Quick Binary Guide

Derek Chiu

1 Bases

When working in binary, we are working in base 2 (0 .. 1). Typically, we work in base 10 (0 .. 9). When working in decimal, we add another digit whenever we would get a number bigger than we could represent on the current amount of digits we are using (e.g. we cannot represent ten on 1 digit in decimal, so we represent it as 10), the same holds for binary and all other bases.

1.1 Decimal

When we look at the a decimal number like 159, we can read it as: $1*10^2+5*10^1+9*10^0$. Note that becuase we are working in base 10, all numbers are multiplied by a power of 10. This leads to a general formula of:

$$D = d_n * 10^n + d_{n-1} * 10^{n-1} + d_{n-2} * 10^{n-2} + ... + d_2 * 10^2 + d_1 * 10^1 + d_0 * 10^0$$

Where D represents the total number and d_i represents our individual digits, with d_n being the most significant digit. This can also be described as:

$$D = \sum_{i=0}^{n} d_i * 10^i.$$

1.2 Binary

In binary, we work in base 2, so all our values are multiplied in base 2 otherwise, everything works the same way. This leads us to the general formula:

$$B = \sum_{i=0}^{n} b_i * 2^i$$

Where B is our number in binary and b_i is a given digit or bit. It should be noted, that when working in binary everything is typically measured in bits, where a binary number of n bits can represent values from $0 - 2^n - 1$, similar to how a decimal number with n digits can represent values $0 - 10^n - 1$.

Binary to Decimal Example :
The number 101101_2 in binary can be represented as:

$$1 * 2^5 + 0 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0$$

which is equal to 45_{10}. This is the basic method for converting binary values into decimal.

2 Conversion From Decimal to Binary

Converting between bases is also an important skill we need to have. In the previous section, we demonstrated the basic method for converting a binary value into decimal. Now, we will demonstrate how to convert a decimal value into binary.

2.1 Method 1: With Division

Our first method uses division by 2 to convert the decimal value into binary. To convert the our number D from decimal into binary, we will begin by dividing D by 2. We take the remainder of $\frac{D}{2}$ and use that as our least significant bit, or b_0. Next, we take the resulting whole number from $\frac{D}{2}$ and divide it by 2. We take the remainder of this number as our value for b_1 and continue this pattern until we are left with 0 as the result of our continuous division.

Example of Decimal to Binary Conversion with Division:
Convert the number 159_{10} from Decimal to Binary.

Bit	Decimal Value	Remainder
n/a	159	n/a
0	79	1
1	39	1
2	19	1
3	9	1
4	4	1
5	2	0
6	1	0
7	0	1

This gives us a value of 10011111_2 as the equivalent binary value of 159_{10}. We can double-check our answer by using the method of converting binary numbers into decimal. This gives us:

$$159_{10} = 10011111_2 = 1 * 2^7 + 0 * 2^6 + 0 * 2^5 + 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 1 * 2^1 + 1 * 2^0$$

2.2 Method 2: With Subtraction

Our second method takes advantage of the method we use for converting binary values into decimal by continuously subtracting powers of 2 from our starting decimal number until we are left with 0. For this method, we begin by subtracting from our starting number D, the largest power of 2 we can. This gives us our most significant bit, or b_n where n is the power we raise 2 to to get the value we subtracted by. We repeat this process with the result of our previous calculation until we once again are left with 0. Any of the powers of 2 we subtracted by will have a value of 1 on their corresponding bit, where any bit between $b_0$and b_n that we did not use in our subtraction is set to 0.

Example of Decimal to Binary Conversion with Subtraction:
Convert the number 379_10 from Decimal to Binary.

n/a	n/a	379
b_8	256	123
b_6	64	59
b_5	32	27
b_4	16	11
b_3	8	3
b_1	2	1
b_0	1	0

This results in the number 101111011_2 as the equivalent binary value for 379_10. We can once again double-check the value by using the method of converting binary numbers to decimal:

$$379_{10} = 101111011_2 = 1*2^8 + 0*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0$$

A Table of notable binary values

Decimal Value	Binary Value	Decimal Value(Cont.)	Binary Value (Cont.)
1023	1111111111	10	1010
512	1000000000	9	1001
256	100000000	8	1000
128	10000000	7	111
64	1000000	6	101
32	100000	5	101
16	10000	4	100
15	1111	3	11
14	1110	2	10
13	1101	1	1
12	1100	0	0
11	1011		