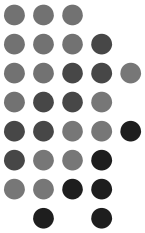
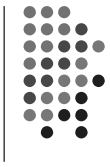
Trees 6B

Heaps & Other Trees



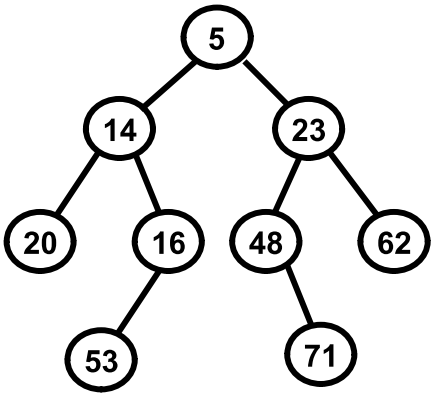


Heap

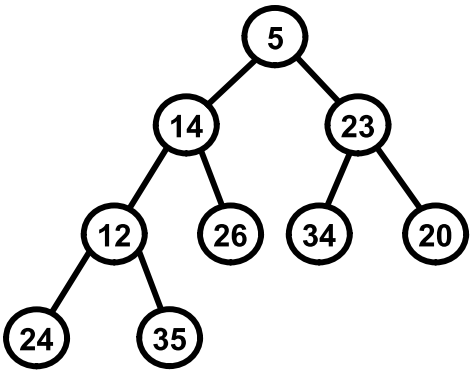
- A min-heap is a binary tree such that
  - the data contained in each node is less than (or equal to) the data in that node's children.
  - the binary tree is complete

- A max-heap is a binary tree such that
  - the data contained in each node is greater than (or equal to) the data in that node's children.
  - the binary tree is complete

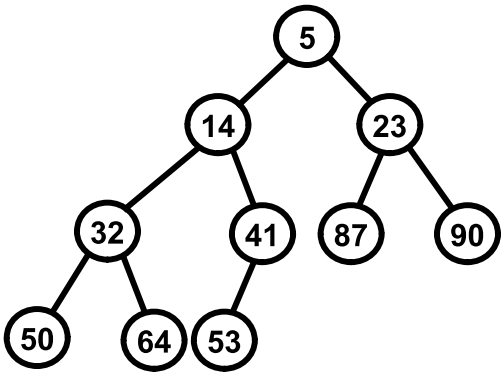
Is it a min-heap?



Is it a min-heap?



Is it a min-heap?

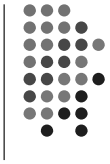


Using heaps

What are min-heaps good for?
(What operation is extremely fast when using a min-heap?)

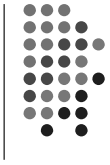
The difference in level between any two leaves in a heap is at most what?

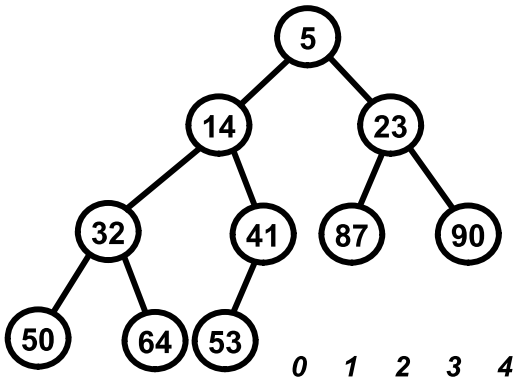
Storage of a heap



- Use an array to hold the data.
- Store the root in position 1.
  - We won't use index 0 for this implementation.
- For any node in position i,
  - its left child (if any) is in position 2i
  - its right child (if any) is in position 2i + 1
  - its parent (if any) is in position i/2
    (use integer division)

Storage of a heap

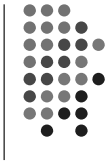




For node at i:
Left child is at 2i
Right child is at 2i12
Parent is at ⌊i/2⌋

0	1	2	3	4	5	6	7	8	9	10
	5	14	23	32	41	87	90	50	64	53

Inserting into a min-heap



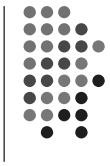
- Place the new element in the next available position in the array.
- Compare the new element with its parent. If the new element is smaller, than swap it with its parent.
- Continue this process until either

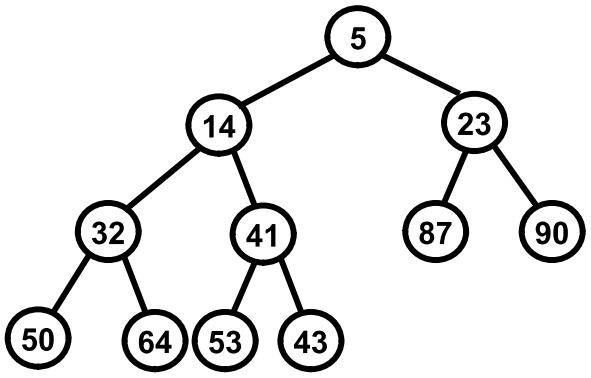
  - the new element's parent is smaller than or equal to the new element, or

  - the new element reaches the root (index 0 of the array)

Inserting into a min-heap

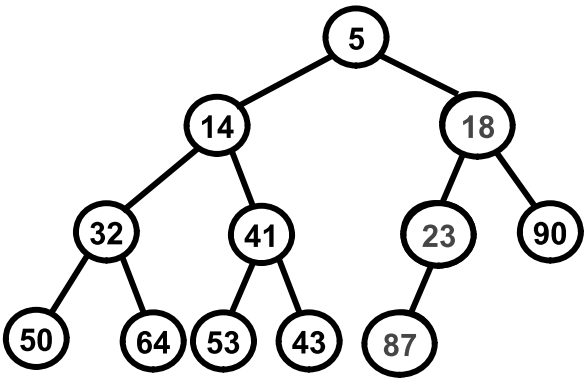
Insert 43





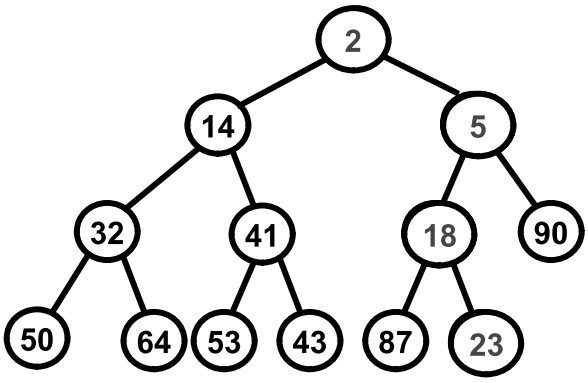
Inserting into a min-heap

Insert 18

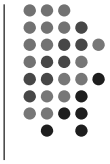


Inserting into a min-heap

Insert 2



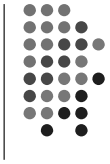
Removing from a heap

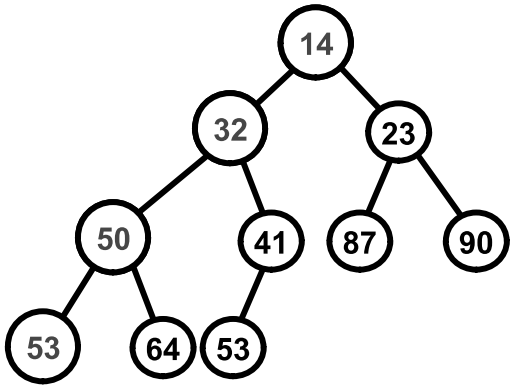


- Place the root element in a variable to return later.
- Remove the last element in the deepest level and move it to the root.
- While the moved element has a value greater than at least one of its children, swap this value with the smaller-valued child.
- Return the original root that was saved.

Removing from a min-heap

Remove min

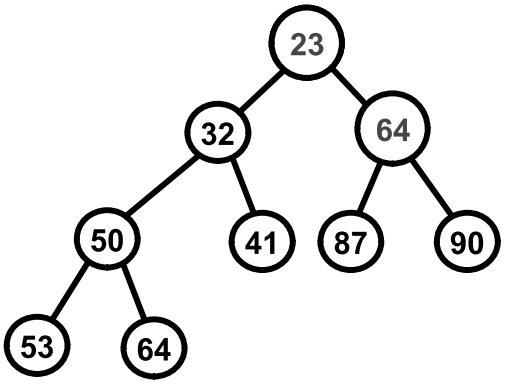




returnValue 5

Removing from a min-heap

Remove min



returnValue 14

Efficiency of heaps

Assume the heap has N nodes.

Then the heap has $\lceil \log_2(N+1) \rceil$ levels.

- Insert

  Since the insert swaps at most once per level, the order of complexity of insert is O(log N)
- Remove

  Since the remove swaps at most once per level, the order of complexity of remove is also O(log N)