Leibniz Universität Hannover

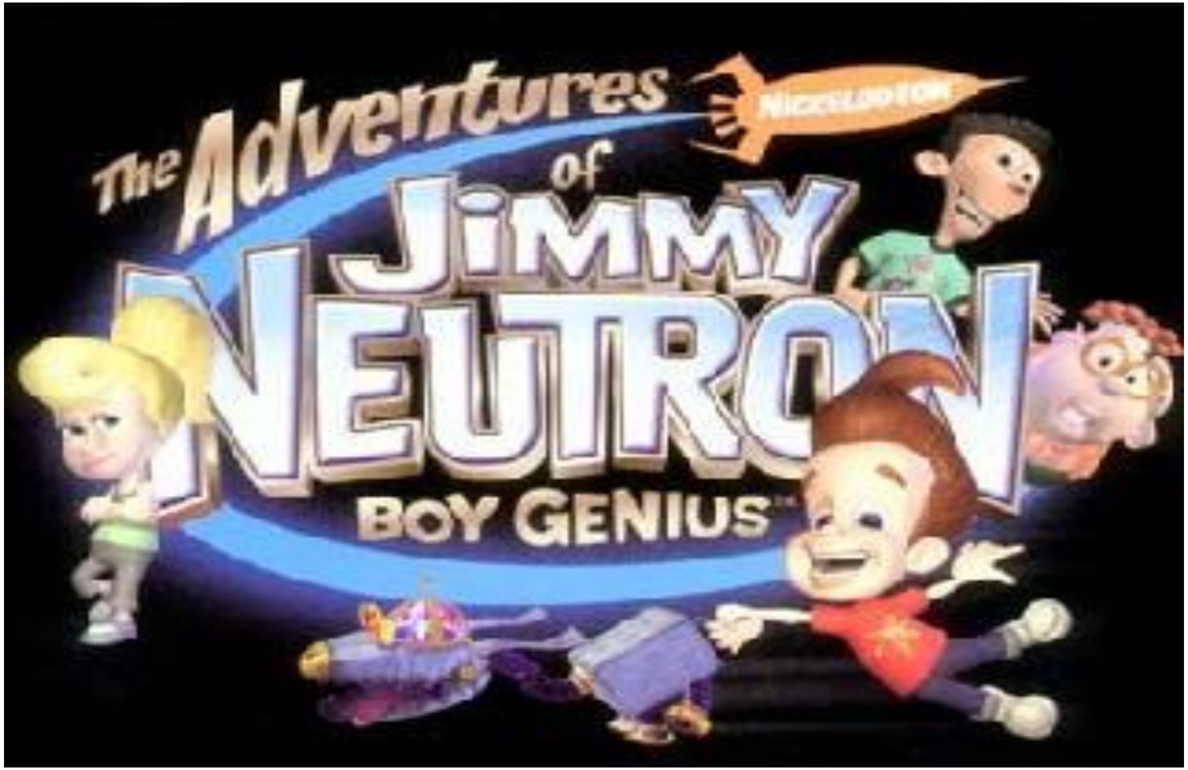
IRS- Institut für Radioökologie und Strahlenschutz

Physik IV
07
Neutronen

SS 2016

Clemens Walther

Neutronen



Geschichte

- Bethe & Becker (1930) beobachteten eine durchdringende nicht-ionisierende Strahlung wenn Be mit ^{210}Po α-Teilchen beschossen wurde (E_α = 5,3 MeV). γ-Strahlung ?

- Curie und Joliot: Wechselwirkung dieser Strahlung mit Wasserstoff-haltigem Paraffin produziert Protonen mit E_p = 5,7 MeV.

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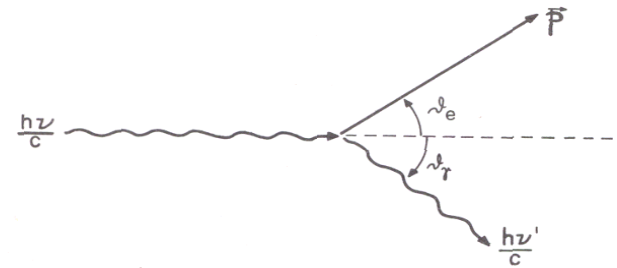
  Erklärungsversuch:
  Rückstoß Protonen mittels
  Compton Effekt

Wie war die maximale Energie von Compton Elektronen?



A. $$E_e = h\nu - \frac{h\nu}{1+2\varepsilon} \quad \text{mit} \quad \varepsilon = \frac{hv}{m_e c^2}$$

B. $$E_e = \frac{h\nu}{1+2\varepsilon} \quad \text{mit} \quad \varepsilon = \frac{hv}{m_e c^2}$$

C. $$E_e = h\nu - \frac{2h\nu}{1+\varepsilon} \quad \text{mit} \quad \varepsilon = \frac{hv}{m_e c^2}$$

D. $$E_e = 2h\nu - \frac{h\nu}{1+\varepsilon} \quad \text{mit} \quad \varepsilon = \frac{hv}{m_e c^2}$$

SMART Response Question

To set the properties right click and select
SMART Response Question Object->Properties...

Compton Energie von γ-Strahlung

$$E_e = h\nu - \frac{h\nu}{1+2\varepsilon} \quad \text{mit} \quad \varepsilon = \frac{h\nu}{m_e c^2}$$

$$E_e = \frac{2h\nu}{2-\frac{1}{\varepsilon}}$$

$$E_p = \frac{2E_\gamma}{2+\frac{M_p \cdot c^2}{E_\gamma}}$$

$$E_p = 5{,}7\ \text{MeV} \longrightarrow E_\gamma = 55\ \text{MeV}$$

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- **Chadwick (1932): Das NEUTRON, Protonen sind Rückstoß Protonen**

- Heisenberg (1932): Der Kern besteht aus Protonen und Neutronen

Geschichte

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$$\boxed{{}^{9}\mathrm{Be}(\alpha,n)^{12}\mathrm{C} \qquad Q = 5{,}7\ \mathrm{MeV}}$$

312 NATURE [FEBRUARY 27, 1932

Letters to the Editor

[The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, nor to correspond with the writers of, rejected manuscripts intended for this or any other part of NATURE. *No notice is taken of anonymous communications.]*

Possible Existence of a Neutron

IT has been shown by Bothe and others that beryllium when bombarded by α-particles of polonium emits a radiation of great penetrating power, which has an absorption coefficient in lead of about 0·3 (cm.)$^{-1}$. Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionisation produced by this beryllium radiation in a vessel with a thin window, that the ionisation increased when matter containing hydrogen was placed in front of the window. The effect appeared to be due to the ejection of protons with velocities up to a maximum of nearly 3×10^9 cm. per sec. They suggested that the transference of energy to the proton was by a process similar to the Compton effect, and estimated that the beryllium radiation had a quantum energy of 50×10^6 electron volts.

I have made some experiments using the valve counter to examine the properties of this radiation excited in beryllium. The valve counter consists of a small ionisation chamber connected to an amplifier, and the sudden production of ions by the entry of a particle, such as a proton or α-particle, is recorded by the deflexion of an oscillograph. These experi-

This again receives a simple explanation on the neutron hypothesis.

If it be supposed that the radiation consists of quanta, then the capture of the α-particle by the Be^9 nucleus will form a C^{13} nucleus. The mass defect of C^{13} is known with sufficient accuracy to show that the energy of the quantum emitted in this process cannot be greater than about 14×10^6 volts. It is difficult to make such a quantum responsible for the effects observed.

It is to be expected that many of the effects of a neutron in passing through matter should resemble those of a quantum of high energy, and it is not easy to reach the final decision between the two hypotheses. Up to the present, all the evidence is in favour of the neutron, while the quantum hypothesis can only be upheld if the conservation of energy and momentum be relinquished at some point.

J. CHADWICK.

Cavendish Laboratory,
Cambridge, Feb. 17.

The Oldoway Human Skeleton

A LETTER appeared in NATURE of Oct. 24, 1931, signed by Messrs. Leakey, Hopwood, and Reck, in which, among other conclusions, it is stated that "there is no possible doubt that the human skeleton came from Bed No. 2 and not from Bed No. 4". This must be taken to mean that the skeleton is to be considered as a natural deposit in Bed No. 2, which is overlaid by the later beds Nos. 3 and 4, and that all consideration of human interment is ruled out.

ments have shown that the radiation ejects particles from hydrogen, helium, lithium, beryllium, carbon, air, and argon. The particles ejected from hydrogen behave, as regards range and ionising power, like protons with speeds up to about $3 \cdot 2 \times 10^9$ cm. per sec. The particles from the other elements have a large ionising power, and appear to be in each case recoil atoms of the elements.

If we ascribe the ejection of the proton to a Compton recoil from a quantum of 52×10^6 electron volts, then the nitrogen recoil atom arising by a similar process should have an energy not greater than about 400,000 volts, should produce not more than about 10,000 ions, and have a range in air at N.T.P. of about 1·3 mm. Actually, some of the recoil atoms in nitrogen produce at least 30,000 ions. In collaboration with Dr. Feather, I have observed the recoil atoms in an expansion chamber, and their range, estimated visually, was sometimes as much as 3 mm. at N.T.P.

These results, and others I have obtained in the course of the work, are very difficult to explain on the assumption that the radiation from beryllium is a quantum radiation, if energy and momentum are to be conserved in the collisions. The difficulties disappear, however, if it be assumed that the radiation consists of particles of mass 1 and charge 0, or neutrons. The capture of the α-particle by the Be^9 nucleus may be supposed to result in the formation of a C^{12} nucleus and the emission of the neutron. From the energy relations of this process the velocity of the neutron emitted in the forward direction may well be about 3×10^9 cm. per sec. The collisions of this neutron with the atoms through which it passes give rise to the recoil atoms, and the observed energies of the recoil atoms are in fair agreement with this view. Moreover, I have observed that the protons ejected from hydrogen by the radiation emitted in the opposite direction to that of the exciting α-particle appear to have a much smaller range than those ejected by the forward radiation.

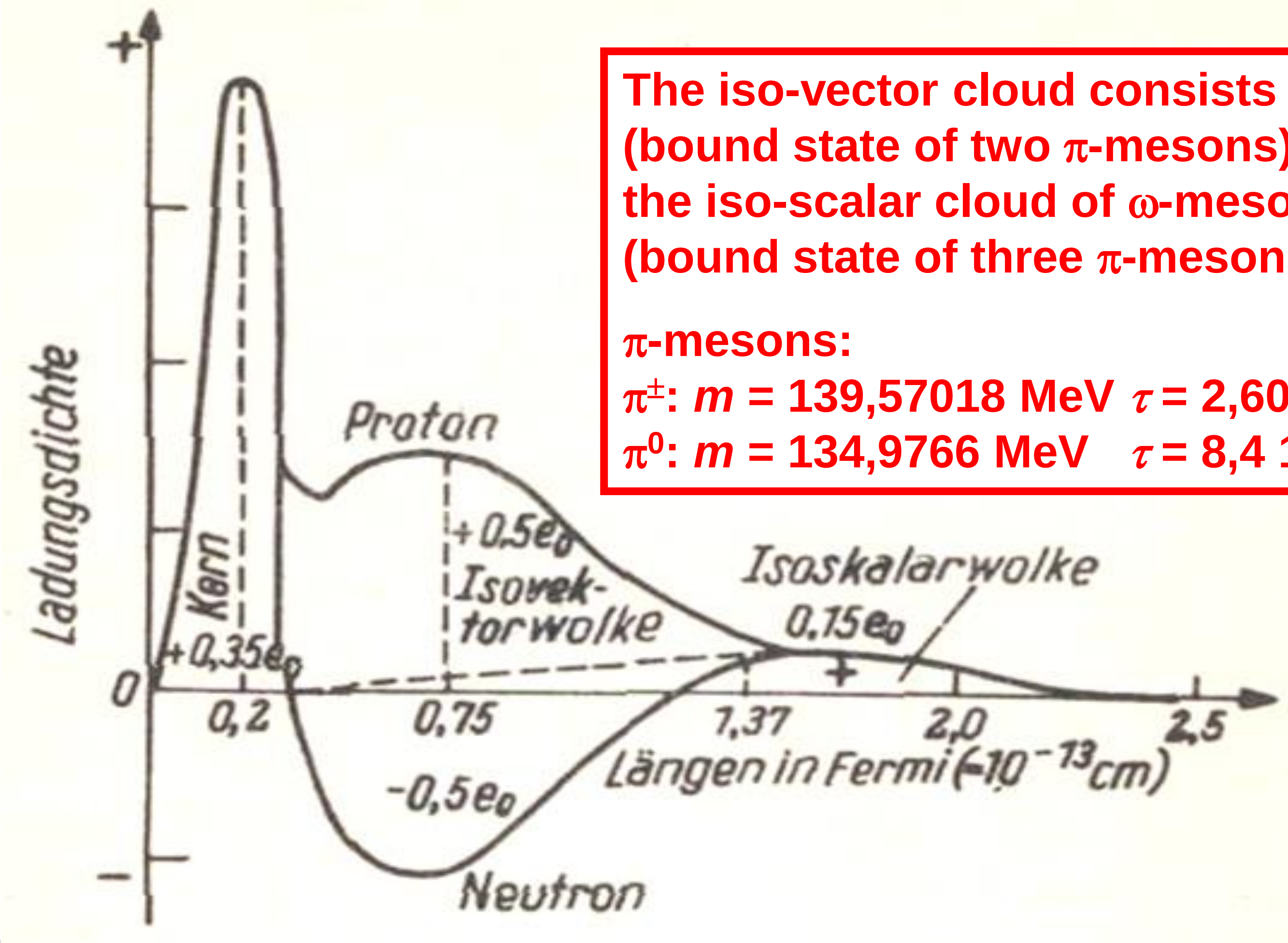
If this be true, it is a most unusual occurrence. The skeleton, which is of modern type, with filed teeth, was found completely articulated down even to the phalanges, and in a position of extraordinary contraction. Complete mammalian skeletons of any age are, as field palæontologists know, of great rarity. When they occur, their perfection can usually be explained as the result of sudden death and immediate covering by volcanic dust. Many of the more or less perfect skeletons which may be seen in museums have been rearticulated from bones found somewhat scattered as the result of death from floods, or in the neighbourhood of drying water-holes. We know of no case of a perfect articulated skeleton being found in company with such broken and scattered remains as appear to be abundant at Oldoway. Either the skeletons are all complete, as in the *Stenomylus* quarry at Sioux City, Nebraska, or are all scattered and broken in various degrees, as in ordinary bone beds. The probability, therefore, that the Oldoway skeleton represents an artificial burial is thus one that will occur to palæontologists.

The skeleton was exhumed in 1913, and published photographs show that the excavation made for its disinterment was extensive. It is, therefore, very difficult to believe that in 1931 there can be reliable evidence left at the site as to the conditions under which it was deposited. If naturally deposited in Bed No. 2, the skeleton is of the highest possible importance, because it would be of pre-Mousterian age, and would be in the company of *Pithecanthropus* and the Piltdown, Heidelberg, and Peking men, all of whose remains are fragmentary to the last degree. Of the few other human remains for which such antiquity is claimed, the Galley Hill skeleton and the Ipswich skeleton are, or apparently were, complete. The first of these was never seen *in situ* by any trained observer, and the latter has, we believe, been withdrawn by its discoverer. The other fragments, found long ago, are entirely without satisfactory evidence as to their mode of occurrence.

Das Neutron

- hat keine Ladung
- hat eine Masse von 939,5 MeV/c^2
- ist nicht stabil, $T_{1/2}$ = 10,6 min
- wechselwirkt hauptsächlich durch die starke Kernkraft
- ist ein Fermion (S=1/2)
- wechselwirkt (wenig) über sein magnetisches Moment.

Das Neutron hat eine Ladungsverteilung im Innern



The iso-vector cloud consists of ρ-mesons (bound state of two π-mesons), the iso-scalar cloud of ω-mesons (bound state of three π-mesons).

π-mesons:
$\pi^{\pm}$: m = 139,57018 MeV τ = 2,6033 10^{-8} s
π^{0}: m = 134,9766 MeV τ = 8,4 10^{-17} s

$$\boxed{\mathbf{x + X \;\rightarrow\; (X+x)^{*} \rightarrow Y + y}}$$

Reaktion: $X(x,y)Y \Leftrightarrow (\alpha,\beta)$

Eingangskanal: $\alpha = X(x,$

Ausgangskanal: $y)Y = \beta$

Kinetische Energie (CMS): $\varepsilon_\alpha, \varepsilon_\beta$

Neutronenquellen

- Radionuklid Quellen: (α,n), Spontanspaltung
- Beschleuniger: (γ,n), $T(d,n)\alpha$, (p,n), (d,n), (d,d), (t,t)
- Spalt- und Fusionsreaktoren: 14,7 MeV & Spaltspektrum
- Spallationsneutronquellen: weißes Spektrum

Neutronendetektion -> NuA

- BF_3
- Rückstoßprotonenmonitor
- Aktivierung
- Emulsionen
- andere

Radionuklid Neutronenquellen

$^{9}Be(\alpha,n)^{12}C$ $Q = 5,7$ MeV

70 Neutronen pro
1Mio ^{241}Am α-Partikel

Charakteristiken von Be(α,n) Neutronenquellen

Source	Half-Life	E_α (MeV)	Neutron Yield per 10^6 Primary Alphas: Calculated	Neutron Yield per 10^6 Primary Alphas: Experimental	Percent of Yield with $E_n < 1.5$ MeV: Calculated	Percent of Yield with $E_n < 1.5$ MeV: Experimental
^{239}Pu/Be	24000 y	5.14	65	57	11	9–33
^{210}Po/Be	138 days	5.30	73	69	13	12
^{238}Pu/Be	87.4 y	5.48	79[b]	—	—	—
^{241}Am/Be	433 y	5.48	82	70	14	15–23
^{244}Cm/Be	18 y	5.79	100[a]	—	18	29
^{242}Cm/Be	162 days	6.10	118	106	22	26
^{226}Ra/Be + daughters	1602 y	multiple	502	—	26	33–38
^{227}Ac/Be + daughters	21.6 y	multiple	702	—	28	38

[a]Does not include a 4 percent contribution from spontaneous fission of ^{244}Cm.
[b]From Anderson and Hertz[14]. All other data as calculated or cited in Geiger and Van der Zwan[15].

Radionuklid Neutronenquellen

$^{9}Be(\alpha,n)^{12}C$	Q = 5,7 MeV	70 Neutronen pro 1Mio ^{241}Am α-Partikel
$^{7}Li(\alpha,n)^{10}B$	Q = - 2,79 MeV	
$^{10}B(\alpha,n)^{13}N$	Q = 1,07 MeV	
$^{11}B(\alpha,n)^{14}N$	Q = 0,158 MeV	

^{210}Po-Be	$T_{1/2}$ = 140 d	E_n = 5,7 MeV (Chadwick)
^{226}Ra-Be	$T_{1/2}$ = 1600 a	$1{,}35 \cdot 10^{7}\ s^{-1}\ (g\ Ra)^{-1}$ starker; γ-Untergrund
^{239}Pu-Be	$T_{1/2}$ = 24 110 a	
^{241}Am-Be	$T_{1/2}$ = 433 a	
^{252}Cf(sf)	$T_{1/2}$ = 2,645 a	3,5 Neutronen pro Spaltung $2{,}7 \cdot 10^{9}\ s^{-1}\ (g\ Cf)^{-1}$

Neutronen-Induzierte Reaktionen

$^{A}Z \rightarrow n_{th},\ n_{epi},\ n_{fast}$

$[^{A+1}Z]^{*}$

$\rightarrow n' \rightarrow {}^{A}Z$

$\rightarrow \gamma \rightarrow {}^{A+1}Z$

$\rightarrow p \rightarrow {}^{A}Z\text{-}1$

$\rightarrow \alpha \rightarrow {}^{A-4}Z\text{-}2$

$\rightarrow 2n \rightarrow {}^{A-1}Z$

$\rightarrow f \rightarrow$ *F.P.*

Neutronen Energien

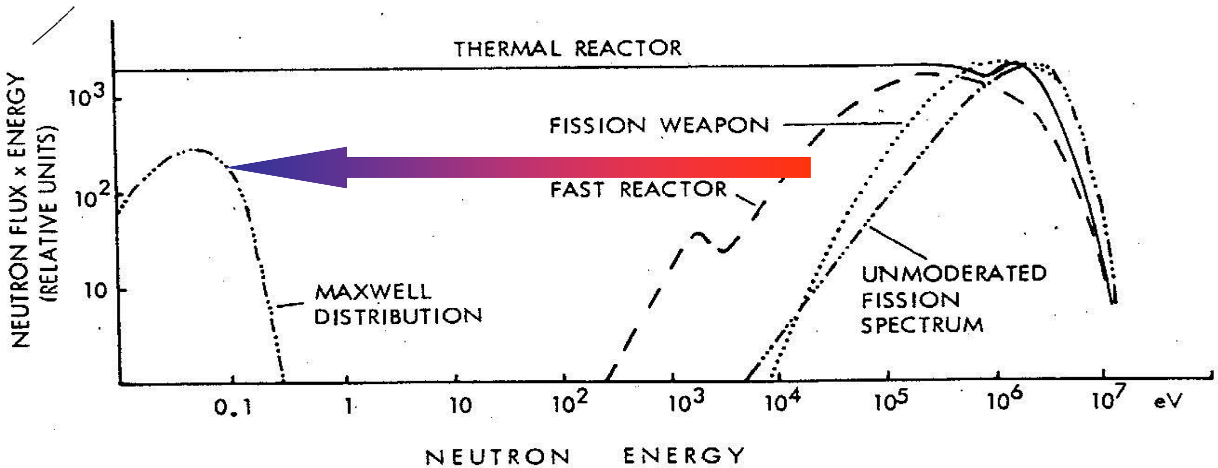
Langsame Neutronen	$E_n < 1$ keV
Mittlere Neutronen	1 keV $< E_n <$ 500 keV
Schnelle Neutronen	500 keV $< E_n <$ 10 MeV
Sehr schnelle Neutronen	10 MeV $< E_n <$ 50 MeV
Mittelschnelle Neutronen	50 MeV $< E_n <$ 10 GeV
Relativistische Neutronen	10 GeV $< E_n$

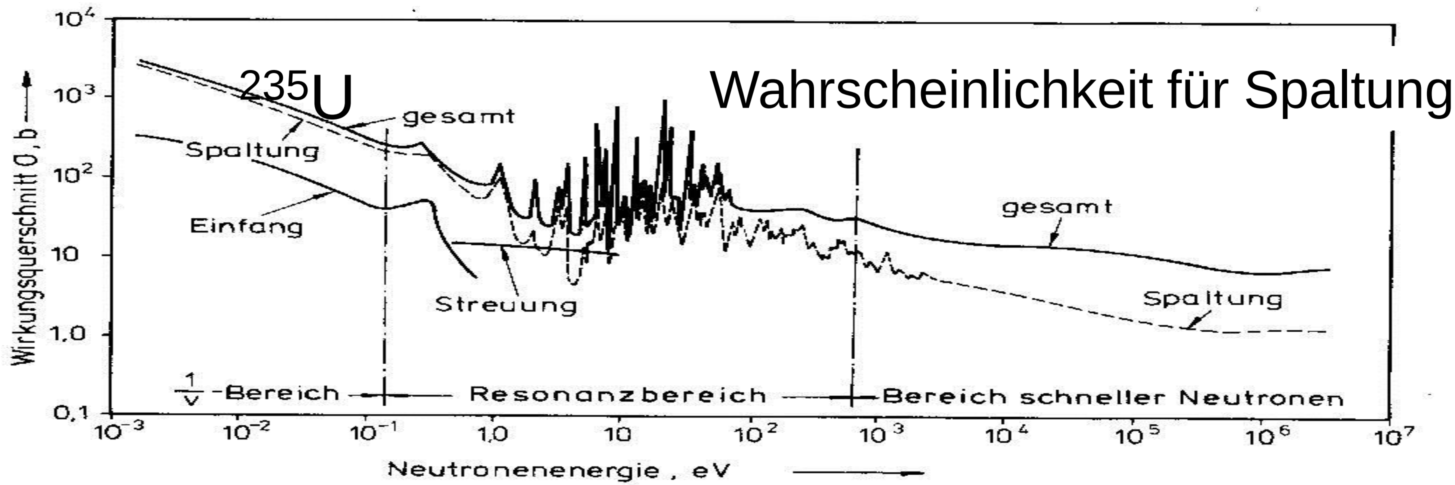
Reaktorphysik:

Kalte Neutronen	0 MeV $< E_n <$ 2 meV
Thermische Neutronen	2 meV $< E_n <$ 0,6 eV
Epithermische Neutronen	0,6 eV $< E_n <$ 1 keV
Schnelle Neutronen	1 keV $< E_n$

$$\boxed{\mathbf{20\ ^oC \cong kT = 1/40\ eV \cong \mathit{v} = 2500\ m\ s^{-1}}}$$

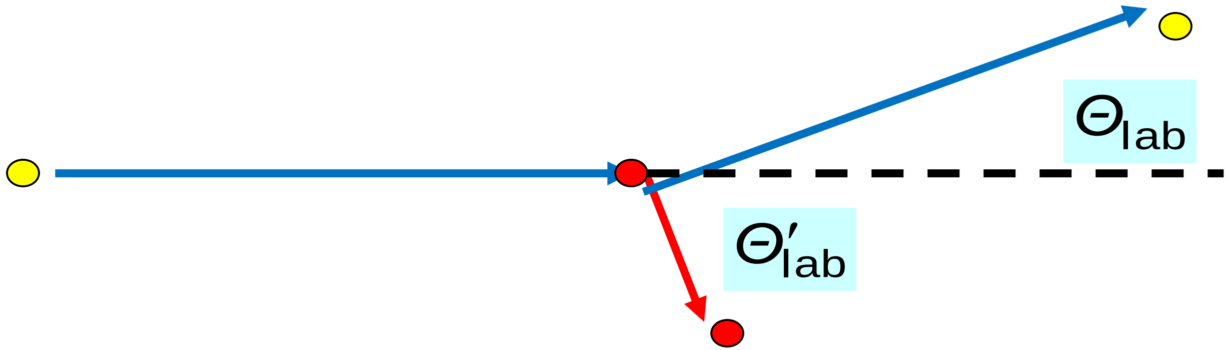
Neutronenenergie





Neutronenmoderation I

Abbremsen von Neutronen durch elastische Stöße mit Atomkernen



Labor System

- Neutron mit Masse m = 1; v_0 vor; v nach Stoß
- Kern mit Masse M = A; V_0 = 0 vor; V nach Stoß

wir lassen jetzt
ca 50 Folien weg
☺

Einige Abbremsparameter oder: was ist der beste Moderator ?

	H	D	He	C	O	U
A	1	2	4	12	16	238
α	0	0,111	0,360	0,716	0,778	0,983
ξ	1,0	0,725	0,425	0,158	0,120	0,00838
$\xi \cdot \Sigma_s / \Sigma_a$	*75*	9300	-	142	265	
n (2 MeV → 0,025 eV)	18	25	43	114	150	2172

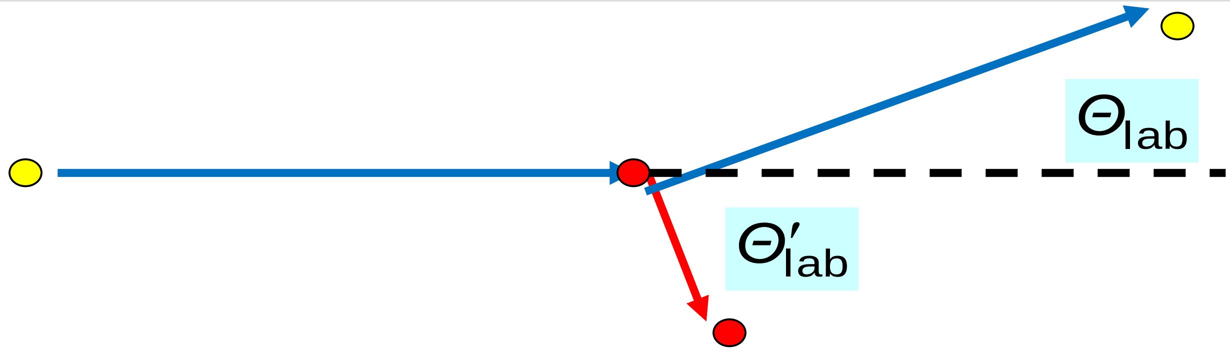
$$n \cdot \xi = \ln \frac{E_0}{E} \qquad n = \frac{\ln(E_0 / E)}{\xi}$$

$\xi \cdot \Sigma_s / \Sigma_a$ **Moderatorverhältnis / Abbremsverhältnis**

$\xi \cdot \Sigma_s$ **Abbremsstärke**

Wer versteht das alles?

Na, dann eben doch die restlichen Folien



Labor System

- Neutron mit Masse $m = 1$; vor Stoß v_0 nach Stoß v
- Kern mit Masse $M = A$; vor Stoß $V_0 = 0$ nach Stoß V

Energie
$$m \cdot v_0^2 = M \cdot V^2 + m \cdot v^2$$

Impuls
$$m \cdot v_0 = M \cdot V \cdot \cos\Theta'_{lab} + m \cdot v \cdot \cos\Theta_{lab}$$
$$0 = M \cdot V \cdot \sin\Theta'_{lab} + m \cdot v \cdot \sin\Theta_{lab}$$

Energie $m \cdot v_0^2 = M \cdot V^2 + m \cdot v^2$

Impuls $m \cdot v_0 = M \cdot V \cdot \cos\Theta'_{lab} + m \cdot v \cdot \cos\Theta$

$0 = M \cdot V \cdot$

Neutron mit M … Stoß v

Kern mit M … oß V

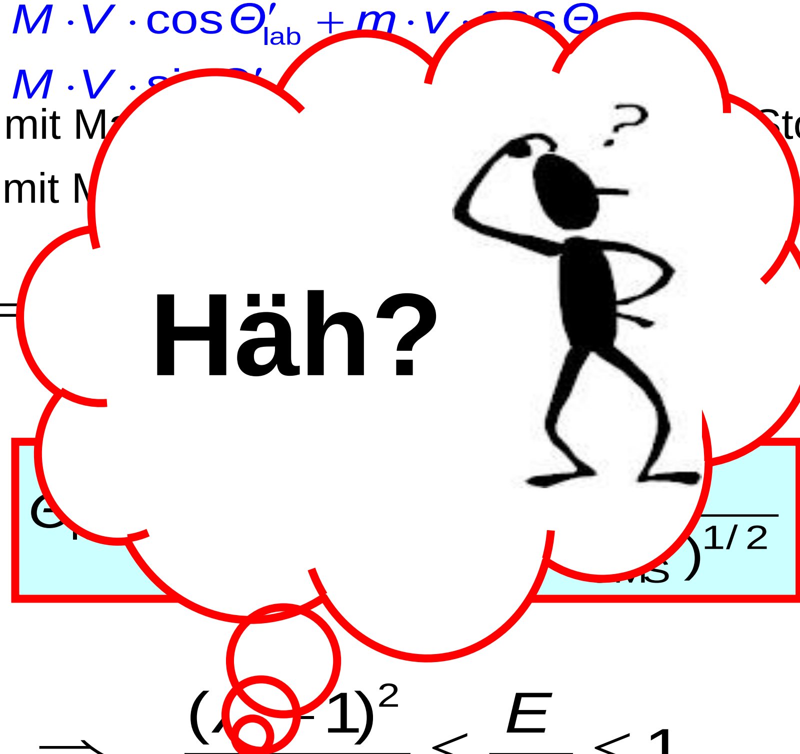
Mit $\mu = \cos\Theta_{lab}$ $M = A$ $m =$

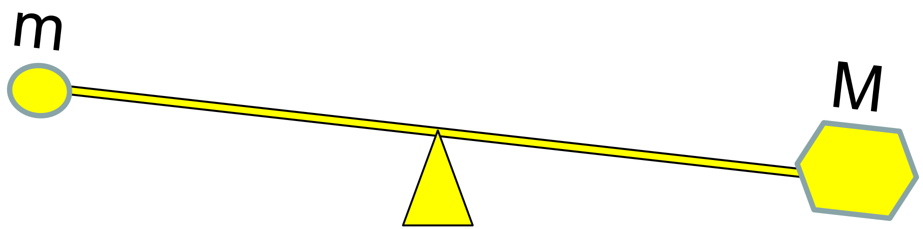
Energie E nach dem Stoß:

$$\frac{E}{E_0} = \frac{\left[(A^2 - 1 + \mu^2)^{1/2} + \mu\right]^2}{(A+1)^2}$$

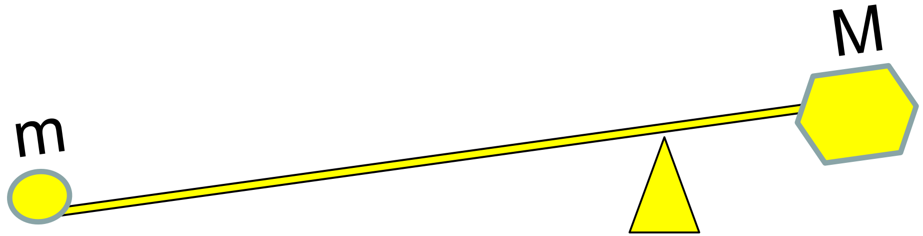
Θ … $)^{1/2}$

$$\frac{E}{E_0} = \frac{\left[A^2 + 1 + 2A \cdot \cos\Theta_{CMS}\right]}{(A+1)^2} \Rightarrow \frac{(A-1)^2}{(A+1)^2} \le \frac{E}{E_0} \le 1$$

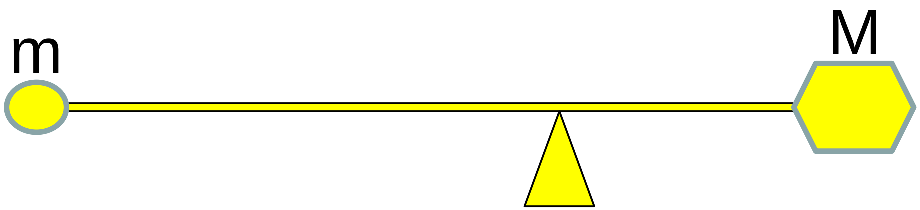



m
M

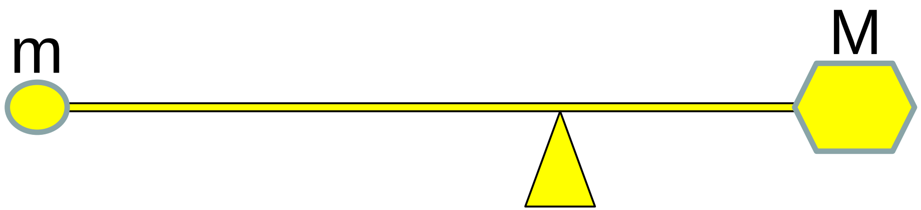
Center of Mass (Schwerpunktsystem)



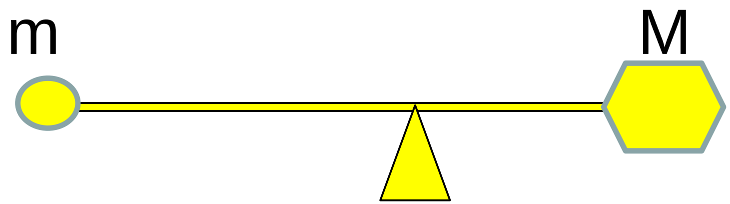
Center of Mass (Schwerpunktsystem)



$$\frac{m}{M+m}$$


m
M

Center of Mass (Schwerpunktsystem)



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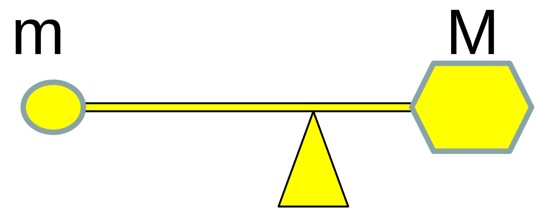
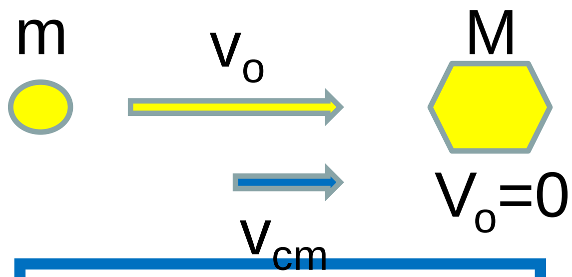
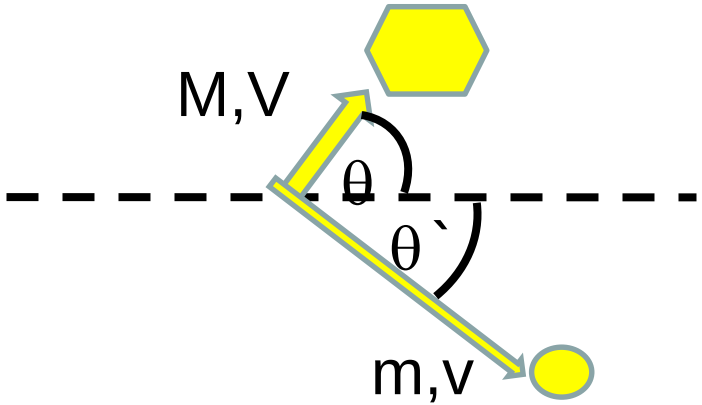


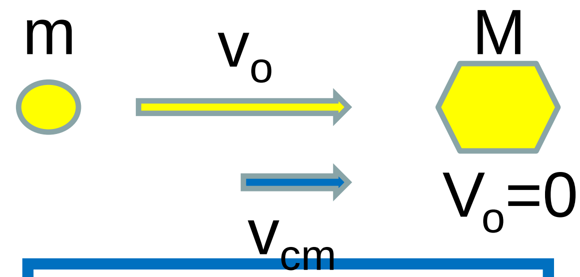
Stoß im Schwerpunktsystem (CMS)

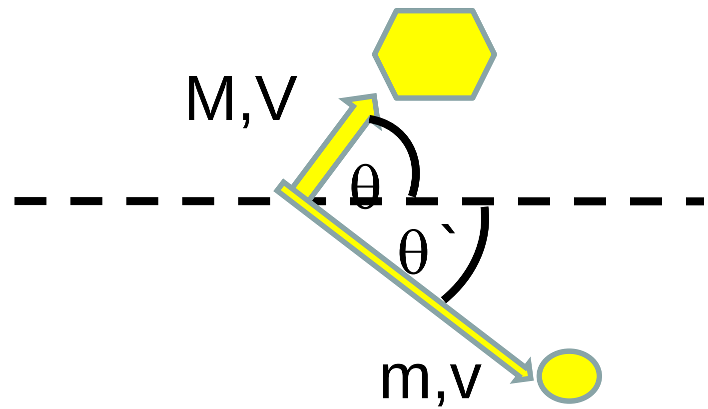


$$c.m.: v_{cm} = \frac{m}{M+m} v_o$$

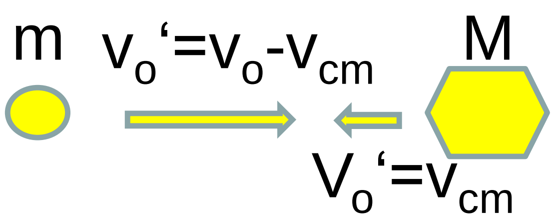


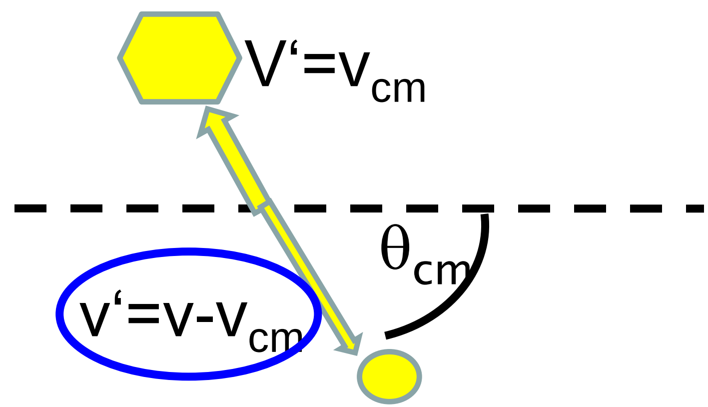
Stoß im Schwerpunktsystem (CMS)





$$c.m.: v_{cm} = \frac{m}{M+m} v_0$$





$$E_{kin} = \tfrac{1}{2} m v^2 = \tfrac{1}{2} m (v' + v_{cm})^2 = \tfrac{1}{2} m (v'^2 + v_{cm}^2 + 2 v' v_{cm} \cos\theta_{cm})$$

Stoß im Schwerpunktsystem (CMS) II

$$E_{kin} = \tfrac{1}{2} m v^2 = \tfrac{1}{2} m (v' + v_{cm})^2 = \tfrac{1}{2} m (v'^2 + v_{cm}^2 + 2v' v_{cm} \cos\theta_{cm})$$

$$E_{kin}(\max) = \tfrac{1}{2} m (v'^2 + v_{cm}^2 + 2v' v_{cm}) = \tfrac{1}{2} m v_0^2$$

Wegen: v'=v-v_{cm}

$$E_{kin}(\min) = \tfrac{1}{2} m (v'^2 + v_{cm}^2 - 2v' v_{cm}) = \tfrac{1}{2} m (v' - v_{cm})^2$$

$$= \tfrac{1}{2} m (v_0 - 2v_{cm})^2$$

$$= \tfrac{1}{2} m {v_0}^2 \left(\frac{M - m}{M + m} \right)^2$$

$$\boxed{c.m.: v_{cm} = \frac{m}{M + m} v_0}$$

$$\boxed{\begin{matrix} M = A \\ m = 1 \end{matrix} \quad \Rightarrow \quad \left(\frac{A - 1}{A + 1} \right)^2}$$

AO

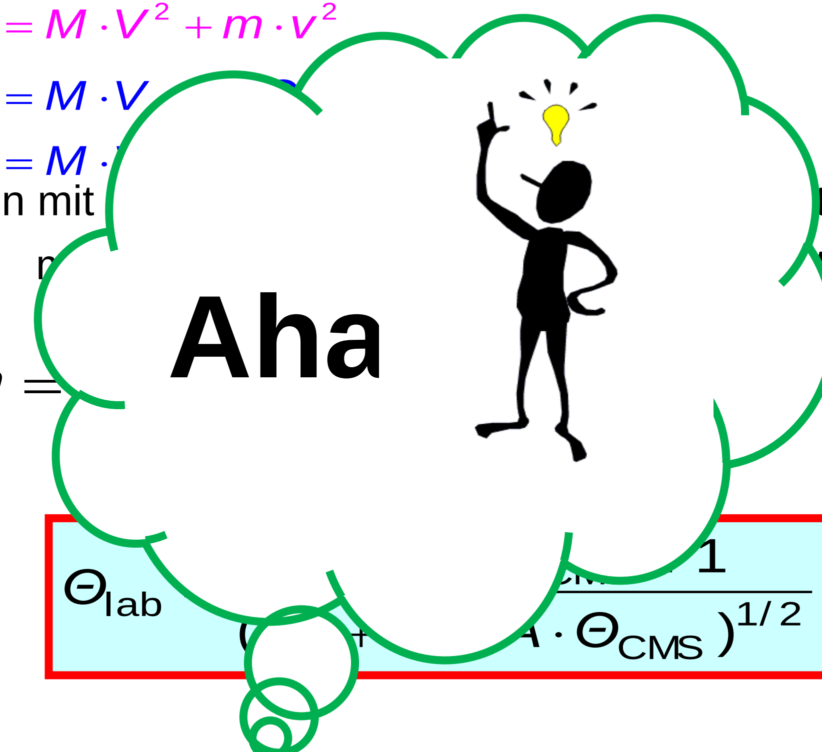
Energie $m\cdot v_0^2 = M\cdot V^2 + m\cdot v^2$

Impuls $m\cdot v_0 = M\cdot V$

$0 = M\cdot$

Neutron mit … toß v

Kern m … toß V



Mit $\mu = \cos\Theta_{\mathrm{lab}}$ $M = A$ $m =$

Energie E nach dem Stoß:

$$\frac{E}{E_0} = \frac{\left[(A^2 - 1 + \mu^2)^{1/2} + \mu\right]^2}{(A+1)^2}$$

Θ_{lab} … 1 … $\cdot\,\Theta_{\mathrm{CMS}})^{1/2}$

$$\frac{E}{E_0} = \frac{\left[A^2 + 1 + 2A\cdot\cos\Theta_{\mathrm{CMS}}\right]}{(A+1)^2} \quad \Rightarrow \quad \frac{(A-1)^2}{(A+1)^2} \le \frac{E}{E_0} \le 1$$

Neutronen Moderation III

Für **s-Wellen**, ist elastische Streuung im **Schwerpunktsystem (CMS) isotrop.** Höhere Drehimpulse werden bei hoher Neutronenenergie wichtig.

de Broglie Wellenlänge

$$\lambda = \frac{h}{m \cdot v} = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{4{,}05 \cdot 10^{-9}}{\sqrt{2mE}}\ \text{cm}$$

m in amu

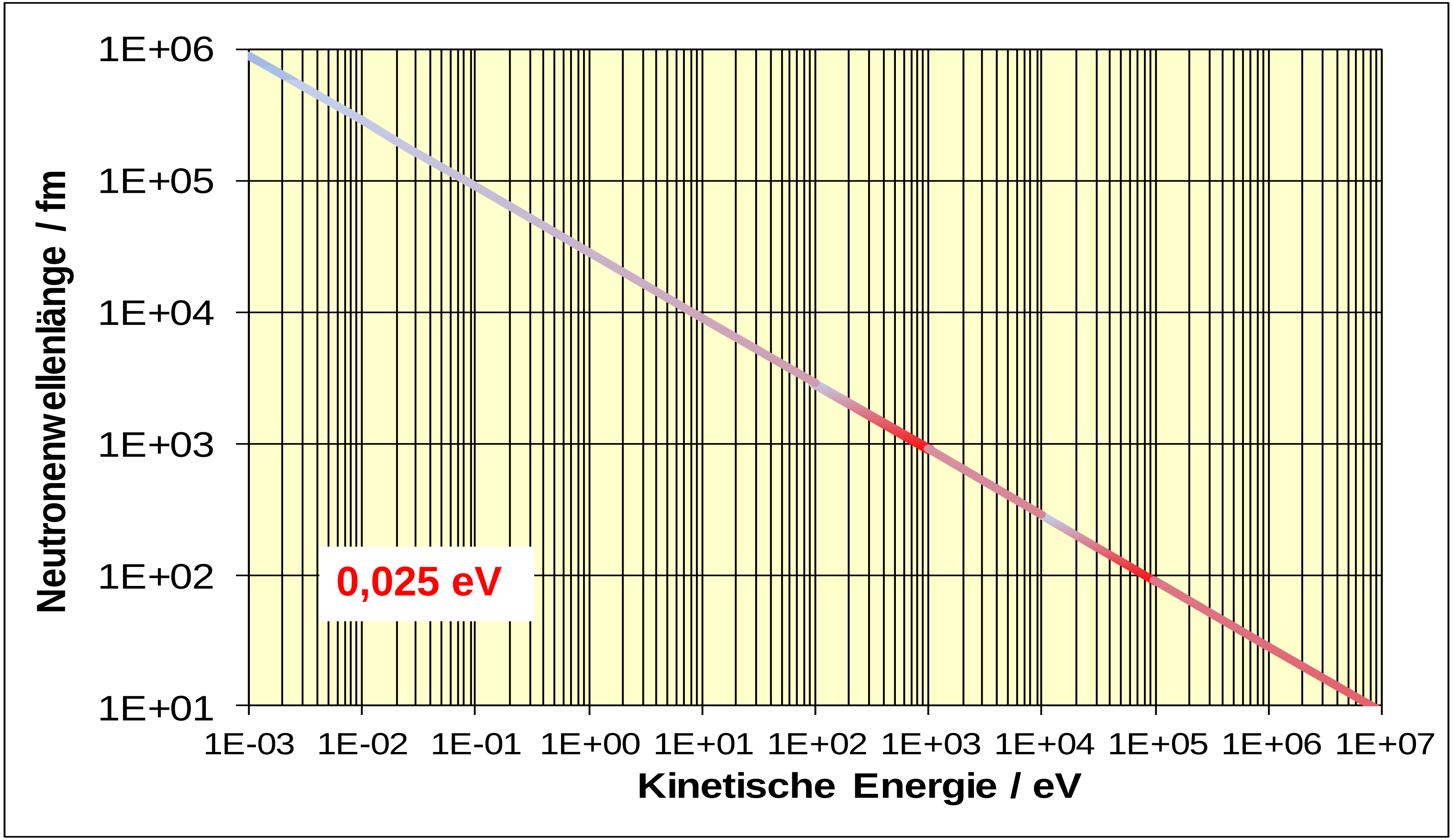
E in eV

Im Falle von Neutronen:

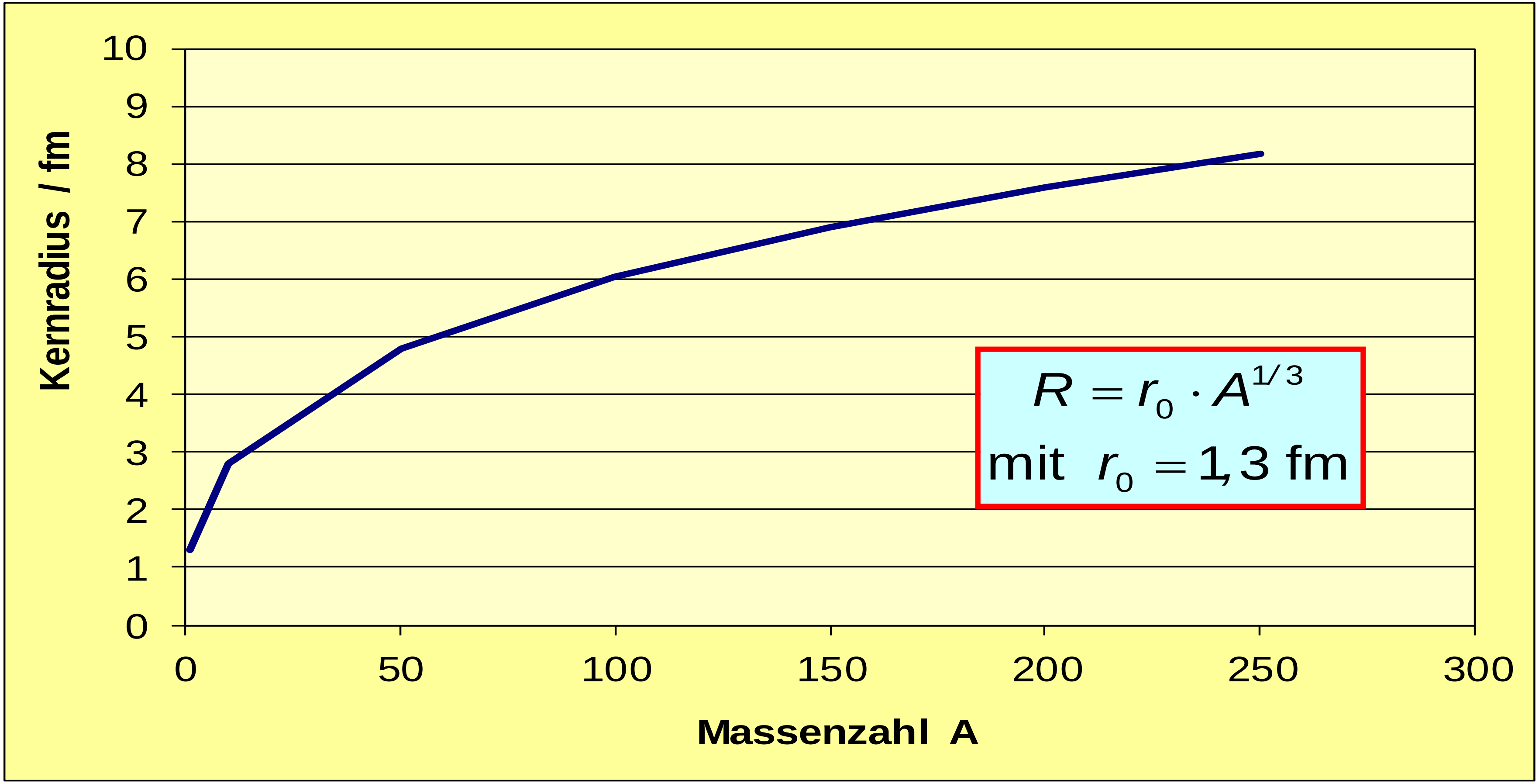
$$\lambda = \frac{2{,}86 \cdot 10^{-9}}{\sqrt{E}}\ \text{cm} \quad \text{für } E_{\text{n}} \text{ in eV}$$

$$E_{\text{n}} = 0{,}025\ \text{eV}: \lambda = 1{,}7 \cdot 10^{-8}\ \text{cm}$$

De Broglie Wellenlänge für Neutronen



Kernradii



Kernradius / fm
Massenzahl A
0
1
2
3
4
5
6
7
8
9
10
0
50
100
150
200
250
300
$R = r_0 \cdot A^{1/3}$
mit $r_0 = 1{,}3$ fm

- **Wissen Sie was eine Besselfunktion ist?**

Ja ✓ **Nein** ✗

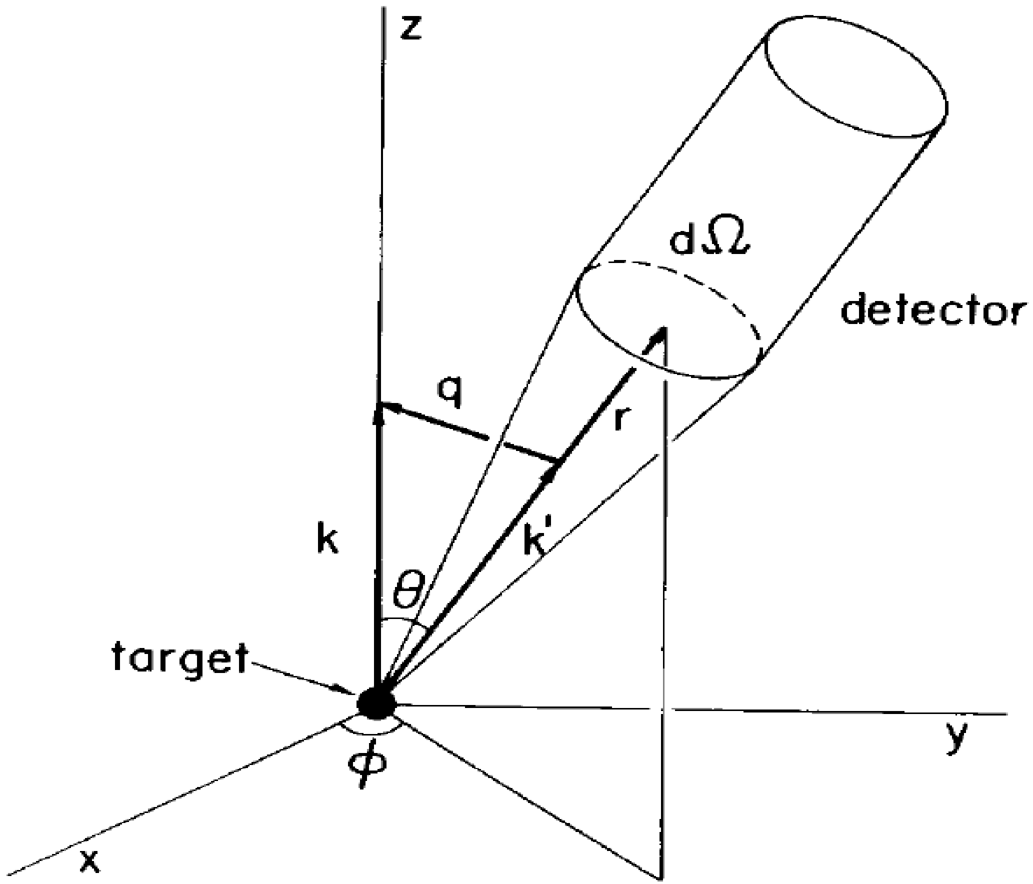
SMART Response Question

To set the properties right click and select
SMART Response Question Object->Properties...

$$dx\,dy\,dz = r^2 dr\, d\phi \sin\theta\, d\theta$$

$$\rightarrow d\omega = 2\pi \sin\theta\, d\theta$$

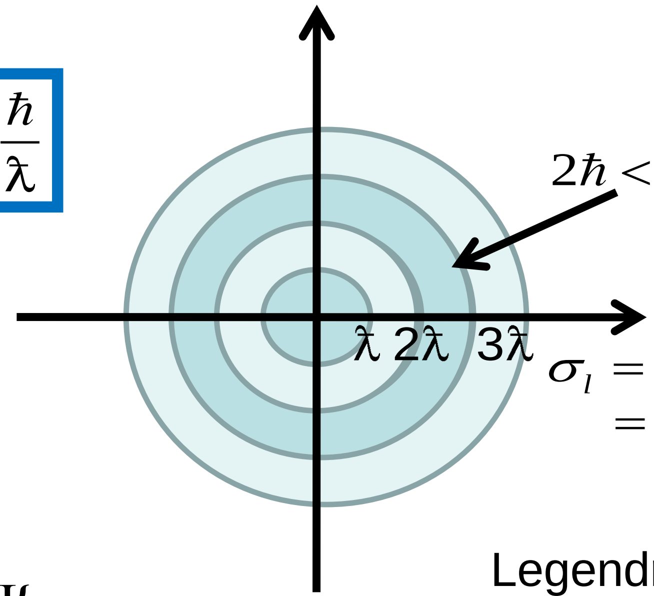
Streuung



s-Wellen Streuung

Einfallendes Neutron:
Ebene Welle

$$\boxed{p = \frac{\hbar}{\lambdabar}}$$



$2\hbar < l < 3\hbar$

$$\sigma_l = (l+1)^2 \lambdabar^2 \pi - l^2 \lambdabar^2 \pi$$
$$= (2l+1)\lambdabar^2 \pi$$

$$SEq.: \quad E\Psi = \left[\frac{-\hbar^2}{2m} \Delta + V(r) \right] \Psi$$

Legendre Polynome
Eigenfunktionen zum
Bahndrehimpuls l

$$\Psi(\vec{r}) = R(r) Y_{l,m}(\vartheta, \phi) = R(r) c_{l,m} P_l^m \cos(\theta) e^{\pm im\varphi}$$

Kugelflächenfunktionen

$$SEq.: \quad E\Psi = \left[\frac{-\hbar^2}{2m}\Delta + V(r) \right]\Psi$$

$$u(r) = rR(r)$$

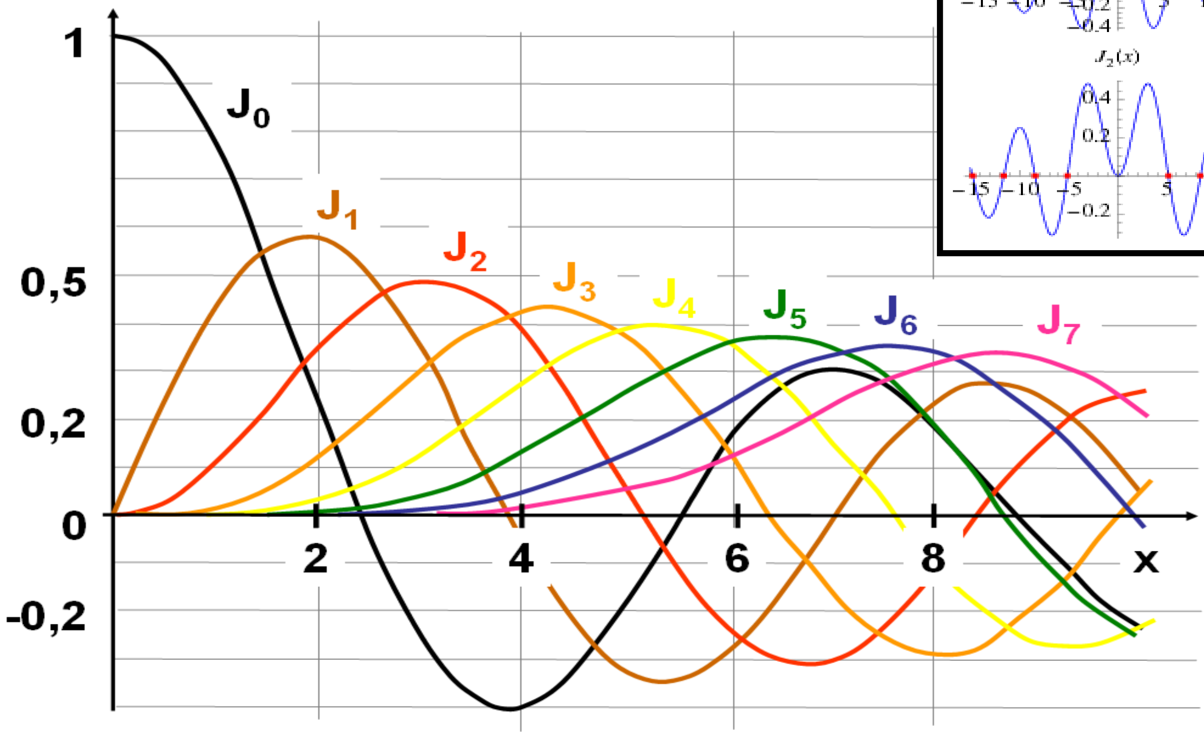
$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}\left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2} \right]u = 0$$

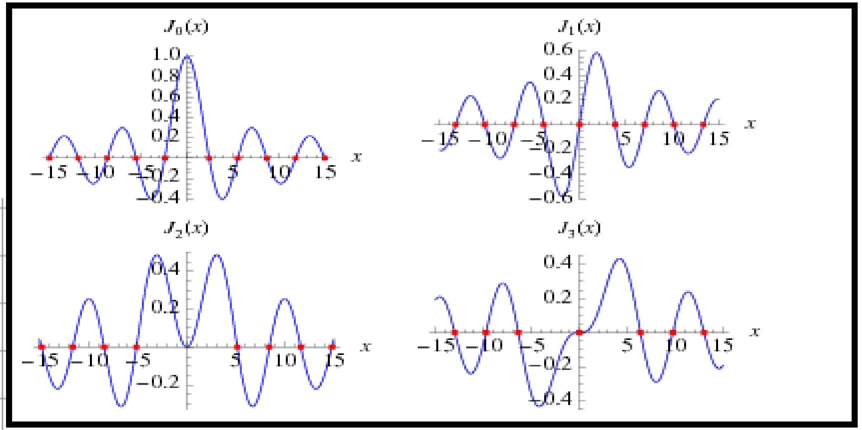
$$\frac{d^2u}{dr^2} + \left[k^2 - V(r)\frac{2m}{\hbar^2} - \frac{l(l+1)}{r^2} \right]u = 0 \quad ; E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$Lsg.: u_l(kr) = rj_l(kr)$$

Sphärische Besselfunktionen

Besselfunktionen





Ebene Welle nach Eigenfunktionen des Drehimpulses zerlegt

$$e^{ikz} = e^{ikr\cos\theta} = \sum_{l=0}^{\infty} (2l+1) i^l j_l(kr) P_l(\cos\theta)$$

$$j_l(kr) \rightarrow \frac{\sin\left(kr - \frac{1}{2} l\pi\right)}{kr} \text{für} \quad kr >> l$$

$$j_l(kr) \rightarrow \frac{i}{2kr}\left[\exp\left(-i\left(kr - \frac{1}{2} l\pi\right)\right) - \exp\left(i\left(kr - \frac{1}{2} l\pi\right)\right)\right]$$

$$\Psi_e = e^{ikz} = \frac{1}{2kr}\sum_{l=0}^{\infty} (2l+1) i^{l+1}\left[\exp\left(-i\left(kr - \frac{1}{2} l\pi\right)\right) - \exp\left(i\left(kr - \frac{1}{2} l\pi\right)\right)\right] P_l(\cos\theta)$$

$$\Psi_e = e^{ikz} = \frac{1}{2kr}\sum_{l=0}^{\infty}(2l+1)i^{l+1}\left[\exp\left(-i\left(kr-\frac{1}{2}l\pi\right)\right)-\exp\left(i\left(kr-\frac{1}{2}l\pi\right)\right)\right]P_l(\cos\theta)$$

$$\Psi_T = e^{ikz} = \frac{1}{2kr}\sum_{l=0}^{\infty}(2l+1)i^{l+1}\left[\exp\left(-i\left(kr-\frac{1}{2}l\pi\right)\right)-\eta_l\exp\left(i\left(kr-\frac{1}{2}l\pi\right)\right)\right]P_l(\cos\theta)$$

$$\Psi_T = \Psi_e + \Psi_{Str} \quad mit \quad \Psi_{Str} = f(\Theta)\frac{e^{ikr}}{r}$$

$$\Psi_{Str} = \Psi_T - \Psi_e$$

$$f(\theta) = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)(1-\eta_l)\ P_l(\cos\theta)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_\Theta = f^*(\theta)f(\theta) = \frac{1}{4k^2}\left|\sum_{l=0}^{\infty}(2l+1)(1-\eta_l)P_l(\cos\theta)\right|^2$$

$$wegen: \quad \int P_l(\cos\theta)P_{l'}(\cos\theta)d\Omega = \frac{4\pi}{2l+1}\delta_{l,l'}$$

$$folgt: \quad \sigma_{s,l} = (\pi/k^2)\sum_l(2l+1)\left|1-\eta_l\right|^2$$

Für **s-Wellen**, ist elastische Streuung im **Schwerpunktsystem (CMS) isotrop.** Höhere Drehimpulse werden bei hoher Neutronenenergie wichtig.

Für s-Wellen ist die Wahrscheinlichkeit dW im CMS in den Winkel $d\omega$ gestreut zu werden:

$$\frac{dW}{4\pi} = \frac{\sin\Theta_{CMS}\, d\Theta_{CMS}}{2} = -\frac{d(\cos\Theta_{CMS})}{2}$$

$$dxdydz = r^2 dr\, d\phi\, \sin\theta\, d\theta$$

$$\rightarrow d\omega = 2\pi \sin\theta\, d\theta$$

Neutron Moderation III

Für **s-Wellen**, ist elastische Streuung im **Schwerpunktsystem (CMS) isotrop.** Höhere Drehimpulse werden bei hoher Neutronenenergie wichtig.

Für s-Wellen ist die Wahrscheinlichkeit dW im CMS in den Winkel dω gestreut zu werden:

$$\frac{dW}{4\pi} = \frac{\sin\Theta_{CMS}\, d\Theta_{CMS}}{2} = -\frac{d(\cos\Theta_{CMS})}{2}$$

Die Wahrscheinlichkeit, nach dem Stoß ein bestimmtes Intevall $\cos\Theta_{CMS}$ zu erreichen hängt nicht von Θ_{CMS} ab..

$$\frac{E}{E_0} = \frac{\left[A^2 + 1 + 2A \cdot \cos\Theta_{CMS}\right]}{(A+1)^2} \Rightarrow \frac{dE}{E_0} = \frac{2A}{(A+1)^2} d\cos\Theta_{CMS}$$

Neutronen Moderation IV

Beim zentralen Stoß ist der maximale Energieverlust:

$$\left[\frac{E}{E_0}\right]_{\max} = \frac{(A-1)^2}{(A+1)^2} = \alpha$$

Θ und $\mu = \cos\theta$ folgen den Gesetzen für Streuung: isotrop im CMS System

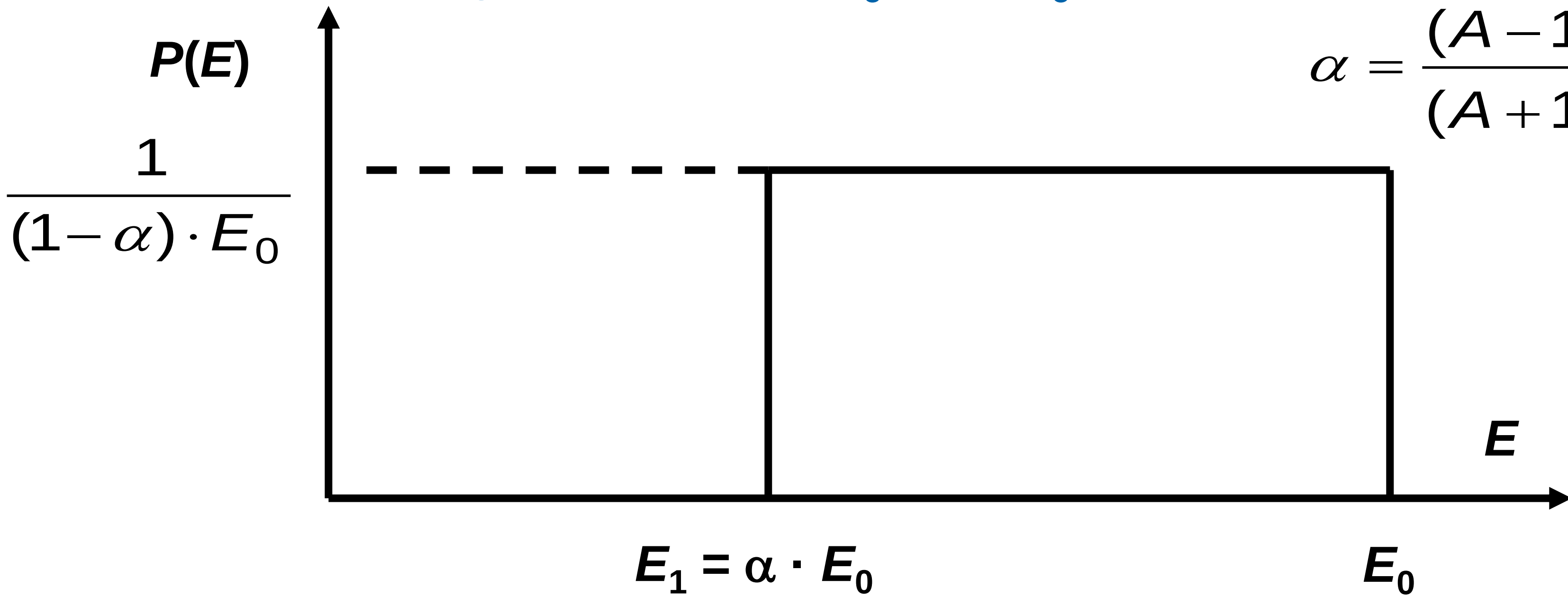
Warum?

Dann ist das mittlere logarithmische Energie Dekrement gegeben durch:

$$\bar{\xi} = \overline{\ln\frac{E_0}{E_1}} = 1 + \frac{(A-1)^2}{2A} \cdot \ln\frac{A-1}{A+1}$$

Konstante Wahrscheinlichkeit, nach einem Stoß die Energie E zu erreichen im Energieintervall [E_0, $\alpha \cdot E_0$]

$$\alpha = \frac{(A-1)^2}{(A+1)^2}$$



Für ^1H wird jede Energie zwischen E_0 und 0 mit gleicher Wahrscheinlichkeit erreicht:

$$\Rightarrow \qquad < E_n > = \frac{1}{2^n} \cdot E_0$$

Lethargie *u*

Mittlerer Wert von *u* nach einem Stoß: $\bar{\xi} = <u>$ $\quad u = \log\frac{E_0}{E}$

$$\bar{\xi} = <u> = \frac{\int_{\alpha\cdot E_0}^{E_0} \log\frac{E_0}{E_1}\cdot\frac{dW_1}{dE_1}\,dE_1}{\int_{\alpha\cdot E_0}^{E_0}\frac{dW_1}{dE_1}\,dE_1} = \frac{(A+1)^2}{4A\cdot E_0}\int_{\alpha\cdot E_0}^{E_0}\log\frac{E_0}{E_1}\,dE_1$$

$$= 1 + \frac{(A-1)^2}{2A}\log\frac{A-1}{A+1}$$

In Reaktorphysik: Konvention E_0=10 MeV.

Einige Moderator Parameter

	H	D	He	C	O	U
A	1	2	4	12	16	238
α	0	0,111	0,360	0,716	0,778	0,983
ξ	1,0	0,725	0,425	0,158	0,120	0,00838

$$n \cdot \xi = \ln \frac{E_0}{E}$$

Welcher Moderator ?

	H_2O	D_2O	He	C	O	U
Σ_C [b]	**0.66**	**0.00092**		**0.0045**		
$\Sigma_s^{epitherm}$ [b]	49	**10.6**		**5.9**		

Einige Moderator Parameter

	H	D	He	C	O	U
A	1	2	4	12	16	238
α	0	0,111	0,360	0,716	0,778	0,983
ξ	1,0	0,725	0,425	0,158	0,120	0,00838
$\xi \cdot \Sigma_s / \Sigma_a$	*75*	9300	-	142	265	
n (2 MeV $\rightarrow$ 0,025 eV)	18	25	43	114	150	2172

$$n \cdot \xi = \ln \frac{E_0}{E} \qquad n = \frac{\ln(E_0 / E)}{\xi}$$

$\xi \cdot \Sigma_s / \Sigma_a$ Moderator Verhältnis, Bremsverhältnis

$\xi \cdot \Sigma_s$ „slowing down power“

Neutronen Moderation VI

Um ein Neutron von der Energie E_0 zur finalen Energie E_n abzubremsen, werden im Mittel n Stöße benötigt

$$n = \frac{\ln(E_0 / E_n)}{\overline{\xi}}$$

Das bedeutet: Auf einer log Skala bewegt sich das Neutron im Mittel bei jedem Stoß um das selbe Intervall zu niedrigeren Energien mit dem logarithmischen Energiedekrement

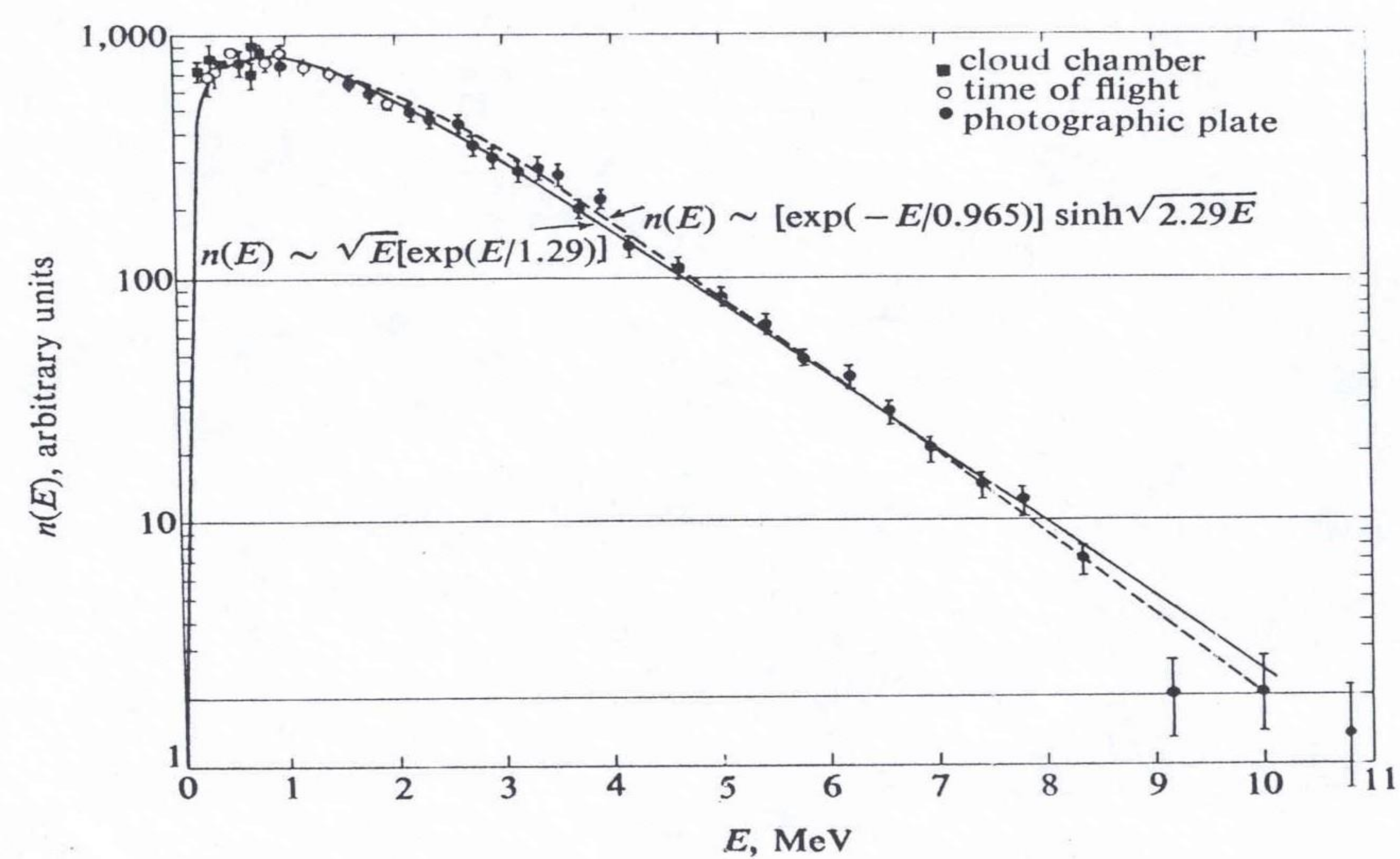
$$< \log E > = \log E_0 - n \cdot \overline{\xi}$$

Die Moderation endet, wenn das Neutron mit dem Moderator im thermischen Gleichgewicht ist. Die Energieverteilung der Neutronen folgt einer Maxwell-Boltzmann Verteilung:

$$n(E)\,\mathrm{d}E = \frac{2\pi}{(\pi k T)^{3/2}} n_0 \cdot e^{-E/kT} \cdot \sqrt{E}\,\mathrm{d}E$$

n_0 Gesamtzahl der Neutronen

Spaltneutronenspektrum von U-235

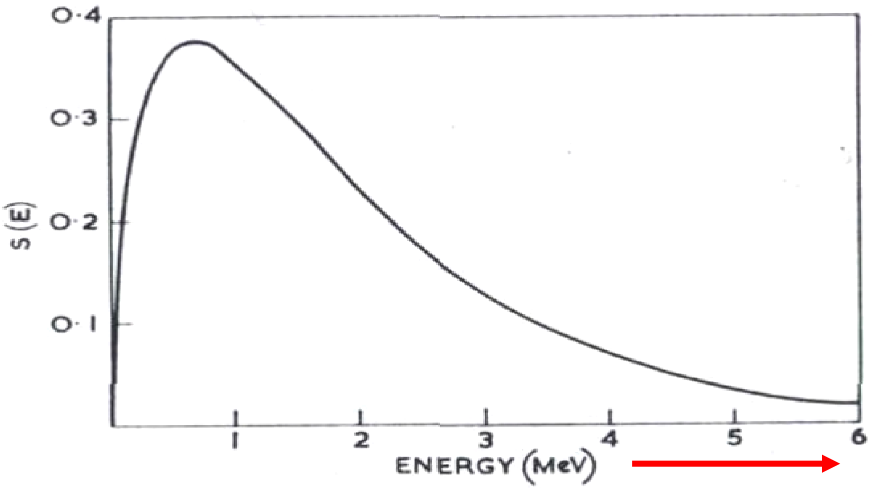


Reaktor Neutronenspektrum

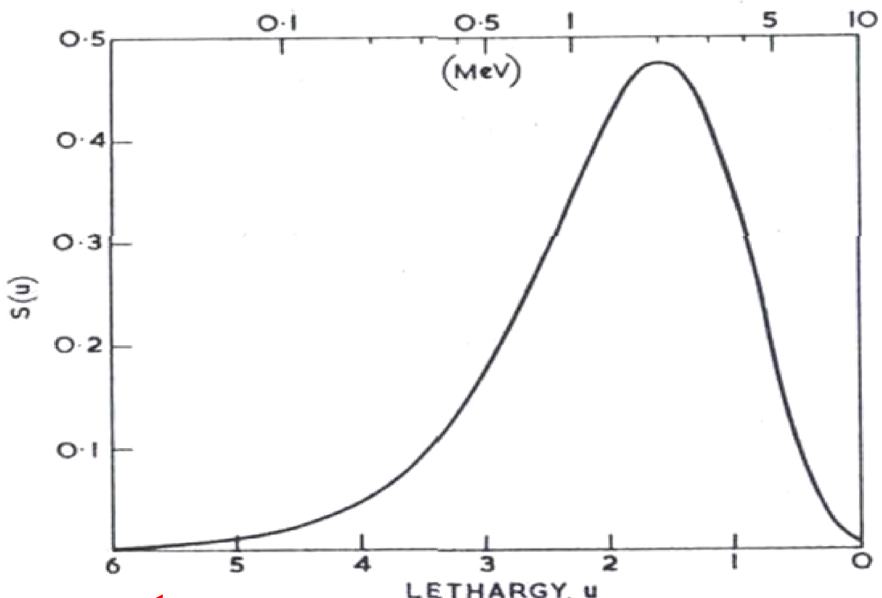
E_n > 0,5 MeV: $$\Phi(E) \propto e^{-E} \cdot \sinh(\sqrt{E})$$

0,2 eV < E_n < 0,5 MeV: $$\Phi(E)\,\mathrm{d}E = \frac{\varphi_{\mathrm{epi}}}{E}\,\mathrm{d}E$$

E_n < 0,2 eV: $$\Phi(E)\,\mathrm{d}E = \Phi_0 \cdot e^{-E/kT} \cdot \sqrt{E}\,\mathrm{d}E$$



a) ENERGY SPACE



b) LETHARGY SPACE

Spalt Neutronen Spektrum

im Energie

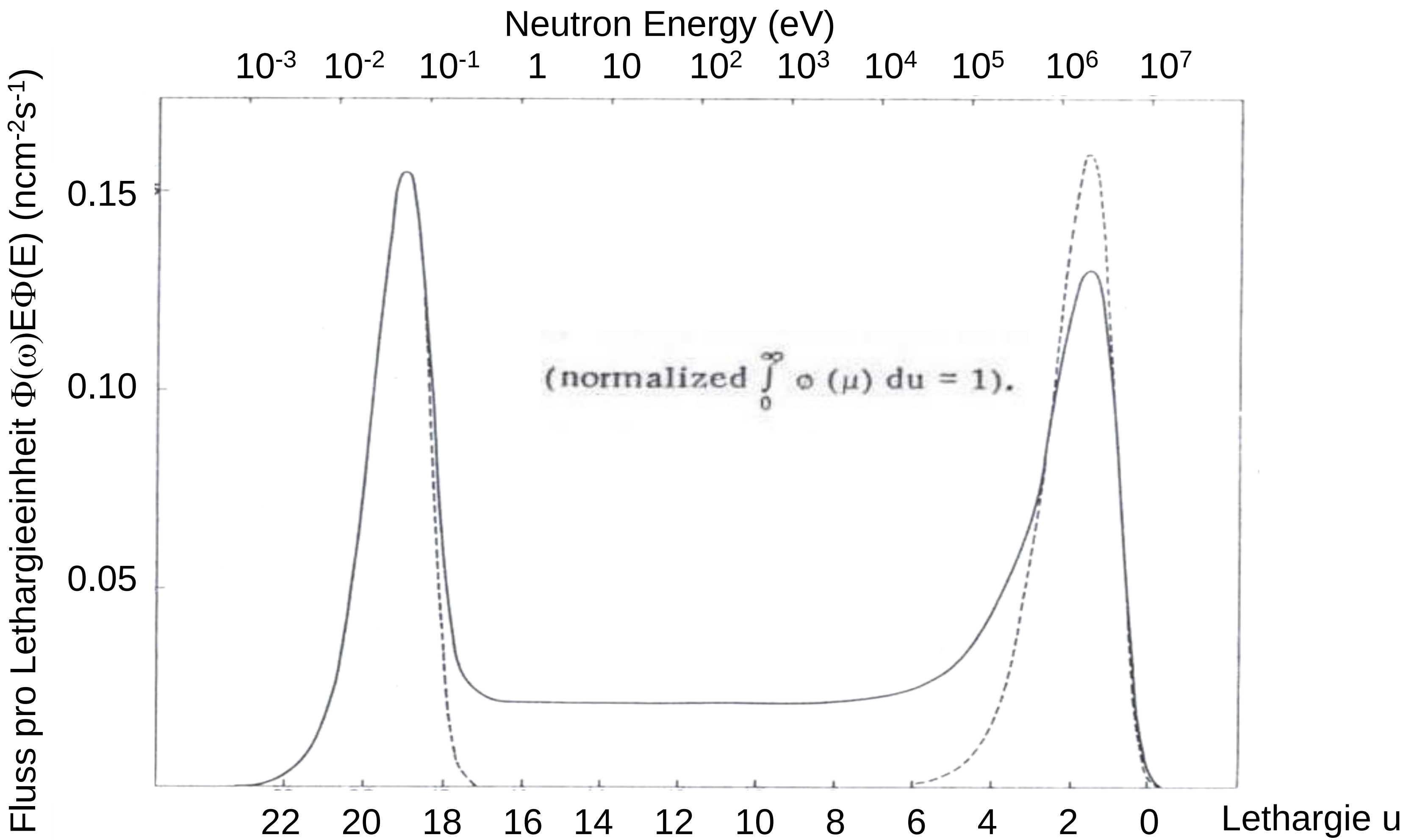
und Lethargie Raum

Lethargie *u*

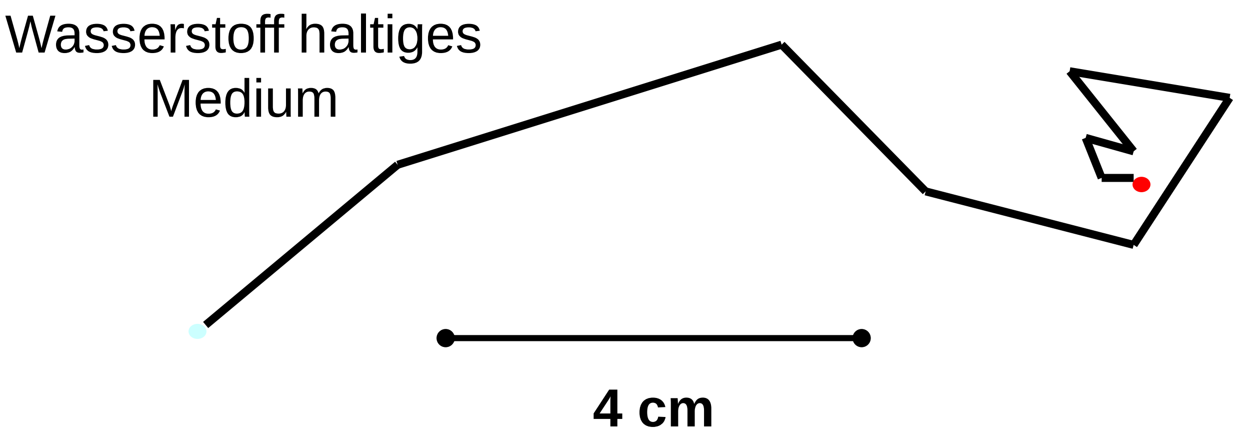
$$u = \ln (E_0/E)$$

$$E_0 = 10\ \mathrm{MeV}$$

Typisches Neutronenspektrum eines Leichtwasserreaktors

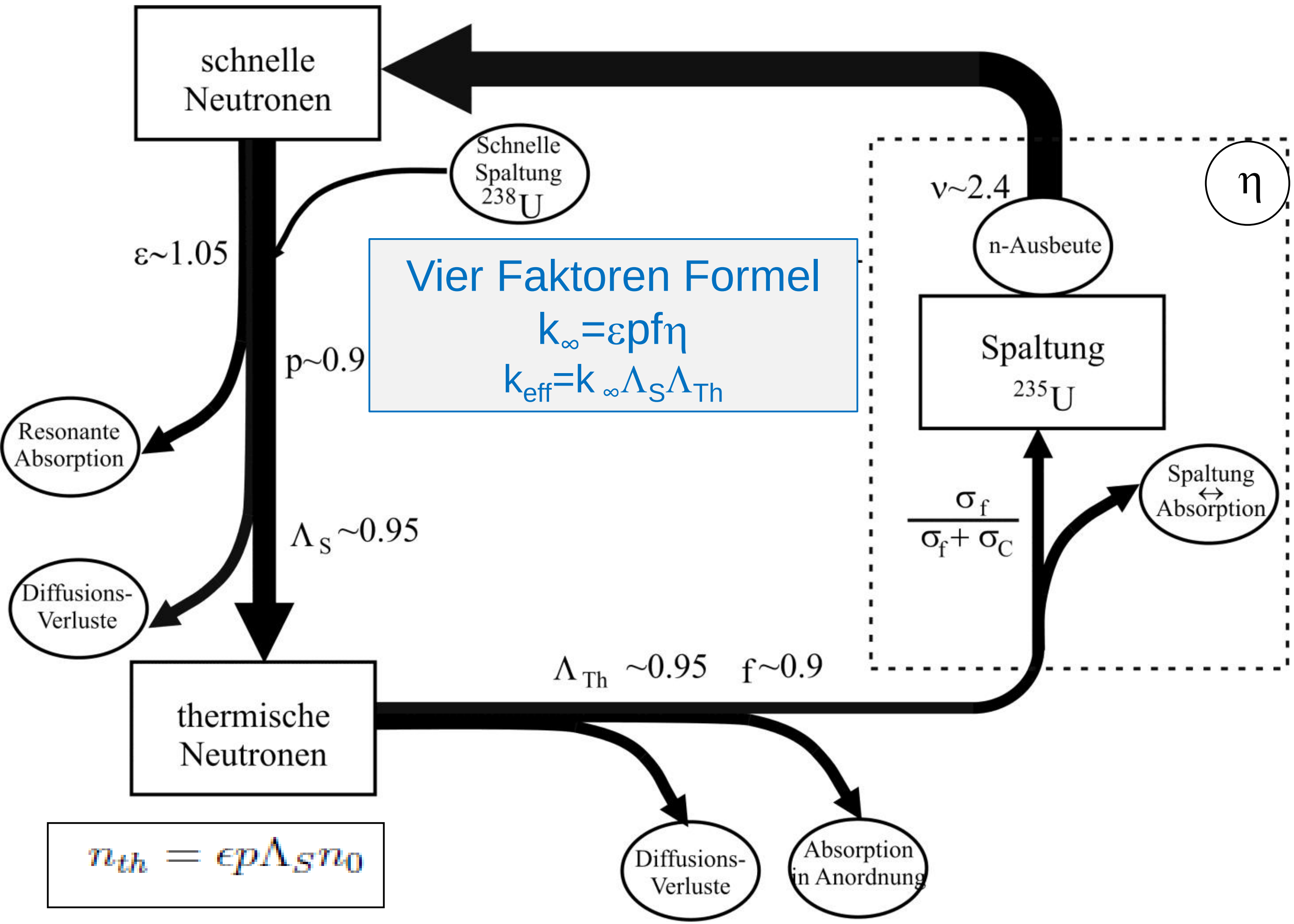


- Elastische Streuung an Kernen



- Bei der Spaötung entstehen schnelle Neutronen, die durch elastische Stöße moderiert werden, bis sie thermische Energien erreichen
- Dann diffundieren sie durch Materie bis sie eingefangen werden.
- Moderation ist gefährlich wegen Resonanzeinfang.

Neutronen Lebenszyklus im Reaktor



Vier Faktoren Formel

$k_\infty=\varepsilon p f \eta$

$k_{eff}=k_\infty \Lambda_S \Lambda_{Th}$

$$n_{th} = \epsilon p \Lambda_S n_0$$

- ε: Schnellspaltfaktor
- p: Resonanzentkomm-wahrscheinlichkeit
- f: thermische Nutzung
- η: Regenerationsfaktor

- Λ_S: schneller Verbleibefaktor
- Λ_{th}: thermischer Verbleibfaktor

- ν: Spaltausbeute

Lebensdauer einer Neutronengeneration

τ: Abbremszeit (H_2O: 40µs)

$$\frac{dN}{dt} = \frac{(k-1)}{\tau} \cdot N$$

mit dem *Multiplikationsfaktor*

$$k = \frac{\text{Zahl der Neutronen in Generation n}}{\text{Zahl der Neutronen in Generation n}-1}.$$

Daraus folgt die einfache Lösung

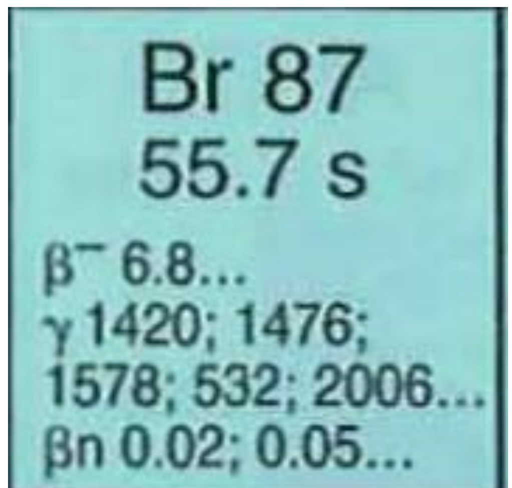
$$N = N_0\, e^{[(k-1)/\tau]t}.$$

k=1,001 → Faktor e in t/(k-1)=40 ms
In 1 s um Faktor 7,2E10 !

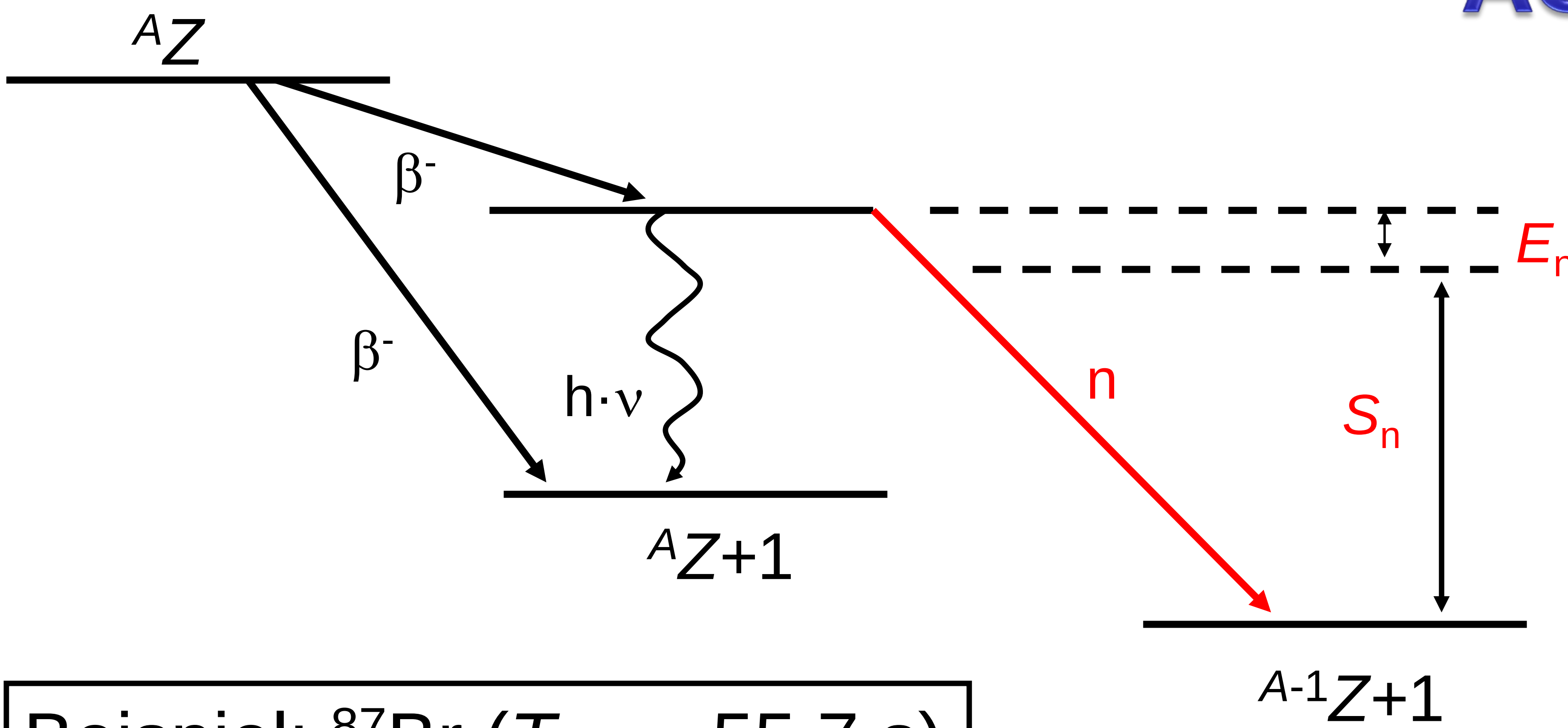
Verzögerte Neutronen

Nach 10^{-14} s sind alle prompten Neutronen emittiert. Aber es gibt einen kleinen **Anteil β verzögerter** Neutronen:

β = 0,26 % für U-233, β = 0,65 % für U-235, β = 0,21 % für Pu-239.



Verzögerter Beta Zerfall



Beispiel: ^{87}Br ($T_{1/2}$ = 55,7 s)

Lebensdauer einer Neutronengeneration

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Ende