Advanced Topics in Computational Complexity

Exercise Session 5

Due 16.11.2015.

Exercise 1

Prove Proposition 14 in the lecture notes. (Induction for cases p, $\vee$, and $\Diamond$ suffices.)

Exercise 2

Prove direction (2) $\rightarrow$ (1) of Theorem 12 in the lecture notes.

Exercise 3

Let $K = (W, R, V)$ be a Kripke model such that $W = \{1, 2, 3, 4, 5\}$, $R = \{(i, j) \in W^2 \mid i + j \leq 5\}$, $V(p) = \{1, 2, 4\}$, and $V(q) = \{4, 5, 6\}$.

Which of the following claims hold?

1. $K, \emptyset \models (p \wedge \neg p)$
2. $K, \{1, 2, 4, 6\} \models \mathrm{dep}(\Diamond p) \vee \mathrm{dep}(\Diamond p)$
3. $K, \{1\} \models \mathrm{dep}(q, p)$
4. $K, \{3, 5\} \models \mathrm{dep}(q, p)$
5. $K, \{1, 2, 4\} \models \mathrm{dep}(q, p)$
6. $K, \{1, 3, 5\} \models \mathrm{dep}(\Diamond q, p)$

Which of the teams of K satisfy the formula $\mathrm{dep}(p)$ or the formula $(p \wedge \mathrm{dep}(\Diamond q))$?

Exercise 4

Give the smallest generated submodels $K_1 = (W_1, R_1, V_1)$ and $K_2 = (W_2, R_2, V_2)$ of K (of Exercise 3) such that $3 \in W_1$ and $1, 5 \in W_2$. Give the disjoint union of the models K_1 and K_2.