User Modeling and Personalization
Exercise 3: Bayesian Networks

Patrick Siehndel, Eelco Herder

L3S Research Center / Leibniz University of Hanover
Hannover, Germany

12 May 2014

Exercise 1: Bayesian Networks / Reasoning under uncertainty

Given are the following CPTs (= Conditional Probability Tables):

P(likesClassics)
0.4

P(likesBlues)
0.1

likesBlues	likesClassics	P(likesJazz)
yes	yes	0.05
yes	no	0.45
no	yes	0.2
no	no	0.15

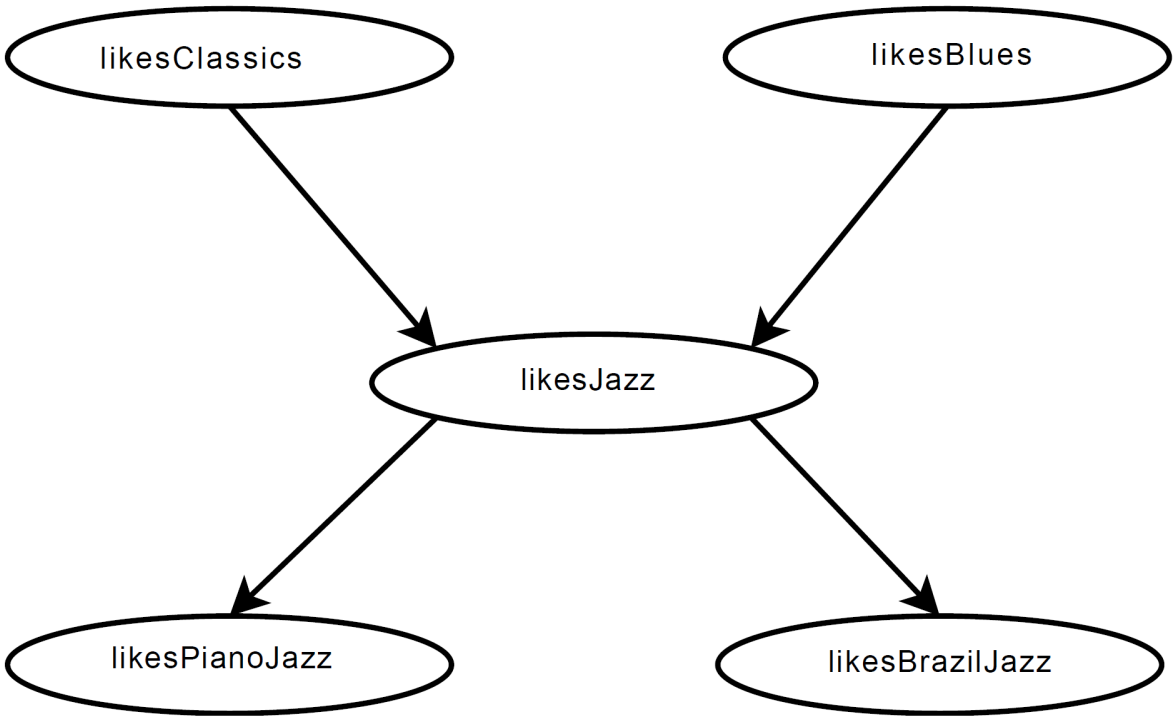
likesJazz	P(likesPianoJazz)
yes	0.85
no	0.3

likesJazz	P(likesBrazilJazz)
yes	0.9
no	0.08

Exercise 1: Bayesian Networks / Reasoning under uncertainty

1. Draw the corresponding Bayesian Network.
2. Calculate the probability that a user...
   - 2.1 likes Classics, Blues and Jazz.
   - 2.2 does not like Classics and Blues, but Jazz and Piano Jazz.
   - 2.3 does not like Classics and Brazil Jazz, but Blues, Jazz and Piano Jazz.

Solution 1: Bayesian Networks / Reasoning under uncertainty



Solution 1: Bayesian Networks / Reasoning under uncertainty

1.2 - a) $P(Classics \wedge Blues \wedge Jazz) =$
$P(Classics) * P(Blues) * P(Jazz|Classics, Blues) =$
$0,4 * 0,1 * 0,05 = 0,002$

1.2 - b) $P(\neg Classics \wedge \neg Blues \wedge Jazz \wedge PianoJazz) =$
$(1 - P(Classics)) * (1 - P(Blues)) *$
$P(Jazz|\neg Classics, \neg Blues) * P(PianoJazz|Jazz) =$
$0,6 * 0,9 * 0,15 * 0,85 = 0,06885$

1.2 - c)

$$P(\neg Classics \wedge Blues \wedge Jazz \wedge PianoJazz \wedge \neg BrazilJazz) =$$
$$(1 - P(Classics)) * P(Blues) * P(Jazz|\neg Classics, Blues) *$$
$$P(PianoJazz|Jazz) * (1 - P(BrazilJazz|Jazz)) =$$
$$0,6 * 0,1 * 0,45 * 0,85 * 0,1 = 0,002295$$

Exercise 2: Bayesian Networks / Reasoning under uncertainty

Given are the following CPTs (= Conditional Probability Tables):

P(likesIce)
0.9

likesLemons	likesIce	P(likesLemonice | likesLemons, likesIce)
yes	yes	0.9
yes	no	0.3
no	yes	0.2
no	no	0.05

likesFruit	P(likesLemons | likesFruit)
yes	0.8
no	0.1

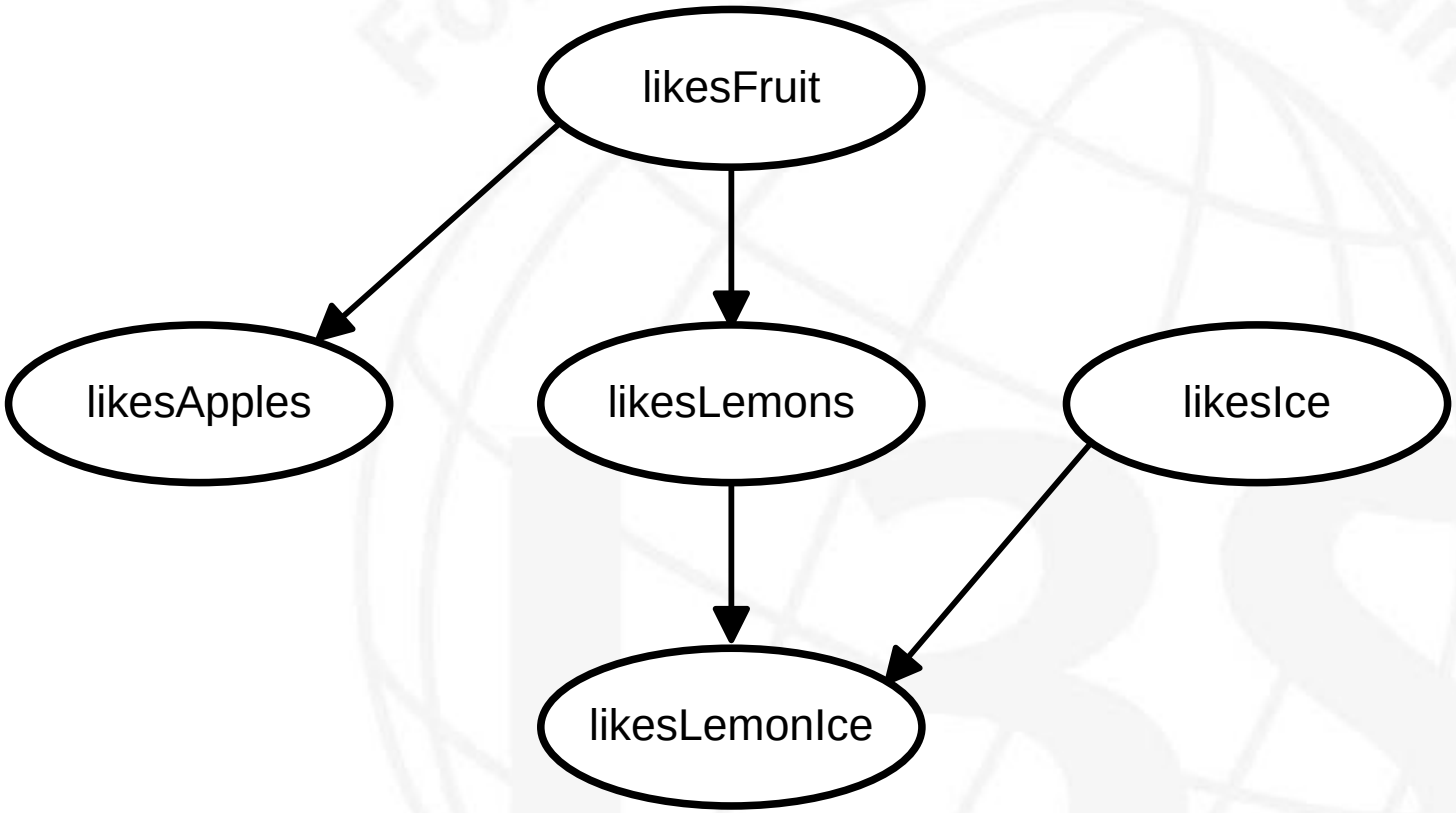
P(likesFruit)
0.7

likesFruit	P(likesApples | likesFruit)
yes	0.8
no	0.05

Exercise 2: Bayesian Networks / Reasoning under uncertainty

1. Draw the corresponding Bayesian Network.
2. Calculate the probability that a user...
   - 2.1 likes Fruit and Lemons, doesn't like Ice and Apple, and likes LemonIce.
   - 2.2 likes Lemon Ice, if the user likes Lemons
3. What is the probability that a user likes Fruit, if he likes Apples?

Solution 2: Bayesian Networks / Reasoning under uncertainty



Solution 2: Bayesian Networks / Reasoning under uncertainty

2.1) User likes Fruit and Lemons, doesn't like Ice and Apple, and likes LemonIce.

$$P(F, L, \neg I, \neg A, LI)$$

$$= P(F) * P(L \mid F) * P(\neg I) * P(\neg A \mid F) * P(LI \mid L, \neg I)$$

$$= 0.7 * 0.8 * (1 - 0,9) * (1 - 0.8) * 0,3 = 0,00336$$

Solution 2: Bayesian Networks / Reasoning under uncertainty

2.2) User likes Lemon Ice, if the user likes Lemons.

$$P(LI|L) = P(LI|L,I)P(I) + P(LI|L,\neg I)P(\neg I)$$
$$= 0.9 * 0.9 + 0.3 * 0.1$$
$$= 0.81 + 0.03 = 0.84$$

Note that $P(L) = 1$, as it is given that the user likes lemons (if we had asked for the probability that the user likes Lemon Ice *and* likes Lemons, you had to include $P(L) = P(L|F)P(F) + P(L|\neg F)P(\neg F)$.

Solution 2: Bayesian Networks / Reasoning under uncertainty

3) User likes Fruit, if he likes Apple.
Hint: for this, we need to make use of *Bayes' Rule*.

$$P(F|A) = \frac{P(A|F)}{P(A)} * P(F)$$

continued from previous slide

$$P(F|A) = \frac{P(A|F)}{P(A)} * P(F)$$

$$= \frac{P(A|F)}{P(A|F)P(F) + P(A|\neg F)(P(\neg F)} * P(F)$$

$$= \frac{0.8}{0.8 * 0.7 + 0.05 * 0.3} * 0.7$$

$$= \frac{0.8}{0.56 + 0.015} * 0.7 = 0.97$$