UNIVERSITY OF ALBERTA

Wide angle AVO/AVAz inversion

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Outline

- Introduction
  - Introduction to AVO/AVAz
  - Introduction to inversion

- Theory
  - Forward modeling
  - Inversion

- Methods
  - Overcoming ambiguity of the AVO/AVAz inverse problem
  - Overcoming the Ill-conditioned nature of the AVO/AVAz inverse problem
  - Practical implementation of the AVO/AVAz problem

- Results
  - Aki Richards synthetic data
  - Rüger synthetic data
  - Real data inversion: Puesto Peter Field

- Discussion
  - Interpretation with Vp/Vs ratio.
  - Strengths and Weaknesses.

- Conclusions

Introduction to AVO/AVAz

- **AVO stands for amplitude variation with offset.**
- **AVO looks at changes in the amplitude of the traces as a function of offset or incident angle to estimate material properties in the ground.**
- **Common practices of AVO look at things like fluid factor angle to find anomalous AVO responses that can be indicators of economic resources like hydrocarbons. (Gidlow and Smith, 2003)**

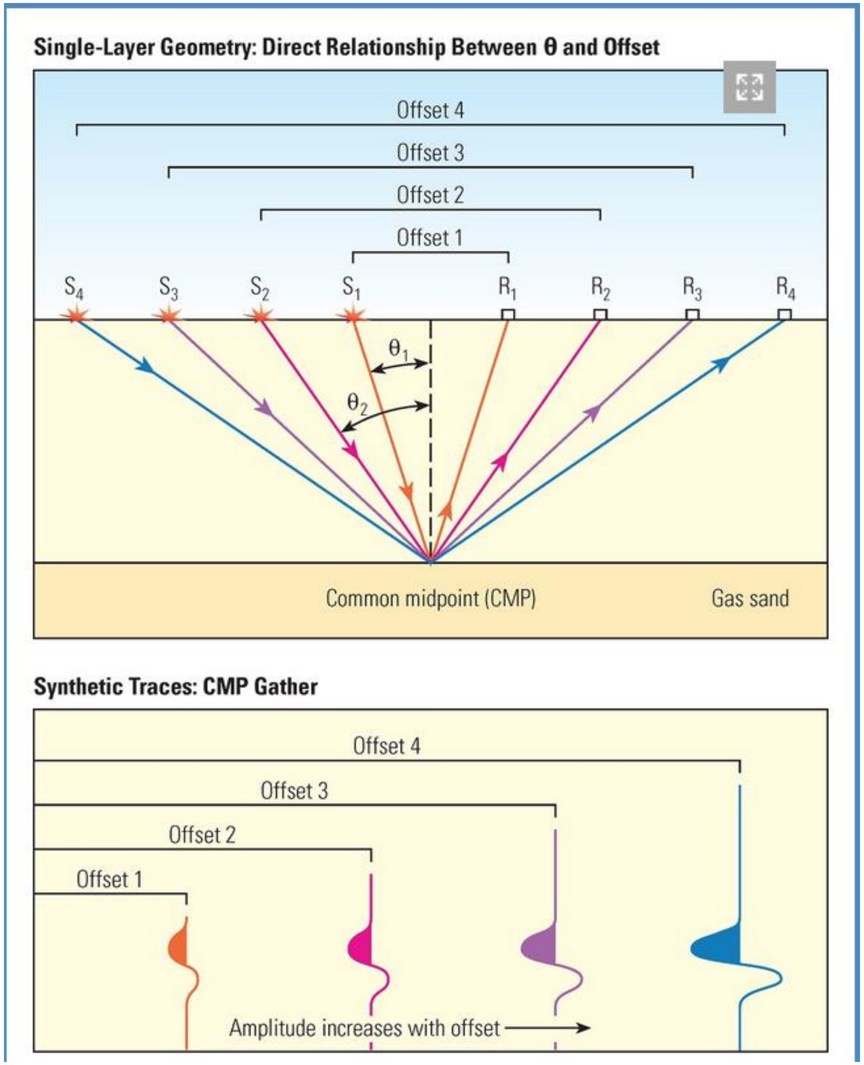


Figure from slb glossary illustrating the geometry for the AVO problem.

Introduction to AVO/AVAz

- **AVAz stands for amplitude variation with azimuth**
- **AVAz in common practice assumes vertical fractures in the subsurface.**
- **The vertical fractures cause the medium to be anisotropic in the x-y plane causing reflectivity to vary based on azimuth. (Close et al. 2010)**
- **Azimuth is measured with respect to the strike of the vertical fractures as seen in the top right figure.**

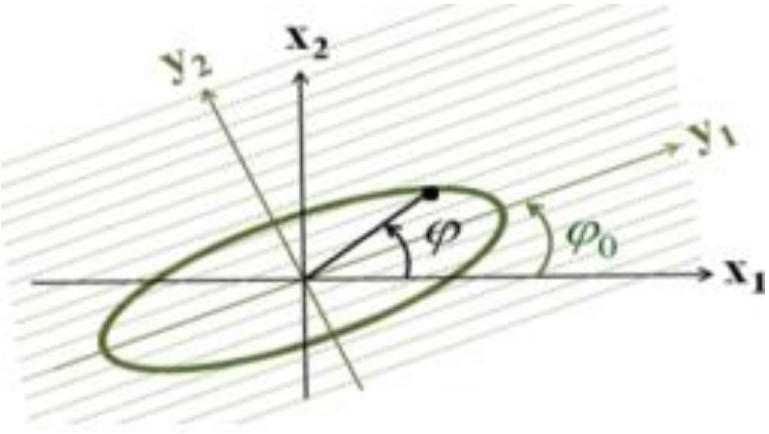


Figure showing AVAz geometry, the green lines are vertical fractures. The azimuth is the angle between the incoming wave and the fractures. Figure from Mahmoudian et al. 2012

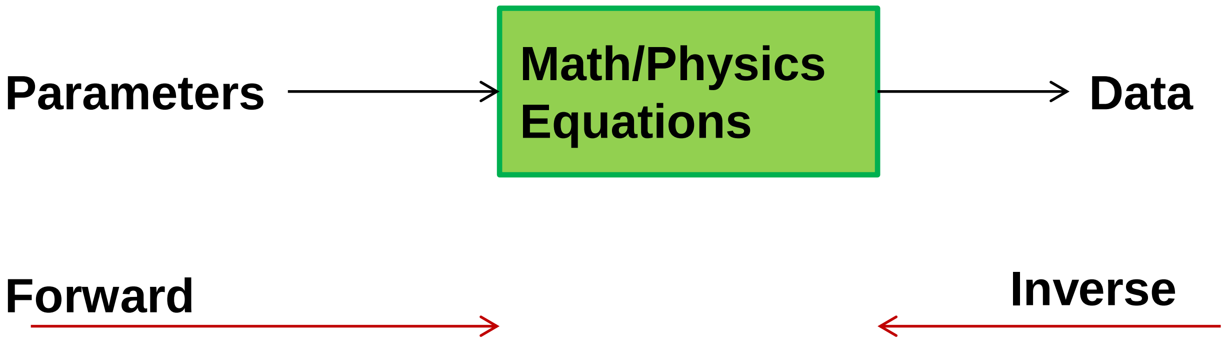
Figure from Close et al. 2010 demonstrating the fast and slow directions for vertical fractures.

Introduction to inversion

Two definitions that are important to inversion:

- **Forward modeling: Using parameters for governing physical or mathematical equations to predict data.**

- **Inverse modeling: using data generated by governing equations to predict parameters.**



- **Inverse problems often suffer from ambiguity, and are ill-conditioned. So it is often necessary to use outside knowledge to help constrain the solution.**

Theory: Forward modeling

The fractional percent of energy reflected in the subsurface is called the reflectivity. A linearized approximation of the equations governing reflectivity is shown here:

$$R_{PP} = A(\theta)\frac{\partial}{\partial t}ln(V_p(t)) + B(\theta)\frac{\partial}{\partial t}ln(V_s(t)) + C(\theta)\frac{\partial}{\partial t}ln(\rho(t)) + D(\theta,\phi)\frac{\partial}{\partial t}\delta(t) + E(\theta,\phi)\frac{\partial}{\partial t}\epsilon(t) + F(\theta,\phi)\frac{\partial}{\partial t}\gamma(t)$$

Where:

- Vp, Vs, and ρ are the P – wave velocity, S – wave velocity, and density.
- δ , ϵ, γ are Thomsen's anisotropy parameters
- ϴ is the angle of incidence to the point in the subsurface.
- Φ is the azimuth with respect to the vertical fractures.
- A, B, C, D, E, F are known coefficients that are functions of ϴ and Φ and calculated before hand. (See Aki-Richards, 1980 and Rüger 1997 for coefficients.)

Generating seismic traces from a ground model can be represented by three matrix operations.
G = WRD

D is the discrete derivative operator, R is the matrix of coefficients in the above equation, and W is a convolutional wavelet operator.

The above equation assumes the change in material properties are small and ϴ is less then the critical angle.

Theory: Inversion

Inversion problems are typically set up in the form of a cost function, in this presentation we use a cost function similar to Theune et al. (2010):

$$J = ||\mathbf{Gm} - \mathbf{d}||_2^2 + \mu(\mathbf{m} - \mathbf{m_0})^T \mathbf{W_m^T W_m}(\mathbf{m} - \mathbf{m_0})$$

Where:

- G is the forward AVO/AVAz operator.
- m is the parameter vector m = [ln(Vp(t)), ln(Vs(t)), ln(ρ(t)), δ(t), ϵ(t), γ(t)] ^{T}
- d is the observed data from the field.
- m_0 is an average value based on a priori knowledge. (Explained in the methods section)
- W_m contains a priori information about the model parameters. (Will be discussed in the methods section)
- μ is the trade off parameter giving relative weighting between the data and model parts. As a general rule the more noisy the data the bigger μ should be.

The goal is then to minimize the cost function. The solution that minimized the coast function is below:

$$\mathbf{m} = \mathbf{m_0} + \left(\mathbf{G^T G} + \mu \mathbf{W_m^T W_m}\right)^{-1} (\mathbf{d_{obs}} - \mathbf{Gm_0})$$

This result can also be found in Menke, 2012.

Methods: Ambiguity of the AVO problem.

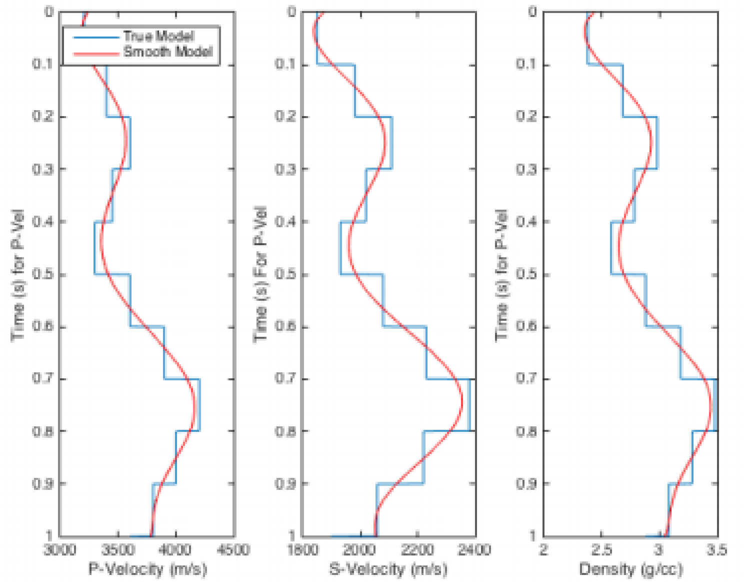


Figure showing a 1D isotropic earth model.

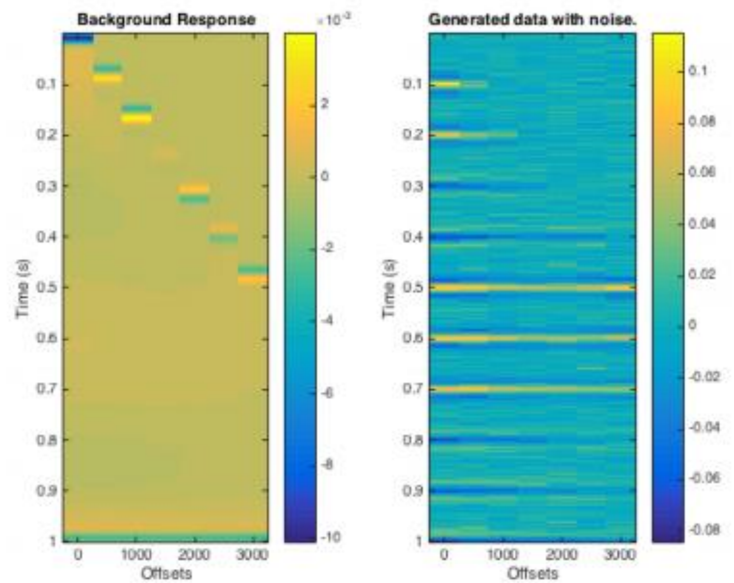


Figure showing AVO response of a 1D isotropic earth model. Left is of the smooth background model, right is of the true model.

- **The figure on the right shows the AVO response for the Red and blue models respectively. We can see the red gives almost no seismic response.**
- **The above figures demonstrate a short coming in seismic: any smooth or constant sections in the subsurface will be not be seen.**
- **We supplement this by ta**
- **This background model corresponds to m_0 shown in the earlier equations.**

Methods: Ill-conditioned nature of AVO problem

- **The AVO problem is known as Ill – conditioned.**
- **This means small changes in data can give large changes in model parameters. inverted for giving unreasonable results. See Lines, 1999 for an example with density.**
- **So far we have assume Vp, Vs, and ρ are mathematically independent. This is not the case.**
- **We want to capture geological information from the well logs to help stabilize the inversion.**
- **We do this with by finding covariance between the three parameters.**
- **Covariance will capture large scale geological effects. Examples include carbonate cementation, porosity changes, and fluid filled zones.**

$$\mathbf{W}_{\mathrm{m}}^{\mathbf{T}}\mathbf{W}_{\mathrm{m}} = \begin{bmatrix} \Sigma_{0_1} & 0 & 0 & \dots \\ 0 & \Sigma_{0_2} & 0 & \vdots \\ \vdots & \dots & \ddots & 0 \\ 0 & 0 & 0 & \Sigma_{0_n} \end{bmatrix}^{-1} = \begin{bmatrix} \Sigma_{0_1}^{-1} & 0 & 0 & \dots \\ 0 & \Sigma_{0_2}^{-1} & 0 & \vdots \\ \vdots & \dots & \ddots & 0 \\ 0 & 0 & 0 & \Sigma_{0_n}^{-1} \end{bmatrix}$$

$$\Sigma_0 = \begin{bmatrix} \tau_{V_P}^2 & \nu_{V_P,V_S} & \nu_{V_P,\rho} \\ \nu_{V_P,V_S} & \tau_{V_S}^2 & \nu_{V_S,\rho} \\ \tau_{V_P,\rho} & \nu_{V_S,\rho}^2 & \tau_\rho^2 \end{bmatrix}$$

Figure showing the matrix representation of the covariance between the model parameters for the isotropic case.

Shale Zone: Porosity decreases causing all parameters to increase slightly.
Gas Zone: Vp and ρ decrease drastically, Vs decrease slightly.

Carbonate Cement Zone: all parameters increase dramatically.

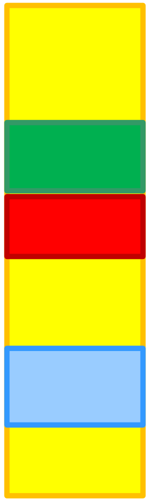


Figure illustrating a basic example of the information we want to use for constraining our inversion.

Methods: Practical implementation

AVO/AVAz problems are large in nature. Constructing the matrices and solving the system as written is to costly. In all practicality we use iterative methods to solve the problem.

We have a generic iterative conjugate gradient least squares algorithm to solve a cost function in the following form:

$$J = ||\mathbf{A}\mathbf{z} - \mathbf{d}'||_2^2 + \mu||\mathbf{z}||_2^2$$

The algorithm just needs a linear operator A, its transpose operation A^T, a data vector d' and an initial guess z_0.

Our cost function is in the following form:

$$J = ||\mathbf{G}\mathbf{m} - \mathbf{d}||_2^2 + \mu(\mathbf{m} - \mathbf{m_0})^T \mathbf{W_m^T} \mathbf{W_m} (\mathbf{m} - \mathbf{m_0})$$

To get our cost function into the generic form we use a change of variables. Using singular value decomposition on $W_m^T W_m$ we can get W_m. Then by assigning d' = (d – Gm_0), A = G W_m^{-1} and z = W_m(m - m_0) we can reformulate our initial cost function into the above form. So we solve for z and change back to m to get the solution.

Results: Aki Richards synthetic data

Synthetic data was generated using the Aki-Richard's formulation (isotropic case).

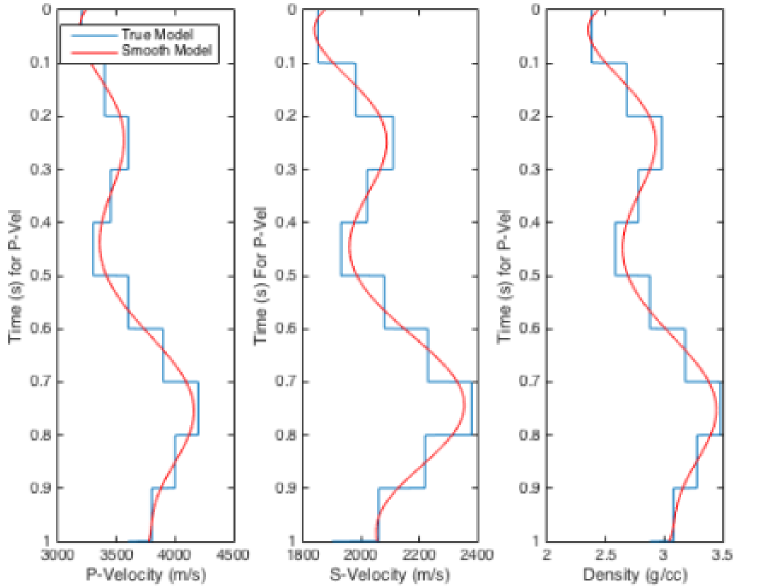


Figure showing the generated synthetic model for a 1D earth model. The red line is the smooth background model used for inversion.

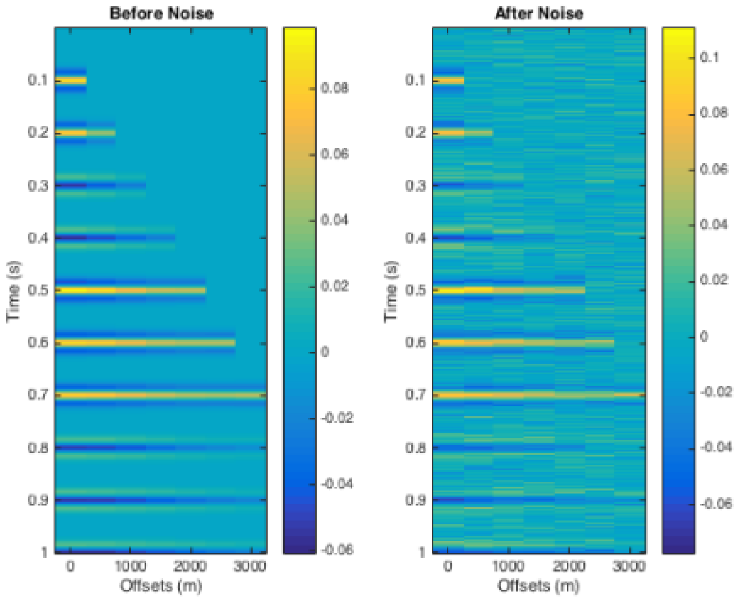


Figure showing the generated synthetic data for a 1D earth model.

- **Data was generated using a 25hz zero phase ricker wavelet with time steps of 0.5ms.**
- **The red line represents the smooth background model used for inversion.**
- **Noise was added to the data to give a signal to noise ratio of three.**

Results: Aki Richards synthetic data

Using the methods outlined earlier we can invert the data using a perfect forward model to outline the best possible case for inversion.

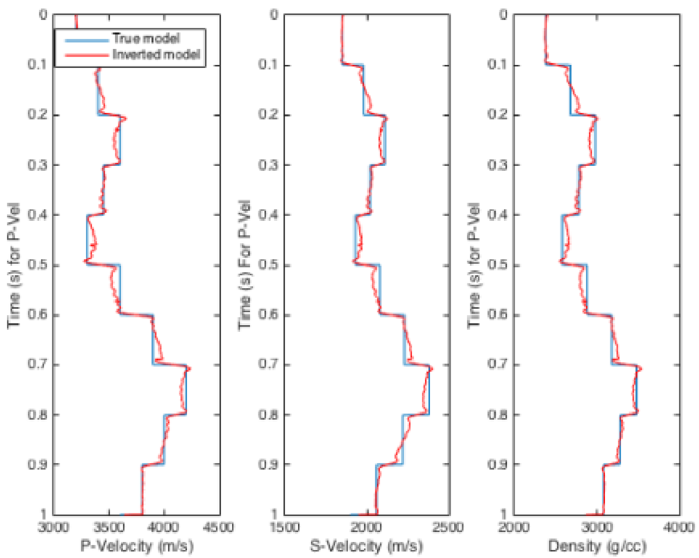


Figure showing the true subsurface model in blue and the inverted subsurface model in red.

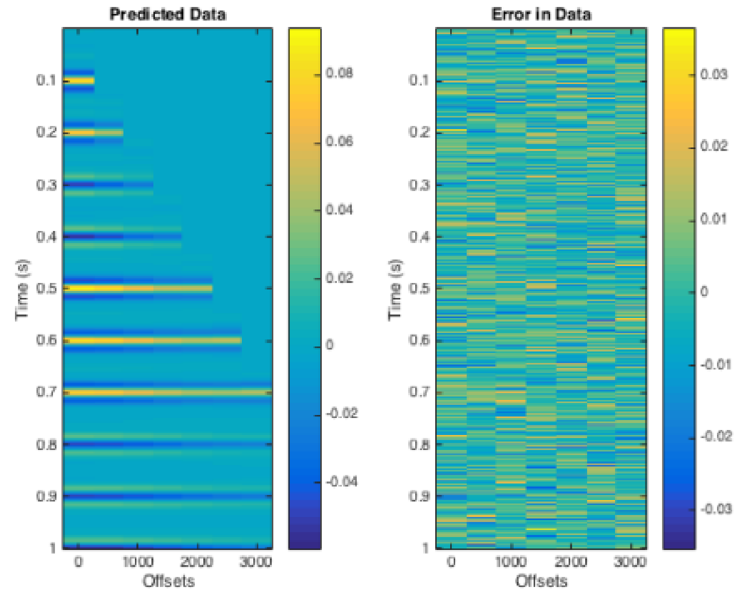


Figure showing AVO response from the inverted model. All of the error appears uncorrelated.

- **Inversion using a perfect forward model that gives near perfect results.**
- **No forward model will ever be perfect in real life, but this gives a theoretical limit to see how well our regularization is stabilizing the inversion.**
- **Covariance does a very good job. It is giving blocky solutions naturally.**
- **Results show that no more regularization is needed.**

Results: Aki Richards synthetic data

With the perfect forward model working what happens when the forward model becomes imperfect. The results below show the inverted models with a 15hz and 35hz wavelet.

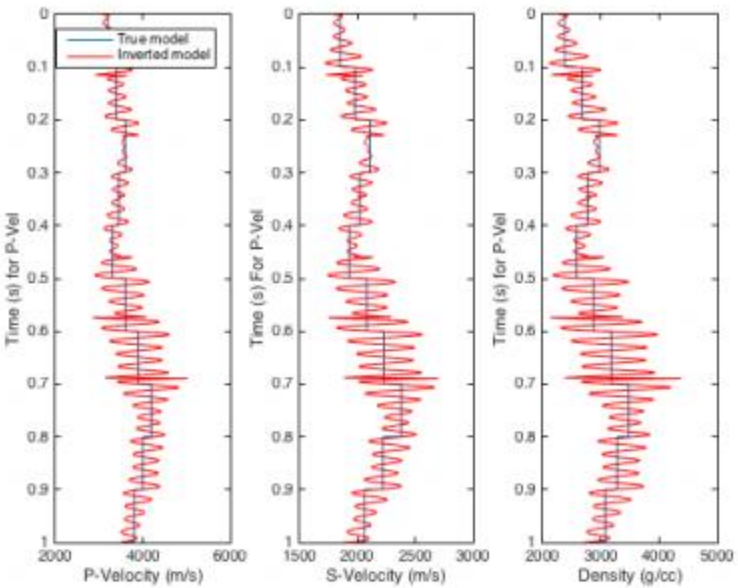


Figure showing the true subsurface model in blue and the inverted subsurface model in red when a 15hz wavelet is used.

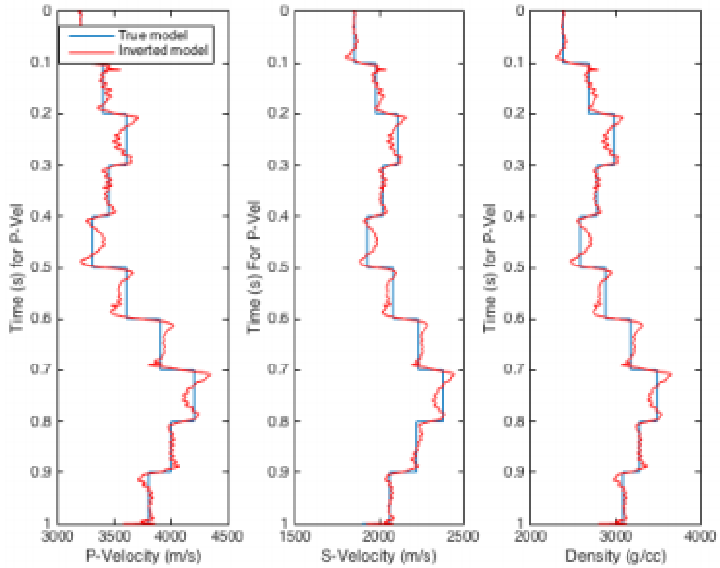


Figure showing the true subsurface model in blue and the inverted subsurface model in red when a 35hz wavelet is used.

- **Both inverted models predict the data just as well as the perfect forward model, but clearly not realistic.**

- **This test demonstrates how sensitive the inversion is to the wavelet, so it is important to take a care when doing the wavelet estimations.**

Results: Aki Richards synthetic data

The next variable the inversion algorithm is sensitive to is how the depth is estimated, which is used to calculate ϴ. P wave RMS velocity is to estimate depth is shown on the left, while the smooth background velocity is shown on the right.

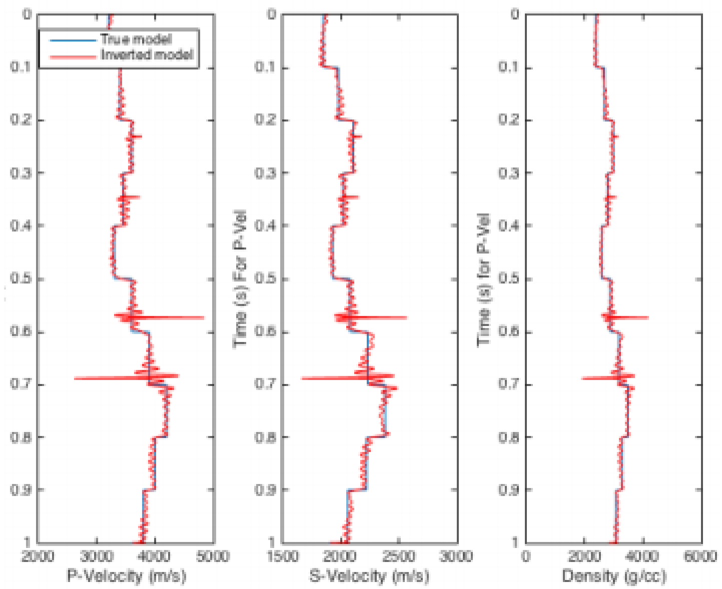


Figure showing the true subsurface model in blue and the inverted subsurface model in red when the depth is estimated from the Vp RMS velocity

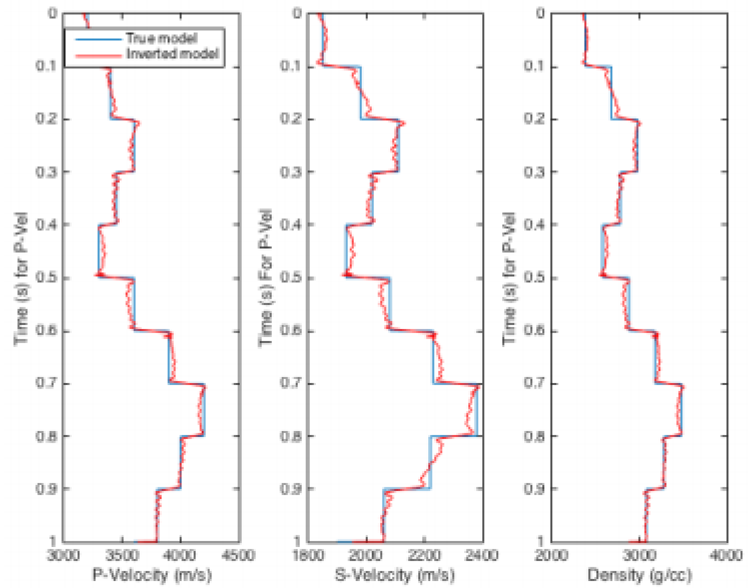


Figure showing the true subsurface model in blue and the inverted subsurface model in red when a the background velocity model to estimate depth is used.

- **Both predict the data the same as in the perfect case.**
- **This test demonstrates that is background velocity model does a better job than using the RMS of the P-wave velocity from the well logs, however the data is still interpretable and could be smoothed if that was the only option.**

Results: Aki Richards synthetic data

Here is a demonstration of using a wrong background model. The results are as expected.

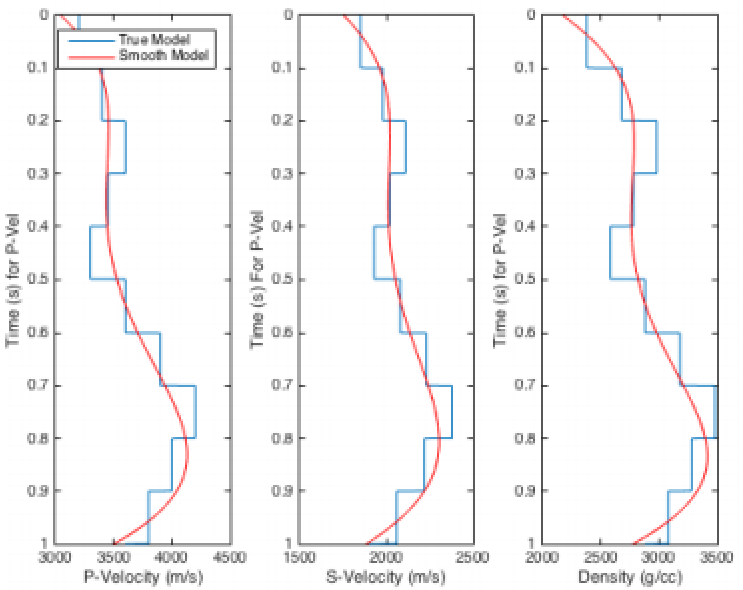


Figure showing the true subsurface model in blue and the wrong background model used for inversion.

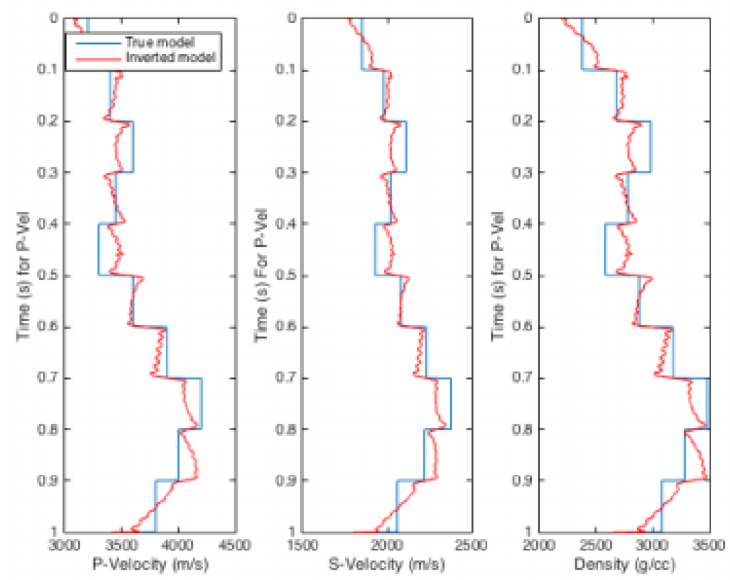


Figure showing the true subsurface model in blue and the inverted subsurface model in red when a fourth order polynomial is fit instead of a 10th order.

- **Both predict the data the same as in the perfect case.**
- **This test demonstrates how important it is to use a good background model from the well logs you want to capture the general trends.**

Results: Rüger synthetic data

Synthetic data was generated for the 3D anisotropic case. Below shows the synthetic data for four different Azimuths.

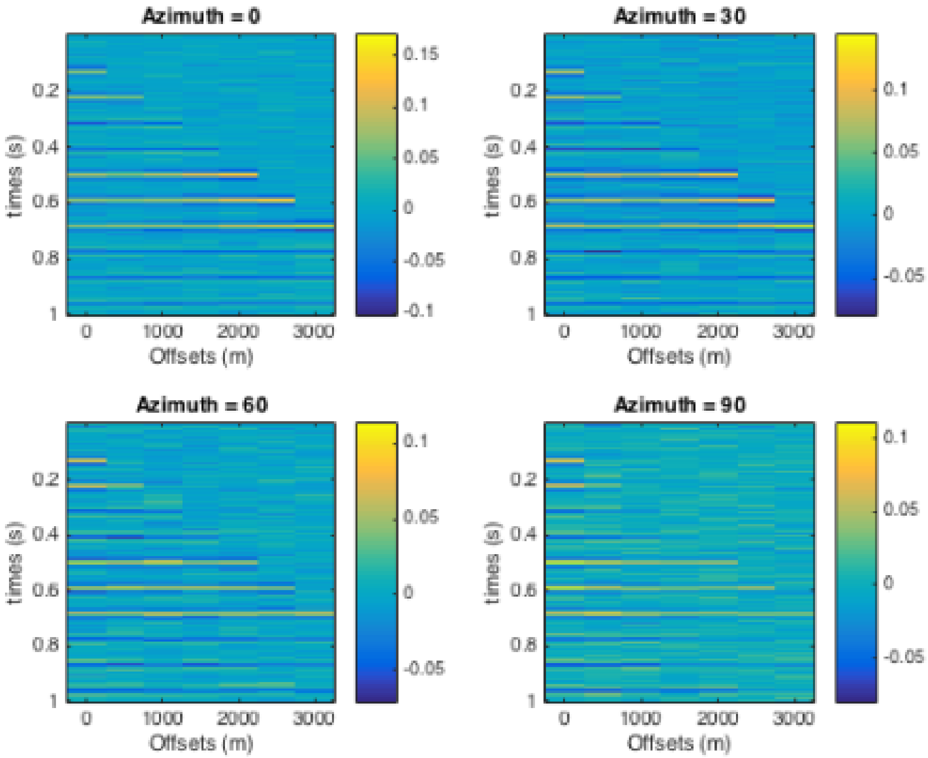


Figure showing the AVAz response for an anisotropic vertically fractured synthetic ground model.

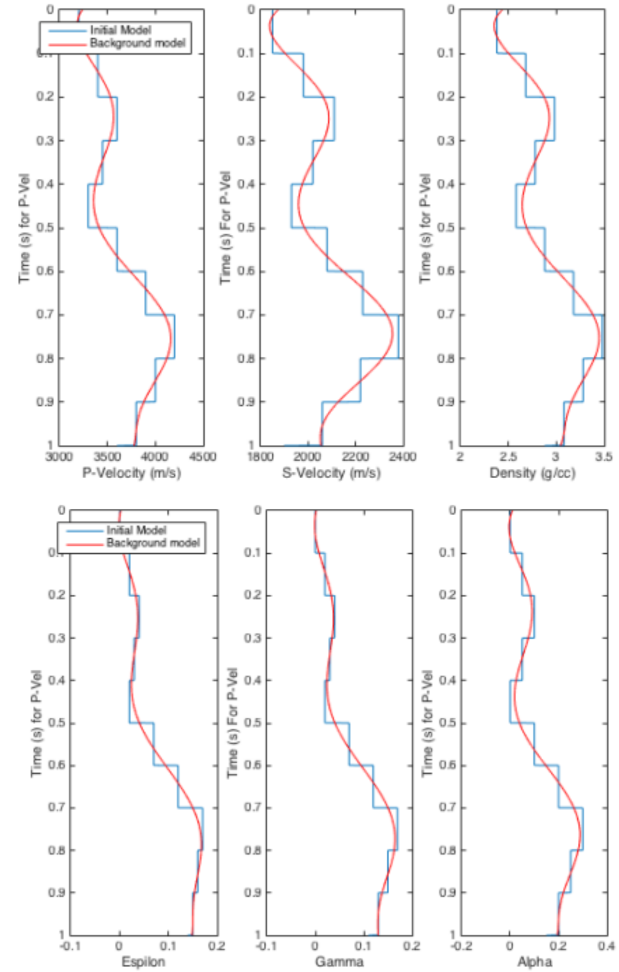
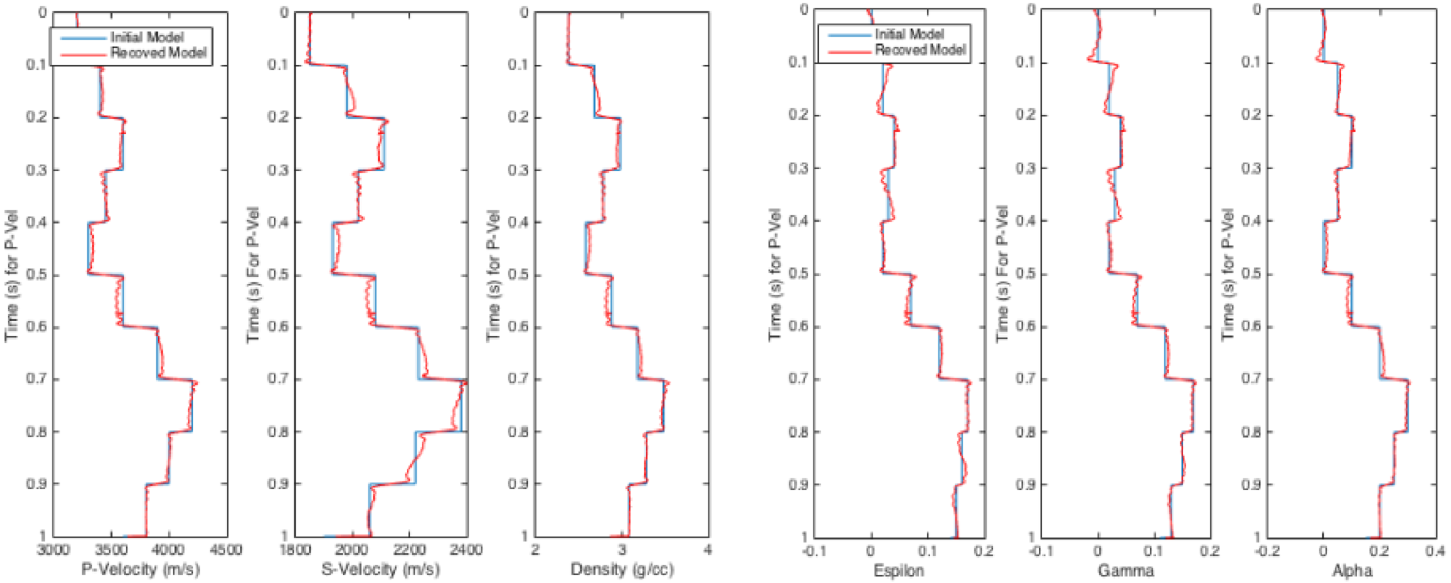


Figure shows the six parameter model used to generate the data.

Results: Rüger synthetic data

Inversion results using a perfect forward model. Similar signatures are shown as in the isotropic case as the model becomes imperfect.



- **The inverted models fit the data extremely well. The thompson parameters can be estimated from the well logs as shown in Sil, 2013 and Prioul et al. 2007. So the method does not change in cases of anisotropy as we can generate well logs for the anisotropic parameters.**

- **Unfortunately no real data could be obtained with a vertically fractured sub-surface model to try this method, but the synthetic results give hope real data could be inverted with good results.**

Results: Real data inversion Puesto Peter Field

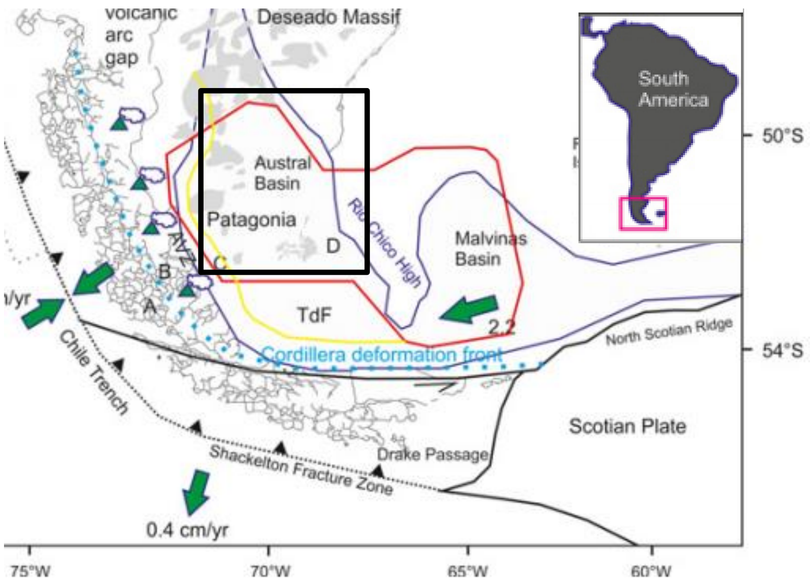
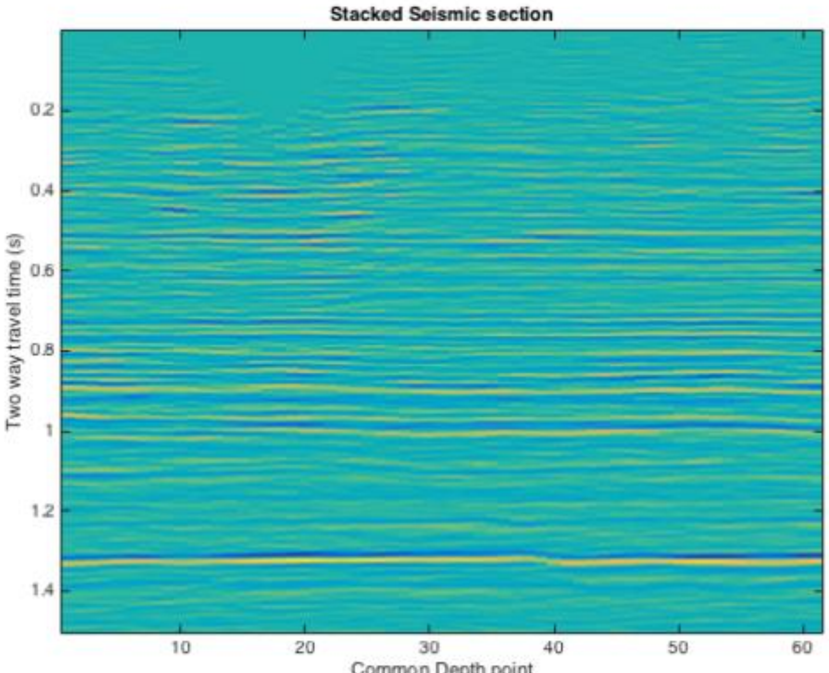


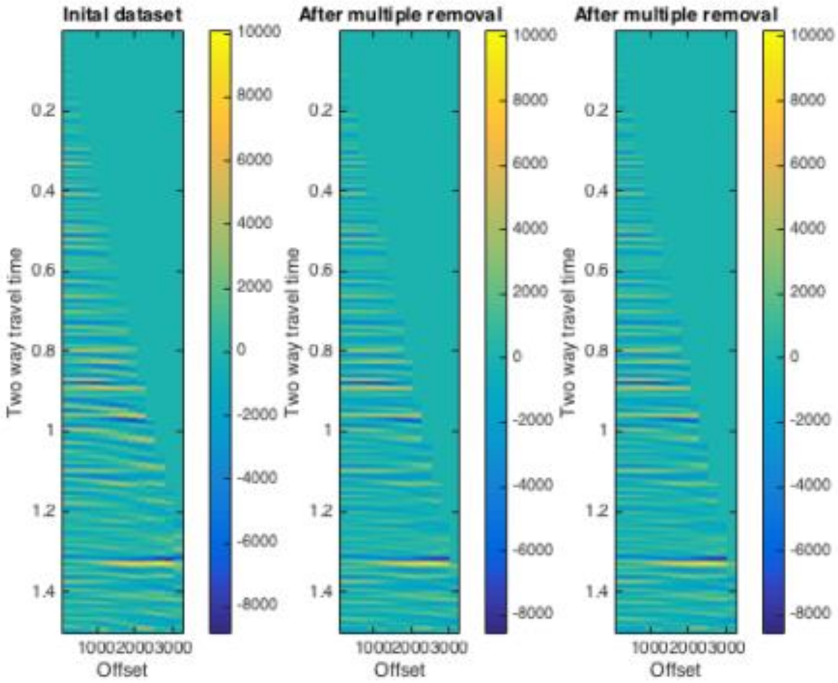
Figure showing location of Austral basin figure from Sachse et al. 2015

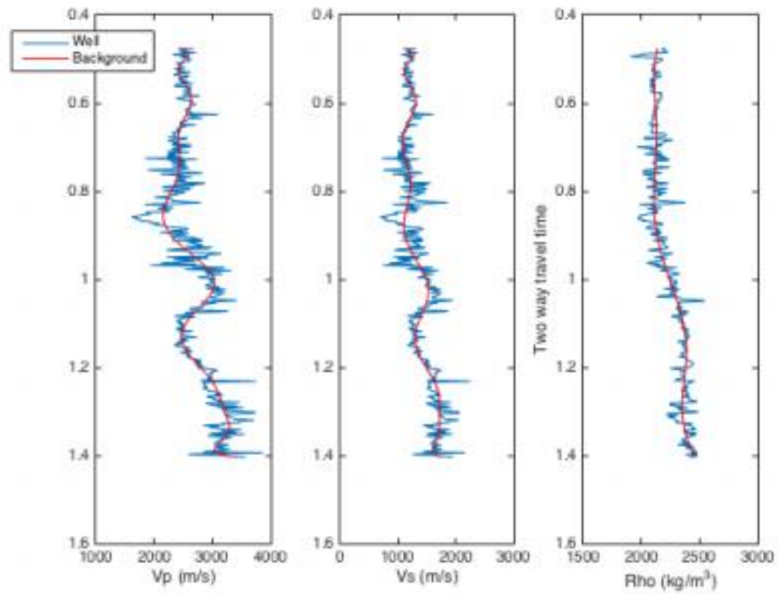


Stacked section for Puesto Peter Field, Austral Basin, Argentina

- **Puesto Peter field is in the retro-arc foreland basin known as the Austral Basin in Argentina. (Sachse et al. 2015)**
- **Main reservoir is called "M3" in Soldo 2009 with good petrophysical qualities giving about 500mD of permeability. AVO anomolys are classified as a type III.**
- **M3 is the bright reflector at about 1.3s.**

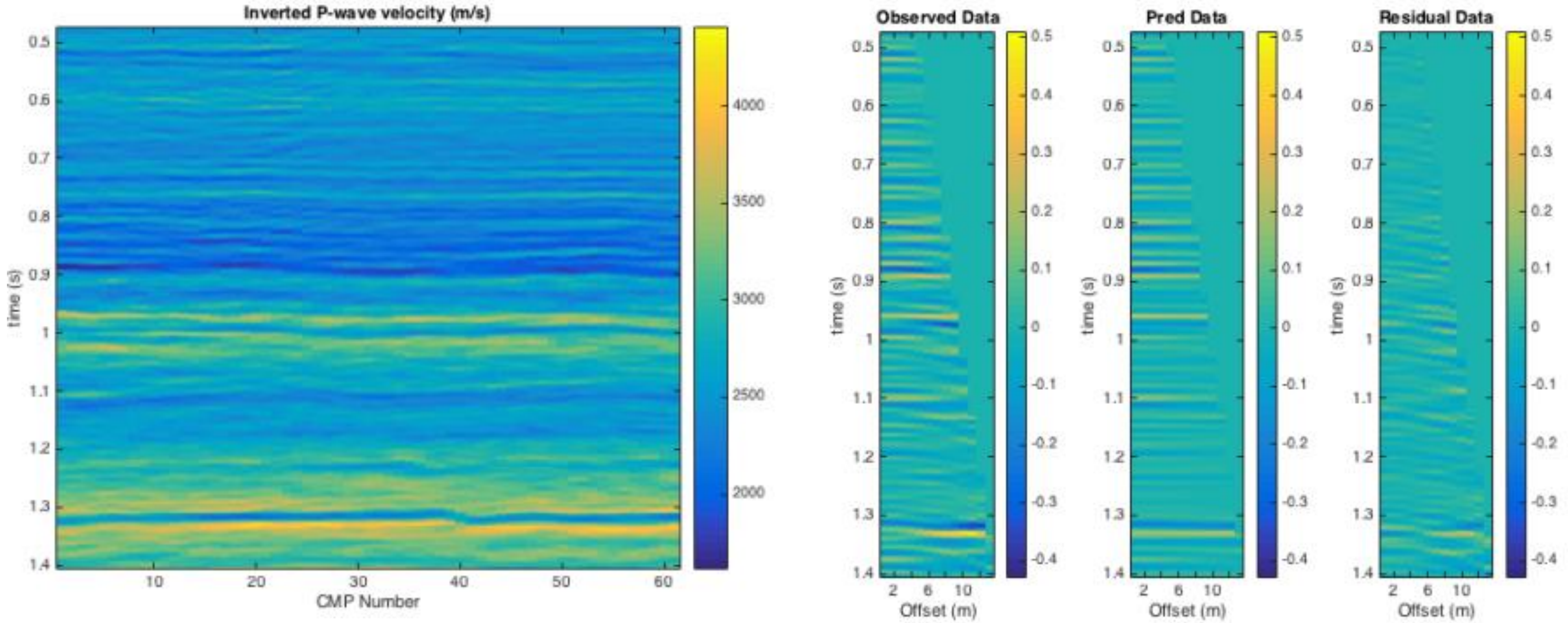
Results: Real data inversion Puesto Peter Field





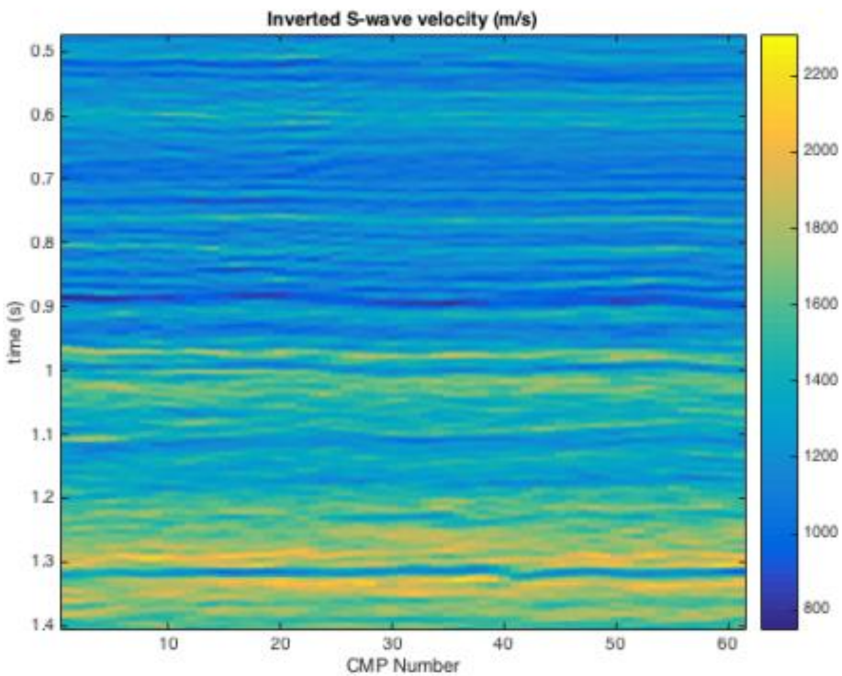
- **Data set was good data quality with some processing that needed to be done.**
- **Parabolic Radon de-multiple tool from SeismicLab was run on the dataset to remove multiples that would produce errors in inversion.**
- **The smooth background model used is shown on the right.**

Results: Real data inversion Puesto Peter Field

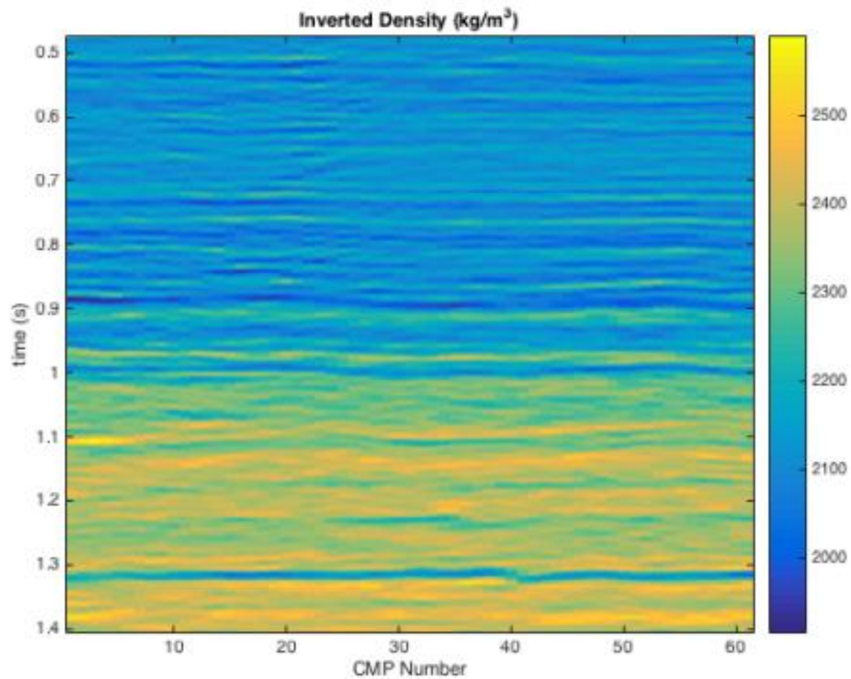


- The Inversion is successful with the M3 reservoir clearly seen in the inverted P wave velocity.
- The predicted data shows a good fit, but there does appear to be some anisotropic effects in the reflectors that the forward model is unable to account for.

Results: Real data inversion Puesto Peter Field



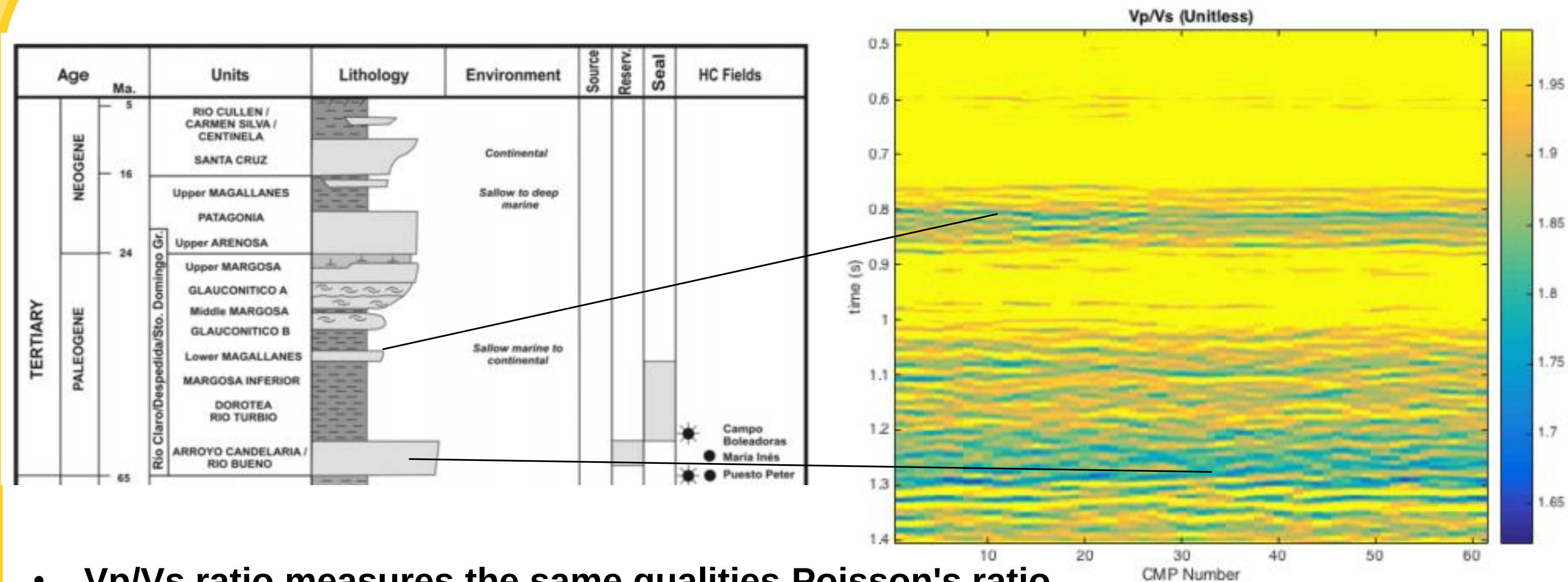
Inverted S – Wave velocity from Puesto Peter Field.



Inverted density of Puesto Peter Field.

- **The depositional setting of this area changes from shallow marine, to deep marine, to shallow marine/continental.**
- **The harmonic fluctuations in velocities is likely an indicator of sea level. High velocities are likely lower porosity shales, while lower velocities are likely higher porosity sandstones for a shallow/deep marine environment.**

Discussion: Interpretation with Vp/Vs ratio



- **Vp/Vs ratio measures the same qualities Poisson's ratio.**

- **Low Possion's ratio is often an indicator of gas zones. This is shown from our known reservoir (Chopra and Castagna, 2014; Goodway, 2006).**

- **Well lithified sandstone is generally Vp/Vs = 1.6 – 1.7 but will be higher otherwise, so it also helps lithology discrimination (Chopra and Castagna, 2014) .**
- **Magallanes is also a formation known to have hydrocarbons (Soldo, 2009), so based on the complete information I would think it is at about 0.8s.**

Discussion: Strengths and weaknesses.

- This method is different than what is typically done in industry. Most industry applications use a two term approach which makes an assumption about density or only uses small incidence angles.

- This method relies on the wells being representative of the geology in the area looking to be inverted. In areas shown in Puesto Peter Field this is a very good assumption. If the wells are far away or large amounts of unconformities are present this method may run into problems.

- This method has been published for isotropic cases (see Buland et al. 2003, and Theune et al. 2010), but I have not seen this applied to anisotropic cases. Synthetic inversion suggest good results are possible.

- The hydrocarbons in this field are very light (0.5g/cc for oil and 0.2 to 0.3 g/cc for gas as indicated by Soldo, 2009). So using density in combination with other seismic attributes is a very powerful tool for assessing the reservoir.

- Chopra and Castagna, 2014 mention that seismic attributes do not add any information to the data, but change the way we look at it. I would extend that to say it is true for all inversion. Taking all of our known information to generate a likely model doesn't add any new information, but makes it much easier to interpret. Which is why we do inversion! for ease of interpretation.

Conclusion

- This presentation showed theory, methods, results for solving the AVO/AVAz inversion problem with using well logs to constrain the inversion.
- It has the advantage of being able to invert for all three material properties for the isotropic case, and all six for the anisotropic case.
- This method relies on the well logs being representative of the geology for the area to be inverted.
- The method inverts data by minimizing a cost function, which minimizes the misfit between the predicted and observed data, while making sure the model is as close to our a priori knowledge about the area.
- AVO/AVAz inversion shown to be a very useful tool to aid interpretation of the area, but it is very important be understand our assumptions, or else we will introduce error.
- AVO inversion has been shown to use to well log data to for three term inversion but I have found no papers that use the same method for six terms.
- Unfortunately AVAz data could not be acquired but results of synthetic data give indication for reasonable results.

Acknowledgments

I would like to thank my supervisor Dr. Mauricio Sacchi for the excellent guidance he has provided over the course of this project. I would also like to thank the members of the Signal Analysis and imaging group for being enthusiastic with help and questions that I had with this project. Most importantly, I would like to thank my Father for his endless support and love. This would all not be possible without him.

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36
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