Reading Notes Chapter 3

3.1 Simple Harmonic Motion

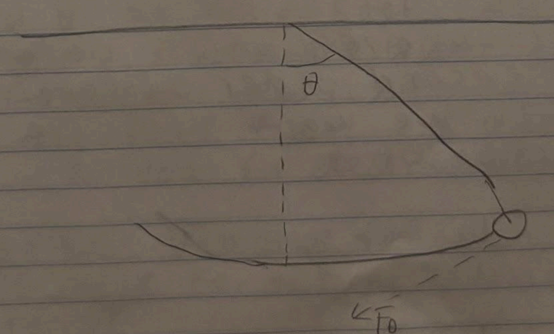
- One example of a simple pendulum is a particle of mass m connected by a massless string to a rigid support.

- We let θ be the angle that the string makes with vertical and assume the string is always taut.

- There are also two forces acting on a particle, gravity and tension.

- Parallel forces equal zero assuming the string does not break or stretch while the force perpendicular is:

$$F_\theta = -mg\sin\theta$$



- Newton's 2nd Law tells us that this force is equal to the mass times the acceleration of the particle along circular arc:

$$F_\theta = m\frac{d^2 s}{dt^2}$$

- If we assume that θ is always small so $\sin\theta \simeq \theta$, we obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\theta$$

- General/central equation:

$$\theta = \theta_0 \sin(\Omega t + \phi)$$

- Where $\Omega = \sqrt{g/\ell}$ and θ_0 and ϕ are constants that depend on the initial displacement and velocity of pendulum.

- Our basic equation of motion is the second order differential equation which we want to solve for θ as a function of t.

$$\frac{d\omega}{dt} = -\frac{g}{\ell}\theta$$

$$\frac{d\theta}{dt} = \omega$$

- Where ω is the angular velocity of the pendulum.

- We convert into difference eqns using a time step Δt so that time is discreted with $t = i\Delta t$, where i is an integar.

- Letting θ_i and ω_i be the numerically approximated angular displacement and velocity of the pendulum, we get:

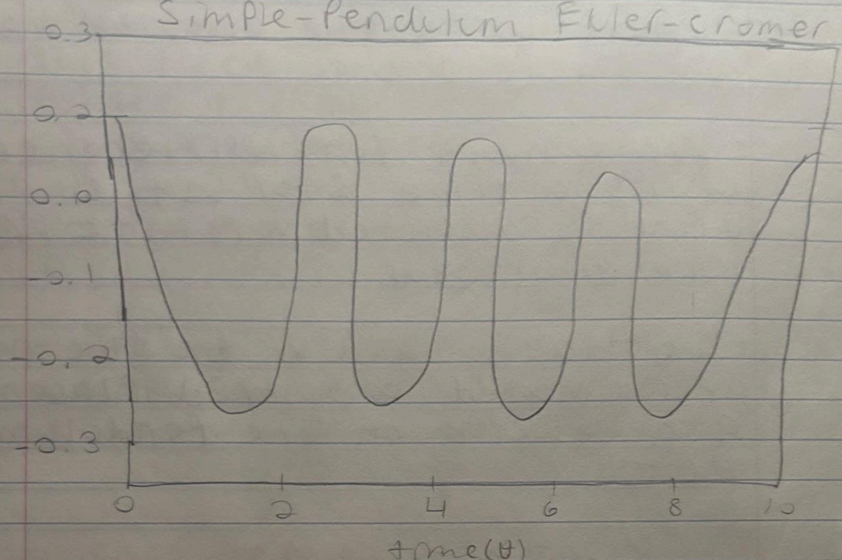
$$\omega_{i+1} = \omega_i - \frac{g}{\ell} \theta_i \Delta t$$

$$\theta_{i+1} = \theta_i + \omega_i \Delta t$$

Ex. 3.2

- For each time step i calculate ω and θ at time step $i+1$.
  - $\omega_{i+1} = \omega_i - (g/\ell)\theta_i \Delta t$
  - $\theta_{i+1} = \theta_i + \omega_{i+1} \Delta t$
  - $t_{i+1} = t_i + \Delta t$
  - Repeat for desired number.

Fig. (3.3) θ as a function of time for a simple pendulum, using Euler-Cromer.



3.2 Making Pendulum more Interesting

- The frictional force we will employ thus has the form $-q(d\theta/dt)$ since the velocity is $l(d\theta/dt)$.

- Thus the equation of our pendulum becomes:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt}$$

- Where the second term is the friction

- The first regime, called underdamped occurs for sufficiently small friction

$\theta(t) = \theta_0 e^{-qt/2} \sin(\sqrt{\Omega^2 - q^2/4}\, t + \phi)$

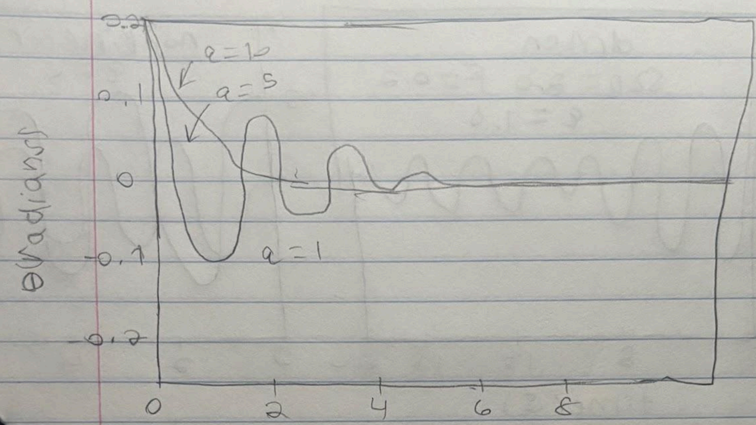


Figure 3.4: θ as a function of time for a damped pendulum.

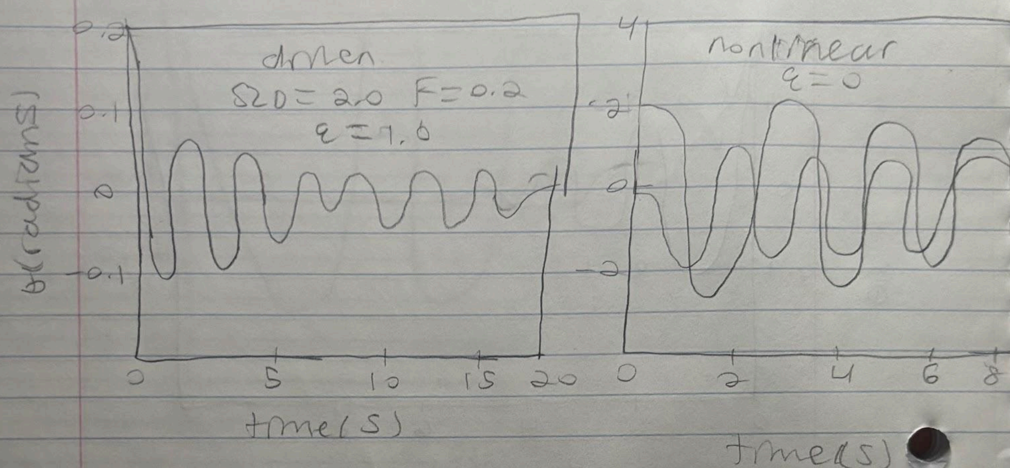
- This shows an oscillatory behavior with frequency $\sqrt{\Omega^2 - q^2/4}$, where $\Omega = \sqrt{g/l}$ and the amplitude decays with time.

- When damping is large (overdamped)

$\theta(t) = \theta_0 e^{-(q/2 \pm \sqrt{q^2/4 - \Omega^2})t}$

which is a monatomic, exponential decay.

$\theta(t) = (\theta_0 + C t) e^{-qt/2}$



3.3 Chaos in Driven Nonliner Pendulum

- We add a sinusoidal driving force:
$F_D \sin(\Omega_D t)$:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t)$$

- Again we write:

$$\frac{d\omega}{dt} = -\frac{g}{l}\sin(\theta) - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t)$$

$$\frac{d\theta}{dt} = \omega$$