Finding Similar Items

Shingling, Min-Hashing, LSH

credits:www.mmds.org

Logistics: Communication

- **For e-mailing us, always use:**
  - anand@l3s.de,singh@l3s.de
  - Use this prefix in the subject field **[lsdm16]**
  - You can message via Stud.ip as well

- **We will post course announcements to the course webpage - via twitter (make sure you check it regularly)**

Work for the Course

- **7-8 Assignments:**
  - Theoretical and programming questions
  - **Assignments take time. Start early!!**
  - **5-6 questions per assignment, online on the lecture day**
  - **Due date Tuesday after the lecture 11:59 am**

- **How to submit?**
  - **Homework write-up:**
    - **Submit @ Jaspreet's office or (11:59 am),**
    - **Scan and send to singh@l3s.de (11:59 pm)**

Tutorials

- After the lecture (with Jaspreet)
- Successfully evaluated assignments will be offered for solution presentation
- Each presentation (whiteboard) should be ~10 minutes..max 15 minutes
- 3 successful presentations give you 1 bonus point
- 1 bonus point = 0.3 grade improvement in your final exam

Final Exam

- Written Exam
- Duration : 2 hours
- 1 bonus point = 0.3 grade improvement in your final exam
  - 1.3 + 1 bonus point = 1.0
  - 5.0 + 1 bonus point = 5.0
- Modelled on Assignments
- More applied and algorithmic aspects rather than memorising

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Near Duplicate News Stories

REUTERS Business Markets World Politics Tech Opinion Breakingviews Money Life

World | Tue Apr 12, 2016 8:23am EDT

Related: WORLD, AFF

Iceland finance minister says won't resign over Panama Papers leaks

Sigurdur Ingi Johannsson (L), minister of fisheries and agriculture of the Progressive Party who was named as new prime minister by two government coalition parties attends press conference together with finance minster Bjarni Benediktsson in Reykjavik, Iceland on April 6,...
REUTERS/SIGTRYGGUR JOHANNSSON

Iceland's Finance Minister Bjarni Benediktsson said on Tuesday he would not resign over the Panama Papers leaks, which showed he once had a stake in an offshore investment firm in the Seychelles.

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Tuesday, April 12, 2016, 11:36 by **Reuters**

Iceland finance minister says he won't resign over Panama Papers

Iceland's Finance Minister Bjarni Benediktsson said today that he would not resign over the Panama Papers leaks, which showed he once had a stake in an offshore investment firm in the Seychelles.

Asked by reporters in London whether he would quit, Benediktsson answered: "No".

The statement came a week after Sigmundur David Gunnlaugsson resigned as prime minister over the leaked documents which showed his wife owned an offshore company that held debt from failed Icelandic banks.

The International Consortium of Investigative Journalists, which saw the leaked papers from Panamanian lawyers Mossack Fonseca, said they showed Benediktsson and two Icelandic businessmen had power of attorney over a shell company called Falson & Co. created in 2005 in the Seychelles.

Near Duplicate Images

Similar Items

- **Finding similar news stories**
- **Finding near duplicate images**
- **Plagiarism detection**
- **Duplications in Web crawls**

Find near-neighbors in high-dim. space

We formally define near neighbors as points that are a small distance apart

Distance Measures

- For each application, we first need to define what "**distance**" means

Distance Measures

- For each application, we first need to define what "**distance**" means
    - Eg. : cosine similarity, manhattan distance, jaccard's distance
- **Is the distance a Metric ?**
    - **Non-negativity: d(x,y) >= 0**
    - **Symmetric: d(x,y) = d(y,x)**
    - **Identity: d(x,y) = 0 iff x = y**
    - **Triangle inequality: d(x,y) + d(y,z) >= d(x,z)**
- **Metric distance leads to better pruning power**

Distance Measures

- **Today: Jaccard distance/similarity**
- The **Jaccard similarity** of two **sets** is the size of their intersection divided by the size of their union:
  $sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$

Distance Measures

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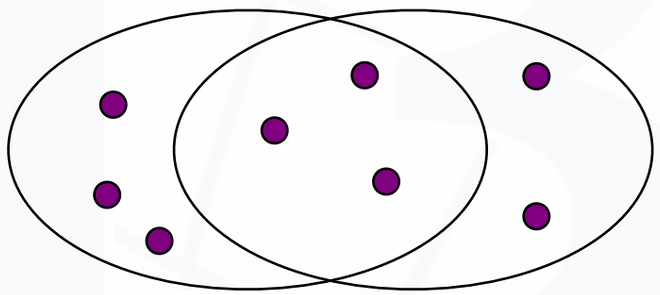
  $sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$
- **Jaccard distance:** $d(C_1, C_2) = 1 - |C_1 \cap C_2| / |C_1 \cup C_2|$



3 in intersection
8 in union
Jaccard similarity= 3/8
Jaccard distance = 5/8

Challenges

Challenges

- If the number of documents are **N** and the number of dimensions are **D**
- What if N and D are small ?
    - quadratic in N and D
- What if N is large and D is small ?
    - Quadratic in N but OK if input fits in memory
    - KD-trees are used for low dimensionality
- What if D and N are both large ?

Task: Finding Similar Documents

Given a large number (N in millions or even billions) of documents find near duplicate pairs

- **Problem of Size:** **Document collection doesn't fit in memory**

Task: Finding Similar Documents

Given a large number (N in millions or even billions) of documents find near duplicate pairs

- **Problem of Size:** **Document collection doesn't fit in memory**
- **Computational complexity:**
  - **Cannot afford quadratic complexity of D**
  - **Cannot afford quadratic complexity of N**
- **Need approximate methods to solve this in reasonable time**

3 Essential Steps for Similar Docs

1. ***Shingling:*** Convert documents to sets

2. ***Min-Hashing:*** Convert large sets to short signatures, while preserving similarity

3. ***Locality-Sensitive Hashing:*** Focus on pairs of signatures likely to be from similar documents

   - **Candidate pairs!**

- **Step 1:** ***Shingling:*** Convert documents to sets

Documents as High-Dim. Data

- **Step 1: *Shingling:* Convert documents to sets**

- **Simple approaches:**
  - Document = set of words appearing in document
  - Document = set of "important" words
  - Don't work well for this application. Why?

- **Need to account for ordering of words!**

- A different way: **Shingles!**

Define: Shingles

- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters for examples

- **Example: k=2**; document $\mathbf{D_1}$ = abcab

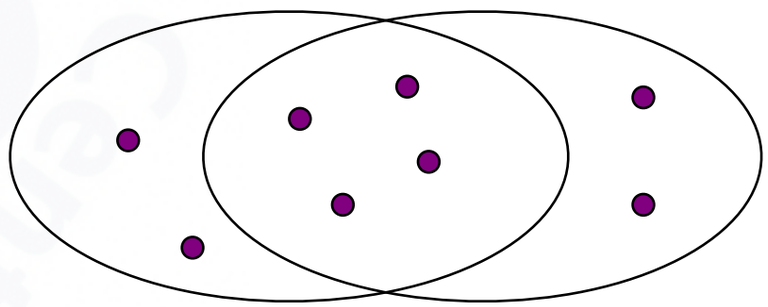
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}
  - **Option:** Shingles as a bag (multiset), count ab twice: $\mathbf{S'(D_1)}$ = {ab, bc, ca, ab}

Similarity Metric for Shingles

- **Document D_1 is a set of its k-shingles $C_1=S(D_1)$**

- Equivalently, each document is a 0/1 vector in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse

- **A natural similarity measure is the Jaccard similarity:**



$$sim(D_1, D_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$$

Working Assumption

- **Documents that have lots of shingles in common have similar text, even if the text appears in different order**

- **Caveat:** You must pick ***k*** large enough, or most documents will have most shingles
  - ***k*** = 5 is OK for short documents
  - ***k*** = 10 is better for long documents

Motivation for Minhash/LSH

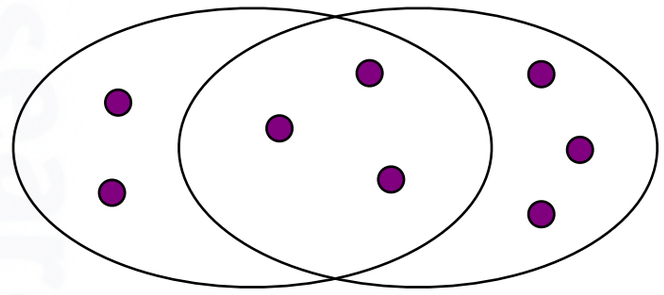
- **Suppose we need to find near-duplicate documents among $N = 1$ million documents**
- Naïvely, we would have to compute **pairwise Jaccard similarities** for **every pair of docs**
  - $N(N-1)/2 \approx 5*10^{11}$ comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take **5 days**
- For $N = 10$ million, it takes more than a year…

- **Step 2:** ***Minhashing:*** Convert **large sets** to **short signatures**, while **preserving similarity**

Encoding Sets as Bit Vectors

- Many similarity problems can be formalized as **finding subsets that have significant intersection**

- **Encode sets using 0/1 (bit, boolean) vectors**
  - One dimension per element in the universal set



- Interpret set intersection as bitwise **AND**, and set union as bitwise **OR**

- **Example:** $\mathbf{C_1}$ = 10111; $\mathbf{C_2}$ = 10011
  - Size of intersection **= 3**; size of union **= 4**,
  - **Jaccard similarity** (not distance) **= 3/4**
  - **Distance: d(C_1,C_2) = 1 – (Jaccard similarity) = 1/4**

From Sets to Boolean Matrices

- **Rows** = elements (shingles)
- **Columns** = sets (documents)
  - 1 in row ***e*** and column ***s*** if and only if ***e*** is a member of ***s***
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1)*
  - **Typical matrix is sparse!**

From Sets to Boolean Matrices

- **Rows** = elements (shingles)
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  - 1 in row ***e*** and column ***s*** if and only if ***e*** is a member of ***s***
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1)*
  - **Typical matrix is sparse!**
- **Each document is a column:**
  - **Example: sim(C_1 ,C_2) = ?**
    - Size of intersection = 3; size of union = 6, Jaccard similarity (not distance) = 3/6
    - **d(C_1,C_2) = 1 – (Jaccard similarity) = 3/6**

Documents **(N)**

Shingles (D)

1	1	1	0
1	1	0	1
0	1	0	1
0	0	0	1
1	0	0	1
1	1	1	0
1	0	1	0

Outline: Finding Similar Columns

- **So far:**
  - Documents → Sets of shingles
  - Represent sets as boolean vectors in a matrix

- **Next goal: Find similar columns while computing small signatures**
  - **Similarity of columns == similarity of signatures**

Outline: Finding Similar Columns

- **Next Goal: Find similar columns, Small signatures**
- **Naïve approach:**
  - **1) Signatures of columns:** small summaries of columns
  - **2) Examine pairs of signatures** to find similar columns
    - **Essential:** Similarities of signatures and columns are related
  - **3) Optional:** Check that columns with similar signatures are really similar
- **Warnings:**
  - Comparing all pairs may take too much time: **Job for LSH**
    - These methods can produce false negatives, and even false positives (if the optional check is not made)

Hashing Columns (Signatures)

- **Key idea:** "hash" each column ***C*** to a small ***signature h(C)***, such that:
  - **(1)** ***h(C)*** is small enough that the signature fits in RAM
  - **(2)** ***sim(C_1, C_2)*** is the same as the "similarity" of signatures ***h(C_1)*** and ***h(C_2)***

Hashing Columns (Signatures)

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- **Goal: Find a hash function *h(·)* such that:**
  - If ***sim(C_1,C_2)*** is high, then with high prob. ***h(C_1) = h(C_2)***
  - If ***sim(C_1,C_2)*** is low, then with high prob. ***h(C_1) ≠ h(C_2)***

- **Hash docs into buckets. Expect that "most" pairs of near duplicate docs hash into the same bucket!**

Min-Hashing

- **Goal: Find a hash function *h(·)* such that:**
  - if ***sim*(C_1,C_2)** is high, then with high prob. ***h*(C_1) = *h*(C_2)**
  - if ***sim*(C_1,C_2)** is low, then with high prob. ***h*(C_1) ≠ *h*(C_2)**

- **Clearly, the hash function depends on the similarity metric:**
  - Not all similarity metrics have a suitable hash function

- **There is a suitable hash function for the Jaccard similarity:** It is called **Min-Hashing**

Min-Hashing

- Imagine the rows of the boolean matrix permuted under **random permutation** π

- Define a **"hash" function $h_\pi(C)$** = the index of the **first** (in the permuted order π) row in which column ***C*** has value **1**:

$$h_\pi(C) = min_\pi \pi(C)$$

- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column

Example

Permutation π

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Example

Permutation π

2
3
7
6
1
5
4

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
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Signature matrix *M*

2	1	2	1

Example

2nd element of the permutation is the first to map to a 1

Permutation π

2
3
7
6
1
5
4

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1	0	1	0
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0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1

Example

2nd element of the permutation is the first to map to a 1

Permutation π

2	4
3	2
7	1
6	3
1	6
5	7
4	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1

Example

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Permutation π

2	4
3	2
7	1
6	3
1	6
5	7
4	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1

4th element of the permutation is the first to map to a 1

Example

2nd element of the permutation is the first to map to a 1

Permutation π

Input matrix (Shingles x Documents)

Signature matrix *M*

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation is the first to map to a 1

Example

Note: Another (equivalent) way is to store row indexes:

1	5	1	5
2	3	1	3
6	4	6	4

2nd element of the permutation is the first to map to a 1

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation is the first to map to a 1

The Min-Hash Property

- **Choose a random permutation π**
- **<u>Claim:</u> $Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$**
- **Why?**

0	0
0	0
1	**1**
0	0
0	1
1	0

One of the two cols had to have 1 at position ***y***

The Min-Hash Property

- **Choose a random permutation π**
- **Claim:** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
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  - Let **X** be a doc (set of shingles), $y \in X$ is a shingle

0	0
0	0
1	**1**
0	0
0	1
1	0

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- **Why?**
  - Let **X** be a doc (set of shingles), $y \in X$ is a shingle
  - **Then:** $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $y \in X$ is mapped to the ***min*** element

0	0
0	0
1	**1**
0	0
0	1
1	0

One of the two cols had to have 1 at position ***y***

The Min-Hash Property

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    - It is equally likely that any $y \in X$ is mapped to the ***min*** element
  - Let ***y*** be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - **Then either:** $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, **or**

    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$

0	0
0	0
1	**1**
0	0
0	1
1	0

One of the two cols had to have 1 at position ***y***

The Min-Hash Property

- **Choose a random permutation π**
- **Claim:** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- **Why?**
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  - **Then either:** $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, **or**

    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that **both** are true is the prob. $y \in C_1 \cap C_2$
  - $\Pr[\min(\pi(C_1))=\min(\pi(C_2))]=|C_1 \cap C_2|/|C_1 \cup C_2| = sim(C_1, C_2)$

0	0
0	0
1	**1**
0	0
0	**1**
1	0

One of the two cols had to have 1 at position ***y***

Similarity for Signatures

- We know: **Pr[$h_\pi(C_1) = h_\pi(C_2)$] = $sim(C_1, C_2)$**

- Now generalize to multiple hash functions

- **The *similarity of two signatures* is the fraction of the hash functions in which they agree**

- **Note:** Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures

Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Min-Hash Signatures

- **Pick K=100 random permutations of the rows**
- Think of ***sig*(C)** as a column vector
- ***sig*(C)[i]** = according to the *i*-th permutation, the index of the first row that has a 1 in column *C*

$$sig(C)[i] = \min(\pi_i(C))$$

- **Note:** The sketch (signature) of document *C* is small **~100 bytes!**
- **We achieved our goal! We "compressed" long bit vectors into short signatures**

- **Step 3:** ***Locality-Sensitive Hashing:***
  Focus on pairs of signatures likely to be from similar documents

LSH: First Cut

2	1	4	1
1	2	1	2
2	1	2	1

- **Goal:** Find documents with Jaccard similarity at least ***s*** (for some similarity threshold, e.g., ***s***=0.8)

- **LSH – General idea:** Use a function ***f(x,y)*** that tells whether ***x*** and ***y*** is a ***candidate pair:*** a pair of elements whose similarity must be evaluated

- **For Min-Hash matrices:**
  - Hash columns of signature matrix ***M*** to many buckets
  - Each pair of documents that hashes into the same bucket is a **candidate pair**

Candidates from Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- **Pick a similarity threshold *s* (0 < s < 1)**
- Columns ***x*** and ***y*** of ***M*** are a **candidate pair** if their signatures agree on at least fraction ***s*** of their rows: ***M* (*i*, *x*) = *M* (*i*, *y*)** for at least frac. ***s*** values of ***i***
  - We expect documents ***x*** and ***y*** to have the same (Jaccard) similarity as their signatures

LSH for Min-Hash

- **Big idea: Hash columns of signature matrix *M* several times**

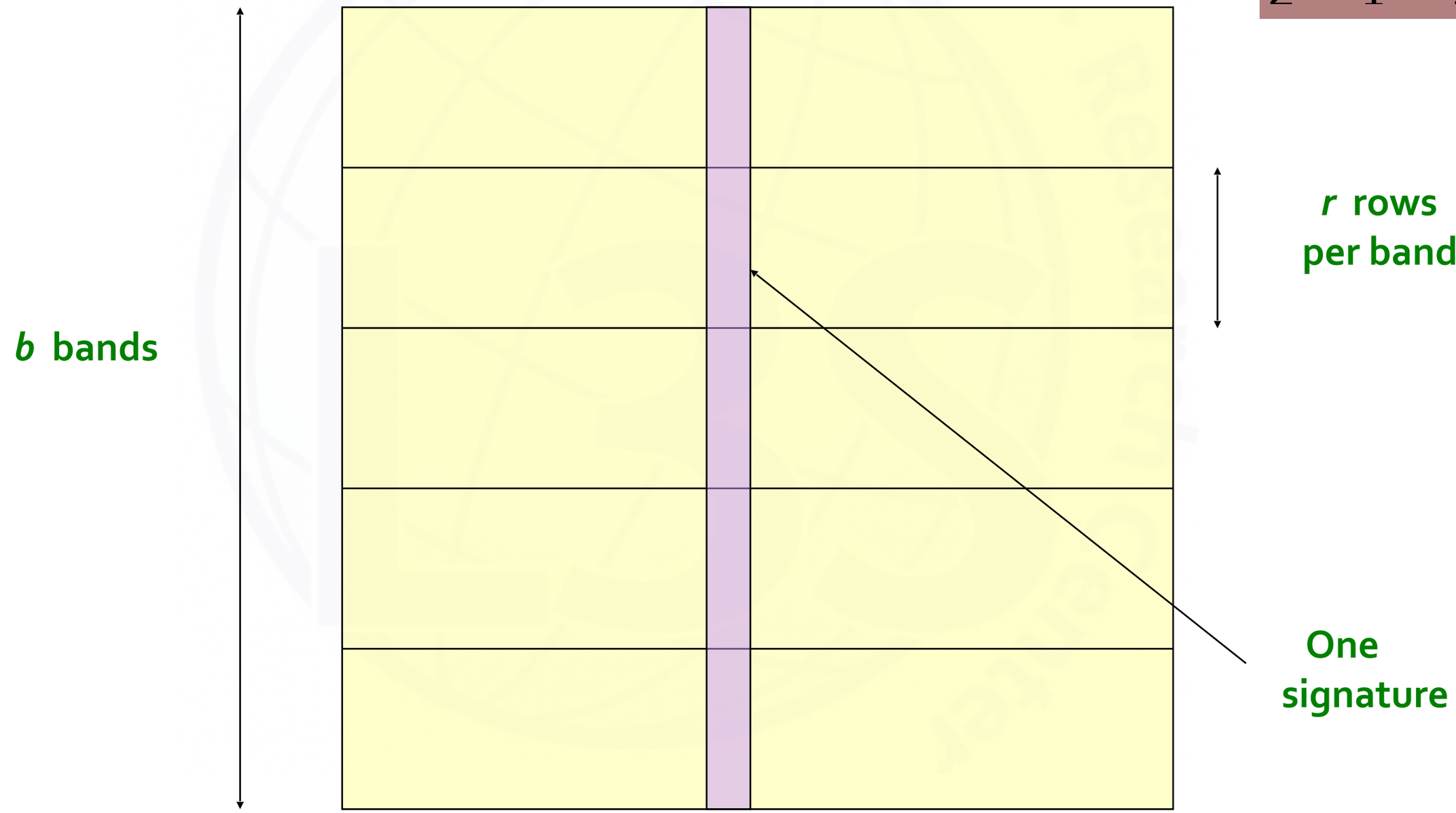
2	1	4	1
1	2	1	2
2	1	2	1

- Arrange that (only) **similar columns** are likely to **hash to the same bucket**, with high probability

- **Candidate pairs are those that hash to the same bucket**

Partition *M* into *b* Bands

2	1	4	1
1	2	1	2
2	1	2	1



Signature matrix *M*

Partition M into Bands

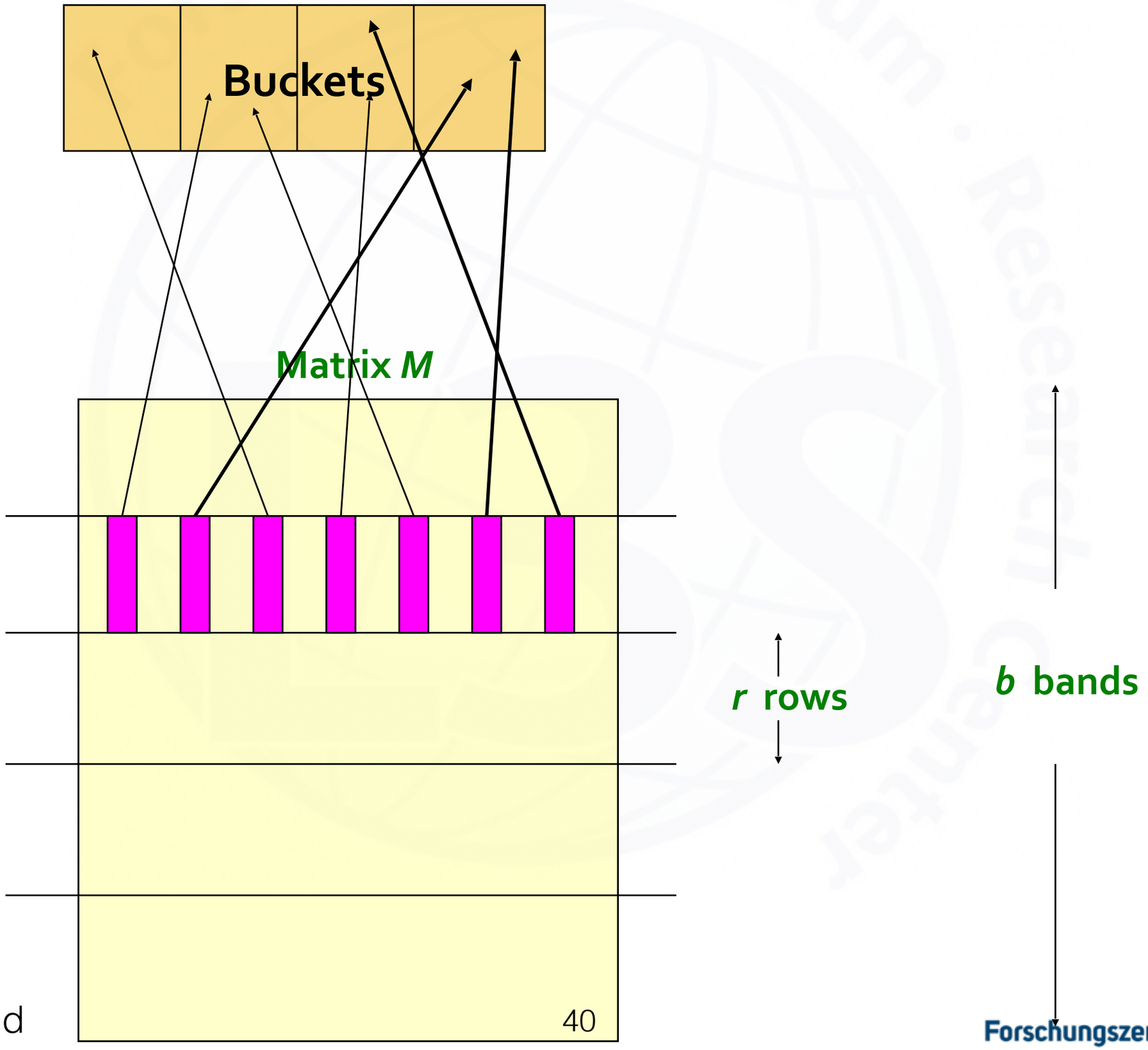
- Divide matrix ***M*** into ***b*** bands of ***r*** rows

- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - Make ***k*** as large as possible

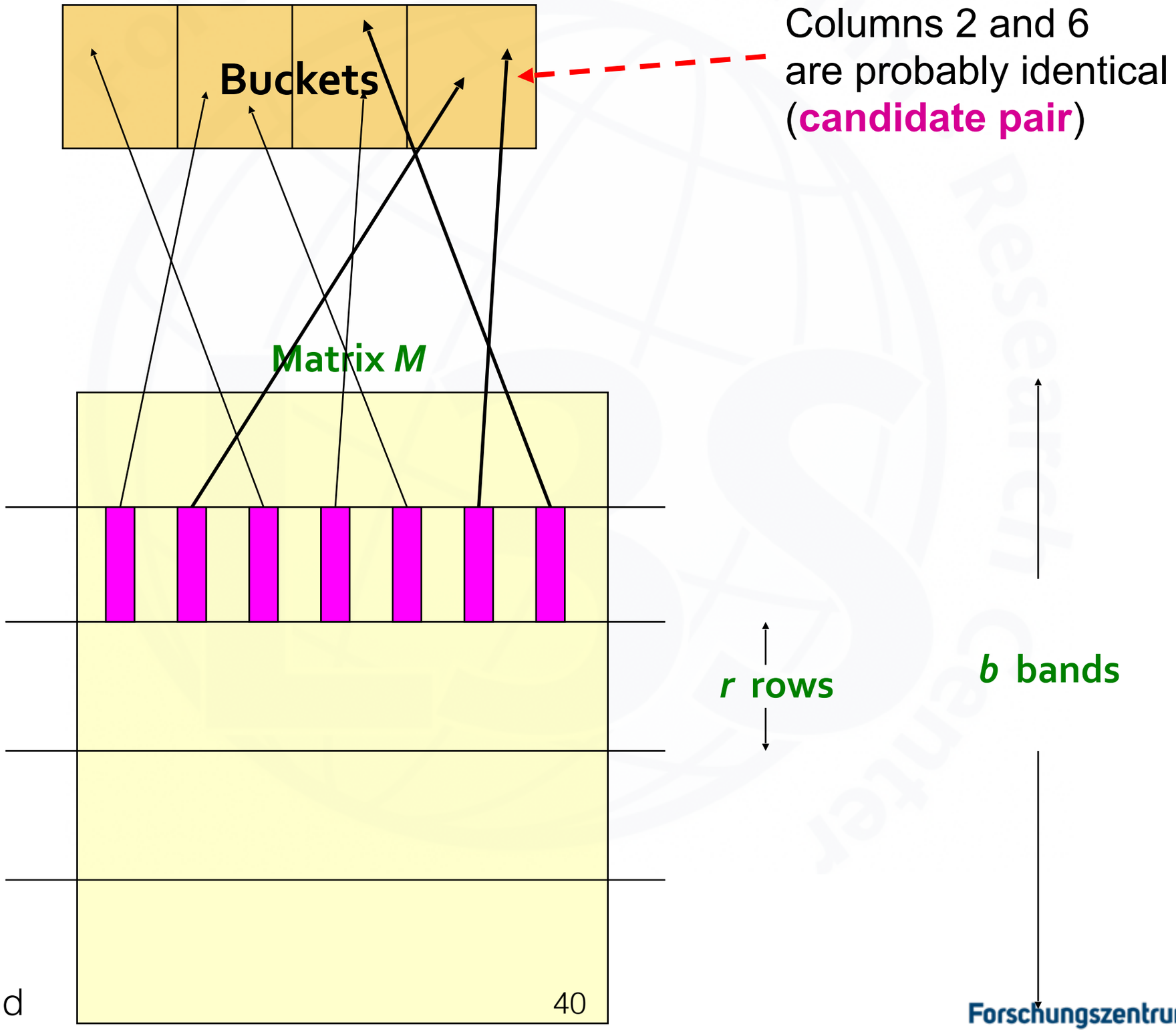
- ***Candidate*** column pairs are those that hash to the same bucket for $\geq$ **1** band

- Tune ***b*** and ***r*** to catch most similar pairs,
  but few non-similar pairs

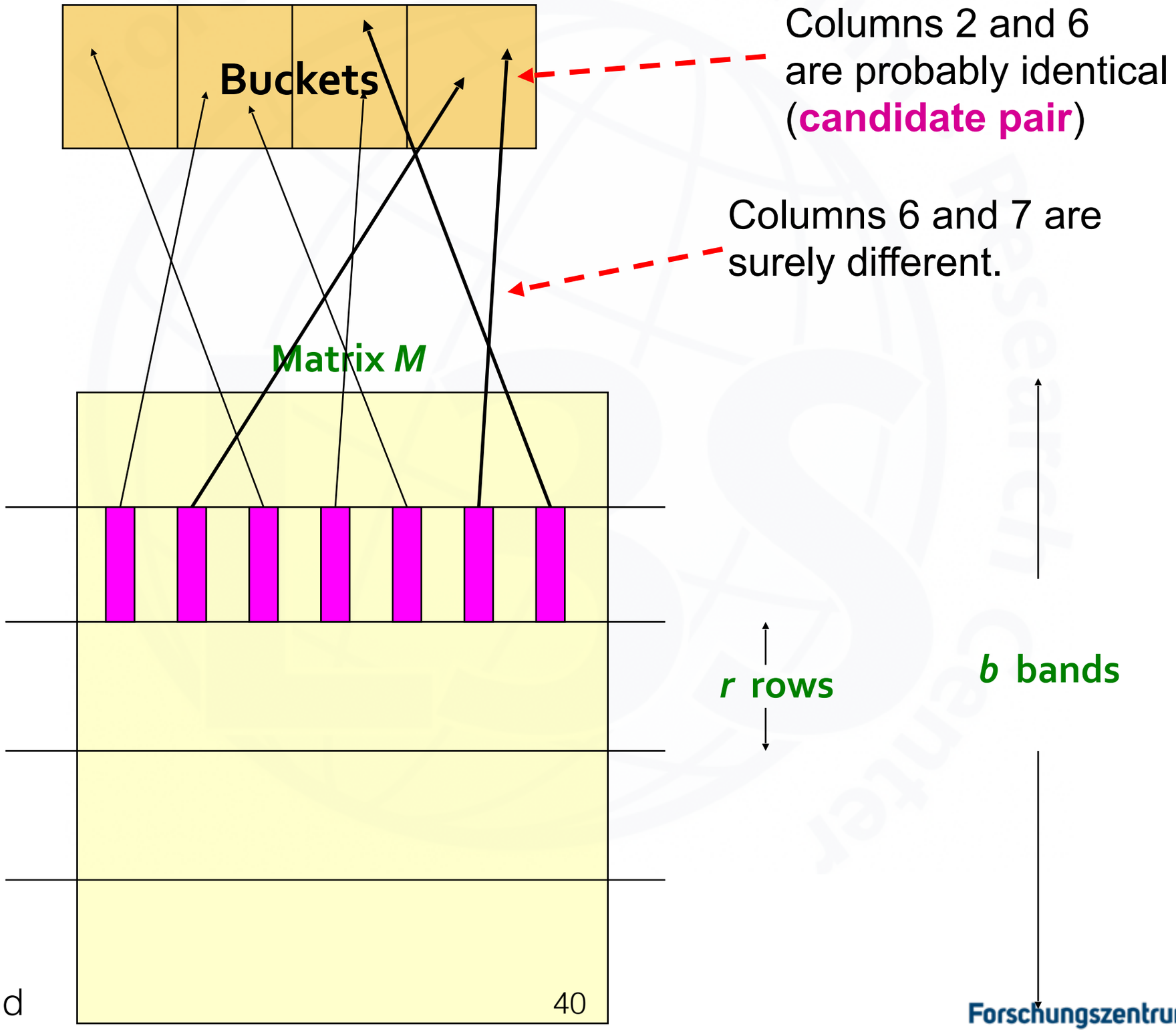
Hashing Bands



Hashing Bands



Hashing Bands



Simplifying Assumption

- There are **enough buckets** that columns are unlikely to hash to the same bucket unless they are **identical** in a particular band

- Hereafter, we assume that "**same bucket**" means "**identical in that band**"

- Assumption needed only to simplify analysis, not for correctness of algorithm

Example of Bands

Assume the following case:

2	1	4	1
1	2	1	2
2	1	2	1

- Suppose 100,000 columns of ***M*** (100k docs)
- Signatures of 100 integers (rows)
- Therefore, signatures take 40Mb
- Choose ***b*** = 20 bands of ***r*** = 5 integers/band
- **Goal:** Find pairs of documents that are at least ***s*** = *0.8* similar

C_1, C_2 are 80% Similar

2	1	4	1
1	2	1	2
2	1	2	1

- **Find pairs of ≥ *s*=0.8 similarity, set b=20, r=5**
- **Assume:** sim(C_1, C_2) = 0.8
  - Since sim(C_1, C_2) ≥ **s**, we want C_1, C_2 to be a **candidate pair**: We want them to hash to at **least 1 common bucket** (at least one band is identical)

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- **Probability C_1, C_2 identical in one particular band:** $(0.8)^5 = 0.328$
- Probability C_1, C_2 are ***not*** similar in all of the 20 bands: $(1-0.328)^{20}$ = 0.00035
  - i.e., about 1/3000th of the 80%-similar column pairs are **false negatives** (we miss them)
  - **We would find 99.965% pairs of truly similar documents**

C_1, C_2 are 30% Similar

- **Find pairs of ≥ *s*=0.8 similarity, set b=20, r=5**
- **Assume:** sim(C_1, C_2) = 0.3
  - Since sim(C_1, C_2) < **s** we want C_1, C_2 to hash to **NO common buckets** (all bands should be different)

2	1	4	1
1	2	1	2
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- **Find pairs of ≥ *s*=0.8 similarity, set b=20, r=5**
- **Assume:** sim(C_1, C_2) = 0.3
  - Since sim(C_1, C_2) < **s** we want C_1, C_2 to hash to **NO common buckets** (all bands should be different)
- **Probability C_1, C_2 identical in one particular band:** $(0.3)^5 = 0.00243$
- Probability C_1, C_2 identical in at least 1 of 20 bands: $1 - (1 - 0.00243)^{20} = 0.0474$
  - In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming **candidate pairs**
    - They are **false positives** since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold **s**

LSH Involves a Tradeoff

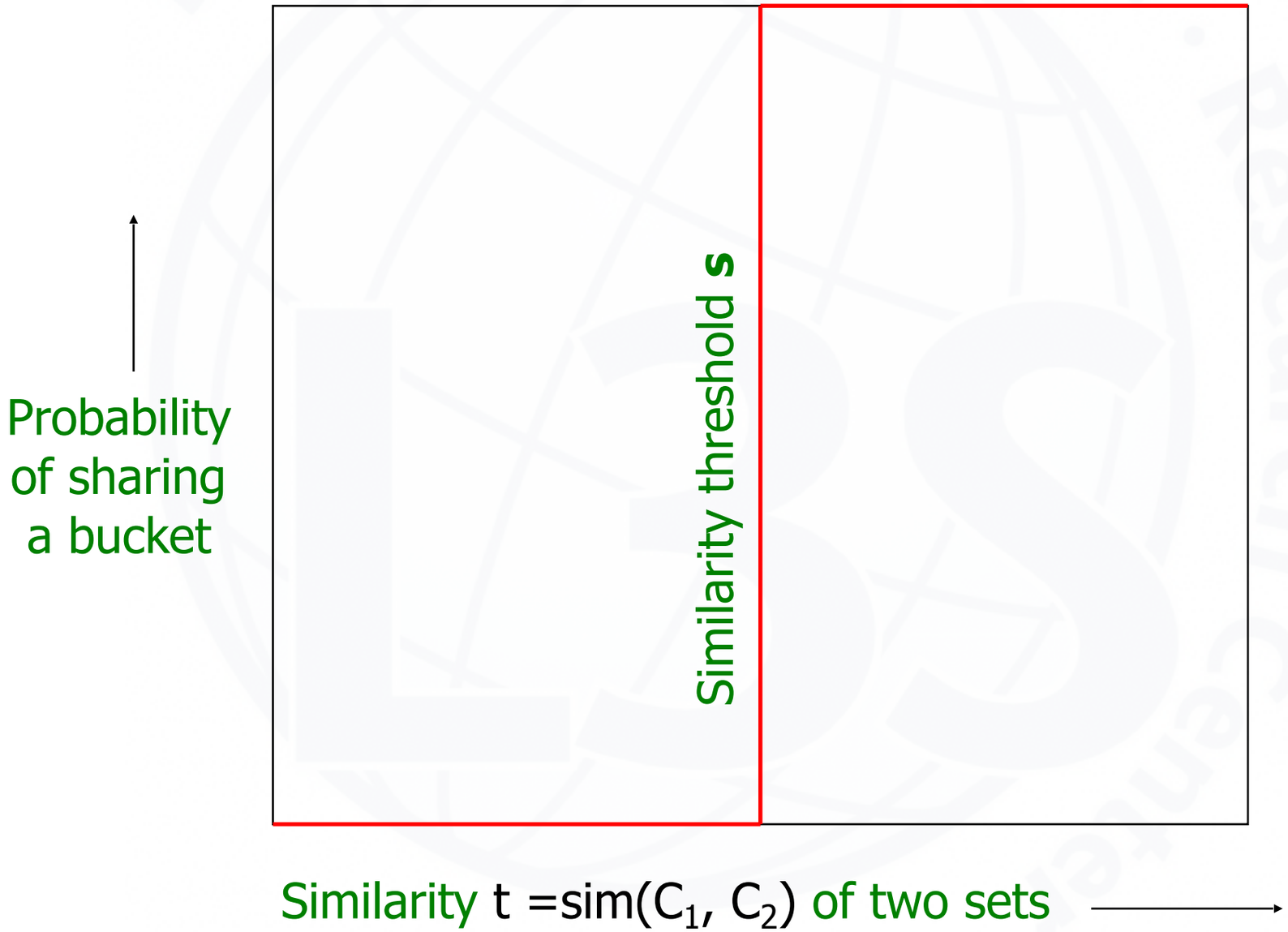
2	1	4	1
1	2	1	2
2	1	2	1

- **Pick:**
  - The number of Min-Hashes (rows of ***M***)
  - The number of bands ***b***, and
  - The number of rows ***r*** per band

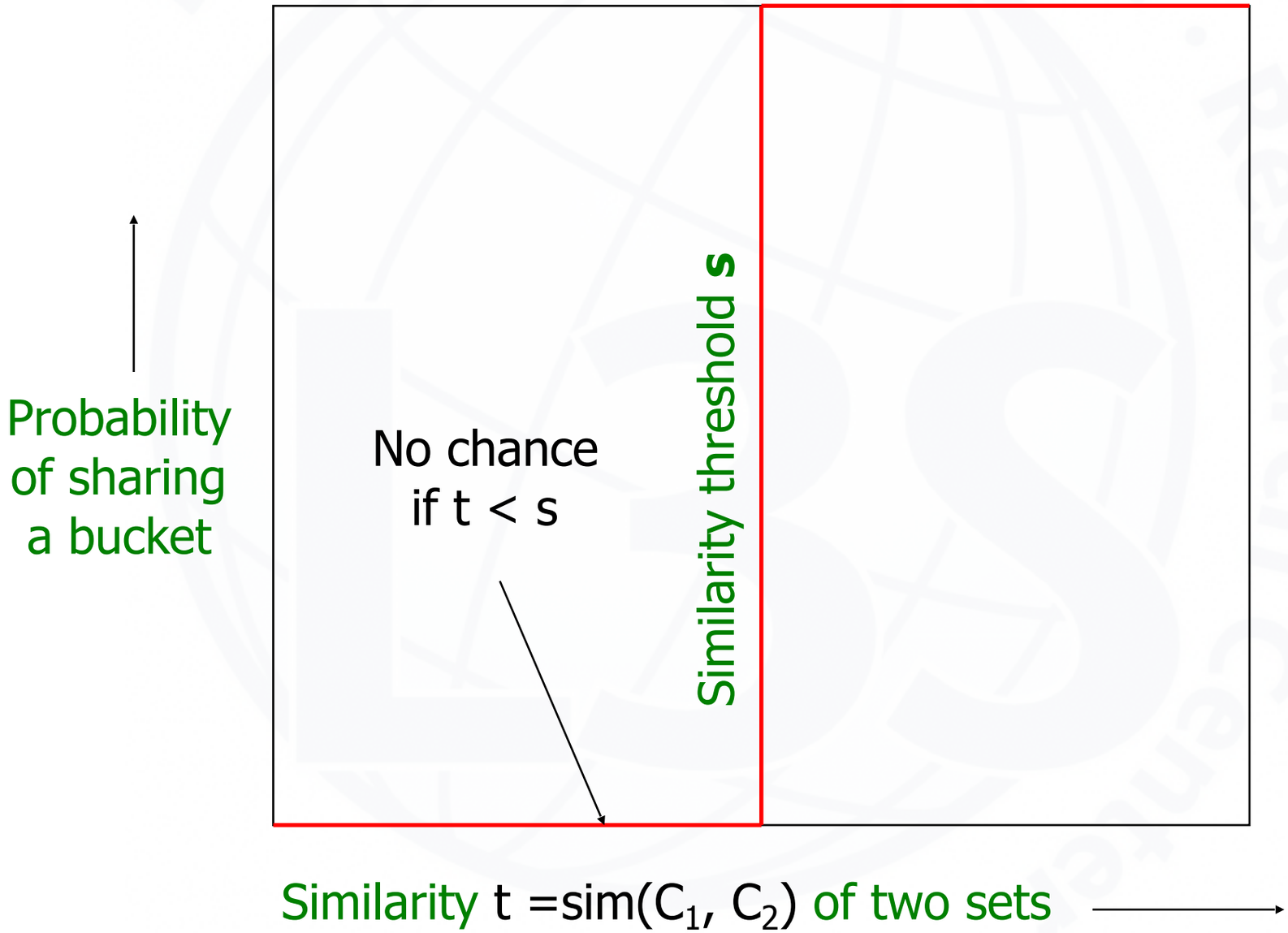
  to balance false positives/negatives

- **Example:** If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

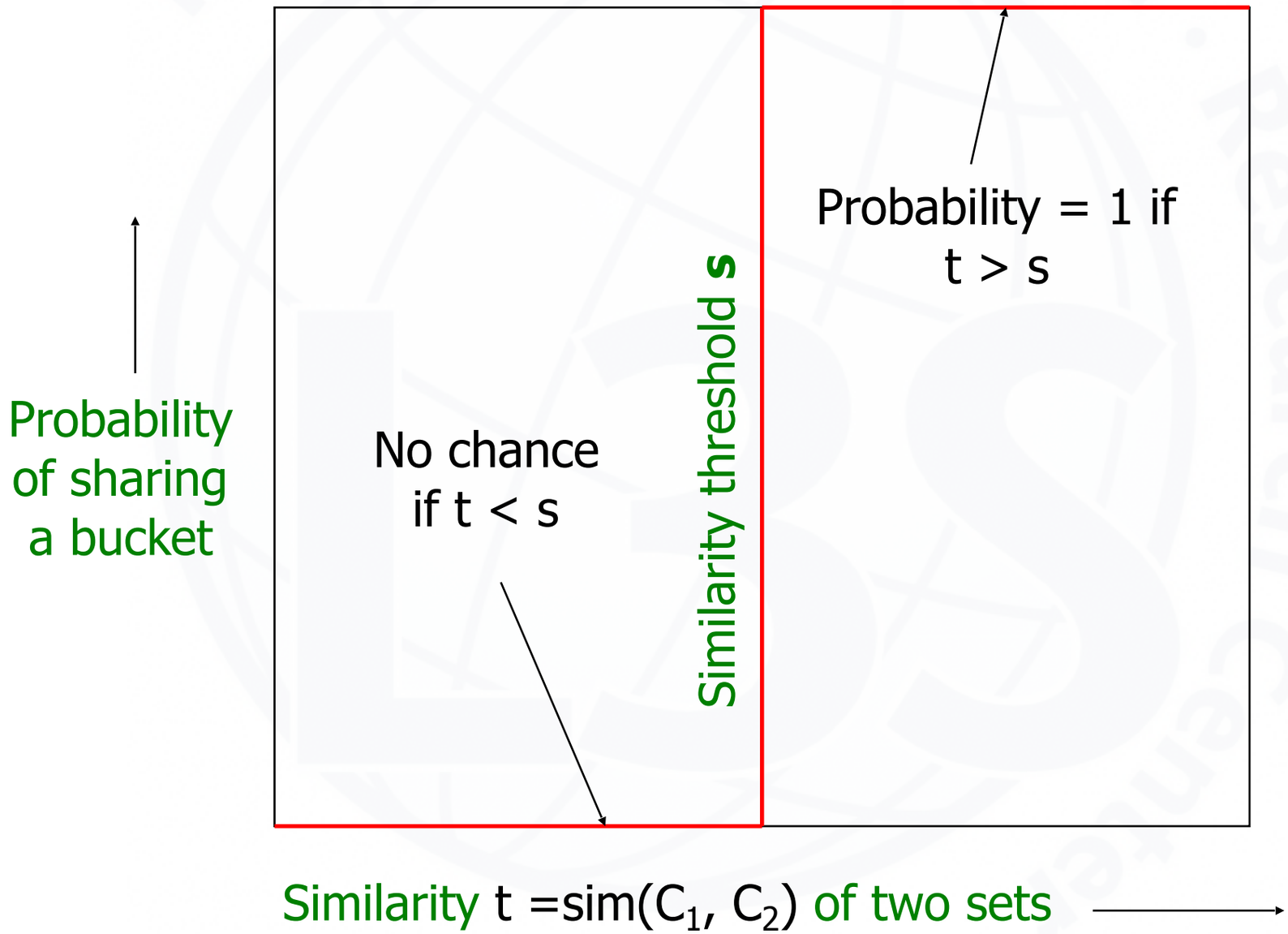
Analysis of LSH – What We Want



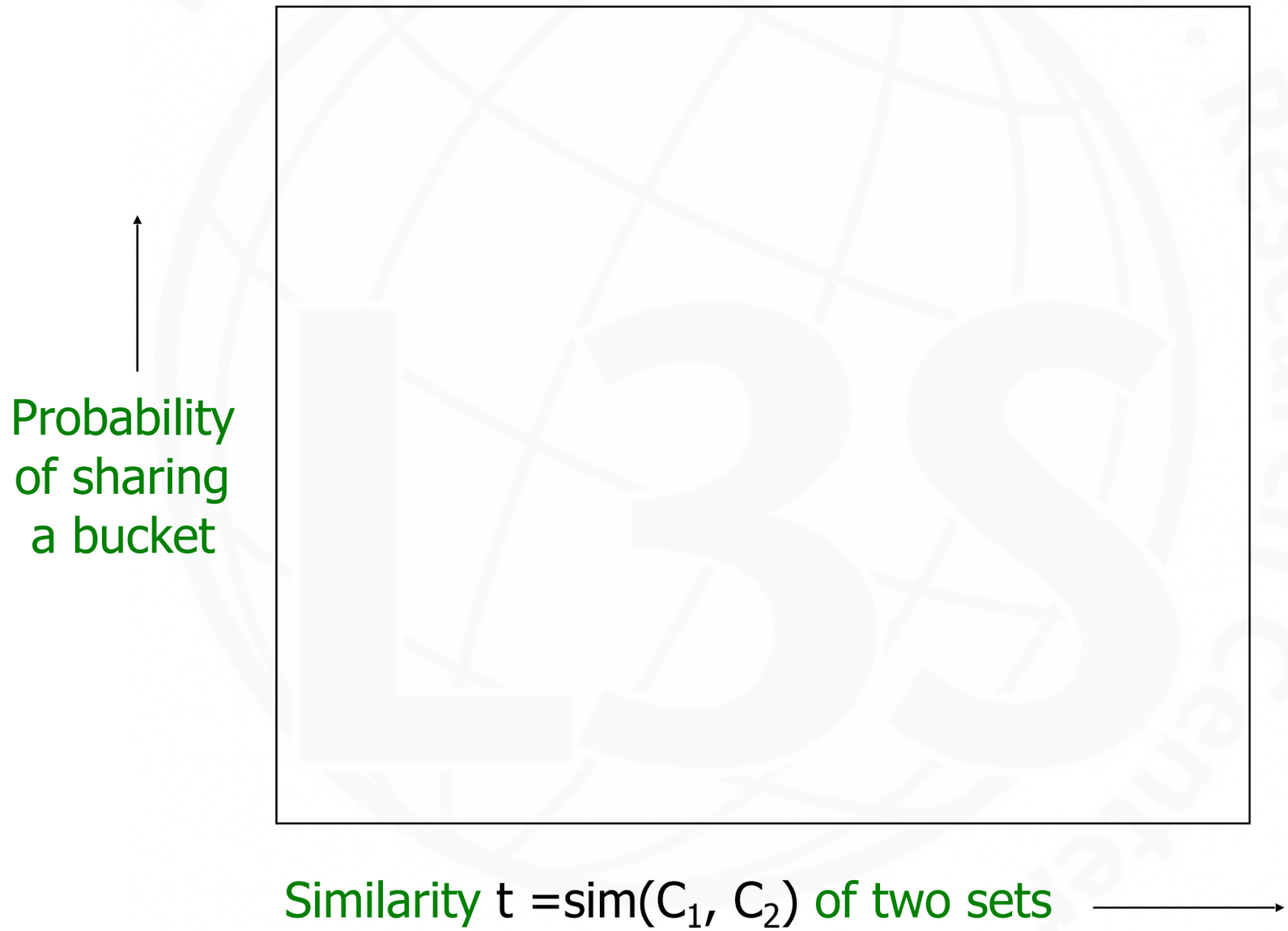
Analysis of LSH – What We Want



Analysis of LSH – What We Want



What 1 Band of 1 Row Gives You



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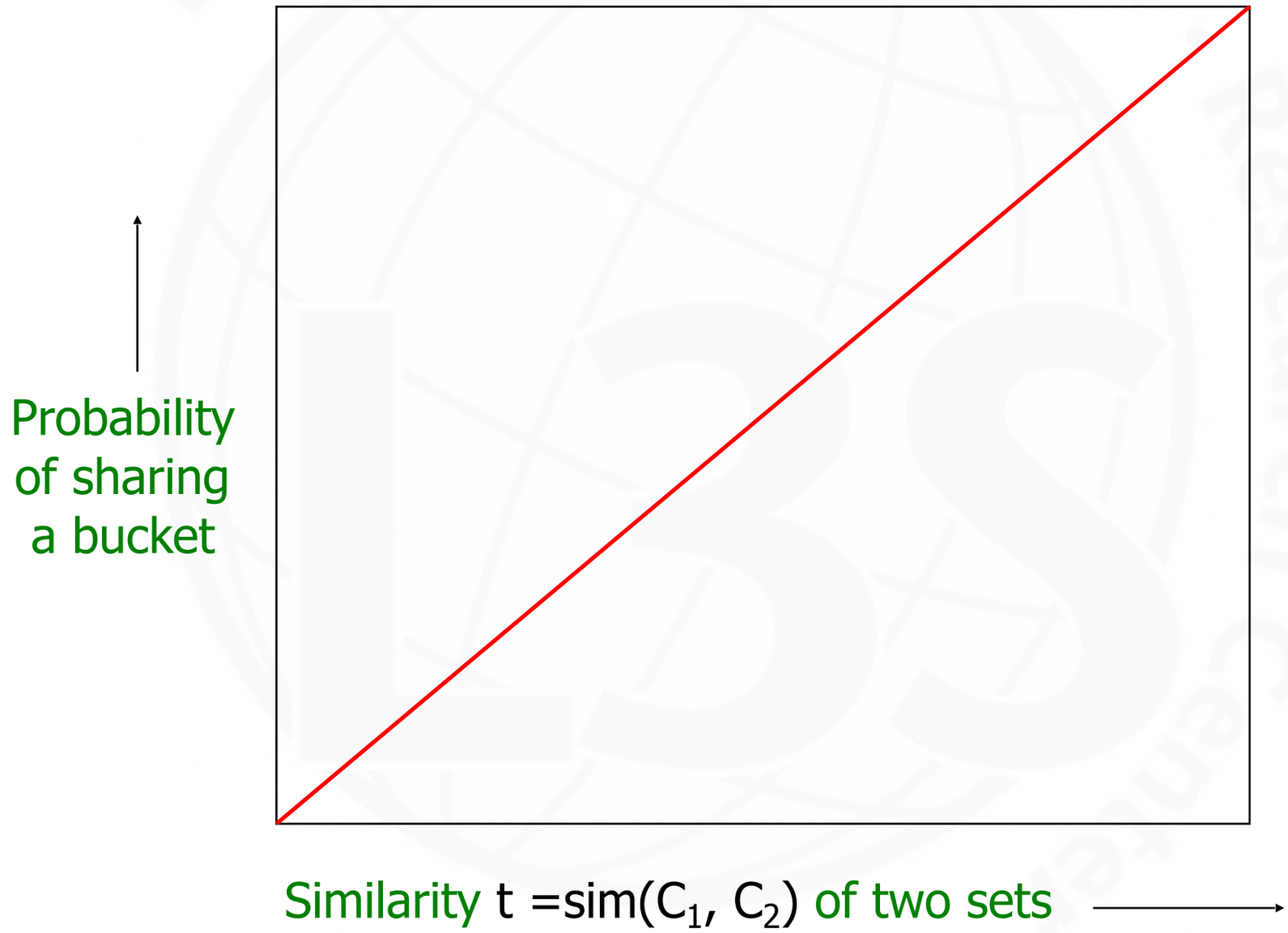

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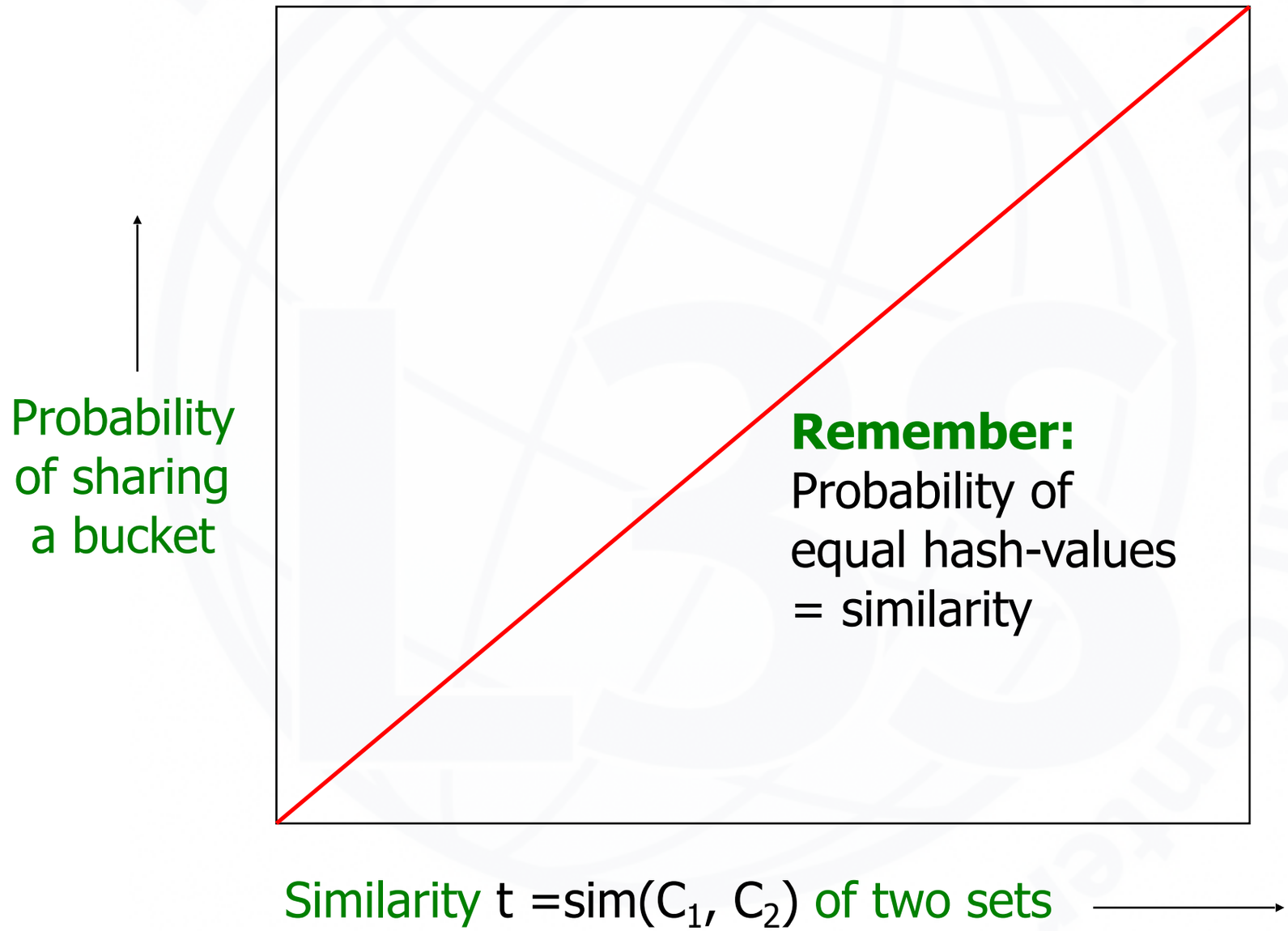
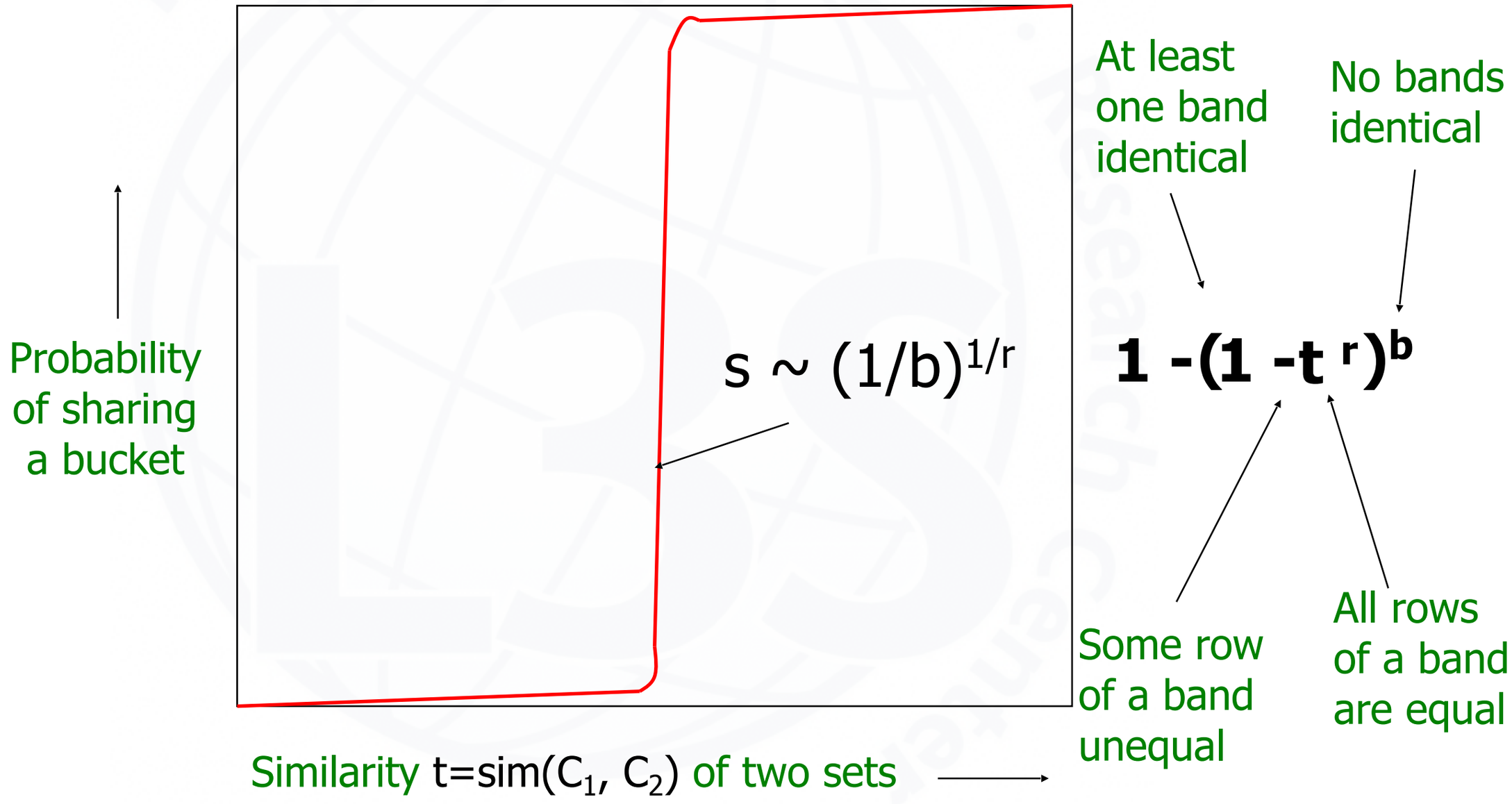


b bands, *r* rows/band

- Columns C_1 and C_2 have similarity ***t***
- Pick any band (***r*** rows)
  - Prob. that all rows in band equal = t^r
  - Prob. that some row in band unequal = $1 - t^r$
- Prob. that no band identical = $(1 - t^r)^b$
- Prob. that at least 1 band identical = $1 - (1 - t^r)^b$

What *b* Bands of *r* Rows Gives You

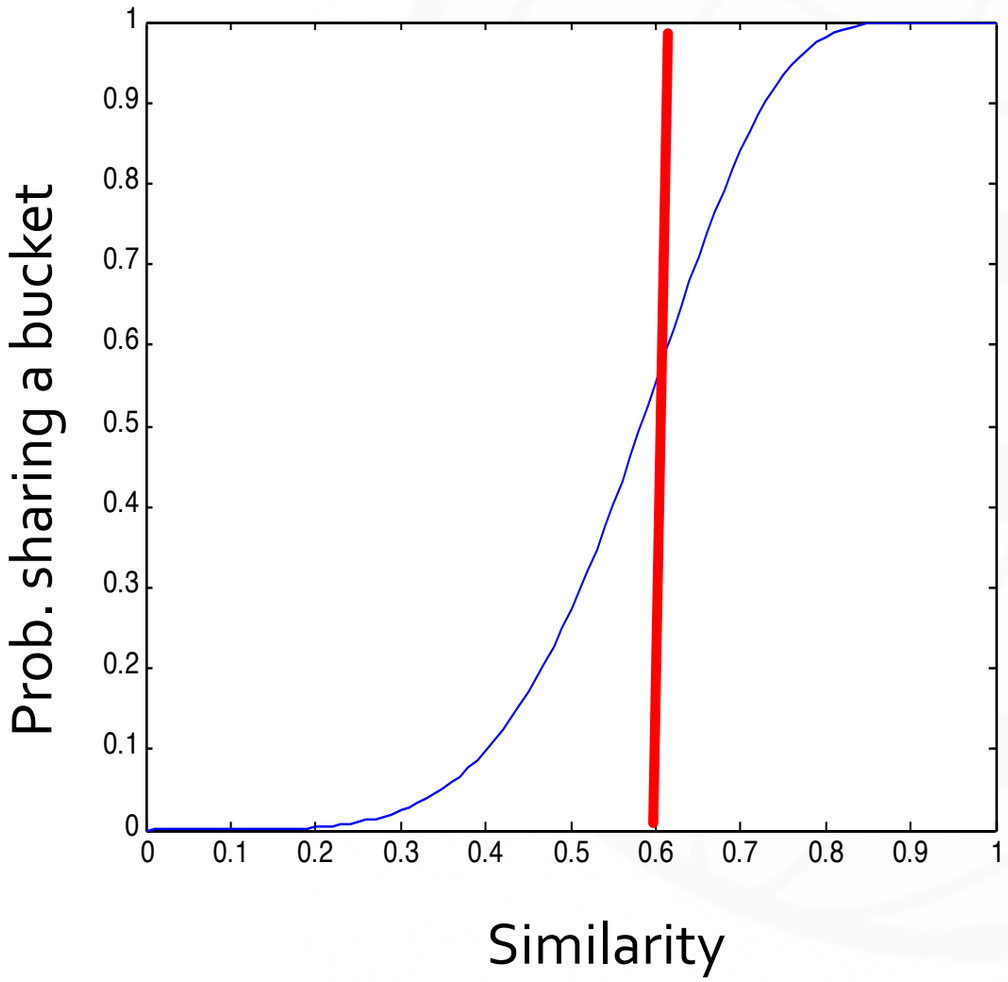


Example: $b = 20; r = 5$

- **Similarity threshold s**
- **Prob. that at least 1 band is identical:**

s	$1-(1-s^r)^b$
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

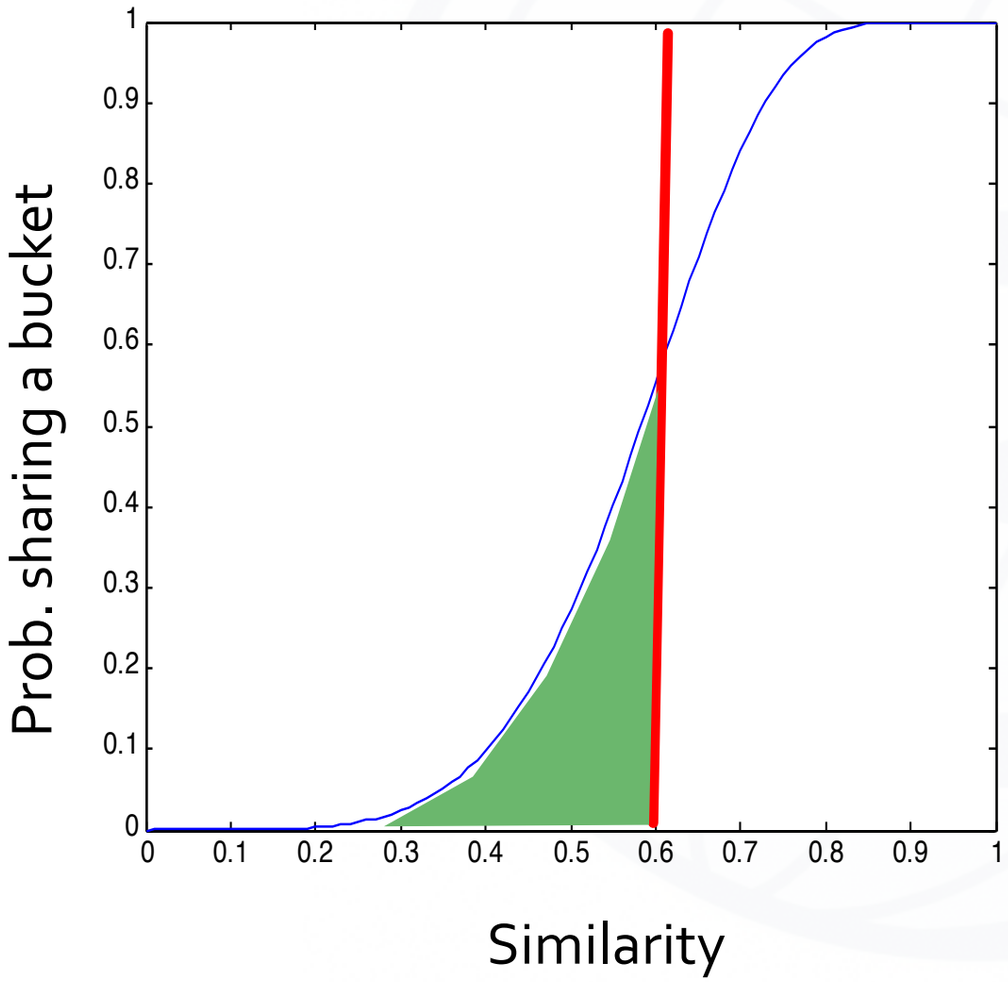
Picking *r* and *b*: The S-curve



- **Picking *r* and *b* to get the best S-curve**
  - 50 hash-functions (r=5, b=10)

Blue area: False Negative rate
Green area: False Positive rate

Picking *r* and *b*: The S-curve



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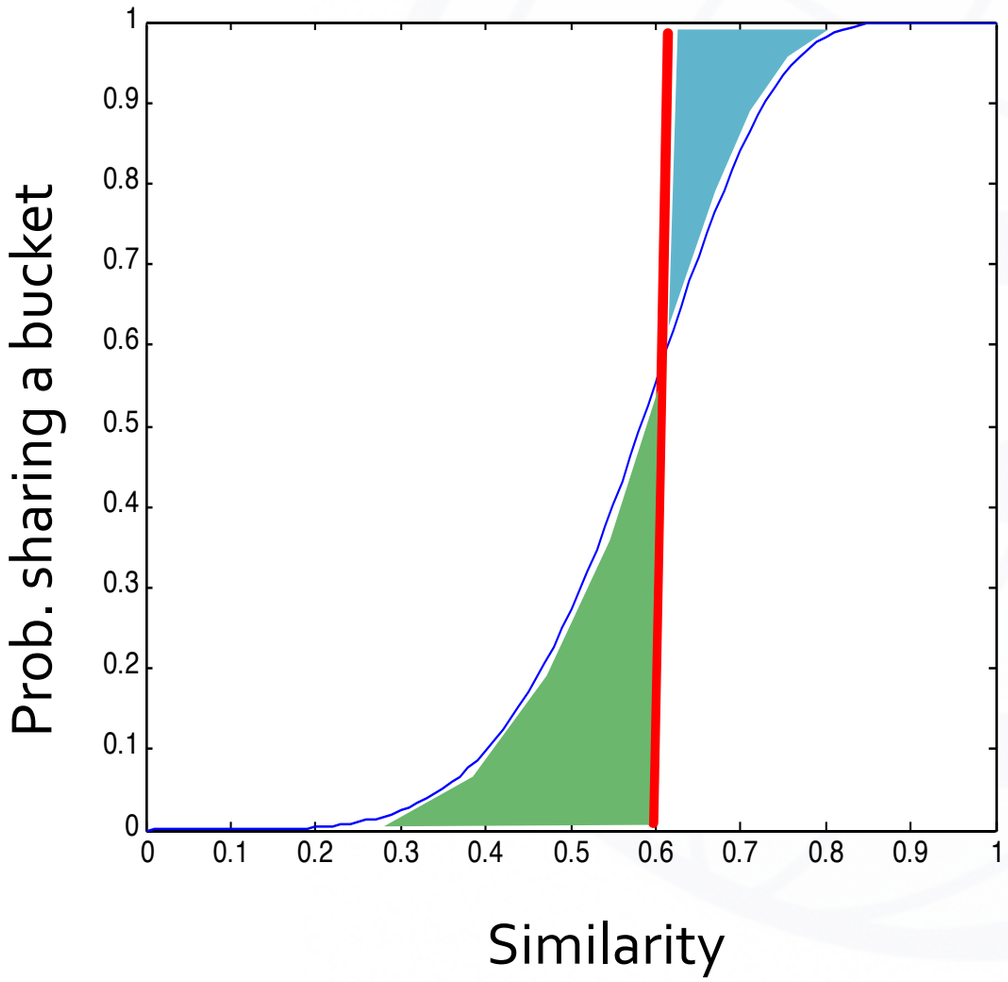

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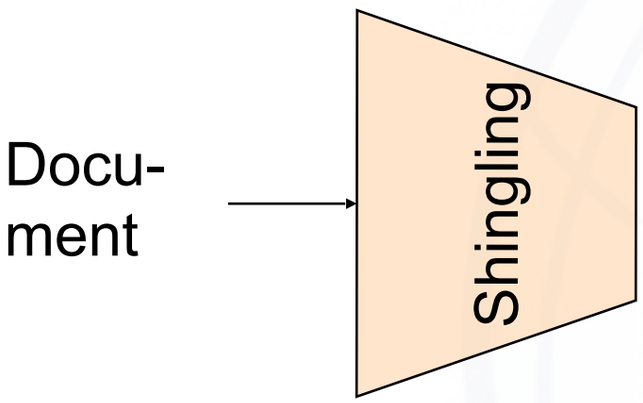
LSH Summary

- Tune ***M, b, r*** to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures

- Check in main memory that **candidate pairs** really do have **similar signatures**

- **Optional:** In another pass through data, check that the remaining candidate pairs really represent similar documents

The Big Picture



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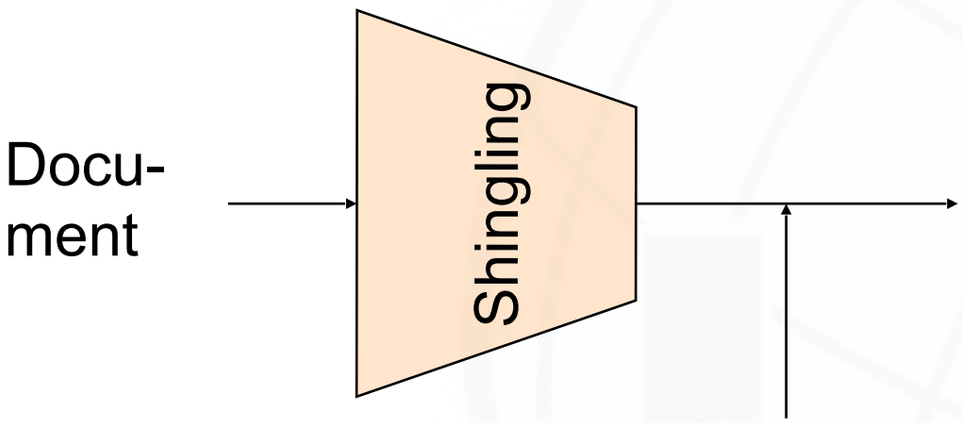
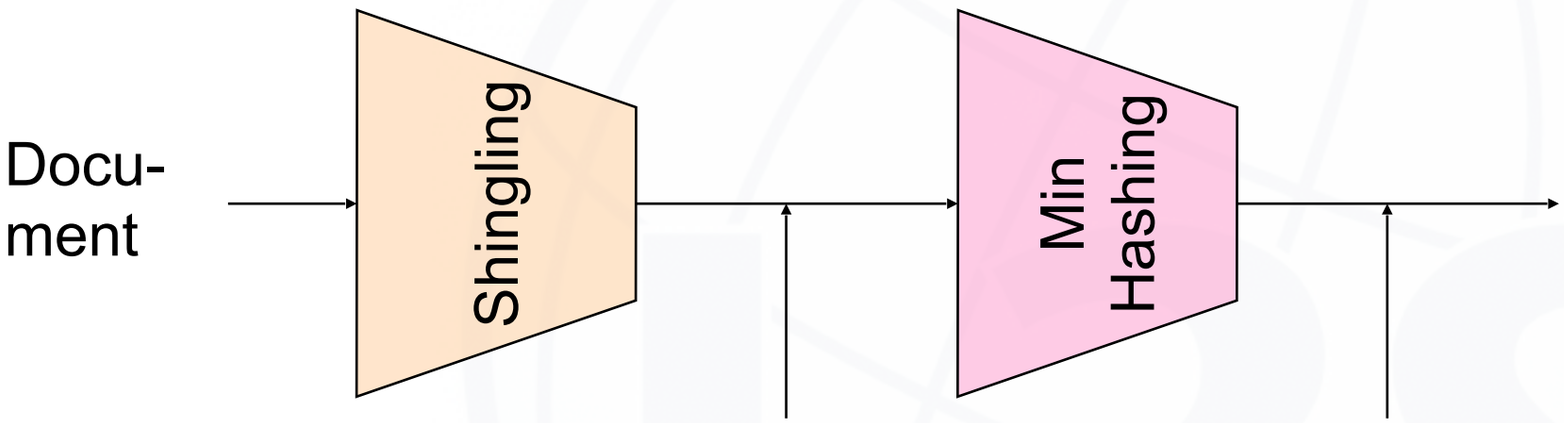


The set of strings of length ***k*** that appear in the doc-ument

The Big Picture



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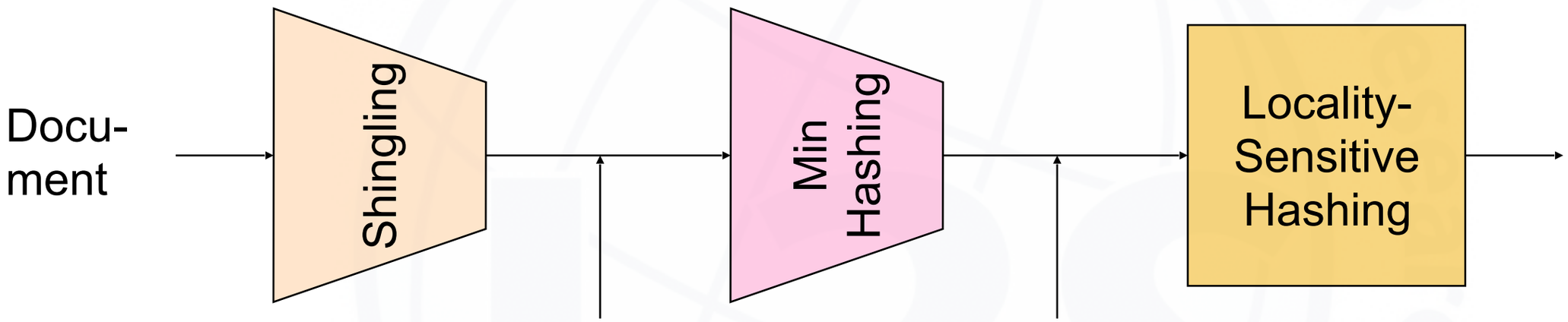
Signatures: short integer vectors that represent the sets, and reflect their similarity

The Big Picture



The set of strings of length ***k*** that appear in the doc-ument

Signatures: short integer vectors that represent the sets, and reflect their similarity

Candidate pairs: those pairs of signatures that we need to test for similarity

Summary: 3 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID
- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr}[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
  - We used hashing to get around generating random permutations
- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity ≥ **s**

Quiz

- **Anonymous**
- **Tear off your Quiz Number on the top right**
  - Save it with you if you want to know your scores
- **Duration : 20 minutes**
- After the test: Shuffle the deck and redistribute
- Crowdsourcing

Implementation Trick

- **Permuting rows even once is prohibitive**
- **Row hashing!**
  - Pick **K = 100** hash functions k_i
  - Ordering under k_i gives a random row permutation!
- **One-pass implementation**
  - For each column ***C*** and hash-func. k_i keep a "slot" for the min-hash value
  - Initialize all ***sig(C)[i] =*** ∞
  - **Scan rows looking for 1s**
    - Suppose row ***j*** has 1 in column ***C***
    - Then for each k_i:
      - If $k_i(j) < sig(C)[i]$, then $sig(C)[i] \leftarrow k_i(j)$

How to pick a random hash function h(x)?
Universal hashing:
$h_{a,b}(x)=((a \cdot x+b) \bmod p) \bmod N$
where:
a,b … random integers
p … prime number (p > N)