Actuary Exam P Practice Solutions

(1) G = gymnastics, B = baseball, S = soccer

$$P(G \cup B \cup S)^c = 1 - P(G \cup B \cup S)$$
$$= 1 - [P(G) + P(B) + P(S) - P(G \cap B) - P(G \cap S) - P(B \cap S) + P(G \cap B \cap S)]$$
$$= 1 - [.28 + .29 + .19 - .14 - .12 - .10 + .08]$$
$$= 1 - .48$$
$$= .52 \quad \text{(D)}$$

(2) L = required lab work, S = referred to a specialist

$P(L \cup S)^c = .35$, $P(S) = .30$, $P(L) = .40$

$$P(L \cap S) = P(S) + P(L) - P(L \cup S)$$
$$= .30 + .40 - [1 - .35)$$
$$= .7 - .65$$
$$= .05 \quad \text{(A)}$$

(3) $P(A \cup B) = .7$, $P(A \cup B^c) = .9$

$P(A \cup B)$ $\qquad$ $P(A \cup B^c)$

$$\Rightarrow P(A) = P(A \cup B^c) - P(A \cup B)^c$$
$$= .9 - .3$$
$$= .6 \quad \text{(D)}$$

(4) R = red, B = blue, R_1 = red in urn 1, R_2 = red in urn 2, B_1 = blue in urn 1, B_2 = blue in urn 2

$$P[(R_1 \cap R_2) \cup (B_1 \cap B_2)] = .44$$
$$\Rightarrow \frac{2}{5} \cdot \frac{16}{16+x} + \frac{3}{5} \cdot \frac{x}{16+x} = .44 \quad \text{(By independence)}$$
$$\Rightarrow 32 + 3x = .44(80 + 5x)$$
$$\Rightarrow 32 + 3x = 35.2 + 2.2x$$
$$\Rightarrow .8x = 3.2$$
$$\Rightarrow x = 4 \quad \text{(A)}$$

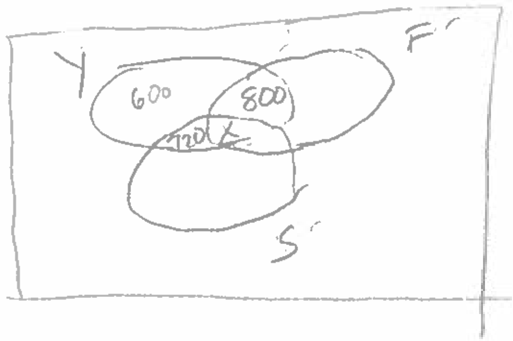
⑤ Y = young, M = male, X = married

$N(Y) = 3000$, $N(M) = 4600$, $N(X) = 7000$

$N(Y \cap M) = 1320$, $N(M \cap X) = 3010$, $N(Y \cap X) = 1400$

$N(Y \cap M \cap X) = 600$

$N(Y \cap M^c \cap X^c)$



$N(Y \cap F^c \cap S^c) = 600$

$N(Y \cap S^c) = 1400$

$N(Y \cap F^c) = 1320$

$\Rightarrow N(Y \cap F \cap S) = 3000 - [600 + 800 + 720]$

$= 3000 - 2120$

$= 880$ (D)

⑥

H: death from causes related to heart disease

S: had at least one parent suffered from heart disease

$P(H) = \frac{210}{937}$, $P(S) = \frac{312}{937}$, $P(H \cap S) = \frac{102}{312}$

$$P(H \mid S^c) = \frac{P(H \cap S^c)}{P(S^c)} = \frac{P(H) - P(H \cap S)}{1 - P(S)} = \frac{\frac{210}{937} - \frac{102}{937}}{1 - \frac{312}{937}} = .1728 \text{ (B)}$$

⑦ A: auto policy, H: homeowners policy

P(renew at least one policy) = P(renew auto only) + P(renew homeowners only) + P(renew both)

$= .4\, P(A \cap H^c) + .6\, P(H \cap A^c) + .8\, P(A \cap H)$

$= .4(.65 - .15) + .6(.50 - .15) + .8(.15)$

$= .2 + .21 + .12$

$= .53$ (D)

⑧ T = physical therapist C = chiropractor

$P(C \cap T) = .22$

$P(C \cup T)^c = .12$

$P(C) = P(T) + .14$

* $P(C \cup T) = 1 - .12 = P(C) + P(T) - P(C \cap T)$

$\Rightarrow .88 = P(T) + .14 + P(T) - .22$

$\Rightarrow 2P(T) = .96$

$\Rightarrow P(T) = .48$ (D)

9) i) C_1 = insure at least one car

$P(C_1) = 1$

ii) C_2 = insure more than one car

$P(C_2) = .7$

iii) S = insure a sports car

$P(S) = .2$

iv) $P(S|C_2) = .15$

$$P(C_2^c \cap S^c) = P(C_2 \cup S)^c$$
$$= 1 - P(C_2 \cup S)$$
$$= 1 - [P(C_2) + P(S) - P(C_2 \cap S)]$$
$$= 1 - [P(C_2) + P(S) - P(S|C_2) \cdot P(C_2)]$$
$$= 1 - [.7 + .2 - .15(.7)]$$
$$= 1 - .795$$
$$= .205 \quad \text{B}$$

10) Duplicate of Q9

11) C = collision coverage D = disability coverage

i) $P(C) = 2P(D)$

ii) C and D are independent events

iii) $P(C \cap D) = .15$

$$* \; P(C \cap D) = P(C) \cdot P(D) = .15$$
$$\Rightarrow 2P(D) \cdot P(D) = .15$$
$$\Rightarrow [P(D)]^2 = .075$$
$$\Rightarrow P(D) = 0.274$$
$$\Rightarrow P(C) = 0.548$$

$$P(C \cup D)^c = 1 - P(C \cup D)$$
$$= 1 - [P(C) + P(D) - P(C \cap D)]$$
$$= 1 - [0.548 + 0.274 - .15] \quad \text{(From *)}$$
$$= 1 - [.672]$$
$$= .328 \quad \text{B}$$

(12) H = high blood pressure, L = low blood pressure, I = irregular heart beat, R = regular heartbeat, N = normal blood pressure

	H	L	N	Total
R	.09	.2	.56	.85
I	.05	.02	.08	.15 (iii)
Total	.14 (i)	.22 (ii)	.64	1

(iv) $P(H|I) = \frac{1}{3}$

$\Rightarrow \frac{1}{3} = \frac{P(H \cap I)}{.15}$

$\Rightarrow P(H \cap I) = .05$

(v) $P(I|N) = \frac{1}{8}$

$\Rightarrow \frac{1}{8} = \frac{P(I \cap N)}{.64}$

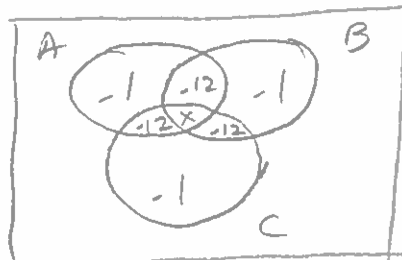
$\Rightarrow P(I \cap N) = .08$

From table,

$P(R \cap L) = .2$ (E)

(13) $P(\text{one risk factor}) = .1$, $P(\text{two risk factors}) = .12$

$P(A \cap B \cap C | A \cap B) = \frac{1}{3}$



$$P[(A \cup B \cup C)^c | A^c) = \frac{P(A \cup B \cup C)^c}{P(A^c)}$$

$$= \frac{1 - P(A \cup B \cup C)}{1 - P(A)}$$

(From *)

$$= \frac{1 - [.3 + .36 + .06]}{1 - [.24 + .1 + .06]}$$

$$= \frac{1 - .72}{1 - .4}$$

$$= \frac{.28}{.6}$$

$$= .4\overline{6}$$ (C)

$* \; P(A \cap B \cap C | A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)}$

$\Rightarrow \frac{1}{3} = \frac{x}{x + .12}$

$\Rightarrow x + .12 = 3x$

$\Rightarrow 2x = .12$

$\Rightarrow x = .06$

(14) For arbitrary $k \geq 0$

$p_k = \frac{1}{5} p_{k-1} = \frac{1}{5}\frac{1}{5} p_{k-2} = \ldots = \left(\frac{1}{5}\right)^n p_0$

$\Rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n p_0 = 1$

$p_0 = 1 - \frac{1}{5} = \frac{4}{5}$

Since $p \sim$ Geom

$\Rightarrow P(N > 1) = 1 - P(N \leq 1) = 1 - \left(\frac{4}{5}\right) + \frac{4}{5}\left(\frac{1}{5}\right) = .04$ (A)

(15) Let x, y, z represent the choices for coverage A, B, and C.

$\Rightarrow x+y=\frac{1}{4},\ x+z=\frac{1}{3},\ y+z=\frac{5}{12}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1/4 \\ 1 & 0 & 1 & 1/3 \\ 0 & 1 & 1 & 5/12 \end{array}\right) \xrightarrow{R_2=r_2-r_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1/4 \\ 0 & -1 & 1 & 1/12 \\ 0 & 1 & 1 & 5/12 \end{array}\right) \xrightarrow{R_3=r_2+r_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1/4 \\ 0 & -1 & 1 & 1/12 \\ 0 & 0 & 2 & 1/2 \end{array}\right)$$

$\Rightarrow 2z=\frac{1}{2}$ $\quad\Rightarrow -y+\frac{1}{4}=\frac{1}{12}$ $\quad\Rightarrow x+\frac{1}{6}=\frac{1}{4}$

$\Rightarrow z=\frac{1}{4}$ $\quad\Rightarrow y=\frac{1}{6}$ $\quad\Rightarrow x=\frac{1}{12}$

Let w be no supplementary coverage s.t. $w=1-(x+y+z)$.

$\Rightarrow w=1-\left(\frac{1}{4}+\frac{1}{6}+\frac{1}{12}\right)=1-\left(\frac{3}{12}+\frac{2}{12}+\frac{1}{12}\right)=1-\frac{1}{2}=\frac{1}{2}$ (C)

(16) Let N_1 = claims received in week 1

N_2 = Claims received in week 2

$$\Rightarrow P(N_1+N_2=7)=\sum_{n=0}^{7} P(N_1=n)\,P(N_2=7-n)$$
$$=\sum_{n=0}^{7} \frac{1}{2^{n+1}}\cdot\frac{1}{2^{7-n+1}}$$
$$=\sum_{n=0}^{7}\frac{1}{2^9}$$
$$=\frac{8}{2^9}$$
$$=\frac{1}{2^6}$$
$$=\frac{1}{64}$$ (D)

(17) E = emergency room, O = operating room

$P(E\cup O)=.85$

$P(E^c)=.25$

E and O are independent

$\Rightarrow P(E\cup O)=.85=P(E)+P(O)-P(E\cap O)$

$\Rightarrow .85=.75+P(O)-.75P(O)$

$\Rightarrow .25P(O)=.10$

$\Rightarrow P(O)=.4$ (D)

(18) Let X_1 be measurement of less accurate instrument s.t.

$X_1 \sim N(\mu = 0, \sigma = .0056h)$

Let X_2 be " " more accurate " "

$X_2 \sim N(\mu = 0, \sigma = .0044h)$

X_1 and X_2 are independent

$$Y = \frac{X_1 + X_2}{2} \sim N\left(0, \sqrt{\frac{.0056^2h^2 + .0044^2h^2}{4}}\right) = N(0, .00365h)$$

$$\Rightarrow P(-.005h \le Y \le .005h) = P(Y \le .005h) - P(Y \le -.005h)$$

$$= P(Y \le .005h) - P(Y \ge .005h)$$

$$= 2P(Y \le .005h) - 1$$

$$= 2P\left(Z \le \frac{.005h}{.00365h}\right) - 1$$

$$= 2P(Z \le 1.4) - 1$$

$$= 2(.9192) - 1$$

$$= .8384 \quad \text{D}$$

(19) Let A = age 16-20, B = age 21-30, C = age 31-65, D = age 66-99,

X = accident

$$P(A|X) = \frac{P(X|A)P(A)}{P(X|A)P(A) + P(X|B)P(B) + P(X|C)P(C) + P(X|D)P(D)}$$

$$= \frac{.06(.08)}{.06(.08) + .03(.15) + .02(.49) + .04(.28)}$$

$$= \frac{.0048}{.0303}$$

$$= .158 \quad \text{B}$$

(20) S = standard, X = preferred, U = ultra-preferred, D = death

$P(S) = .5$, $P(X) = .4$, $P(U) = .1$

$P(D|S) = .010$, $P(D|X) = .005$, $P(D|U) = .001$

$$P(U|D) = \frac{P(D|U)P(U)}{P(D|U)P(U) + P(D|S)P(S) + P(D|X)P(X)}$$

$$= \frac{.001(.1)}{.001(.1) + .010(.5) + .005(.4)}$$

$$= \frac{.0001}{.0071}$$

$$= .014 \quad \text{D}$$

(21) (i) C = critical ER patients

$P(C) = .1$

(ii) S = serious ER patients

$P(S) = .3$

(iii) X = stable ER patients

$P(X) = .6$

(iv) D: death

$P(D|C) = .4$

(v) $P(D|S) = .1$

(vi) $P(D|X) = .01$

$$P(S|D^c) = \frac{P(D^c|S)P(S)}{P(D^c|S)P(S) + P(D^c|C)P(C) + P(D^c|X)P(X)}$$

$$= \frac{.9(.3)}{.9(.3) + .6(.1) + .99(.6)}$$

$$= \frac{.27}{.924}$$

$$= .292 \quad \text{B}$$

(22) H = heavy smokers, L = light smokers, N = non-smokers, D = death

$P(H) = .2 \quad P(L) = .3 \quad P(N) = .5$

$P(D|L) = 2P(D|N)$

$P(D|L) = \frac{1}{2}P(D|H)$

$$P(H|D) = \frac{P(D|H)P(H)}{P(D|H)P(H) + P(D|L)P(L) + P(D|N)P(N)}$$

$$= \frac{.2 \cdot 2P(D|L)}{.2 \cdot 2P(D|L) + .3P(D|L) + .5 \cdot .5P(D|L)}$$

$$= \frac{.4P(D|L)}{.95P(D|L)}$$

$$= .421 \quad D$$

(23) T = teen, Y = young adult, M = midlife, S = senior, C = at least one collision

$$P(Y|C) = \frac{P(C|Y)P(Y)}{P(C|Y)P(Y) + P(C|M)P(M) + P(C|T)P(T) + P(C|S)P(S)}$$

$$= \frac{.08(.16)}{.08(.16) + (.08)(.15) + .45(.04) + .31(.05)}$$

$$= \frac{.0128}{.0583}$$

$$= .220 \quad D$$

(24) N: # injury claims per month which $P(N) = \frac{1}{(n+1)(n+2)}$

$$P(N \geq 1 \mid N \leq 4) = \frac{P(1 \leq N \leq 4)}{P(N \leq 4)} = \frac{\sum_{n=1}^{4} \frac{1}{(n+1)(n+2)}}{\sum_{n=0}^{4} \frac{1}{(n+1)(n+2)}}$$

$$= \frac{\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}}{\frac{1}{2} + \sum_{n=1}^{4} \frac{1}{(n+1)(n+2)}}$$

$$= \frac{\frac{10}{60} + \frac{5}{60} + \frac{3}{60} + \frac{2}{60}}{\frac{1}{2} + \sum_{n=1}^{4} \frac{1}{(n+1)(n+2)}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{2} + \frac{1}{3}}$$

$$= \frac{\frac{1}{3}}{\frac{5}{6}}$$

$$= \frac{1}{3}\left(\frac{6}{5}\right)$$

$$= \frac{2}{5} \quad B$$

(25) D = has the disease

X = positive test

$$P(D|X) = \frac{P(X|D)P(D)}{P(X|D)P(D) + P(X^c|D^c)P(D^c)}$$

$$= \frac{.95(.01)}{.95(.01) + (.005)(.99)}$$

$$= .657$$ (B)

(26) C = blood circulation problem

S = smoker

$P(C) = .25$

$P(S|C) = 2P(S|C^c)$

$$P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|C^c)P(C^c)} = \frac{.25P(S|C)}{.25P(S|C) + .75 \cdot .5P(S|C)}$$

$$= \frac{.25P(S|C)}{.625P(S|C)}$$

$$= .4$$ (C)

(27) $X_{14} = 2014$, $X_{13} = 2013$, $X_{12} = 2012$, A = accident

X_0 = other

$$P(X_{14}|A) = \frac{P(A|X_{14})P(X_{14})}{P(A|X_{14})P(X_{14}) + P(A|X_{13})P(X_{13}) + P(A|X_{12})P(X_{12})}$$

$$= \frac{.05(.16)}{.05(.16) + .18(.02) + .20(.03)}$$

$$= \frac{.008}{.0176}$$

$$= .45$$ (D)

(28) X = vaccine shipments from company X

Y = every other company

I = vial ineffective

$$P(X|I) = \frac{P(I|X)P(X)}{P(I|X)P(X)+P(I|Y)P(Y)} = \frac{.2\binom{30}{1}.1^1(.9^{29})}{.2\binom{30}{1}.1(.9^{29}) + .8\binom{30}{1}.02^1(.98)^{29}}$$

$$= \frac{.0283}{.0283+.267}$$

$$= .096 \quad \text{(A)}$$

(29) # of days of A $\sim$ Expo

$$P(X \le 50) = \int_0^{50} \lambda e^{-\lambda x}\,dx = .3 \Rightarrow -e^{-\lambda x}\Big|_0^{50} = .3$$

$$\Rightarrow 1-e^{-50\lambda} = .3$$

$$\Rightarrow e^{-50\lambda} = .7$$

$$-50\lambda = \ln .7$$

$$\lambda = .0071$$

$$\Rightarrow P(X \le 80) = \int_0^{80} \lambda e^{-\lambda x}\,dx = \int_0^{80} .0071 e^{-.0071x}\,dx$$

$$= -e^{-.0071x}\Big|_0^{80}$$

$$= 1-e^{-.0071(80)}$$

$$= .433 \quad \text{(C)}$$

(30) $P(X=2) = 3P(X=4)$, $X \sim$ # claims filed

$X \sim Pois(\lambda)$

$$\Rightarrow P(X=2) = 3P(X=4)$$

$$\Rightarrow \cancel{e^{-\lambda}}\frac{\lambda^2}{2!} = 3\cancel{e^{-\lambda}}\frac{\lambda^4}{4!}$$

$$\Rightarrow \frac{\lambda^2}{2} = \frac{\lambda^4}{8}$$

$$\Rightarrow \lambda^2 = 4$$

$$\Rightarrow \lambda = 2$$

Since $X \sim Pois$, then $Var(X) = 2$ (D)

(31) P(payments inadequate to cover for high performance) $< .01$

$\Rightarrow$ P(payments adequate to cover for high performance) $> .99$

Let X be # of employees achieving high performance.

$P(X=0) = \binom{20}{0} .02^0 .98^{20} = .668$

$P(X=1) = \binom{20}{1} .02^1 .98^{19} = .272$

$P(X=2) = \binom{20}{2} .02^2 .98^{18} = .0528$

$\sum_{i=0}^{2} P(X=i) = .668 + .272 + .0528 = .9928 > .99$

$\Rightarrow 2C = 120$

$\Rightarrow C = 60$ (D)

(32) L = low risk drivers, M = moderate, H: high

$P(H \geq L+2) = p(0,0,4) + (0,1,3) + p(0,2,2) + (1,0,3)$

$= .2^4 + 4(.3)(.2)^3 + \frac{4!}{2!2!}(.3)^2(.2)^2 + 4(.5)(.2)^3$

$= .0016 + .0096 + .0216 + .016$

$= .0488$ (D)

(33) $P(X>16 \mid X>8) = \frac{P(X>16)}{P(X>8)} = \frac{\int_{16}^{20} .005(20-x)\,dx}{\int_{8}^{20} .005(20-x)\,dx}$

$= \frac{\int_{16}^{20}(20-x)\,dx}{\int_{8}^{20}(20-x)\,dx}$

$= \frac{20x - \frac{1}{2}x^2 \Big|_{16}^{20}}{20x - \frac{1}{2}x^2 \Big|_{8}^{20}}$

$= \frac{200-192}{200-128}$

$= \frac{8}{72}$

$= \frac{1}{9}$ (B)

(34) $f(x) \propto \frac{1}{(10+x)^2} \Rightarrow f(x) = C \cdot \frac{1}{(10+x)^2}$

$\Rightarrow$ Find C

$$\Rightarrow 1 = \int_0^{40} C \frac{1}{(10+x)^2} dx = C \int_0^{40} u^{-2} du = C\left(-\frac{1}{u}\Big|_0^{40}\right)$$

$u = 10+x \quad du = dx$

$$= C\left(-\frac{1}{10+x}\Big|_0^{40}\right)$$

$$= C\left(\frac{1}{10} - \frac{1}{50}\right)$$

$$\Rightarrow C = \frac{25}{2}$$

Thus

$$P(X<6) = \int_0^6 \frac{25}{2} (10+x)^{-2} dx = \frac{25}{2}\left(-\frac{1}{10+x}\Big|_0^6\right)$$

$$= \frac{25}{2}\left(\frac{1}{10} - \frac{1}{16}\right)$$

$$= \frac{25}{2}\left(\frac{8}{80} - \frac{5}{80}\right)$$

$$= \frac{25}{2}\left(\frac{3}{80}\right)$$

$$= \frac{5}{2} \frac{3}{16}$$

$$= \frac{15}{32} \quad C$$

(35) Duplicate of Q34

(36) $V = 100000Y$

$$P(V>40000 \mid V>10000) = P(Y>.4 \mid Y>.1)$$

Find k

$$\Rightarrow 1 = \int_0^1 k(1-y)^4 dy = \int_0^1 -k(-u)^4 dy = -k \cdot \frac{1}{5}(1-y)^5\Big|_0^1 = \frac{1}{5}k$$

$u = 1-y \quad du = -dy$

$$\Rightarrow k = 5$$

Thus

$$P(V>40000 \mid V>10000) = \frac{P(Y>.4)}{P(Y>.1)} = \frac{\int_{.4}^1 5(1-y)^4 dy}{\int_{.1}^1 5(1-y)^4 dy} = \frac{-\frac{1}{5}(1-y)^5\Big|_{.4}^1}{-\frac{1}{5}(1-y)^5\Big|_{.1}^1}$$

$$= \frac{.015552}{.118098}$$

$$= .132 \quad B$$

(37) X = life of printer

$$P(X\le 1)=\int_0^1 \frac{1}{2}e^{-1/2x}\,dx = -e^{-1/2x}\Big|_0^1 = 1-e^{-1/2} = .393$$

$$P(1\le X\le 2)=\int_1^2 \frac{1}{2}e^{-1/2x}\,dx = -e^{-1/2x}\Big|_1^2 = e^{-1/2}-e^{-1} = .239$$

For expected value for 1 printer,

$$\Rightarrow E(X)=200(.393)+100(.239)=102.5$$

Thus, expected value for 100 printers,

$$\Rightarrow 100\,E(X)=100(102.5)=10250 \quad \text{(D)}$$

(38)
$$P(X<2\mid X\ge 1.5)=\frac{P(1.5\le X<2)}{P(X\ge 1.5)} = \frac{P(1.5\le X<2)}{1-P(X<1.5)}$$

$$=\frac{\int_{1.5}^{2} 3x^{-4}\,dx}{1-\int_1^{1.5} 3x^{-4}\,dx}$$

$$=\frac{-\frac{1}{x^3}\Big|_{1.5}^{2}}{1+\frac{1}{x^3}\Big|_1^{1.5}}$$

$$=\frac{.171}{.296}$$

$$=.577 \quad \text{(A)}$$

(39) Let X be # of hurricanes

$$P(X<3)=\sum_{i=0}^{2} P(X=i) = P(X=0)+P(X=1)+P(X=2)$$

$$=\binom{20}{0}.05^0.95^{20}+\binom{20}{1}.05^1.95^{20}+\binom{20}{2}.05^2.95^{11}$$

$$=.358+.358+.1887$$

$$=.905 \quad \text{(E)}$$

(40) Deductible, let $Y = \begin{cases} 0 & 0 < X \le c \\ X - c & c < X < 1 \end{cases}$ which Y = insurance payment
c = deductible

$$P(Y < .5 \mid X) = .64 = P(0 < X < .5 + c) = \int_0^{.5+c} 2x\,dx$$
$$= x^2 \Big|_0^{.5+c}$$
$$= (.5+c)^2$$

$\Rightarrow .64 = (.5+c)^2$
$\Rightarrow .8 = .5 + c$
$\Rightarrow c = .3$ (B)

(41) A = ≥ 9 in group A completes study
B = ≥ 9 in group B completes study

$* P(A) = \binom{10}{9}(.8)^9(.2)^1 + \binom{10}{10}.8^{10}.2^0 = .376$

$$P([A \cap B^c] \cup [A^c \cap B]) = P(A \cap B^c) + P(A^c \cap B)$$
$$= P(A)P(B^c) + P(A^c)P(B)$$
$$= P(A)P(A^c) + P(A)P(A^c) \quad (\text{since } P(A) = P(B))$$
$$= 2P(A)P(A^c)$$
$$= 2(.376)(1 - .376) \quad (\text{from } *)$$
$$= .469 \quad \text{(E)}$$

(42) X = A's total claim amount, Y = B's total claim amount ≥ 1 claim
$X \sim N(10000, 2000)$

1)* A had no claim and B got at least one claim
2)* Both got at least one claim and $Y > X$

P(B's total claim > A's) = P(1st scenario) + P(2nd scenario)
P(2nd scenario) = P[(A at least one claim) ∩ (B at least one claim) ∩ P(Y > X)]
P(1st scenario) = P[A no claim ∩ B at least one claim]

$\Rightarrow P(1\text{st}) = .6[(1 - .7)] = .18$
$P(2\text{nd}) = .4(.3)P(Y > X) = .0436$

$* P(Y > X) = P(Y - X > 0) = .3632$
since $(Y - X) \sim N(-1000, 8000000)$

Thus $P(B > A) = P(1\text{st}) + P(2\text{nd})$
$= .18 + .0436$
$= .224$ (D)

(43) k = # of failures before 4th success, $k \sim$ NegBin

$p_X(k) = P(X=k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}, \quad k = r, r+1, \ldots$

Calculate probability that there will be at least x failure before the 4th success.

$X \sim$ Neg. Bin $(r=4)$

Failure defined as no accident, while success defined as at least one accident occurs.

Thus, $P(X \geq 4) = 1 - P(X \leq 3) = 1 - \sum_{k=0}^{3} \binom{3+k}{k} \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^k$

$= 1 - \left[\binom{3}{0}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^0 + \binom{4}{1}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^1 + \binom{5}{2}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^2 + \binom{6}{3}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^3\right]$

$= 1 - (.1296 + .2074 + .2074 + .1659)$

$= .2897$ (D)

(44) $E(X) = 100\left(\frac{6-1}{15}\right) + 200\left(\frac{6-2}{15}\right) + 300\left(\frac{6-3}{15}\right) + 350\left(\frac{6-4}{15}\right) + 400\left(\frac{6-5}{15}\right)$

$= \frac{500 + 800 + 900 + 700 + 400}{15}$

$= 220$ (C)

(45) $E(X) = \int_{-2}^{4} x \frac{|x|}{10} dx = \int_{-2}^{0} x \frac{-x}{10} dx + \int_{0}^{4} x \frac{x}{10} dx$

$= \int_{-2}^{0} \frac{-x^2}{10} dx + \int_{0}^{4} \frac{x^2}{10} dx$

$= \frac{-x^3}{30}\Big|_{-2}^{0} + \frac{x^3}{30}\Big|_{0}^{4}$

$= -\frac{8}{30} + \frac{64}{30}$

$= \frac{56}{30}$

$= \frac{28}{15}$ (D)

(46) $X = \max(T,2) = \begin{cases} 2 & T \le 2 \\ T & T > 2 \end{cases}$

$* $ Tabular integration

t	$+$	$\frac{1}{3}e^{-t/3}$
1	$-$	$-e^{-t/3}$
0		$3e^{-t/3}$

$$E(X) = E[\max(T,2)] = \int_0^2 2f(t)\,dt + \int_2^\infty t\,f(t)\,dt$$
$$= \int_0^2 \frac{2}{3}e^{-t/3}dt + \int_2^\infty \frac{t}{3}e^{-t/3}dt$$
$$= 2(1-e^{-2/3}) + \left[t(-e^{-t/3}) - 3e^{-t/3}\right]_2^\infty$$
$$= 2 - \cancel{2e^{-2/3}} + \cancel{2e^{-2/3}} + 3e^{-2/3}$$
$$= 2 + 3e^{-2/3} \quad (D)$$

(47) T = time until failure of equipment $\quad T \sim \text{Expo}(\theta = 10)$

P = payment $\quad P = \begin{cases} x & \text{if } T \le 1 \\ .5x & \text{if } 1 \le T \le 3 \\ 0 & \text{if } T > 3 \end{cases}$

If $E[P] = 1000$, find x -

$$\Rightarrow E[P] = xP(T \le 1) + .5x\,P(1 < T \le 3) + 0$$
$$= x\,F_T(1) + .5x(F_T(3) - F_T(1))$$
$$\Rightarrow 1000 = .0952x + .5x(.1640)$$
$$\Rightarrow x = 5644.23 \quad (D)$$

(48)

x	P(failing)	Y
1	$.4$	4000
2	$.6(.4)$	3000
3	$.6^2(.4)$	2000
4	$.6^3(.4)$	1000

Thus $E(Y) = 4000(.4) + 3000(.6)(.4) + 2000(.6)^2(.4) + 1000(.6^3).4$

$$= 2694 \quad (E)$$

(49) Question duplicates Q44

(50) $E(10000(N-1)) = 10000E(N) - 10000 = 5000$

$$\sum_{n=1}^\infty 10000\frac{e^{-1.5}1.5^n}{n!} = -10000(0-1)e^{-1.5} + \sum_{n=1}^\infty 10000\frac{e^{-1.5}1.5^n}{n!}$$
$$= 10000e^{-1.5} + E(10000(N-1))$$
$$= 2231 + 5000$$
$$= 7231 \quad (C)$$

(51) $E(Y)=\int_{.6}^{2} 2.5(.6)^{2.5}x^{-2.5}dx+\int_{2}^{\infty}5(.6)^{2.5}x^{-3.5}dx$

$= 2.5(.6)^{2.5}\left[-\frac{1}{1.5x^{1.5}}\right]_{.6}^{2} + 5(.6)^{2.5}\left[-\frac{1}{2.5x^{2.5}}\right]_{2}^{\infty}$

$= 2.5(.6)^{2.5}(1.4344-.2357)+5(.6)^{2.5}(.0707)$

$= .8357+.0986$

$= .93$ (C)

(52) Net Premium $= E[\text{Payment}]$

$P(N \mid \text{loss incurred}) = \frac{K}{N}$

$\Rightarrow K=\frac{60}{137}$

$P(N)=\frac{60}{137N}(.05)=\frac{3}{137N}$

$\Rightarrow E(P)= 1\cdot\frac{3}{137(3)}+2\cdot\frac{3}{137(4)}+3\cdot\frac{3}{137(5)}=.031$ (A)

(53) Benefit $= \begin{cases} y, & 1<y\le 10 \\ 10, & y>10 \end{cases}$

$\Rightarrow E(\text{Benefit})=\int_{1}^{10}2y\cdot y^{-3}dy+\int_{10}^{\infty}20y^{-3}dy$

$=\int_{1}^{10}2y^{-2}dy+\int_{10}^{\infty}20y^{-3}dy$

$= -\frac{2}{y}\Big|_{1}^{10}-\frac{10}{y^2}\Big|_{10}^{\infty}$

$= 2-\frac{1}{5}+\frac{1}{10}$

$= 1.9$ (D)

(54) Y = claim payment

	Y	Prob.
Undamaged	0	.94
Partial Damage	max(0, x−1)	.04
Total	14	.02

$E(Y)=E(0)\cdot.94+E[\max(0,x-1)]\cdot.04+E(14)\cdot.02$

$E[\max(0,x-1)]=\int_{1}^{15}(x-1).5003e^{-x/2}dx$

$E(Y)=.328$

$\Rightarrow E(\text{claim payment})=1000(.328)=328$ (B)

(55) $f(x) \propto (1+x)^{-4} \quad 0 < x < \infty \Rightarrow f(x) = k(1+x)^{-4}$

First find k,

$$\Rightarrow 1 = \int_0^\infty k(1+x)^{-4}\,dx = k \cdot -\frac{1}{3u^3}\Big|_0^\infty = k \cdot -\frac{1}{3(1+x)^3}\Big|_0^\infty = \frac{1}{3}k$$

$u = 1+x \quad du = dx$

$\Rightarrow k = 3$

$\Rightarrow f(x) = 3(1+4)^{-4}$

$\Rightarrow F(X) = \int_0^x f(t)\,dt = \int_0^x 3(1+t)^{-4}\,dt = 1-(1+x)^{-3}$

Since $S_X(x) = (1+x)^{-3}$,

$$\Rightarrow E(X) = \int_0^\infty S_X(x)\,dx = \int_0^\infty (1+x)^{-3}\,dx$$

$$= -\frac{1}{2(1+x)^2}\Big|_0^\infty$$

$$= \frac{1}{2} \quad \text{(C)}$$

(56) $X \sim Unif(0,1000) \qquad Y = \begin{cases} 0 & \text{if } 0 < X \le d \\ X-d & \text{if } d < X \le 1000 \end{cases}$

$E(X) = \frac{a+b}{2} = 500$

$$\Rightarrow E(Y) = \int_d^{1000} \frac{1}{1000}(x-d)\,dx = \frac{(x-d)^2}{2000}\Big|_d^{1000} = \frac{(1000-d)^2}{2000}$$

Since $E(Y) = .25\,E(X) = 125$

$\Rightarrow 125 = \frac{(1000-d)^2}{2000}$

$\Rightarrow (1000-d)^2 = 250000$

$\Rightarrow 1000-d = \pm 500$

$\Rightarrow d = 500$ or $d = 1500$

$\Rightarrow d = 500$ (C)

(57) $M_X(t) = (1-2500t)$

$\Rightarrow M_X'(t) = -4(1-2500t)^{-5} \cdot -2500 = 10000(1-2500t)^{-5}$

$\Rightarrow M_X''(t) = -50000(1-2500t)^{-6} \cdot -2500 = 125000000(1-2500t)^{-6}$

$\Rightarrow Var(X) = E(X^2) - (E(X))^2$

$\Rightarrow E(X^2) = M_X''(0) = 125000000$

$\Rightarrow E(X) = M_X'(0) = 10000$

$\Rightarrow Var(X) = 125000000 - 10000^2 = 25000000$

$\Rightarrow SD(X) = \sqrt{Var(X)} = 5000$ (B)

(58) $M_X(t) = M_J(t) \cdot M_K(t) \cdot M_L(t)$

$= (1-2t)^{-3} \cdot (1-2t)^{-2.5} (1-2t)^{-4.5}$

$= (1-2t)^{-10}$

$E(X^3) = M_X'''(0)$

$M_X'(t) = -10(1-2t)^{-11} \cdot -2 = 20(1-2t)^{-11}$

$M_X''(t) = -220(1-2t)^{-12} \cdot (-2) = 440(1-2t)^{-12}$

$M_X'''(t) = -5280(1-2t)^{-13} \cdot -2 = 10560(1-2t)^{-13}$

$\Rightarrow E(X^3) = 10560$ (E)

(59) ★ $\pi_{.7} - \pi_{.3}$

$F(\pi_{.7}) = 1 - \frac{(200)^{2.5}}{\pi_{.7}^{2.5}} = .7$

$\Rightarrow \frac{(200)^{2.5}}{\pi_{.7}^{2.5}} = .3$

$\Rightarrow \pi_{.7} = 323.73$

$\Rightarrow \pi_{.7} - \pi_{.3} = 93.06$ (B)

$F(\pi_{.3}) = 1 - \frac{(200)^{2.5}}{\pi_{.3}^{2.5}} = .3$

$\Rightarrow \pi_{.3} = 230.67$

(60) Y = annual cost after tax introduced

$\Rightarrow Var(Y) = Var(1.2X) = 1.2^2 Var(X) = 1.2^2(260) = 374$ (E)

(61) Duplicate of Q60

(62) $f(x) = \begin{cases} 1/2 & \text{if } x=1 \\ x-1 & \text{if } 1<x<2 \\ 0 & \text{else} \end{cases}$ *Note: since $F'(x)=0$ at $x=1$, $F(1)=\frac{1}{2}$

$$E(X^2) = 1^2 P(X=1) + \int_1^2 x^2(x-1)\,dx = \frac{1}{2} + \int_1^2 x^3 - x^2\,dx$$

$$= \frac{1}{2} + \left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_1^2$$

$$= \frac{1}{2} + \left[4 - \frac{8}{3} - \frac{1}{4} + \frac{1}{3}\right]$$

$$= \frac{1}{2} + \left[\frac{48}{12} - \frac{32}{12} - \frac{3}{12} + \frac{4}{12}\right]$$

$$= \frac{6}{12} + \frac{17}{12}$$

$$= \frac{23}{12}$$

$$E(X) = 1P(X=1) + \int_1^2 x(x-1)\,dx = \frac{1}{2} + \int_1^2 x^2 - x\,dx$$

$$= \frac{1}{2} + \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_1^2$$

$$= \frac{1}{2} + \left[\frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2} + \left[\frac{16}{6} - \frac{12}{6} - \frac{2}{6} + \frac{3}{6}\right]$$

$$= \frac{3}{6} + \frac{5}{6}$$

$$= \frac{4}{3}$$

$$\Rightarrow Var(X) = E(X^2) - (E(X))^2$$

$$= \frac{23}{12} - \left(\frac{4}{3}\right)^2$$

$$= \frac{23}{12} - \frac{16}{9}$$

$$= \frac{69}{36} - \frac{64}{36}$$

$$= \frac{5}{36} \quad \text{(C)}$$

(63) $Y = \begin{cases} x & \text{for } 0<x<4 \\ 4 & \text{for } 4\le x\le 5 \end{cases}$

$$E(Y) = \int_0^4 x\cdot\frac{1}{5}\,dx + \int_4^5 4\cdot\frac{1}{5}\,dx = \int_0^4 \frac{1}{5}x\,dx + \int_4^5 \frac{4}{5}\,dx$$
$$= \frac{1}{10}x^2\Big|_0^4 + \frac{4}{5}x\Big|_4^5$$
$$= \frac{8}{5} + 4 - \frac{16}{5}$$
$$= \frac{8}{5} + \frac{20}{5} - \frac{16}{5}$$
$$= \frac{12}{5}$$

$$E(Y^2) = \int_0^4 x^2\cdot\frac{1}{5}\,dx + \int_4^5 4^2\cdot\frac{1}{5}\,dx = \int_0^4 \frac{1}{5}x^2\,dx + \int_4^5 \frac{16}{5}\,dx$$
$$= \frac{1}{15}x^3\Big|_0^4 + \frac{16}{5}x\Big|_4^5$$
$$= \frac{64}{15} + 16 - \frac{64}{5}$$
$$= \frac{64}{15} + \frac{240}{15} - \frac{192}{15}$$
$$= \frac{112}{15}$$

$$\Rightarrow Var(Y) = E(Y^2) - (E(Y))^2 = \frac{112}{15} - \left(\frac{12}{5}\right)^2 = \frac{112}{15} - \frac{144}{25} = 1.7 \quad (C)$$

(64)

Claim Size	Probability	$xP(X=x)$	$x^2P(X=x)$
20	.15	3	60
30	.10	3	90
40	.05	2	80
50	.20	10	500
60	.10	6	360
70	.10	7	490
80	.30	24	1920
		55	3500

$Var(X) = 3500 - 55^2 = 475$

$SD(X) = \sqrt{Var(X)} = 21.79$

$E(X) + SD(X) = 55 + 21.79 = 76.79$

$E(X) - SD(X) = 55 - 21.79 = 33.21$

43.58 (A)

(65) $E(Y) = \int_{250}^{1500} \frac{1}{1500}(x-250)\,dx = \frac{1}{1500} \frac{(x-250)^2}{2}\Big|_{250}^{1500}$

$= \frac{1}{1500} \frac{(1500-250)^2}{2}$

$= 520.83$

$E(Y^2) = \int_{250}^{1500} \frac{1}{1500}(x-250)^2\,dx = \frac{1}{1500} \frac{(x-250)^3}{3}\Big|_{250}^{1500} = 434027.78$

$Var(Y) = 434027.78 - 520.83^2 = 162763.11$

$SD(Y) = \sqrt{Var(Y)} = 403.4$ (B)

(66) Deleted

(67) Let Y = payment

$$Y = \begin{cases} 0 & \text{if } x=0 \\ 1000 & \text{if } x=1 \\ 2000 & \text{if } x=2 \end{cases}$$

$E(Y) = 0 \cdot P(X=0) + 1000P(X=1) + 2000P(X \geq 2)$

$P(X=0) = e^{-.6}$

$P(X=1) = .6e^{-.6}$

$P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - 1.6e^{-.6}$

$E(Y) = 0 + 1000(.6e^{-.6}) + 2000(1 - 1.6e^{-.6}) = 573.09$

$E(Y^2) = 0 + 1000^2P(X=1) + 2000^2P(X \geq 2) = 816892.51 \Rightarrow SD(Y) = 699$ (B)

(68) Since $X \sim Expo(\mu = \frac{1}{.004})$, then $c = .004$

Thus for the median benefit,

$\Rightarrow .5 = \int_0^m .004e^{-.004x}\,dx$

$\Rightarrow -e^{-.004x}\Big|_0^m = .5$

$\Rightarrow 1 - e^{-.004m} = .5$

$\Rightarrow e^{-.004m} = .5$

$\Rightarrow m = \frac{\ln(.5)}{-.004}$

$\Rightarrow m = 173.3$ (C)

(69) $.5=\int_0^4 \frac{1}{u} e^{-1/u\, x}\, dx = -e^{-1/u\, x}\Big|_0^4 = 1-e^{-4/u}$

$\Rightarrow e^{-4/u} = .5$

$\Rightarrow u = 5.77$

$\Rightarrow P(X\geq 5) = \int_5^\infty \frac{1}{5.77} e^{-1/5.77\, x}\, dx = -e^{-1/5.77\, x}\Big|_5^\infty = e^{-5/5.77} = .42$ (D)

(70) $P(X\leq \pi_p) = p\%$ $0\leq p\leq 100$

$(X|X>d)$ = losses that exceed the deductible

$\Rightarrow P(X\leq \pi_{95}|X>100) = .95 = \frac{P([X\leq \pi_{95}]\cap[X>100])}{P(X>100)} = \frac{P(100<X\leq \pi_{95})}{P(X>100)}$

$\Rightarrow .95 = \frac{F_X(\pi_{95}) - F_X(100)}{1-F_X(100)}$

$\Rightarrow .95 = \frac{1-e^{-\pi_{95}/300} - (1-e^{-1/3})}{1-(1-e^{-1/3})} = \frac{e^{-1/3} - e^{-\pi_{95}/300}}{e^{-1/3}} = 1-e^{1/3}e^{-\pi_{95}/300}$

$\Rightarrow e^{-\pi_{95}} = .05e^{-1/3}$

$\Rightarrow \pi_{95} = -300\ln(.05e^{-1/3}) = 998.7$ (E)

(71) $G(y) = P(Y\leq y) = P(T^2\leq y) = P(T\leq\sqrt{y}) = F(\sqrt{y}) = 1-\frac{4}{y}$

$\Rightarrow g(y) = \frac{4}{y^2}$ for $y>4$ (A)

(72) $F(v) = P(V\leq v) = P(10000e^R\leq v) = P(R\leq \ln v - \ln 10000)$

$= \frac{1}{.04}\int_{.04}^{\ln v - \ln 10000} dr$

$= 25r\Big|_{.04}^{\ln v - \ln 10000}$

$= 25\ln v - 25\ln 10000 - 1$

$= 25\left[\ln\left(\frac{v}{10000}\right) - .04\right]$ (E)

(73) $F(y) = P(Y\leq y) = P(10X^{.8}\leq y) = P(X\leq (\frac{y}{10})^{5/4})$

$= 1-e^{-(y/10)^{1.25}}$

$\Rightarrow f(y) = F'(y) = -e^{-(y/10)^{1.25}}\cdot -1.25(\frac{y}{10})^{.25}\cdot .1$

$= .125(\frac{y}{10})^{.25}e^{-(y/10)^{1.25}}$ (E)

(74) $R = \frac{10}{T}$

$P(R \le r) = P(\frac{10}{T} \le r) = P(T \ge \frac{10}{r}) = 1 - P(T < \frac{10}{r})$

$= 1 - F(\frac{10}{r})$

$= 1 - \frac{\frac{10}{r} - 8}{12 - 8}$

$= 1 - \frac{5}{2r} + 2$

$= 3 - \frac{5}{2r}$

$\Rightarrow f(r) = F'(r) = \frac{10}{(2r)^2} = \frac{10}{4r^2} = \frac{5}{2r^2}$ (E)

(75) X = profit for I Y = profit for II, $Y = 2X$

$\Rightarrow P(Y \le y) = P(2X \le y) = P(X \le \frac{y}{2}) = F(\frac{y}{2})$

$\Rightarrow G(Y) = F(\frac{y}{2})$

$\Rightarrow g(y) = F'(\frac{y}{2}) \cdot \frac{d}{dy}(\frac{y}{2}) = f(\frac{y}{2}) \cdot \frac{1}{2} = f(\frac{x}{2}) \cdot \frac{1}{2}$ (A)

(76) Let $Y = \max(X_1\ X_2\ X_3)$

Let $G(y) = P(Y \le y)$

$\Rightarrow G(y) = P(Y \le y) = P(\max(X_1 X_2 X_3) \le y) = [P(X \le y)]^3$

$F(x) = \int_1^x \frac{3}{t^4} dt = 1 - \frac{1}{x^3}$ for $x > 1$

$\Rightarrow G(y) = (1 - \frac{1}{y^3})^3$ for $y > 1$

$\Rightarrow g(y) = G'(y) = 3(1 - \frac{1}{y^3})^2 \cdot (\frac{3}{y^4}) = \frac{9}{y^4}(1 - \frac{1}{y^3})^2 \quad y > 1$

Thus $E(Y) = \int_1^\infty \frac{9}{y^3}(1 - \frac{1}{y^3})^2 dy = \int_1^\infty \frac{9}{y^3}(1 - \frac{2}{y^3} + \frac{1}{y^6}) dy$

$= 9\int_1^\infty y^{-3} - 2y^{-6} + y^{-9}\, dy$

$= 9\left[-\frac{1}{2y^2} + \frac{2}{5y^5} - \frac{1}{8y^8}\right]_1^\infty$

$= 9(\frac{1}{2} - \frac{2}{5} + \frac{1}{8})$

$= 2.025$ (in thousands) (A)

(77) $P((X<1)\cup(Y<1)) = 1-P((X>1)\cap(Y>1))$

$$= 1-\int_1^2\int_1^2 \tfrac{1}{8}x+\tfrac{1}{8}y \; dy\,dx$$

$$= 1-\int_1^2 \tfrac{1}{8}xy+\tfrac{1}{16}y^2\Big|_1^2 \, dx$$

$$= 1-\int_1^2 \tfrac{1}{4}x+\tfrac{1}{4}-\tfrac{1}{8}x-\tfrac{1}{16}\, dx$$

$$= 1-\int_1^2 \tfrac{1}{8}x+\tfrac{3}{16}\, dx$$

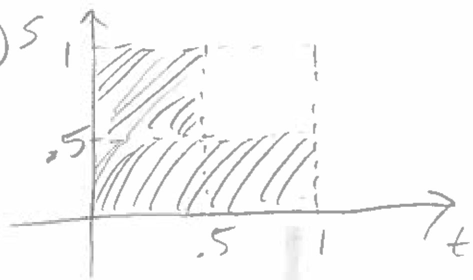
$$= 1-\left[\tfrac{1}{16}x^2+\tfrac{3}{16}x\right]_1^2$$

$$= 1-\left(\tfrac{1}{4}+\tfrac{3}{8}-\tfrac{1}{16}-\tfrac{3}{16}\right)$$

$$= .625 \quad \text{(D)}$$

(78) Duplicates Q77

(79)



(E)

(80) $E(X_i) = 3125$ $\qquad$ Y = total contributions

$Var(X_i) = 250^2 = 62500$ $\qquad$ $Y = X_1 + X_2 + \dots + X_{2025}$

$$Z_{90} = 1.282 \Rightarrow \frac{\pi_{Y,90} - n E(X_i)}{\sqrt{n\, Var(X_i)}} = 1.282 \quad \text{(CLT)}$$

$$\frac{\pi_{Y,90} - 2025(3125)}{\sqrt{2025(62500)}} = 1.282$$

$$\pi_{Y,90} = 6342547.5 \quad \text{(C)}$$

(81) $\bar{X} = \frac{\sum X_i}{25}$ $\qquad$ $\bar{X} \sim N\left(\mu = 19400, \sigma^2 = \frac{1}{n} Var(X)\right)$

$$\Rightarrow P(\bar{X} > 20000) = P\left(Z > \frac{20000-19400}{1000000}\right)$$

$$= P(Z > .6)$$

$$= 1 - .7257$$

$$= .2743 \quad \text{(C)}$$

(82) $X_i \sim Pois(\lambda = 2)$, S = total # of claims = $X_1 + X_2 + \dots + X_{1250}$

$S \sim N(nE(X_i), nVar(X_i))$

$$\Rightarrow P\left(\frac{2450-2500}{\sqrt{2500}} \le Z \le \frac{2600-2500}{\sqrt{2500}}\right) = P(-1 \le Z \le 2)$$

$$= P(Z \le 2) - P(Z < -1)$$

$$= P(Z \le 2) - (1 - P(Z < 1))$$

$$= .9772 - (1 - .8413)$$

$$= .8185 \quad B$$

(83) $X_i \sim N(3,1)$ $\quad S \sim N(3n, n)$

$\Rightarrow P(S \ge 40) \ge .9772$

$\Rightarrow P\left(Z \ge \frac{40-3n}{\sqrt{n}}\right) \ge .9772$

$\Rightarrow P\left(Z < -\frac{40-3n}{\sqrt{n}}\right) \ge .9772$

$\Rightarrow -\frac{40-3n}{\sqrt{n}} \ge 2$

$\Rightarrow n = 16 \quad B$

(84) Let W_i = total hours by "i" s.t. $W_i = X_i + Y_i$

$T = W_1 + W_2 + \dots + W_{100}$

$T \sim N(100E(W), 100Var(W))$

$E(W) = E(X+Y) = E(X) + E(Y) = 70$

$Var(W) = Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y) = 100$

$\Rightarrow P(T < 7100) = P\left(Z < \frac{7100-7000}{\sqrt{10000}}\right)$

$= P(Z < 1)$

$= .8413 \quad B$

85 Mean = 1000 Var = 1000000
(claim)
Premium Mean = 1100 Var = 1210000 P(Claims > Premiums)

$P\left(\frac{X-1000}{\sqrt{100000}} \geq \frac{X-1100}{\sqrt{1210000}}\right)$

Claim ~ Expo $\mu = \sigma = 1000$
Premium $\Rightarrow \mu = 1100, \sigma = 10000$
Total Premiums = 100(1100) = 110000

$\Rightarrow P\left(Z > \frac{110000-100000}{10000}\right) = P(Z>1) = 1 - .8413 = .1587$

B

86 X_i = # pensions to recruit i
S = total pensions to 100 recruits
$P(S \leq 90)$
$S \sim N(100E(X_i), 100Var(X_i))$

$X_i = \begin{cases} 0 & \text{if recruit doesn't stay, Prob} = .6 \\ 1 & \text{if recruit stays but not married, Prob} = .1 \\ 2 & \text{if stays \& married, Prob} = .3 \end{cases}$

$E(X_i) = .7$
$E(X_i^2) = 1.3$
$Var(X_i) = .81$

Thus by Continuity Correction,

$\Rightarrow P(S \leq 90) = P(S \leq 90.5) = P\left(Z \leq \frac{90.5-70}{9}\right) = .9887$

E

87 T = true ages, $\overline{T}$ = mean of true ages, R = rounded ages, $\overline{R}$ = mean of rounded ages

$\Rightarrow P(\overline{T} - .25 < \overline{R} < \overline{T} + .25) = P(-.25 < \overline{R} - \overline{T} < .25)$

$* \ \overline{R} - \overline{T} = \frac{X_1 + X_2 + \ldots + X_{48}}{48} = \overline{X}, \quad E(\overline{X}) = 0 \quad Var(\overline{X}) = \frac{1}{48}\left(\frac{.25+.25}{12}\right)^2 = \frac{25}{576}$

Thus $P\left(-\frac{.25}{\sqrt{\frac{25}{576}}} < Z < \frac{.25}{\sqrt{\frac{25}{576}}}\right) = P(-1.2 < Z < 1.2) = .77$ D

(88) X = time until first claim from good driver
Y = " " " bad

$F(x) = 1 - e^{-x/6}$, $G(y) = 1 - e^{-y/3}$

$\Rightarrow P(X \le 3 \cap Y \le 2) = P(X \le 3) \cdot P(Y \le 2)$

$= 1 - e^{-3/6} \cdot (1 - e^{-2/3})$

$= 1 - e^{-2/3} - e^{-1/2} + e^{-7/6}$ (C)

(89) (B)

(90) Want $P(Y < X)$

Let Y = time until Deluxe

$f_{X,Y}(x,y) = \frac{1}{6} e^{-x/2} e^{-y/3}$ $x > 0, y > 0$

Thus $P(Y < X) = \int_0^\infty \int_0^x \frac{1}{6} e^{-x/2} e^{-y/3} \, dy \, dx$

$= \int_0^\infty -\frac{1}{2} e^{-x/2} e^{-y/3} \Big|_0^x \, dx$

$= \int_0^\infty \frac{1}{2} e^{-x/2} - \frac{1}{2} e^{-5/6 x} \, dx$

$= -e^{-1/2 x} + \frac{3}{5} e^{-5/6 x} \Big|_0^\infty$

$= 1 - \frac{3}{5}$

$= \frac{2}{5}$ (C)

(91) $P(X + Y \ge 1) = \int_0^1 \int_{1-x}^{2} \frac{2x + 2 - y}{4} \, dy \, dx$

$= \int_0^1 \int_{1-x}^{2} \frac{1}{2}x + \frac{1}{2} - \frac{1}{4}y \, dy \, dx$

$= \int_0^1 \frac{1}{2}xy + \frac{1}{2}y - \frac{1}{8}y^2 \Big|_{1-x}^{2} \, dx$

$= \int_0^1 x + 1 - \frac{1}{2} - \frac{1}{2}x(1-x) - \frac{1}{2}(1-x) + \frac{1}{8}(1-x)^2 \, dx$

$= \int_0^1 x + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}x} + \frac{1}{2}x^2 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}x} + \frac{1}{8} - \frac{1}{4}x + \frac{1}{8}x^2 \, dx$

$= \int_0^1 \frac{5}{8}x^2 + \frac{3}{4}x + \frac{1}{8} \, dx$

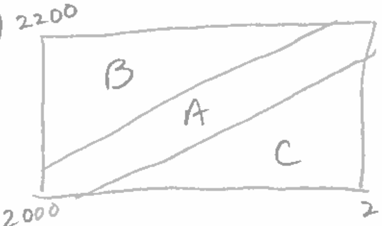
$= \frac{5}{24}x^3 + \frac{3}{8}x^2 + \frac{1}{8}x \Big|_0^1$

$= \frac{5}{24} + \frac{3}{8} + \frac{1}{8}$

$= \frac{5}{24} + \frac{9}{24} + \frac{3}{24}$

$= \frac{17}{24}$ (D)

(92)

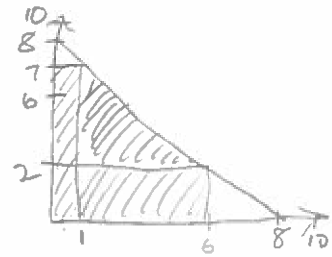


$|X-Y|<20$

$$\Rightarrow P(|X-Y|<20) = \frac{200^2 - 2(\frac{1}{2})(180)^2}{200^2} = .19 \quad \text{(B)}$$

(93) X = loss amount w/ d = 1
Y = " " " " d = 2

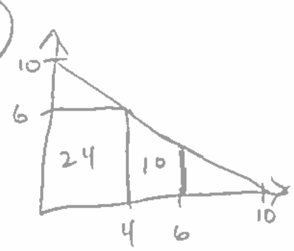
Policy 1: 0 or $X-1$
Policy 2: 0 or $Y-2$



$$\text{Total Benefit} = \begin{cases} 0 \\ X-1 \\ Y-2 \\ X+Y-3 \end{cases} \Rightarrow \begin{array}{l} X-1<5 \Rightarrow X<6 \\ Y-2<5 \Rightarrow Y<7 \\ X+Y-3<5 \Rightarrow X+Y<8 \end{array}$$

$$\text{Prob.} = \frac{6(2)+1(5)+\frac{1}{2}(5)(5)}{100} = \frac{12+5+12.5}{100} = .295 \quad \text{(C)}$$

(94)



$f(t_1, t_2) = \frac{1}{34}$

$$\Rightarrow E(T_1+T_2) = 2E(T_1) \quad \text{(by symmetry)}$$

$$= 2\left(\int_0^4 t_1 \int_0^6 \tfrac{1}{34}\, dt_2\, dt_1 + \int_4^6 t_1 \int_0^{10-t_1} \tfrac{1}{34}\, dt_2\, dt_1\right)$$

$$= 2\left(\int_0^4 t_1 \left(\tfrac{1}{34} t_2 \Big|_0^6\right) dt_1 + \int_4^6 t_1 \left(\tfrac{1}{34} t_2 \Big|_0^{10-t_1}\right) dt_1\right)$$

$$= 2\left(\int_0^4 \tfrac{3}{17} t_1\, dt_1 + \int_4^6 \tfrac{1}{34}(10t_1 - t_1^2)\, dt_1\right)$$

$$= 2\left(\tfrac{3}{34} t_1^2 \Big|_0^4 + \tfrac{1}{34}\left[5t_1^2 - \tfrac{1}{3}t_1^3\right]_4^6\right)$$

$$= 2\left(1.412 + \tfrac{1}{34}(108 - 58.667)\right)$$

$$= 5.726 \quad \text{(C)}$$

(95)
$$M(t_1,t_2) = E(e^{t_1 W + t_2 Z}) = E(e^{t_1(X+Y) + t_2(Y-X)})$$

$$= E(e^{(t_1-t_2)X} e^{(t_1+t_2)Y})$$

$$= E[e^{(t_1-t_2)X}]\, E[e^{(t_1+t_2)Y}]$$

$$= e^{1/2(t_1-t_2)^2} e^{1/2(t_1+t_2)^2}$$

$$= e^{1/2 t_1^2 - t_1 t_2 + \frac{1}{2}t_2^2 + \frac{1}{2}t_1^2 + t_1 t_2 + \frac{1}{2}t_2^2}$$

$$= e^{t_1^2 + t_2^2} \quad \text{(E)}$$

(96) Scenario 1

All 21 show up

$\Rightarrow$ Prob $= .98^{21} = .65$

Revenue $= 21 \cdot 50 - 100$
$= 950$

Scenario 2

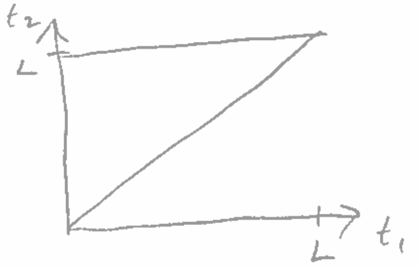
At least 1 tourist does not show up (no penalty)

$\Rightarrow$ Prob $= 1 - .65 = .35$

Revenue $= 21 \cdot 50 = 1050$

$E(\text{revenue}) = 950(.65) + 1050(.35) = 985$ (E)

(97)



Area $= \frac{1}{2}L^2 = \frac{L^2}{2}$

$f(t_1, t_2) = \frac{1}{\text{area}} = \frac{2}{L^2}$

$$\Rightarrow E(T_1^2 + T_2^2) = \int_0^L \int_0^{t_2} (t_1^2 + t_2^2) \cdot \frac{2}{L^2}\, dt_1\, dt_2$$

$$= \frac{2}{L^2} \int_0^L \frac{1}{3}t_1^3 + t_1 t_2^2 \Big|_0^{t_2} dt_2$$

$$= \frac{2}{L^2} \int_0^L \frac{1}{3}t_2^3 + t_2^3 \; dt_2$$

$$= \frac{2}{L^2} \int_0^L \frac{4}{3} t_2^3 \, dt_2$$

$$= \frac{2}{L^2} \left[\frac{1}{3} t_2^4 \right]_0^L$$

$$= \frac{2}{L^2} \left[\frac{1}{3} L^4 \right]$$

$$= \frac{2}{3} L^2 \quad \text{(C)}$$

(98) $Y = X_1 X_2 X_3$

$Y = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$ when $X = 1$

Thus $M_Y(t) = \frac{8}{27}e^t + \frac{19}{27}$ (A)

(99) $Var(X+Y) = 17000$, $Cov(X,Y) = 1000$, $Var(X) = 5000$, $Var(Y) = 10000$

$$\Rightarrow Var(X + 100 + 1.1Y) = Var(X + 1.1Y)$$

$$= Var(X) + 1.1^2 Var(Y) + 2(1.1)\,Cov(X,Y)$$

$$= 5000 + 12100 + 2200$$

$$= 19300 \quad \text{(C)}$$

(100) $E(X) = \sum_{\text{all } y} \sum_{\text{all } x} x\, P(X=x, Y=y) = 0(\frac{1}{6}) + 1(\frac{1}{12}) + 1(\frac{1}{6}) + 2(\frac{1}{12}) + 2(\frac{1}{3}) + 2(\frac{1}{6})$

$= \frac{1}{12} + \frac{1}{6} + \frac{2}{12} + \frac{2}{3} + \frac{2}{6}$

$= \frac{1}{12} + \frac{2}{12} + \frac{2}{12} + \frac{8}{12} + \frac{4}{12}$

$= \frac{17}{12}$

$E(X^2) = 0^2(\frac{1}{6}) + 1^2(\frac{1}{12}) + 1^2(\frac{1}{6}) + 2^2(\frac{1}{12}) + 2^2(\frac{1}{3}) + 2^2(\frac{1}{6})$

$= \frac{1}{12} + \frac{1}{6} + \frac{4}{12} + \frac{4}{3} + \frac{4}{6}$

$= \frac{1}{12} + \frac{2}{12} + \frac{4}{12} + \frac{16}{12} + \frac{8}{12}$

$= \frac{31}{12}$

Thus $Var(X) = E(X^2) - (E(X))^2 = \frac{31}{12} - (\frac{17}{12})^2 = .576$ (B)

(101) $Var(Z) = Var(3X - Y - 5) = 9\,Var(X) + Var(Y)$

$= 9(1) + 2$

$= 11$ (D)

(102) $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

$Var(X) = Var(Y) = 10^2$

$Var(X) + Var(Y) = 200$ (by independence) (E)

(103) $Y = \max(X_1, X_2, X_3)$

$P(Y > 3) = P(\max(X_1, X_2 X_3) > 3) = 1 - P(\max(X_1, X_2 X_3) \le 3)$

$= 1 - F_{X_1}(3) F_{X_2}(3) F_{X_3}(3)$

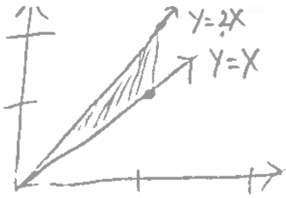
$= 1 - (1 - e^{-3/1})(1 - e^{-3/1.5})(1 - e^{-3/2.4})$

$= .414$ (E)

(104) $f(x,y) = kx \quad 0 < x < 1,\ 0 < y < 1$

$\Rightarrow Cov(X,Y) = 0$ (by independence) (B)

(105)



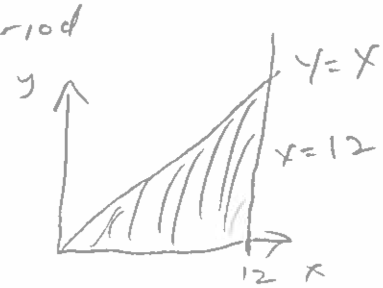
$$E(X)=\int_0^1\int_x^{2x}\frac{8}{3}x^2y\,dy\,dx=\int_0^1\frac{4}{3}x^2y^2\Big|_x^{2x}dx$$
$$=\int_0^1\frac{16}{3}x^4-\frac{4}{3}x^4\,dx$$
$$=\int_0^1 4x^4dx$$
$$=\frac{4}{5}x^5\Big|_0^1$$
$$=\frac{4}{5}$$

$$E(Y)=\int_0^1\int_x^{2x}\frac{8}{3}xy^2\,dy\,dx=\int_0^1\frac{8}{9}xy^3\Big|_x^{2x}dx$$
$$=\int_0^1\frac{64}{9}x^4-\frac{8}{9}x^4\,dx$$
$$=\int_0^1\frac{56}{9}x^4\,dx$$
$$=\frac{56}{45}x^5\Big|_0^1$$
$$=\frac{56}{45}$$

$$E(XY)=\int_0^1\int_x^{2x}\frac{8}{3}x^2y^2\,dy\,dx=\int_0^1\frac{8}{9}x^2y^3\Big|_x^{2x}dx=\int_0^1\frac{64}{9}x^5-\frac{8}{9}x^5\,dx$$
$$=\int_0^1\frac{56}{9}x^5\,dx$$
$$=\frac{56}{54}x^6\Big|_0^1$$
$$=\frac{28}{27}$$

Thus $Cov(X,Y)=E(XY)-E(X)E(Y)$
$$=\frac{28}{27}-\left(\frac{4}{5}\right)\left(\frac{56}{45}\right)$$
$$=.0415 \text{ (A)}$$

(106) X and Y: value of 2 stocks at the end of 5 year period



$X\sim Unif(0,12)\rightarrow f(x)=\frac{1}{12}\quad 0<x<12$

$Y|X\sim Unif(0,X)\rightarrow f(y|x)=\frac{1}{x}\quad 0<y<x$

$f(y|x)=\frac{f(x,y)}{f(x)}\Rightarrow f(x,y)=f(x)f(y|x)=\frac{1}{x}\cdot\frac{1}{12}=\frac{1}{12x}$

$\Rightarrow Cov(X,Y)=E(XY)-E(X)E(Y)$
$$=\int_0^{12}\int_0^x\frac{y}{12}dy\,dx-\frac{0+12}{2}\left(\int_0^{12}\int_0^x\frac{y}{12x}dy\,dx\right)$$
$$=\int_0^{12}\frac{1}{24}y^2\Big|_0^x dx-6\left(\int_0^{12}\frac{1}{24x}y^2\Big|_0^x dx\right)$$
$$=\int_0^{12}\frac{1}{24}x^2dx-6\left(\int_0^{12}\frac{1}{24}x\,dx\right)$$
$$=\frac{1}{72}x^3\Big|_0^{12}-\frac{1}{8}x^2\Big|_0^{12}$$
$$=24-18$$
$$=6 \text{ (C)}$$

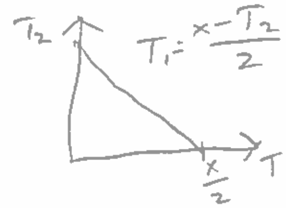
(107) $X \sim$ size of surgical claim $Y \sim$ size of associated hospital claim

$C_1 = X+Y$, $C_2 = X+1.2Y$, $Var(X+Y) = 8$, $Var(X) = Var(Y) = 2.4$

Thus, $\quad * \; (a+b)(c+d) = ac+ad+bc+b$

$$
\begin{aligned}
Cov(C_1, C_2) &= Cov(X+Y, X+1.2Y) \\
&= Cov(X,X) + Cov(X,1.2Y) + Cov(X,Y) + Cov(Y,1.2X) \quad \text{(using logic from *)} \\
&= Cov(X,X) + 1.2Cov(X,Y) + Cov(X,Y) + 1.2Cov(Y,Y) \\
&= Var(X) + 2.2\,Cov(X,Y) + 1.2\,Var(Y) \\
&= 2.4 + 2.2(1.6) + 1.2(2.4) \\
&= 8.8 \quad \text{(A)}
\end{aligned}
$$

(108) $T_1 \sim Expo(\mu=1)$, $T_2 \sim Expo(\mu=1)$



$* \; f_{T_1,T_2}(t_1,t_2) = e^{-t_1}e^{-t_2}$

$X = 2T_1 + T_2$

$$
\begin{aligned}
F_X(x) = P(X \le x) &= P(2T_1 + T_2 \le x) \\
&= P\left(T_1 \le \frac{x-T_2}{2}\right) \\
&= \iint_x f_{T_1,T_2}(t_1,t_2)\,dt_1\,dt_2 \\
&= \int_0^x \int_0^{\frac{x-t_2}{2}} e^{-t_1} e^{-t_2}\,dt_1\,dt_2 \\
&= \int_0^x -e^{-t_1} e^{-t_2} \Big|_0^{\frac{x-t_2}{2}} dt_2 \\
&= \int_0^x e^{-t_2} - e^{-\frac{1}{2}x - \frac{1}{2}t_2}\,dt_2 \\
&= \int_0^x e^{-t_2}\,dt_2 - e^{-1/2x} \int_0^x e^{-\frac{1}{2}t_2}\,dt_2 \\
&= -e^{-t_2}\Big|_0^x + 2e^{-\frac{1}{2}x} e^{-\frac{1}{2}t_2}\Big|_0^x \\
&= 1 - e^{-x} + 2e^{-1/2x}e^{-1/2x} - 2e^{-1/2x} \\
&= 1 + e^{-x} - 2e^{-1/2x}
\end{aligned}
$$

Thus $g(x) = F_X'(x) = e^{-1/2x} - e^{-x}$ (A)

(109) $X=\frac{U}{V}$, $g(u,v)=\frac{1}{2}e^{-u}e^{-v/2}$ for $u>0, v>0$, $u=vx$

$F(x)=P(U\le Vx)=\iint_A g(u,v)\,du\,dv$

$$=\int_0^\infty\int_0^{vx}\frac{1}{2}e^{-u}e^{-v/2}\,du\,dv$$

$$=\int_0^\infty -\frac{1}{2}e^{-v/2}e^{-u}\Big|_0^{vx}\,dv$$

$$=\int_0^\infty \frac{1}{2}e^{-v/2}-\frac{1}{2}e^{-v/2}e^{-vx}\,dv$$

$$=\int_0^\infty \frac{1}{2}e^{-v/2}\,dv-\int_0^\infty\frac{1}{2}e^{-\frac{1}{2}v\left(\frac{2x+1}{2}\right)}\,dv$$

$$=-e^{-v/2}\Big|_0^\infty+\frac{1}{2x+1}e^{-v}\Big|_0^\infty$$

$$=1-\frac{1}{2x+1}$$

Thus $f(x)=F'(x)=\frac{2}{(2x+1)^2}$ (B)

(110) * $f_X(x)=\int_0^{1-x}24xy\,dy \Rightarrow f_X(\frac{1}{3})=\int_0^{1-\frac{1}{3}}24(\frac{1}{3})\,y\,dy$

$$=\int_0^{2/3}8y\,dy$$

$$=4y^2\Big|_0^{2/3}$$

$$=\frac{16}{9}$$

** $f_{Y|X}(y|X=\frac{1}{3})=\frac{24(\frac{1}{3})y}{f_X(\frac{1}{3})}$

$$=\frac{8y}{\frac{16}{9}}$$

$$=\frac{9}{16}\cdot 8y$$

$$=\frac{9}{2}y$$

Thus $P(Y<X\,|\,X=\frac{1}{3})=\int_0^{1/3}\frac{9}{2}y\,dy=\frac{9}{2}\left[\frac{1}{2}y^2\right]_0^{1/3}$

$$=\frac{9}{2}\left(\frac{1}{2}\cdot\frac{1}{9}\right)$$

$$=\frac{9}{2}\cdot\frac{1}{18}$$

$$=\frac{1}{4}$$ (C)

(111) $f(2,y)=\frac{1}{2}y^{-3}$, $f_X(2)=\int_1^\infty\frac{1}{2}y^{-3}dy=-\frac{1}{4y^2}\Big|_1^\infty=\frac{1}{4}$

Thus $P(1<Y<3\,|\,X=2)=\int_1^3\frac{f_{X,Y}(2,y)}{f_X(2)}\,dy=\int_1^3 2y^{-3}\,dy$

$$=-\frac{1}{y^2}\Big|_1^3$$

$$=1-\frac{1}{9}$$

$$=\frac{8}{9}$$ (E)

(112) $f(x,y) = 2(x+y) \quad 0<x<y<1$

$f_X(x) = \int_0^x (2x+2y)\,dy = 2xy + y^2 \Big|_0^x = 3x^2 \quad 0<x<1$

$f(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{2(x+y)}{3x^2} = \frac{2}{3}\left(\frac{1}{x} + \frac{y}{x^2}\right) \quad 0<y<x$

$f(y|x=.10) = \frac{2}{3}\left(\frac{1}{.1} + \frac{y}{.01}\right) = \frac{2}{3}(10+100y) \quad 0<y<.1$

$\Rightarrow P(Y<.05|X=.10) = \int_0^{.05} \frac{2}{3}(10+100y)\,dy$

$= \frac{2}{3}(10y + 50y^2)\Big|_0^{.05}$

$= .4167$ (D)

(113) B = benefit paid

P = profit = $1000 - B$

H = husband survives ≥ 10 years

W = wife " " " "

$* \; P(H) = P(H \cap W^c) + P(H \cap W)$

Thus, $E(P) = 1000 - E(B)$

$= 1000 - 10000 \frac{P(H \cap W^c)}{P(H)}$

$= 1000 - 10000 \left(\frac{.01}{.97}\right)$ (from $*$)

$= 897$ (E)

(114) $Var(Y|X=1) = E(Y^2|X=1) - (E(Y|X=1))^2$

$E(Y|X=1) = \sum_{\text{all } y} y\,P(Y=y|X=1) = 0\,P(Y=0|X=1) + 1\,P(Y=1|X=1)$

$= P(Y=1|X=1)$

$= \frac{P(X=1, Y=1)}{P(X=1)}$

$= \frac{.125}{.125+.05}$

$= \frac{5}{7}$

$E(Y^2|X=1) = \sum_{\text{all } y} y^2 P(Y=y|X=1) = P(Y=1|X=1) = \frac{5}{7}$

Thus $Var(Y|X=1) = \frac{5}{7} - \left(\frac{5}{7}\right)^2 = .2$ (C)

(115) Note: If joint pdf is missing a variable, y, then $Y|X \sim Unif(a,b)$, $a<y<b$

$Var(Y|X=x) = E(Y^2|X=x) - (E(Y|X=x))^2$

$Y|X \sim Unif(x, x+1)$

Thus, $Var(Y|X=x) = \frac{(x+1-x)^2}{12} = \frac{1}{12}$ (A)

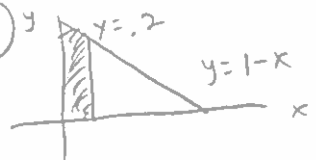
(116) $Var(Y|X=0) = E(Y^2|X=0) - (E(Y|X=0))^2$

$E(Y|X=0) = \frac{1(.06)+2(.05)+3(.02)}{.12+.06+.05+.02} = .88$

$E(Y^2|X=0) = \frac{1^2(.06)+2^2(.05)+3^2(.02)}{.12+.06+.05+.02} = 1.76$

Thus $Var(Y|X=0) = 1.76 - .88^2 = .99$ (D)

(117)



Thus $P(X<.2) = \int_0^{.2}\int_0^{1-x} 6[1-(x+y)]\,dy\,dx$

$= \int_0^{.2}\int_0^{1-x} 6-6x-6y \; dy\,dx$

$= \int_0^{.2} 6y - 6xy - 3y^2 \Big|_0^{1-x} dx$

$= \int_0^{.2} 6(y - xy - \frac{1}{2}y^2)\Big|_0^{1-x} dx$

$= \int_0^{.2} 6[1-x - x(1-x) - \frac{1}{2}(1-x)^2]\,dx$

$= \int_0^{.2} 6[1-x-x+x^2-\frac{1}{2}+x-\frac{1}{2}x^2]\,dx$

$= \int_0^{.2} 6(\frac{1}{2}x^2 - x + \frac{1}{2})\,dx$

$= \int_0^{.2} 3x^2 - 6x + 3 \; dx$

$= x^3 - 3x^2 + 3x \Big|_0^{.2}$

$= .008 - .12 + .6$

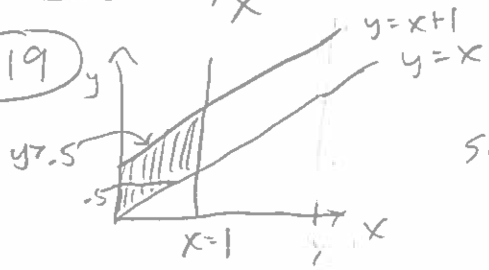
$= .488$ (C)

(118) $g(y) = \int_y^{\sqrt{y}} 15y\,dx = 15xy\Big|_y^{\sqrt{y}} = 15y^{3/2} - 15y^2 = 15y^{3/2}(1-y^{1/2})$

Since $y=x$ at $y=1 \wedge x=1$

then $g(y) = 15y^{3/2}(1-y^{1/2})$, $0<y<1$ (E)

(119)



$\Rightarrow P(Y>.5) = \frac{\text{Area}(Y>.5)}{\text{Area}(\text{Domain})}$

Since area(domain) = 1

$\Rightarrow P(Y>.5) = 1 - P(Y<.5)$

$= 1 - \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$ (D)

(120) Y = time to process claim

$Y|X \sim \text{Unif}(x, 2x)$

$* \; f_{X,Y}(x,y) = f_X(x)\cdot f_{Y|X}(y|X=x)$

$= \frac{3}{8}x^2 \cdot \frac{1}{x}$

$= \frac{3}{8}x$

Thus, $P(Y>3) = \int_{\frac{3}{2}}^{2}\int_3^{2x} \frac{3}{8}x\,dy\,dx$

$= \int_{3/2}^{2} \frac{3}{8}xy\Big|_3^{2x}\,dx$

$= \int_{3/2}^{2} \frac{3}{4}x^2 - \frac{9}{8}x\,dx$

$= \frac{1}{4}x^3 - \frac{9}{16}x^2\Big|_{3/2}^{2}$

$= 2 - \frac{36}{16} - \frac{1}{4}\left(\frac{27}{8}\right) + \frac{9}{16}\left(\frac{9}{4}\right)$

$= .172$ (A)

(121) $E(X) = \frac{1}{64}\int_2^{10}\int_0^1 10x - x^2y^2 \, dy \, dx = \frac{1}{64}\int_2^{10} 10xy - \frac{1}{3}x^2y^3 \Big|_0^1 dx$

$= \frac{1}{64}\int_2^{10} 10x - \frac{1}{3}x^2 \, dx$

$= \frac{1}{64}\left(5x^2 - \frac{1}{9}x^3\right)\Big|_2^{10}$

$= \frac{1}{64}\left(500 - \frac{1000}{9} - 20 + \frac{8}{9}\right)$

$= 5.\overline{7}$ (C)

(122) $f_Y(y) = \int_0^y f(x,y)\,dx = \int_0^y 6e^{-x}e^{-2y}\,dx$

$= 6e^{-2y}\int_0^y e^{-x}\,dx$

$= 6e^{-2y}(-e^{-x})\Big|_0^y$

$= 6e^{-2y} - 6e^{-3y}$

Thus $E(Y) = \int_0^\infty y\, f_Y(y)\,dy = \int_0^\infty 6ye^{-2y} - 6ye^{-3y}\,dy$

$= 3\int_0^\infty 2ye^{-2y}\,dy - 2\int_0^\infty 3ye^{-3y}\,dy$

$= 3\left(\frac{1}{2}\right) - 2\left(\frac{1}{3}\right)$ ($y \sim$ Expo)

$= \frac{3}{2} - \frac{2}{3}$

$= .83$ (D)

(123) $P(4<S<8) = P(4<S<8 \mid N=1)P(N=1) + P(4<S<8 \mid N>1)P(N>1)$

$= (F(8) - F(4))\frac{1}{3} + (G(8) - G(4))\frac{1}{6}$

$= \left[(1-e^{-8/5}) - (1-e^{-4/5})\right]\frac{1}{3} + \left[(1-e^{-1}) - (1-e^{-1/2})\right]\frac{1}{6}$

$= .12$ (C)

(124) X and Y are independent, $f_{X,Y}(x,y) = g(x)\cdot h(y)$, Range of x must not depend on y and vice versa

$Var(Y \mid X>3, Y>3) = Var(Y \mid Y>3)$

$f_{X,Y}(x,y) = f_X(x)f_Y(y) = e^{-x}(2e^{-2y})$

$\Rightarrow Y \sim Expo(\mu = \frac{1}{2})$ $X \sim Expo(\mu = 1)$

By Memoryless Property,

$Y-3 \mid Y>3 \sim Expo(\mu = \frac{1}{2})$

s.t. $Y-3 \mid Y>3 = (Y \mid Y>3) - 3$

Thus, $Var(Y-3 \mid Y>3) = Var(Y \mid Y>3) + 0$

$= Var(Y \mid Y>3)$

$= \left(\frac{1}{2}\right)^2$

$= .25$ (A)

(125) $f(y|x)=\frac{1}{x}$ $\quad 0<y<x$
$f(x)=2x$ $\quad 0<x<1$

$\Rightarrow f(y|x)=\frac{f(x,y)}{f(x)}$

$\Rightarrow \frac{1}{x}=\frac{f(x,y)}{2x}$

$\Rightarrow f(x,y)=2 \quad 0<y<x<1$

Thus $f(x|y)=\frac{f(x,y)}{f(y)}$

$=\frac{2}{\int_y^1 2\,dx}$

$=\frac{2}{2x|_y^1}$

$=\frac{2}{2-2y}$

$=\frac{1}{1-y}$ (E)

(126) * $p_n-p_{n+1}=c$ for $n=0,1,2,3,4$
* $P(N<2)=p_0+p_1=.4$

* $P(N>3)=p_4+p_5$
$n=0$: $p_0-p_1=c \Rightarrow p_1=p_0-c$
$n=1$: $p_1-p_2=c \Rightarrow p_2=p_0-2c$
$n=2$: $p_2-p_3=c \Rightarrow p_3=p_0-3c$

$n=5$ $p_4-p_5=c \Rightarrow p_5=p_0-5c$
* $p_0+p_1+p_2+p_3+p_4+p_5=1$
$\Rightarrow 6p_0-15c=1$

Calculate c
Note that $P(N<2)=p_0+p_1=.4$
$\Rightarrow p_0+p_0-c=.4$
$\Rightarrow 2p_0-c=.4$
Now solve for c using following system of linear equations.
$2p_0-c=.4$
$6p_0-15c=1$

$$\left(\begin{array}{cc|c} 2 & -1 & .4 \\ 6 & -15 & 1 \end{array}\right) \xrightarrow{R_2=r_2-3r_1} \left(\begin{array}{cc|c} 2 & -1 & .4 \\ 0 & -12 & .2 \end{array}\right) \Rightarrow$$

$-12c=.2 \Rightarrow c=\frac{1}{60}$
$2p_0-\frac{1}{60}=.4 \Rightarrow p_0=\frac{5}{24}$

Thus $P(N>3)=p_4+p_5=(p_0-4c)+(p_0-5c)$
$=\frac{5}{24}-\frac{4}{60}+\frac{5}{24}-\frac{5}{60}$
$=.27$ (C)

(127) X = loss amt, $X \sim \text{Unif}(0, 20000)$ S = total payout

Y = payout $Y = \begin{cases} 0 & \text{if } X \le 5000 \\ X-5000 & \text{if } X > 5000 \end{cases}$ $S = Y_1 + \dots + Y_{200} \sim$ Normal

$E(S) = 200E(Y)$

$Var(S) = 200\,Var(Y)$

$E(Y) = \int_d^\infty (x-d) f_X(x)\,dx = \int_{5000}^{20000} (x-5000)\left(\frac{1}{20000}\right) dx$

$= 5625$

$E(Y^2) = 56250000$

$Var(Y) = 26609375$

$\Rightarrow E(S) = 1125000$

$\Rightarrow Var(S) = 1125000$

Thus $P(-1.78 < Z < 1.07) = P(Z < 1.07) - P(Z < -1.78)$

$= .8202$ (D)

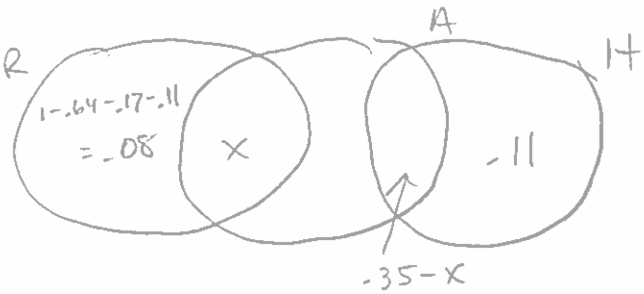
(128) i) $P(A \cup H \cup R)^c = .17$

ii) $P(A) = .64$

iii) $P(H) = 2P(R)$

iv) $P(A \cap H) + P(A \cap R) + P(H \cap R) = .35$

v) $P(H \cap A^c) = .11$



$P(H) = .11 + .35 - x$

$P(R) = .08 + x$

$\Rightarrow .46 - x = 2(.08 + x)$ (from iii)

$\Rightarrow x = .10$ (B)

(129) $X \sim \text{Expo}(\mu = 100)$

Y = reimbursement amount

$Y = \begin{cases} 0 & \text{if } x \le 20 \\ X-20 & \text{if } 20 \le X \le 120 \\ 100 + .5(X-120) & \text{if } X > 120 \end{cases}$

For $Y = 115$,

$115 = 100 + .5(X - 120)$

$\Rightarrow X = 150$ for $Y > 0$, $X > 20$

Thus $G(115) = P(Y \le 115 \mid Y > 0) = P(X \le 150 \mid X > 20)$

$= \dfrac{F_X(150) - F_X(20)}{1 - F_X(20)}$

$= \dfrac{1 - e^{-1.5} - (1 - e^{-.2})}{1 - (1 - e^{-.2})}$

$= \dfrac{.5956}{.8187}$

$= .727$ (B)

(130) * $E[e^{tx}]$

$$E[100(.5)^x] = 100E[(.5)^x]$$
$$= 100E[e^{x\ln(.5)}]$$
$$= 100M_x[\ln(.5)]$$
$$= 100\left(\frac{1}{1-2\ln(.5)}\right)$$
$$= 41.9 \quad C$$

(131) $$p_{N_1}(2) = \sum_{n_2=1}^{\infty} \frac{3}{4}\left(\frac{1}{4}\right)e^{-2}(1-e^{-2})^{n_2-1} = \frac{3e^{-2}}{16}\frac{1}{1-(1-e^{-2})} = \frac{3}{16}$$

$$p_1(n_2 | N_1=2) = \frac{p(2,n_2)}{p_{N_1}(2)} = \frac{3}{4}\frac{1}{4}e^{-2}(1-e^{-2})^{n_2-1}\left(\frac{16}{3}\right)$$
$$= e^{2}(1-e^{-2})^{n_2-1}$$

$\Rightarrow$ Mean is $\frac{1}{e^{-2}} = e^2$ E (Geometric prob. function)

(132) # defective in A = .2·30 = 6
" " B = .08(50) = 4
Total defective = 10

X = # chosen that are defective
X ~ hypergeometric

Thus $P(X=x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$

m = losses
N − m = total failures
n = choose from N = 80 modems

$$\Rightarrow P(X=2) = \frac{\binom{10}{2}\binom{70}{3}}{\binom{80}{5}} = .102 \quad C$$

(133) Payment = $\begin{cases} 5000 & \text{for } T<50 \\ 0 & \text{for } T\geq 50 \end{cases}$

Thus $$E(P) = 5000\,P(T<50 | T>40)$$
$$= 5000\frac{P(40<T<50)}{P(T>40)}$$
$$= 5000\frac{F(50)-F(40)}{1-F(40)}$$
$$= 348 \quad B$$

(134) K = king, Q = queen, T = twin

$K = 3T \quad Q = \frac{1}{4}(K+T) = \frac{1}{4}(3T+T) = T$

$K + Q + T = 1$
$\Rightarrow 3T + T + T = 1$
$\Rightarrow T = \frac{1}{5}$

Since $K = \frac{3}{5}$, $Q = \frac{1}{5}$
$\Rightarrow P(K \cup Q) = \frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ C

(135) $N \sim Pois(\lambda),\ \lambda \sim Unif(0,3)$

$$Var(N) = E[Var[N|\lambda]] + Var[E[N|\lambda]]$$
$$= E(\lambda) + Var(\lambda)$$
$$= \frac{3+0}{2} + \frac{(3-0)^2}{12}$$
$$= 2.25 \quad E$$

(136) $E(X|Y=2) = E(X|X=1)P(X=1|Y=2) + E(X|X\geq 3)\cdot P(X\geq 3|Y=2)$

$$= 1(\tfrac{1}{5}) + E(X|X\geq 3)(\tfrac{4}{5})$$
$$= 1(\tfrac{1}{5}) + (\tfrac{1}{\frac{1}{6}} + 2)\tfrac{4}{5} \qquad (E(X|X\geq 3) \sim Geom(p=\tfrac{1}{6}))$$
$$= \tfrac{1}{5} + \tfrac{32}{5}$$
$$= 6.6 \quad D$$

(137) $M_{X+Y}(t) = e^{-2t}P(X+Y=-2) + \dots$

$\Rightarrow X+Y=-2 \Rightarrow X=Y=-1$ (i.i.d.)

$\Rightarrow P(X+Y=-2) = P(X=-1)\cdot P(Y=-1) = \sqrt{.09} = .3$

Similarly $P(X+Y=2) = .3$

$\Rightarrow P(X=0) = .4$

Thus $P(X\leq 0) = .4 + .3 = .7$ E

(138) $T = \max(X,Y) = \begin{cases} X & \text{if } x>y \\ Y & \text{if } y>x \end{cases}$

$$E(T) = E(X|x>y) + E(Y|y>x) = \int_0^5\int_y^{10-y}\frac{1}{50}x\,dx\,dy + \int_0^5\int_x^{10-x}\frac{1}{50}y\,dy\,dx$$
$$= \int_0^5 \frac{1}{100}x^2\Big|_y^{10-y}dy + \int_0^5\frac{1}{100}y^2\Big|_x^{10-x}dx$$
$$= \int_0^5\frac{1}{100}((10-y)^2-y^2)dy + \int_0^5\frac{1}{100}((10-x)^2-x^2)dx$$
$$= \frac{1}{100}\int_0^5 100-20y\,dy + \frac{1}{100}\int_0^5 100-20x\,dx$$
$$= \frac{1}{100}(100y-10y^2)\Big|_0^5 + \frac{1}{100}(100x-10x^2)\Big|_0^5$$
$$= 2.5 + 2.5$$
$$= 5 \quad D$$

(139) $N = \#$ people hospitalized, $L =$ total loss

$P(L<1 \mid N=0) = 1$
$P(L<1 \mid N=1) = 1$
$P(L<1 \mid N=2) = .5$

$* \; P(L<1) = .49(1) + .42(1) + .09(.5)$
$= .955$

Thus,

$$E(N \mid L<1) = \sum_{n=0}^{2} n \cdot P(N=n \mid L<1)$$
$$= \frac{P(L<1 \mid N=n)P(N=n)}{P(L<1)}$$
$$= 0 \cdot \frac{1(.49)}{.955} + 1 \frac{1(.42)}{.955} + 2 \frac{(.5)(.09)}{.955}$$
$$= .534 \quad \text{(B)}$$

(140) Binomial has fixed # of trials
Neg. Bin has fixed # of successes
$X = \#$ hurricanes it takes for 2 occurences of damage

$$P(X=n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}$$
$$\Rightarrow P(X=n) = \binom{n-1}{1} .4^1 .6^{n-1} (.4)$$

Max occurs at $n=3$ (B)

(141) $\binom{5}{3} 6 \cdot 5 \cdot 4 = 1200$ (C)

(142)

Class	Low Risk	High Risk
# policy	400	600
P(no accidents)	.9	.8
Bonus per year	5(12)	5(12)

$$\Rightarrow E(\text{bonus}) = 400(.9)(60) + 600(.8)(60) = 50400 \quad \text{(B)}$$

(143) $L =$ filing liability claim, $P_1 =$ property claim

$P(L) = .04$, $P(P_1) = .10$, $P(L \cap P_1^c) = .01$

$$P(L \cup P_1) = P(L \cap P_1^c) + P(P_1) = .01 + .10 = .11$$

Thus $P(L \cup P_1)^c = 1 - .11 = .89$ (E)

(144) (E)

(145) $Var(Y|X=.75) = E(Y^2|X=.75) - (E(Y^2|X=.75))^2$

$$f_X(.75) = \int_0^1 f_{X,Y}(X=.75y)\,dy = \int_0^{.5} 1.5\,dy + \int_{.5}^1 .75\,dy$$

$$= 1.5y\Big|_0^{.5} + .75y\Big|_{.5}^1$$

$$= .75 + .375$$

$$= 1.125$$

$$f_{Y|X}(y|X=.75) = \begin{cases} 1.50/1.125 = 4/3 & \text{for } 0<y<.5 \\ .75/1.125 = 2/3 & \text{for } .5<y<1 \end{cases}$$

$$E(Y|X=.75) = \int_0^{.5} \tfrac{4}{3}y\,dy + \int_{.5}^1 \tfrac{2}{3}y\,dy = \tfrac{2}{3}y^2\Big|_0^{.5} + \tfrac{1}{3}y^2\Big|_{.5}^1$$

$$= \tfrac{2}{3}\left(\tfrac{1}{4}\right) + \tfrac{4}{12} - \tfrac{1}{12}$$

$$= \tfrac{5}{12}$$

$$E(Y^2|X=.75) = \int_0^{.5} \tfrac{4}{3}y^2\,dy + \int_{.5}^1 \tfrac{2}{3}y^2\,dy = \tfrac{4}{9}y^3\Big|_0^{.5} + \tfrac{2}{9}y^3\Big|_{.5}^1$$

$$= \tfrac{4}{9}\left(\tfrac{1}{8}\right) + \tfrac{16}{72} - \tfrac{2}{72}$$

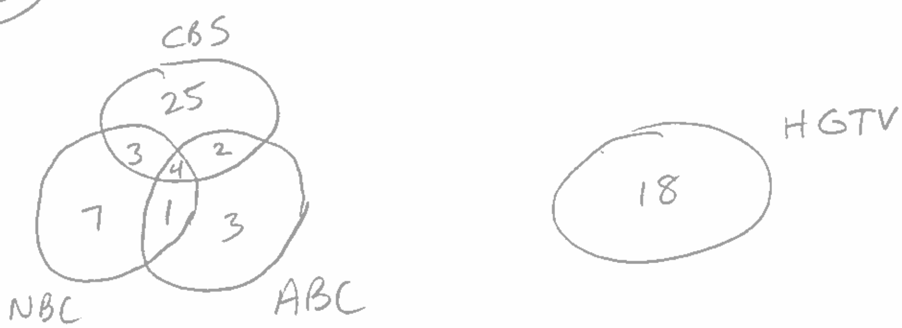
$$= \tfrac{18}{72}$$

$$= \tfrac{1}{4}$$

Thus

$$Var(Y|X=.75) = \tfrac{1}{4} - \left(\tfrac{5}{12}\right)^2 = .076 \quad \text{(C)}$$

(146)



Thus

$$N(CBS \cup NBC \cup ABC \cup HGTV)^c = 100 - (25+3+4+2+7+1+3+18)$$

$$= 100 - 63$$

$$= 37 \quad \text{(B)}$$

(147) X = claim payment w/o deductible $\quad Y = \begin{cases} 0 & \text{if } X \le d \\ X-d & \text{if } X > d \end{cases}$

Y = " " w/ deductible

$E(X) = \theta$

$Var(X) = \theta^2$

$E(X^2) = 2\theta^2$

$E(Y) = 0P(X \le d) + E(X-d \mid X>d) \cdot P(X>d) = \theta P(\theta > d) = (1-.1)E(X)$
$= .9\theta$

$E(Y^2) = .9E(X^2) = 1.8\theta^2$

$Var(Y) = 1.8\theta^2 - (.9\theta)^2 = .99\theta^2$

$\Rightarrow$ 1% reduction (A)

(148) N = # hurricanes, $N \sim Pois(\lambda = 4)$

X_i = loss amount for hurricane i, $X_i \sim Expo(\theta = 1000)$

$S|N = \sum_{i=1}^{N} X_i$

$*\ Var(S) = E(Var(S|N)) + Var(E(S|N))$

$Var(S|N) = Var\left(\sum_{i=1}^{N} X_i\right) = N\,Var(X_i) = N(1000^2) = 1000000N$

$E(S|N) = E\left(\sum_{i=1}^{N} X_i\right) = N\,E(X_i) = 1000N$

From $*$,

$Var(S) = E(1000000N) + Var(1000N) = 1000000(4) + 1000^2(4)$
$= 8000000$ (C)

(149) N = # of accidents, $N \sim Bin(n=3, p=.25)$

X_i : loss amount for i, $X \sim Expo(\theta = .8)$

$S|N = \sum_{i=1}^{N} X_i$

$T = .3S$, T = total unreimbursed loss

$*\ Var(T|N) = Var\left(.3\sum_{i=1}^{N} X_i\right)$
$= .3^2\,Var\left(\sum_{i=1}^{N} X_i\right)$
$= .3^2 N\,Var(X_i)$
$= .8^2 .3^2 N$
$= .0576N$

$E(T|N) = E\left(.3\sum_{i=1}^{N} X_i\right)$
$= .3E(X_i)N$
$= .24N$

Thus,

$Var(T) = E(Var(T|N)) + Var(E(T|N))$
$= E(.0576N) + Var(.24N)$
$= .0576E(N) + .24^2 Var(N)$
$= .0576(3 \cdot .25) + .24^2(3(.25).75)$
$= .0432 + .0324$
$= .0756$ (B)

(150) Y = payout amount

X = unconditional loss amount

K = loss amount given accident occured, $K = X|X>0 \sim \text{Expo}(\theta = 3000)$

* $F_X(x) = P(X=0) + F_K(x)\,P(X>0)$

$\Rightarrow F_X(x) = .8 + F_K(x)$

$\Rightarrow F_X(\pi_{X,.95}) = .8 + F_K(\pi_{X,.95})$

$\pi_{Y,.95} = \pi_{X,.95} - 500$

$\Rightarrow .8 + .2F_K(\pi_{X,.95}) = .95$ since $P(X \le \pi_{X,.95}) = .95$

$\Rightarrow F_K(\pi_{X,.95}) = .75$

$\Rightarrow 1 - e^{-\pi_{X,.95}/3000} = .75$

$\Rightarrow \pi_{X,.95} = 4159$

Thus $Y = 4159 - 500 = 3659$ (B)

(151) X = # pieces of damaged luggage insured, X ~ hypergeometric

$P(X=1) = 2P(X=0)$

$$\Rightarrow \frac{P(X=1)}{P(X=0)} = 2 = \frac{\binom{m}{1}\binom{27-m}{3}\Big/\binom{27}{4}}{\binom{m}{0}\binom{27-m}{4}\Big/\binom{27}{4}} = \frac{\dfrac{m(27-m)!}{3!(24-m)!}}{\dfrac{1\cdot(27-m)!}{4!(23-m)!}} = \frac{m\cancel{(27-m)!}}{3!(24-m)!}\cdot\frac{4!(23-m)!}{\cancel{(27-m)!}}$$

$$\Rightarrow \frac{4m}{24-m} = 2 \qquad\qquad * \; P(X=x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$$

$\Rightarrow 4m = 48 - 2m$

$\Rightarrow 6m = 48$

$\Rightarrow m = 8$

Thus,

$$P(X=2) = \frac{\binom{8}{2}\binom{19}{2}}{\binom{27}{4}} = .27 \text{ (C)} \quad \text{(from *)}$$

(152) $P(\text{Rahul examines } n \text{ polices} \cap \text{Toby examines} > n) = \sum_{\text{all } n} P(\text{Rahul})\cdot P(\text{Toby})$

$$= \sum_{\text{all } n} (1-.1)^{n-1}\cdot .1(1-.2)^n$$

$$= \sum_{\text{all } n} .1\cdot\frac{.9^n}{.9}\cdot\frac{.8^n}{1}$$

$$= \frac{1}{9}\sum_{\text{all } n}(.72)^n$$

$$= \frac{1}{9}\sum_{n=1}^{\infty}(.72)^n$$

$$= \frac{1}{9}\left(\frac{.72}{1-.72}\right)$$

$$= .2857 \text{ (A)}$$

(153) $X, Y \sim Pois$

$E(X) = \lambda_X$

$E(X^2) = \lambda_X + \lambda_X^2$

$E(Y) = \lambda_Y$

$E(Y^2) = \lambda_Y + \lambda_Y^2$

$\lambda_X = \lambda_Y - 8$ (from ii)

$\lambda_X + \lambda_X^2 = .6(\lambda_Y + \lambda_Y^2)$ (from iii)

Since $Var(Y) = \lambda_Y$, solve for λ_Y

By substitution,

$\lambda_Y - 8 + (\lambda_Y - 8)^2 = .6\lambda_Y + .6\lambda_Y^2$

$\lambda_Y - 8 + \lambda_Y^2 - 16\lambda_Y + 64 = .6\lambda_Y + .6\lambda_Y^2$

$.4\lambda_Y^2 - 15.6\lambda_Y + 56 = 0$

$\lambda_Y = \frac{15.6 \pm \sqrt{(-15.6)^2 - 4(.4)(56)}}{2(.4)} = \frac{15.6 \pm 12.4}{.8} = 4, 35$

Since λ_X is negative when $\lambda_Y = 4$, then $\lambda_Y = 35$ (E)

(154) $N = \#$ red sectors

$P(\text{winning}) = \frac{\text{blue area}}{\text{total area}} = 1 - \text{red area}$

$P(\text{winning}) < .2$

$*$ Geom $\sim a + ar + \ldots + ar^n = \frac{a - ar^{n+1}}{1-r}$

$\text{Red Area} = \frac{9}{20} + \left(\frac{9}{20}\right)^2 + \ldots + \left(\frac{9}{20}\right)^N$

$= \frac{\frac{9}{20} - \left(\frac{9}{20}\right)^{N+1}}{1 - \frac{9}{20}}$ (from $*$)

$= \frac{9}{11}\left[1 - \left(\frac{9}{20}\right)^N\right]$

N	P(winning
3	25.6%
4	21.5%
5	19.7%
6	18.9%
7	18.5%

Thus, minimum number of red sectors making chance of player winning less than 20% occurs at $N = 5$. (C)

(155) $E(X^4) = \int_0^{10} \frac{1}{10} x^4 dx = \frac{1}{50} x^5 \Big|_0^{10} = 2000$

For $Y = 0$ and $Y = 10$, $P(Y) = \frac{1}{20}$

For $Y = 1, 2, \ldots, 9$, $P(Y) = \frac{1}{10}$

$E(Y^4) = \frac{1^4 + 2^4 + 3^4 + \ldots + 9^4}{10} + \frac{10^4}{20} = 2033.3$

Thus, $E(Y^4) - E(X^4) = 2033.3 - 2000 = 33.3$ (B)

156 $P(2 \text{ losses in } 2 \text{ years}) = P(X=1, Y=1) + P(X=2, Y=0) + P(X=0, Y=2)$

$P(X=1, Y=1) = P(Y=1 | X=1) P(X=1) = .3(.5)^{1+1} = .075$

$P(X=2, Y=0) = P(Y=0 | X=2) P(X=2) = .25(.5)^3 = .03125$

$P(X=0, Y=2) = P(Y=2 | X=0) P(X=0) = .05(.5)^1 = .025$

Thus $P(2 \text{ losses in } 2 \text{ years}) = .075 + .03125 + .025 = .131$ E

157 $E(X) = \int_1^\infty x^{1-p}(p-1)\,dx = p-1 \int_1^\infty x^{1-p}\,dx$

$$= p-1 \left[\frac{x^{2-p}}{2-p} \right]_1^\infty$$

$$= p-1 \left[-\frac{1}{2-p} \right]$$

$$= \frac{1}{2-p} - \frac{p}{2-p}$$

$$= \frac{1-p}{2-p}$$

thus $E(X) = 2 = \frac{1-p}{2-p}$

$\Rightarrow 2(2-p) = 1-p$

$\Rightarrow 4 - 2p = 1 - p$

$\Rightarrow p = 3$ C

158 $P(X=0) = .5$

$P(0 < X < 2) = 0$

$P(2 < X < 3) = .5$

Thus $E(X) = 0\,P(X=0) + \int_2^3 .5x\,dx = \frac{1}{4}x^2\Big|_2^3 = \frac{9}{4} - \frac{4}{4} = \frac{5}{4}$ D

159 X = abs value of difference of two numbers on the dice

$\boxed{(1,1)}$	$\boxed{(1,2)}$	$\boxed{(1,3)}$	$(1,4)$	$(1,5)$	$(1,6)$
$\boxed{(2,1)}$	$\boxed{(2,2)}$	$\boxed{(2,3)}$	$\boxed{(2,4)}$	$(2,5)$	$(2,6)$
$\boxed{(3,1)}$	$\boxed{(3,2)}$	$\boxed{(3,3)}$	$\boxed{(3,4)}$	$\boxed{(3,5)}$	$(3,6)$
$(4,1)$	$\boxed{(4,2)}$	$\boxed{(4,3)}$	$\boxed{(4,4)}$	$\boxed{(4,5)}$	$\boxed{(4,6)}$
$(5,1)$	$(5,2)$	$\boxed{(5,3)}$	$\boxed{(5,4)}$	$\boxed{(5,5)}$	$\boxed{(5,6)}$
$(6,1)$	$(6,2)$	$(6,3)$	$\boxed{(6,4)}$	$\boxed{(6,5)}$	$\boxed{(6,6)}$

$P(X < 3) = \frac{24}{36} = \frac{2}{3}$ E

(160) $\rho(M,N) = .64 = \dfrac{Cov(M,N)}{\sqrt{Var(M)Var(N)}}$

$\Rightarrow .64 = \dfrac{Cov(M,N)}{\sqrt{1600 \cdot 900}}$

$\Rightarrow Cov(M,N) = 768$

Thus $Var(M+N) = Var(M) + Var(N) + 2\,Cov(M,N)$

$= 1600 + 900 + 2(768)$

$= 4036$ (D)

(161) $E(Y) = \int_d^{d+u} S_X(x)\,dx = \int_1^6 e^{-x/2}\,dx = -2\left(e^{-x/2}\right)\Big|_1^6 = 2e^{-1/2} - 2e^{-3}$ (C)

(162) $Var\left(\frac{X+Y}{2}\right) = E\left(\frac{X^2+2XY+Y^2}{4}\right) - \left(E\left(\frac{X+Y}{2}\right)\right)^2$

$= \frac{1}{4}E(X^2) + \frac{1}{2}E(XY) + \frac{1}{4}E(Y^2) - \left(\frac{1}{2}E(X) + \frac{1}{2}E(Y)\right)^2$

$E(X^2) = \int_0^2\int_0^2 \frac{1}{8}x^3 + \frac{1}{8}x^2y\,dy\,dx = \int_0^2 \frac{1}{8}x^3y + \frac{1}{16}x^2y^2\Big|_0^2 dx$

$= \int_0^2 \frac{1}{4}x^3 + \frac{1}{4}x^2\,dx$

$= \frac{1}{16}x^4 + \frac{1}{12}x^3\Big|_0^2$

$= \frac{5}{3}$

$E(X) = \int_0^2\int_0^2 \frac{1}{8}x^2 + \frac{1}{8}xy\,dy\,dx$

$= \int_0^2 \frac{1}{8}x^2y + \frac{1}{16}xy^2\Big|_0^2 dx$

$= \int_0^2 \frac{1}{4}x^2 + \frac{1}{4}x\,dx$

$= \frac{1}{12}x^3 + \frac{1}{8}x^2\Big|_0^2$

$= \frac{7}{6}$

$E(Y^2) = \int_0^2\int_0^2 \frac{1}{8}xy^2 + \frac{1}{8}y^3\,dy\,dx = \int_0^2 \frac{1}{24}xy^3 + \frac{1}{32}y^4\Big|_0^2 dx$

$= \int_0^2 \frac{1}{3}x + \frac{1}{2}\,dx$

$= \frac{1}{6}x^2 + \frac{1}{2}x\Big|_0^2$

$= \frac{5}{3}$

$E(Y) = \int_0^2\int_0^2 \frac{1}{8}xy + \frac{1}{8}y^2\,dy\,dx$

$= \int_0^2 \frac{1}{16}xy^2 + \frac{1}{24}y^3\Big|_0^2 dx$

$= \int_0^2 \frac{1}{4}x + \frac{1}{3}\,dx$

$= \frac{1}{8}x^2 + \frac{1}{3}x\Big|_0^2$

$= \frac{7}{6}$

$E(XY) = \int_0^2\int_0^2 \frac{1}{8}x^2y + \frac{1}{8}xy^2\,dy\,dx = \int_0^2 \frac{1}{16}x^2y^2 + \frac{1}{24}xy^3\Big|_0^2 dx$

$= \int_0^2 \frac{1}{4}x^2 + \frac{1}{3}x\,dx$

$= \frac{1}{12}x^3 + \frac{1}{6}x^2\Big|_0^2$

$= \frac{4}{3}$

Thus $Var\left(\frac{X+Y}{2}\right) = \frac{1}{4}\left(\frac{5}{3}\right) + \frac{1}{2}\left(\frac{4}{3}\right) + \frac{1}{4}\left(\frac{5}{3}\right) - \left(\frac{1}{2}\left(\frac{7}{6}\right) + \frac{1}{2}\left(\frac{7}{6}\right)\right)^2$

$= \frac{30}{72} + \frac{48}{72} + \frac{30}{72} - \left(\frac{7}{6}\right)^2$

$= \frac{10}{72}$ (A)

(163) $P(X>N)>.1$, Want to find $P(X>N+1)<.1$

Let $Y \sim N(\mu = np = 20, \sigma^2 = np(1-p) = 10)$

By Continuity Correction,

$P(X>N) \approx P(Y>N+.5) = P\left(\frac{Y-\mu}{\sigma} > \frac{N+.5-20}{\sqrt{10}}\right)$

$P(X>N+1) \approx P\left(\frac{Y-\mu}{\sigma} > \frac{N+1+.5-20}{\sqrt{10}}\right)$

$\Rightarrow \frac{N-19.5}{\sqrt{10}} = 1.282$

$\Rightarrow N = 23.554$ (A)

(164) $X \sim Pois(\lambda = 1+1+.5+.5+.5 = 3.5)$

Thus $P(X=2) = \frac{e^{-3.5}3.5^2}{2!} = .185$ (B)

(165) * $SD(X+Y) = \sqrt{Var(X)+Var(Y)+2Cov(X,Y)} = \sqrt{2Var(X)}$ (i.i.d.)

$M_X(t) = (1-1.5t)^{-2}$

$M_X'(t) = -2(1-1.5t)^{-3} \cdot -1.5 = 3(1-1.5t)^{-3}$

$M_X''(t) = -9(1-1.5t)^{-4} \cdot -1.5 = \frac{27}{2}(1-1.5t)^{-4}$

$Var(X) = E(X^2) - (E(X))^2 = M_X''(0) - (M_X'(0))^2 = \frac{27}{2} - 3^2 = 4.5$

Thus $SD(X+Y) = \sqrt{2(4.5)} = 3$ (B)

(166) X = snowfall in inches, Y = payment

Y	0	300	600	700
$P(Y=y)$	$.06+.18+.26+.22=.72$	.14	.06	$.04+.04=.08$

$E(Y) = .72(0) + .14(300) + .06(600) + .08(700) = 134$

$E(Y^2) = .72(0)^2 + .14(300)^2 + .06(600)^2 + .08(700)^2 = 73400$

$Var(Y) = 73400 - 134^2 = 55444$

Thus $SD(Y) = \sqrt{Var(Y)} = 235$ (B)

(167) $P(X>10 | X>2) = \frac{P(X>10 \cap X>2)}{P(X>2)} = \frac{P(X>10)}{P(X>2)} = \frac{c\int_{10}^{20} x^2 - 60x + 800\, dx}{c\int_{2}^{20} x^2 - 60x + 800\, dx}$

$= \frac{\frac{1}{3}x^3 - 30x^2 + 800x \Big|_{10}^{20}}{\frac{1}{3}x^3 - 30x^2 + 800x \Big|_{2}^{20}}$

$= \frac{6666.67 - 5333.33}{6666.67 - 1482.67}$

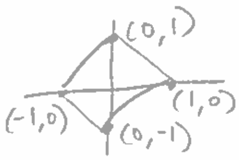
$= .257$ (D)

(168) C is false since A and B are independent.
B is false since odd + odd = even
Mutually independent $\Rightarrow P(C) = P(C \mid A \cap B)$
A is true since $P(A \cap C) = P(A)P(C) = P(B \cap C) = P(B)P(C)$
$\Rightarrow \frac{1}{4} = \frac{1}{2} \cdot \frac{18}{36} = \frac{1}{4}$ ✓ (A)

(169) A = selecting ordinary die, B = selecting 2-4-6 die,
C = selecting 6 only die, D = rolling 2 sixes w/ selected die
$$\Rightarrow P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$$
$$= P(A)P(D \mid A) + P(B)P(D \mid B) + P(C)P(D \mid C)$$
$$= \frac{2}{4}\left(\frac{1}{6} \cdot \frac{1}{6}\right) + \frac{1}{4}\left(\frac{2}{6} \cdot \frac{2}{6}\right) + \frac{1}{4}(1)$$
$$= .292$$ (C)

(170) p = prob. customer bought some insurance
X_1 = # auto only who bought insurance, $X_1 \sim Bin$
X_2 = # homeowners only customers, $X_2 \sim Bin$
X_3 = life only, $X_3 \sim Bin$
Y = total customers who bought, $Y \sim Bin$
$Y = X_1 + X_2 + X_3$
$$\Rightarrow P(X_1 = 2 \cap X_2 = 2 \cap X_3 = 2 \mid Y = 6) = \frac{P(X_1=2) \cdot P(X_2=2) \cdot P(X_3=2)}{P(Y=6)}$$
$$= \frac{\binom{6}{2}\binom{4}{2}\binom{2}{2}}{\binom{12}{6}}$$
$$= .097$$ (D)

(171)



By Symmetry, $E(X) = 0$

(since length $= \frac{1}{2}(1-x-(x-i$
$= \frac{1}{2}(2-2x)$
$= 1-x$ for $x>$
$\Rightarrow 1+x$ for $x<$

$$\text{Thus } Var(X) = \int_{-1}^{0} x^2(1+x)\,dx + \int_0^1 x^2(1-x)\,dx$$
$$= \int_{-1}^{0} x^2 + x^3\,dx + \int_0^1 x^2 - x^3\,dx$$
$$= \frac{1}{3}x^3 + \frac{1}{4}x^4 \Big|_{-1}^{0} + \left(\frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_0^1\right)$$
$$= \frac{1}{3} - \frac{1}{4} + \frac{1}{3} - \frac{1}{4}$$
$$= \frac{2}{3} - \frac{1}{2}$$
$$= \frac{4}{6} - \frac{3}{6}$$
$$= \frac{1}{6}$$ (A)

(172) If no claims, $P(Y<48)=1$ $\qquad$ $Y=$ benefit paid

If one claim, $P(Y<48)=\frac{48}{60}=.8$

If two claims, $P(Y<48)=\frac{48^2/2}{60^2}=.32$

Thus $P(Y<48)=1(.7)+.2(.8)+.1(.32)=.892$ (D)

(173) $X=$ # tornadoes in a week, $Y=$ # in 3 weeks

$X\sim Pois(\lambda=2)$ and $Y\sim Pois(\lambda=6)$

Thus $P(Y<4)=\frac{e^{-6}6^0}{0!}+\frac{e^{-6}6^1}{1!}+\frac{e^{-6}6^2}{2!}+\frac{e^{-6}6^3}{3!}=.151$ (B)

(174) $X=$ # of components that fail

overheating $\Rightarrow \geq 2$ components fail

$X\sim Bin(n=3, p=.05)$

Thus $P(X\geq 2)=\binom{3}{2}(.05)^2(.95)^1+\binom{3}{3}(.05)^3(.95)^0=.00725$ (A)

(175) Let $Y=$ company's profit in a year

$$\Rightarrow P(Y<60|Y>0)=\frac{P(0<Y<60)}{P(Y>0)}=\frac{P\left(\frac{0-100}{20}<Z<\frac{60-100}{20}\right)}{P\left(Z>\frac{0-100}{20}\right)}$$

$$=\frac{F(-2)-F(-5)}{1-F(-5)}$$

$$=\frac{1-F(2)-(1-F(5))}{1-(1-F(5))}$$

$$=\frac{F(5)-F(2)}{F(5)}$$ (E)

(176) $B=$ high blood pressure, $C=$ high cholesterol

$P(B)=.2$, $P(C)=.3$, $P(C|B)=.25$

Thus $P(B|C)=\frac{P(B\cap C)}{P(C)}=\frac{P(B)\cdot P(C|B)}{P(C)}=\frac{.25(.2)}{.3}=\frac{\frac{1}{4}\left(\frac{1}{5}\right)}{\frac{3}{10}}=\frac{1}{20}\cdot\frac{10}{3}=\frac{1}{6}$ (A)

(177) $P(\text{exactly one of 2 selected factory workers low risk})=\frac{\binom{20}{1}\binom{5}{1}}{\binom{25}{2}}=.333$ (D)

(178) $\operatorname{Var}\left(\frac{x}{1-x}\right) = E\left(\frac{x}{(1-x)^2}\right) - \left(E\left(\frac{x}{1-x}\right)\right)^2$

$$= \int_0^1 60x^3\cancel{(1-x)^2}\cdot\frac{x^2}{\cancel{(1-x)^2}}\,dx - \left[\int_0^1 60x^3(1-x)^{\cancel{2}}\cdot\frac{x}{\cancel{1-x}}\,dx\right]^2$$

$$= \int_0^1 60x^5\,dx - \left[\int_0^1 60x^4 - 60x^5\,dx\right]^2$$

$$= 10x^6\Big|_0^1 - \left[12x^5 - 10x^6\Big|_0^1\right]^2$$

$$= 10 - 2^2$$

$$= 6 \quad \text{(C)}$$

(179) E = at least one ER visit
H = at least one hospital stay
$P(H) = .15$, $P(E) = .3$

thus $P(E \cap H) = P(E) + P(H) - P(E \cup H)^c$

$= .45 - .39$

$= .06$ (B)

(180) (A)

(181) M = male, O = insured over 25 $\quad$ * $P(M \cap O) = P(O) - P(M^c \cap O)$

$P(O) = \frac{395}{900} = \frac{79}{180}$, $P(O|M^c) = .43$

Thus $P(M|O^c) = \frac{P(M \cap O^c)}{P(O^c)} = \frac{P(M) - P(M \cap O)}{1 - P(O)}$ (from *)

$= \frac{.2989}{1 - \frac{79}{180}}$

$= .53$ (B)

(182)
P(red car | claim exceeds deductible) = $\frac{\text{\# red exceeds deductible}}{\text{total exceeding deductible}}$

$= \frac{300(.1)(.9)}{300(.1)(.9) + 700(.05)(.8)}$

$= .491$ (C)

(183) $P(T \le t) = P(X^2 \le t) = P(-\sqrt{t} \le X \le \sqrt{t})$

$= P(-\sqrt{t} \le X < 0) + P(0 < X \le \sqrt{t})$

$= \int_{-\sqrt{t}}^{0} 2e^{4x}\,dx + \int_{0}^{\sqrt{t}} e^{-2x}\,dx$

$= \frac{1}{2}e^{4x}\Big|_{-\sqrt{t}}^{0} + -\frac{1}{2}e^{-2x}\Big|_{0}^{\sqrt{t}}$

$= \frac{1}{2} - \frac{1}{2}e^{-4t^{1/2}} - \frac{1}{2}e^{-2t^{1/2}} + \frac{1}{2}$

$= 1 - \frac{1}{2}e^{-4t^{1/2}} - \frac{1}{2}e^{-2t^{1/2}}$

Thus $f(t) = F'(t) = -\frac{1}{2}e^{-4t^{1/2}} \cdot -2t^{-1/2} - \left[\frac{1}{2}e^{-2t^{1/2}} \cdot -t^{-1/2}\right]$

$= \frac{e^{-2\sqrt{t}}}{2\sqrt{t}} + \frac{e^{-4\sqrt{t}}}{\sqrt{t}}$ (A)

(184) X = George, Y = Paul

Paul wins = (two #'s differ > 3)^c = two numbers differ ≤ 3

1: (1,2) (1,3) (1,4)
2: (2,1) (2,2) (2,3) (2,4) (2,5)
3: (3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
4: (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (4,7)
5: (5,2) (5,3) (5,4) (5,5) (5,6) (5,7) (5,8)
6: (6,3) (6,4) (6,5) (6,6) (6,7) (6,8) (6,9)
⋮
17: (17,14) (17,15) (17,16) (17,17) (17,18) (17,19) (17,20)
18: (18,15) (18,16) (18,17) (18,18) (18,19) (18,20)
19: (19,16) (19,17) (19,18) (19,19) (19,20)
20: (20,17) (20,18) (20,19) (20,20)

Thus
$P(\text{Paul wins}) = P(|X-Y|=0) + P(|X-Y|=1) + P(|X-Y|=2) + P(|X-Y|=3)$

$= \frac{20}{400} + \frac{38}{400} + \frac{36}{400} + \frac{34}{400}$

$= .32$ (B)

(185) K = question that student knows answer, C = answers correctly

$P(K) = \frac{N}{20}$, $P(C|K) = 1$, $P(C|K^c) = .5$, $P(K|C) = .824$

$\Rightarrow P(K|C) = \frac{P(K)P(C|K)}{P(K)P(C|K) + P(K^c)P(C|K^c)} = .824 = \frac{\frac{N}{20}(1)}{\frac{N}{20}(1) + (1-\frac{N}{20}).5}$

$\Rightarrow \frac{\frac{N}{20}}{\frac{N}{20} + \frac{10}{20} - \frac{.5N}{20}} = .824 \Rightarrow \frac{\frac{N}{20}}{\frac{.5N+10}{20}} = .824 \Rightarrow \frac{N}{20} = .824\left(\frac{.5N+10}{20}\right) \Rightarrow N = 14$ (C)

(186) Let Y = cable not breaking for an applied force

$\Rightarrow P(Y > 12400) = P\left(Z > \frac{12400-12432}{25}\right) = P(Z > -1.28) = .9$

Let N = # of cables, $N \sim Bin(n = 400, p = .9)$

$N \sim$ approx. Normal $(\mu = 360, \sigma = 6)$

Thus by Continuity Correction,

$P(N \geq 349) = P\left(Z \geq \frac{348.5-360}{6}\right) = P(Z \geq -1.9167) = .97$ (D)

(187) Since mode occurs at $X=2$ and $X=3$

$\Rightarrow P(X=2) = P(X=3)$

$\Rightarrow \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-\lambda}\lambda^3}{3!}$

$\Rightarrow \lambda = 3$

Want to find $P(X=K)$ in which prob. of selling more than the number of policies is less than .25. Need to find K that the sum exceeds prob. = .75.

$P(X=0) = \frac{e^{-3}3^0}{0!} = .05$

$P(X=1) = \frac{e^{-3}3^1}{1!} = .149$

$P(X=2) = \frac{e^{-3}3^2}{2!} = .224$

$P(X=3) = \frac{e^{-3}3^3}{3!} = .224$

$P(X=4) = \frac{e^{-3}3^4}{4!} = .168$

Since $P(X \leq 4) = .815 > .75$, then $K = 4$ (D)

(188) R = red, G = green

Red \ Green	1	2	3	4	5	6
1	E	O	E	O	E	O
2	O	E	O	E	O	E
3	E	O	E	O	E	O
4	O	E	O	E	O	E
5	E	O	E	O	E	O
6	O	E	O	E	O	E

Thus $P(\text{sum is odd} \mid R > G) = \frac{9}{15} = \frac{3}{5}$ (E)

(189) $X = 1982$ SAT Scores, $X \sim N(\mu = 503, \sigma^2 = 9604)$
$Y = 2008$ SAT Scores, $Y \sim N(\mu = 521, \sigma^2 = 10201)$
$\pi = 93$rd percentile of X
$P(X \le \pi) = .93$, Need to find $P(Y \le \pi)$

$$P(X \le \pi) = P\left(Z \le \frac{\pi - 503}{\sqrt{9604}}\right) = .93$$

$$\Rightarrow \frac{\pi - 503}{\sqrt{9604}} = 1.48$$

$$\Rightarrow \pi = 648.04$$

$\Rightarrow \pi = 650$ (multiples of 10)

Thus $P(Y \le 650) = P\left(Z \le \frac{650 - 521}{\sqrt{10201}}\right) = P(Z \le 1.28) = .8997$ (B)

(190) $X_1 \quad X_2 \quad X_3$ are refrigerator life spans

$$P(X_1 + X_2 > 1.9X_3) = P(X_1 + X_2 - 1.9X_3 > 0)$$

$$E(S) = E(X_1) + E(X_2) - 1.9E(X_3) = 10 + 10 - 1.9(10) = 1$$

$$Var(S) = Var(X_1) + Var(X_2) + (-1.9)^2 Var(X_3) = 9 + 9 + (-1.9)^2(9) = 50.49$$

Thus $P(S > 0) = P\left(Z > \frac{0-1}{\sqrt{50.49}}\right) = P(Z > -.14) = P(Z < .14) = .556$ (C)

(191) $var(X \mid Y = 28.5) = 57 = \sigma_X^2(1 - \rho^2)$

$$\Rightarrow 1 - \rho^2 = \frac{57}{76}$$

$$\Rightarrow \rho^2 = \frac{1}{4}$$

$$\Rightarrow \rho = \frac{1}{2}$$

Thus, $Var(Y \mid X = 25) = \sigma_Y^2(1 - \rho^2) = 32(1 - \frac{1}{2}^2) = 32(\frac{3}{4}) = 24$ (B)

(192) $P(\text{insurer must pay at least } 1.2) = P(\text{loss} \ge 1.2 + d)$

$$\Rightarrow .3 = \frac{2 - 1.2 - d}{2 - 0}$$

$$\Rightarrow .6 = .8 - d$$

$$\Rightarrow d = .2$$

Thus $P(\text{insurer must pay at least } 1.44) = P(\text{loss} \ge 1.44 + d) = \frac{2 - 1.44 - .2}{2}$

$= .18$ (C)

(193) $P(X>4) = .3 = \int_4^\infty \lambda e^{-\lambda x}\,dx = -e^{-\lambda x}\Big|_4^\infty = e^{-4\lambda}$

$\Rightarrow \lambda = -\frac{\ln .3}{4}$

$\Rightarrow f(x) = -\frac{\ln .3}{4} e^{\frac{\ln .3}{4}x} = -\frac{\ln .3}{4}(.3)^{x/4}$ (E)

(194) $f(x) \propto \frac{x^2}{1+x^3} \Rightarrow f(x) = c\left(\frac{x^2}{1+x^3}\right)$

To find mode, set $f'(x) = 0$ and solve.

$\Rightarrow f'(x) = \frac{(1+x^3)2xc - cx^2(3x^2)}{(1+x^3)^2} = 0$

$\Rightarrow \frac{2xc + 2cx^4 - 3cx^4}{(1+x^3)^2} = 0$

$\Rightarrow 2 + 2x^3 - 3x^3 = 0$

$\Rightarrow 2 - x^3 = 0$

$\Rightarrow x^3 = 2$

$\Rightarrow x = 1.26$ (C)

(195) $f(x) \propto xe^{-x^2} \Rightarrow f(x) = cxe^{-x^2}$

Using same logic as (194),

$\Rightarrow f'(x) = cx \cdot -2xe^{-x^2} + ce^{-x^2} = 0$

$\Rightarrow -2x^2 + 1 = 0$

$\Rightarrow x^2 = \frac{1}{2}$

$\Rightarrow x = .71$ (C)

(196) Geom prob. dist. w/ mean $= 1.5 \Rightarrow p = \frac{2}{3^n}$ where $n = \#$visits

There are 4 cases where total admissions will be two or less.

Case 1 (X_1): No employees will have hospital admissions $\Rightarrow P(X_1) = .8^5 = .328$

Case 2 (X_2): One employee has one admission & other employees have none

$\Rightarrow P(X_2) = \binom{5}{1}(.2)(.8)^4\left(\frac{2}{3}\right) = .273$

Case 3 (X_3): One employee has 2 admissions & other employees have none.

$\Rightarrow P(X_3) = \binom{5}{1}(.2)(.8)^4\left(\frac{2}{9}\right) = .091$

Case 4 (X_4): Two employees each have one admission & other 3 employees have none.

$\Rightarrow P(X_4) = \binom{5}{2}(.2)^2(.8)^3\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = .091$

Thus $P(\text{Costs in a year} < 50000) = P(X_1) + P(X_2) + P(X_3) + P(X_4) = .783$ (E)

(197) P(3rd malfunction on 5th day | no malfunction in 3 days)

$= \dfrac{P(\text{3rd malfunction on 5th day})}{P(\text{no malfunctions in 3 days})}$ ← Neg Bin

$= \dfrac{P(\text{2 malfunctions in 4 days}) \cdot P(\text{malfunction on 5th day})}{P(\text{no malfunctions in 3 days})}$

$= \dfrac{\binom{4}{2}(.4)^2(.6)^2(.4)}{1-(.4)^3}$

$= .148$ C

(198) s_0 = stage 0, s_1 = stage 1, s_2 = stage 2, s_3 = stage 3, s_4 = stage 4

$P(s_0)+P(s_1)+P(s_2)+P(s_3)+P(s_4)=1$

$P(s_0)+P(s_1)+P(s_2)=.75 \quad (i)$

$P(s_1)+P(s_2)+P(s_3)+P(s_4)=.8 \quad (ii)$

$P(s_0)+P(s_1)+P(s_3)+P(s_4)=.8 \quad (iii)$

To find $P(s_1)$, need to solve linear system of equations.

$$\Rightarrow \left(\begin{array}{ccccc|c} 1&1&1&1&1&1\\ 1&1&1&0&0&.75\\ 0&1&1&1&1&.8\\ 1&1&0&1&1&.8 \end{array}\right) \begin{array}{l} R_2=r_2-r_1 \\ \\ R_4=r_4-r_1 \end{array} \left(\begin{array}{ccccc|c} 1&1&1&1&1&1\\ 0&0&0&-1&-1&-.25\\ 0&1&1&1&1&.8\\ 0&0&-1&0&0&-.2 \end{array}\right) R_3=r_3-r_1$$

$$\left(\begin{array}{ccccc|c} 1&1&1&1&1&1\\ 0&0&0&-1&-1&-.25\\ -1&0&0&0&0&-.2\\ 0&0&-1&0&0&-.2 \end{array}\right) R_2=r_2+r_1 \left(\begin{array}{ccccc|c} 1&1&1&1&1&1\\ 1&1&1&0&0&.75\\ -1&0&0&0&0&-.2\\ 0&0&-1&0&0&-.2 \end{array}\right)$$

$\Rightarrow P(s_2)=.2$

$\Rightarrow P(s_0)=.2$

$\Rightarrow .2+P(s_1)+.2=.75$

$\Rightarrow P(s_1)=.35$ C

(199) $P(k+.75 < X \le k+1 \mid k < X \le k+1) = \dfrac{P(k+.75 < X \le k+1)}{P(k < X \le k+1)}$

$$= \frac{F(k+1) - F(k+.75)}{F(k+1) - F(k)}$$

$$= \frac{1-e^{-(k+1)/2} - 1 + e^{-(k+.75)/2}}{1-e^{-(k+1)/2} - 1 + e^{-k/2}}$$

$$= \frac{e^{-.375} - e^{-.5}}{1-e^{-.5}}$$

$$= .205 \quad D$$

(200) $P(12 \text{ inspected}) = P(3 \text{ damaged out of } 11) \cdot P(\text{damaged on } 12 \mid 3 \text{ damaged out of } 11)$

$$= \frac{\binom{7}{3}\binom{13}{8}}{\binom{20}{11}} \cdot \left(\frac{7-3}{20-11}\right)$$

$$= .119 \quad B$$

(201) M: size of family that visits park

N: members that ride the roller coaster

Thus $P(M=6 \mid N=5) = \dfrac{P(N=5 \mid M=6)P(M=6)}{\sum_{m=1}^{7} P(N=5 \mid M=m)P(M=m)}$

$$= \frac{\frac{1}{6}\left(\frac{2}{28}\right)}{0+0+0+0+\frac{1}{5}\left(\frac{3}{28}\right)+\frac{1}{6}\left(\frac{2}{28}\right)+\frac{1}{7}\left(\frac{1}{28}\right)}$$

$$= .31 \quad E$$

(202) S = default on at least one student loan

C = default on at least one car loan

Want to find $P(C \mid S)$

$P(S \mid C) = \dfrac{P(C \cap S)}{P(C)}$

$\Rightarrow .4 = 1 - \dfrac{.196}{P(C)}$

$\Rightarrow P(C) = .327$

Thus

$P(C \mid S) = \dfrac{P(S \mid C)P(C)}{P(S)} = \dfrac{.4(.327)}{.3} = .436$

C

$P(S) = .3$ $\qquad P(C \mid S^c) = .28$

$P(S \mid C) = .4$

$P(C \mid S^c) = \dfrac{P(C \cap S^c)}{P(S^c)}$

$\Rightarrow .28 = \dfrac{P(C) - P(C \cap S)}{.7}$

$\Rightarrow P(C) - P(C \cap S) = .196$

$\Rightarrow P(C \cap S) = P(C) - .196$

(203) $f_{Y|X}(y|2) = \frac{f_{X,Y}(2,y)}{f_X(2)} = \frac{\frac{1}{18}e^{-(2+y)/6}}{\int_2^{\infty} \frac{1}{18}e^{-(2+y)/6}\,dy} = \frac{\frac{1}{18}e^{-(2+y)/6}}{-\frac{1}{3}e^{-(2+y)/6}\Big|_2^{\infty}} = \frac{1}{6}e^{-(y-2)/6}$ for $y>2$

Thus $Var(Y|X=2) = 6^2 = 36$

(204) Let Y denote time between report and payment

$\Rightarrow f(t,y) = f(y|t)f(t) = \frac{1}{10-2-t}\left(\frac{8t-t^2}{72}\right) = \frac{t}{72}$ for $0<t<6$, $2+t<y<10$

Thus $P(T+Y<4) = \int_0^1 \int_{2+t}^{4-t} \frac{t}{72}\,dy\,dt = \int_0^1 y\frac{t}{72}\Big|_{2+t}^{4-t}\,dt$

$= \int_0^1 \frac{1}{72}(4t-t^2-2t-t^2)\,dt$

$= \int_0^1 \frac{1}{72}(2t-2t^2)\,dt$

$= \frac{1}{72}\left[t^2-\frac{2}{3}t^3\right]_0^1$

$= .0046$ (A)

(205) $W=0$ if $T>8$ or $T<1.5$

$P(W=0) = \frac{2+1.5}{10} = .35$

$P(0<W<79) = P(100e^{-.04T}<79) = P(T>5.893) = \frac{8-5.893}{10} = .211$

Thus $P(W<79) = P(W=0) + P(0<W<79) = .35 + .211 = .561$ (D)

(206) Without deductible, std dev of uniform is $\frac{b}{\sqrt{12}} = .28868b$

Expected payout w/ deductible $\Rightarrow E(Y) = \int_{.1b}^{b}(y-.1b)\frac{1}{b}\,dy = \frac{1}{b}\left[\frac{1}{2}y^2-.1by\right]_{.1b}^{b}$

$= \frac{.5b^2-.1b^2-.005b^2+.01b^2}{b}$

$= .405b$

$E(Y^2) = \int_{.1b}^{b}(y-.1b)^2\left(\frac{1}{b}\right)dy = \frac{1}{b}\int_{.1b}^{b} y^2-.2yb+.01b^2\,dy$

$= \frac{1}{b}\left[\frac{1}{3}y^3-.1y^2b+.01b^2y\right]_{.1b}^{b}$

$= \frac{1}{3}b^2-.1b^2+.01b^2-.00033b^2+.001b^2-.001b^2$

$= .243b^2$

$\Rightarrow Var(Y) = .243b^2-(.405b)^2 = .079b^2$

$SD(Y) = .281$

Thus ratio $= \frac{.281}{.289} = .972$ (E)

207 C

208 D=death, H=high risk, M=medium risk, L=low risk

$P(D) = P(H)P(D|H) + P(M)P(D|M) + P(L)P(D|L)$

$= P(H)P(D|H) + P(M)\left(\frac{1}{2}P(D|H)\right) + P(L)\left(\frac{1}{2}\frac{1}{3}P(D|H)\right)$

$\Rightarrow .009 = P(H)P(D|H) + P(M)\left(\frac{1}{2}P(D|H)\right) + P(L)\left(\frac{1}{2}\frac{1}{3}P(D|H)\right)$

$\Rightarrow .009 = .2P(D|H) + .35\left(\frac{1}{2}P(D|H)\right) + .45\left(\frac{1}{6}P(D|H)\right)$

$\Rightarrow P(D|H) = \frac{.009}{.45} = .02$ B

209 If the deductible is less than 60

$\Rightarrow .10(60-d) + .05(200-d) + .01(3000-d) = 30$

$\Rightarrow d = 100$

Since this can't be the case, suppose the deductible is between 60 and 200.

$\Rightarrow .05(200-d) + .01(3000-d) = 30$

$\Rightarrow d = 166.67$ since $60 < d < 200$

C

210

$$\frac{\binom{3}{k}\binom{7}{3-k}}{\binom{10}{3}} = \frac{1}{120} \Rightarrow k = 3$$

Thus $P(X \le 1) = \frac{1}{120} + \frac{\binom{3}{2}\binom{7}{1}}{\binom{10}{3}} = \frac{11}{60}$ C

211 $P(\text{win}) = \frac{\binom{4}{4}\binom{8}{5}}{\binom{12}{9}} = .255$ B

212 N = # sick days for an employee in 3 months

Sum of independent Poisson variables is also Poisson

$\Rightarrow N \sim Pois(\lambda = 3)$

Thus $P(N > 2) = 1 - P(N \le 2) = 1 - \left(e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right)\right)$

$= 1 - .423$

$= .577$ D

(213) $Q \sim Pois(\lambda=3)$ $\quad A = P(Q > E(Q))$ $\quad B = P(R > E(R))$

$R \sim Pois(\lambda=1.5)$ $\quad A = P(Q>3)$ $\quad B = P(R>1.5)$

$$\Rightarrow B - A = P(R>1.5) - P(Q>3)$$
$$= 1 - P(R \le 1.5) - (1 - P(Q \le 3))$$
$$= \left[1 - (e^{-1.5} + 1.5e^{-1.5})\right] - \left[1 - \left(e^{-3} + 3e^{-3} + \frac{3^2 e^{-3}}{2} + \frac{3^3 e^{-3}}{6}\right)\right]$$
$$= .442 - .353$$
$$= .089 \quad (B)$$

(214) For Policy A

$$\Rightarrow .64 = P(L > 1.44) = e^{-1.44/\mu}$$
$$\Rightarrow \mu = 3.2266$$

Policy B

$$\Rightarrow .512 = P(L > d) = e^{-d/3.2266}$$
$$\Rightarrow d = 2.1599 \quad (E)$$

(215) First find c

$$\int_0^5 cx^a dx = \frac{cx^{a+1}}{a+1}\Big|_0^5 = \frac{c5^{a+1}}{a+1} = 1$$
$$\Rightarrow c = \frac{a+1}{5^{a+1}}$$
$$P(X < 3.75) = \int_0^{3.75} cx^a dx = \frac{cx^{a+1}}{a+1}\Big|_0^{3.75} = \frac{c \cdot 3.75^{a+1}}{a+1} = \frac{a+1}{5^{a+1}} \cdot \frac{3.75^{a+1}}{a+1}$$
$$\Rightarrow \frac{3.75^{a+1}}{5^{a+1}} = .4871$$
$$\Rightarrow a = 1.5$$

Thus
$$P(X>4) = \int_4^5 cx^a dx = \frac{cx^{a+1}}{a+1}\Big|_4^5 = \frac{c5^{a+1}}{a+1} - \frac{c4^{a+1}}{a+1}$$
$$= \frac{a+1}{5^{a+1}} \cdot \frac{5^{a+1} - 4^{a+1}}{a+1}$$
$$= \frac{5^{a+1} - 4^{a+1}}{5^{a+1}}$$
$$= 1 - \frac{4^{a+1}}{5^{a+1}}$$
$$= 1 - \frac{4^{2.5}}{5^{2.5}}$$
$$= .428 \quad (B)$$

(216) X = warranty claim $X \sim Pois(\lambda = c)$

$P(X=0) = .6$

$\Rightarrow e^{-c} = .6$

$\Rightarrow c = -\ln .6$

Payment	Prob.
5000	$P(X=2)$
5000(2)	$P(X=3)$
5000(3)	$P(X=4$

Thus $E(5000(X-1)) = 5000(P(X=2) + 2P(X=3) + 3P(X=4) + \dots)$

$= 5000(P(X=1) + 2P(X=2) + \dots) - 5000(P(X=1) + P(X=2) + \dots$

$= 5000E(X) - 5000(1 - P(X=0))$

$= 5000(-\ln .6) - 5000(.4)$

$= 554.13$ (A)

(217) L = loss

X = unreimbursed loss $\quad X = \begin{cases} L & \text{if } L \le 180 \\ 180 & \text{if } L > 180 \end{cases}$

$\Rightarrow 144 = E(X) = \int_0^{180} \ell\,[f(\ell)]\,d\ell + 180\,P(L>180) = \int_0^{180} \ell\,\frac{1}{b}\,d\ell + 180\,\frac{b-180}{b}$

$\Rightarrow 144 = \frac{1}{2b}\ell^2\Big|_0^{180} + 180 - \frac{180^2}{b}$

$\Rightarrow 144 = \frac{180^2}{2b} + 180 - \frac{180^2}{b}$

$\Rightarrow \frac{180^2}{b} - \frac{180^2}{2b} = 36$

$\Rightarrow \frac{2\cdot 180^2 - 180^2}{2b} = 36$

$\Rightarrow 72b = 180^2$

$\Rightarrow b = 450$ (D)

(218) $X \sim N(\mu = 10, \sigma^2 = 4)$ Let π_{12} = 12th percentile

$\Rightarrow .12 = P(X \le \pi_{12}) = P\left(\frac{X-10}{2} \le \frac{\pi_{12}-10}{2}\right) = P\left(Z \le \frac{\pi_{12}-10}{2}\right)$

Since $P(Z \le -1.175) = .12$

$\Rightarrow \frac{\pi_{12}-10}{2} = -1.175$

$\Rightarrow \pi_{12} = 7.65$ (B)

(219) $\pi_{14} = P(Z \le \frac{x-\mu}{\sigma}) = -1.08$

Since $E(A) = E(B)$, $SD(B) = 1.5\, SD(A)$,

$\Rightarrow$ profit that is 1.08 std dev below the mean for company A is $\frac{1.08}{\sqrt{2.25}} = .72$ std dev below the mean for company B.

Thus a Z-score of $-.72$ is in the 23.6th percentile. (D)

(220) Since $X \sim (X-5 | X>5)$, then by the Memoryless Property,

$Var(X | X>10) = 25$ (C)

(221) X, Y = annual profits for companies A and B

m = common mean

s = common std. dev of Y

Z = std. normal

Since $SD(X) = \frac{1}{2}$

$\Rightarrow P(X<0) = P(\frac{X-m}{.5m} < \frac{0-m}{.5m}) = P(Z<-2) = .0228$

$\Rightarrow$ Company B's prob. of a loss $= .9(.0228) = .02052$

$\Rightarrow .02052 = P(Y<0) = P(\frac{Y-m}{s} < \frac{0-m}{s}) = P(Z < -\frac{m}{s})$

$\Rightarrow -2.04 = -\frac{m}{s}$

$\Rightarrow s = \frac{m}{2.04}$

Thus Ratio of std dev $= \frac{\frac{m}{2.04}}{.5m} = .98$ (C)

(222) $E(X) = M'(t)|_{t=0} = .45e^t + .35(2)e^{2t} + .15(3)e^{3t} + .05(4)e^{4t} = 1.8$

$E(X^2) = M''(t)|_{t=0} = .45e^t + .35(2)^2e^{2t} + .15(3)^2e^{3t} + .05(4^2)e^{4t} = 4$

Thus $SD(X) = \sqrt{E(X^2) - (E(X))^2} = .87$ (B)

(223) $Y \sim N(\mu = 1.04(100) + 5 = 109,\ \sigma = 1.04(25) = 26)$

The average of 25 observations has mean $= 109$ & std dev $= \frac{26}{5} = 5.2$

Thus $P(100 <$ sample mean $< 110) = P(\frac{100-109}{5.2} = 1.73 < Z < \frac{110-109}{5.2} = .19)$

$= .5753 - (1 - .9582)$

$= .5335$ (B)

(224) First find c

$\Rightarrow 1 = \int_0^1 \int_0^{1-x^2} c \, dy \, dx = \int_0^1 c(1-x^2) \, dx = c\left(x - \frac{1}{3}x^3 \Big|_0^1\right) = \frac{2}{3} c$

$\Rightarrow c = \frac{3}{2}$

Thus,

$E(XY) = \int_0^1 \int_0^{1-x^2} \frac{3}{2} xy \, dy \, dx = \int_0^1 \frac{3}{4} x y^2 \Big|_0^{1-x^2} dx$

$= \int_0^1 \frac{3}{4} x (1-x^2)^2 \, dx$

$= \frac{3}{4} \int_0^1 x (1-x^2)^2 \, dx$

$u = 1-x^2 \quad du = 2x$

$= \frac{3}{4} \cdot \frac{1}{2} \int_0^1 u^2 \, du$

$= \frac{3}{8} \cdot \frac{1}{3} u^3 \Big|_0^1$

$= \frac{1}{8}$ (B)

(225) Z = # tornadoes that cause damage < 50 million

X = damages ≥ 50 million

Note that $Y = X + Z \Rightarrow Z = Y - X$

Want to find $E(Z)$

$x \backslash y$	0	1	2	3
0	0	2c	4c	6c
1		3c	5c	7c
2			6c	8c
3				9c

Since $c = \frac{1}{50}$

* $P(Z=1) = 2c + 5c + 8c = .3$

$P(Z=2) = 4c + 7c = .22$

$P(Z=3) = 6c = .12$

Thus $E(Z) = 1P(Z=1) + 2P(Z=2) + 3P(Z=3)$

$= 1(.3) + 2(.22) + 3(.12)$

$= 1.1$ (E)

(226) Let R = Republicans, D = Democrats, I = independent

Ind. $E(|R-D| \mid I) = \frac{4}{9}(0) + \frac{5}{9}(1) = \frac{5}{9}$

Rep. $E(|R-D| \mid R) = \frac{2}{9}(0) + \frac{5}{9}(1) + \frac{2}{9}(2) = 1$

Dem $E(|R-D| \mid D) = \frac{3}{9}(0) + \frac{5}{9}(1) + \frac{1}{9}(2) = \frac{7}{9}$

Thus $E(|R-D|) = \frac{1}{2}\left(\frac{5}{9}\right) + \frac{3}{10}(1) + \frac{1}{5}\left(\frac{7}{9}\right) = \frac{11}{15}$ (D)

227. Let $Z = XY$ and a, b, c be prob. that Z takes on the values of $0, 1,$ and 2 respectively.

$b = p(1,1)$ and $c = p(1,2) \Rightarrow 3b = c$

$\Rightarrow a + b + c = 1$

$\Rightarrow a = 1 - 4b$

$E(Z) = b + 2c = 7b$

$E(Z^2) = b + 4c = 13b$

$Var(Z) = 13b - 49b^2$

Max $Var(XY) =$ Max $Var(Z) = 0 = \frac{d}{db} Var(Z) = 13 - 98b$

$\Rightarrow b = \frac{13}{98}$

Thus $P(X = 0 \cup Y = 0) = P(Z) = 0$

$\Rightarrow a = 1 - 4b = \frac{46}{98} = \frac{23}{49}$ C

228. Marginal Density of $X = \frac{1}{3}$ is $\int_{1/3}^{1} 24(\frac{1}{3})(1-y)\,dy = 8y - 4y^2 \Big|_{1/3}^{1}$

$= 8 - 4 - \frac{8}{3} + \frac{4}{9}$

$= \frac{16}{9}$

Conditional Density of Y given $X = \frac{1}{3}$ is $f_{Y|X=\frac{1}{3}}(y|x=\frac{1}{3}) = \dfrac{24(\frac{1}{3})(1-y)}{\frac{16}{9}} = \frac{9}{16} \cdot 8(1-y)$

$= \frac{9}{2}(1-y)$

Thus $E(Y|X=\frac{1}{3}) = \int_{1/3}^{1} \frac{9}{2} y(1-y)\,dy = \frac{9}{2}\int_{1/3}^{1} y - y^2\,dy$

$= \frac{9}{2}\left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{1/3}^{1}$

$= \frac{9}{2}\left(\frac{1}{2} - \frac{1}{3} - \frac{1}{18} + \frac{1}{81}\right)$

$= \frac{5}{9}$ B

(229) $P(J=j|K=5)=\frac{P(K=5|J=j)P(J=j)}{P(K=5)}$

$P(K=5|J=3)=\frac{1}{6}$ $P(J=3)=\binom{5}{3}.6^3.4^2=.3456$

$P(K=5|J=4)=\frac{1}{3}$ $P(J=4)=\binom{5}{4}.6^4.1^1=.2592$

$P(K=5|J=5)=\frac{1}{2}$ $P(J=5)=\binom{5}{5}.6^5.4^0=.07776$

$P(K=5)=\frac{1}{6}(.3456)+\frac{1}{3}(.2592)+\frac{1}{2}(.07776)=.18288$

$P(J=3|K=5)=\frac{\frac{1}{6}(.3456)}{.18288}=.31496$

$P(J=4|K=5)=\frac{\frac{1}{3}(.2592)}{.18288}=.47244$

$P(J=5|K=5)=\frac{\frac{1}{2}(.07776)}{.18288}=.21260$

Thus $E(J|K=5)=3(.31496)+4(.47244)+5(.21260)$

$=3.898$ (C)

(230) $F_Y(y)=F(1,y)=y+y^2-y^3 \Rightarrow f_Y(y)=1+2y-3y^2$

Thus $E(Y)=\int_0^1 y+2y^2-3y^3\,dy=\frac{1}{2}y^2+\frac{2}{3}y^3-\frac{3}{4}y^4\Big|_0^1$

$=\frac{1}{2}+\frac{2}{3}-\frac{3}{4}$

$=.417$ (C)

(231) Note $N+S=2 \Rightarrow (N,S)=(2,0),(1,1),(0,2)$ w/ prob. .12, .18, and .1 respectively

$P(N=0|N+S=2)=\frac{.12}{.40}=.3$

$P(N=1|N+S=2)=\frac{.18}{.40}=.45$

$P(N=2|N+S=2)=\frac{.10}{.40}=.25$

$E(N|N+S=2)=0(.3)+1(.45)+2(.25)=.95$

$E(N^2|N+S=2)=0^2(.3)+1^2(.45)+2^2(.25)=1.45$

Thus $Var(N|N+S=2)=1.45-.95^2=.5475$ (B)

(232) $F_X(x)=F(x,100)=\frac{100x(x+100)}{2000000}=\frac{100x^2+10000x}{2000000}$

$\Rightarrow f_X(x)=\frac{1}{10000}x+\frac{1}{200}$

$E(X)=\int_0^{100}\frac{1}{10000}x^2+\frac{1}{200}x\,dx=\frac{1}{30000}x^3+\frac{1}{400}x^2\Big|_0^{100}=58.33$

$E(X^2)=\int_0^{100}\frac{1}{10000}x^3+\frac{1}{200}x^2\,dx=\frac{1}{40000}x^4+\frac{1}{600}x^3\Big|_0^{100}=4166.67$

Thus $Var(X)=4166.67-58.33^2=764$ (A)

(233) $f_X(x)=\int_0^{\infty} f(x,y)\,dy = .65e^{-.5x} - .3e^{-x} - .15e^{-.5x} + .3e^{-x} = .5e^{-.5x}$

$\Rightarrow X \sim Expo(\mu = 2)$

$\Rightarrow Var(X) = 4$

$\Rightarrow SD(X) = 2$ (B)

(234) $P(Claim=100) = \frac{.9+.8+.7}{3} = .8$

$P(Claim=500) = \frac{.08+.11+.20}{3} = .13$

$P(Claim=1000) = \frac{.02+.09+.10}{3} = .07$

$E(Claim) = 100(.8)+500(.13)+1000(.07) = 215$

$E(Claim^2) = 100^2(.8)+500^2(.13)+1000^2(.07) = 110500$

Thus $SD(Claim) = 254$ (A)

(235) With each load of coal having mean 1.5 and std. dev .25,
20 loads have mean of $20(1.5)=30$ and variance of $20(.0625)=1.25$

Total Amount removed $\sim N(\mu = 4(7.25)=29,\ \sigma^2 = 4(.25)=1)$

Difference $\sim N(\mu = 30-29=1,\ \sigma = \sqrt{1.25+1} = 1.5)$

If D is the difference,

$\Rightarrow P(W>0) = P(Z > \frac{0-1}{1.5} = -.67) = .7486$ (D)

(236) Let G = # claims for good driver, B = # claims for bad driver

$P(G+B=0) = .5 \cdot .2 = .1$

$P(G+B=1) = P(G=0)P(B=1) + P(G=1) \cdot P(B=0) = .21$

$P(G+B=2) = .33$

$P(G+B=3) = .23$ $\Bigg\}$ Use same logic

$P(G+B=4) = .11$

Thus the mode is 2 claims. (C)

(237) Prob. both treatments $= .9 \cdot .4 = .36$

Radiation only $= .9 - .36 = .54$

Chemo only $= .4 - .36 = .04$

$$P(Y=y) = \begin{cases} .06 & \text{for } y=0 \\ .54 & \text{for } y=2 \\ .04 & \text{for } y=3 \\ .36 & \text{for } y=5 \end{cases}$$

Let S = total insurance payment for 15 policyholders.

Thus, $Var(S) = Var(Y) \cdot 15$

$$= 15\left[\left[0^2(.06) + 2^2(.54) + 3^2(.04) + 5^2(.36)\right] - \left[0(.06) + 2(.54) + 3(.04) + 5(.36)\right]^2\right]$$

$$= 15(11.52 - 3^2)$$

$$= 37.8 \quad \text{(B)}$$

(238) $E(Y) = P(X=0)E(Y|X=0) + P(X\geq 1)E(Y|X\geq 1)$

$E(Y) = n\,p_Y = 6(.1) = .6$

$P(X=0) = \binom{6}{0} .2^0 .8^6 = .8^6$

$P(X\geq 1) = 1 - .8^6$

$P(Y|X=0) = \frac{.1}{.1+.7} = \frac{1}{8}$

$E(Y|X=0) = 6\left(\frac{1}{8}\right) = .75$

Thus $E(Y|X\geq 1) = \dfrac{E(Y) - P(X=0)E(Y|X=0)}{P(X\geq 1)}$

$$= \frac{.6 - .8^6(.75)}{1 - .8^6}$$

$$= .547 \quad \text{(C)}$$

(239) $p(x) = \dfrac{\binom{x-1}{2}(12-x)}{495} = \dfrac{(x-1)(x-2)(12-x)}{990}$ (A)

(240) $P(X<k \mid X>10000) = \dfrac{P(10000<X<k)}{P(X>10000)} = \dfrac{P(X<k)-P(X<10000)}{P(X>10000)}$

$= \dfrac{P(X<k)-(1-P(X>10000))}{P(X>10000)}$

$\Rightarrow .95 = \dfrac{P(X<k)-1+P(X>10000)}{P(X>10000)}$

$\Rightarrow .95\,P(X>10000) = P(X<k)-1+P(X>10000)$

$\Rightarrow P(X<k) = 1-.05\,P(X>10000) = .9582$

$\Rightarrow P(X>10000) = \dfrac{1-.9582}{.05} = .836$

After standardization,

$P(Z>\frac{-2000}{\sigma}) = 1-P(Z>\frac{2000}{\sigma}) = .836$

$\Rightarrow P(Z>\frac{2000}{\sigma}) = .164$

$\Rightarrow \frac{2000}{\sigma} = .98$ (from Z table)

$\Rightarrow \sigma = \frac{2000}{.98} = 2040.8$ (A)