Advanced Topics in Computational Complexity

Exercise Session 1

Due 19.10.2015.

Exercise 1

Give a set of first-order formulae Γ that defines the class of structures $\mathfrak{A}$ in which the size of domain A is either infinite or odd.

Exercise 2

Give a (single) formula of second-order logic that defines the class of infinite structures.

Exercise 3

Proof Lemma 1 in the handout for first-order logic.

Exercise 4

Complete the APTIME algorithm for model checking of FO in the handout.

Exercise 5

A graph is a structure $G = (V, E)$, where V is a finite set and $E \subseteq V \times V$ is a symmetric relation (i.e., if $(x, y) \in E$ then $(y, x) \in E$). Path is sequence of nodes $a_1, \ldots a_m$ such that for every $i < n$, $(a_i, a_{i+1}) \in E$. A graph is connected if there is a path between each two points of the graph.

Give a formula of second-order logic that defines the class of connected graphs. (You may assume that the formula will be evaluated only on graphs.)