

Joshua Mitchell; MATH 5374; Exercises 10.1a, 10.3

10.1a

Determine the eigenvalues of a Householder reflector. Give a geometric as well as an algebraic proof.

The eigenvalues of a Householder reflector F are all 1's.

Algebraic Proof:

A house holder reflector F takes the form:

$$F = I - 2 \frac{vv^*}{v^*v}$$

Notice:

$$Fx = ||x||e_k$$

For all $x \in \mathbb{C}^{m-k+1}$

Since eigenvalues take the form:

$$\begin{aligned} Fx &= \lambda x \\ &= ||x||e_k \end{aligned}$$

That means that e_k must be an eigenvector of F . Since e_k has length 1, that means 1 is an eigenvalue.

Since F is unitary, that implies that there are no eigenvalues that aren't 1.

Geometric proof:

A Householder reflector always reflects across H to align with e_k . The only way it wouldn't change x 's orientation is if x were already a multiple of e_k (which would make $e_k \in H$). Since F is unitary, it doesn't stretch vectors, which makes 1 its only possible eigenvalue, and e_k its only possible eigenvector.

10.3

Let Z be the matrix:

$$Z = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix}_{5 \times 3}$$

Compute 3 reduced QR factorizations of Z in MATLAB: by the Gram-Schmidt routine `mgs` of Exercise 8.2, by the Householder routines `house` and `formQ` of Exercise 10.2, and by MATLAB's built-in command `[Q, R] = qr(Z, 0)`. Compare these three and comment on any differences you see.

They actually seem identical, save a few negative signs. Sometimes the Q column is negative and the corresponding R entries are positive, but sometimes the opposite is true.

There are also a few columns in `mgs Q` that are the negative of the other Q 's (i.e. column 2).

`mgs_Q` =

0.1010	0.3162	0.5420
0.4041	0.3534	0.5162
0.7071	0.3906	-0.5248
0.4041	-0.5580	0.3871
0.4041	-0.5580	-0.1204

`mgs_R` =

9.8995	9.4954	9.6975
0	3.2919	3.0129
0	0	1.9701

`house_Q` =

-0.1010	-0.3162	0.5420
-0.4041	-0.3534	0.5162
-0.7071	-0.3906	-0.5248
-0.4041	0.5580	0.3871
-0.4041	0.5580	-0.1204

house_R =

-9.8995	-9.4954	-9.6975
0	-3.2919	-3.0129
0	0	1.9701
0	0	-0.0000
0	0	0.0000

builtin_Q =

-0.1010	-0.3162	0.5420
-0.4041	-0.3534	0.5162
-0.7071	-0.3906	-0.5248
-0.4041	0.5580	0.3871
-0.4041	0.5580	-0.1204

builtin_R =

-9.8995	-9.4954	-9.6975
0	-3.2919	-3.0129
0	0	1.9701