*

Application

In countable vocabulary

Theorem 59 (Löwenheim-Skolem Theorem of $\mathcal{D}$) *Suppose ϕ is a sentence of dependence logic such that ϕ either has an infinite model or has arbitrarily large finite models. Then ϕ has models of all infinite cardinalities, in particular, ϕ has a countable model and an uncountable model.*

From ESO to D

Theorem 68 ([4],[30]) *For every Σ^1_1-sentence ϕ there is a sentence ϕ^* in dependence logic such that for all $\mathcal{M}$: $\mathcal{M} \models \phi \iff \mathcal{M} \models \phi^*$.*

Current developments

- Also independence atoms.
- See Doctoral Thesis of Pietro Galliani: www.illc.uva.nl/Research/Dissertations/DS-2012-07.text.pdf
- See paper by Kontinen-Väänänen: http://arxiv.org/abs/1208.0176
- See paper by Grädel-Väänänen: http://logic.helsinki.fi/people/jouko.vaananen/graedel_vaananen.pdf