Joshua Mitchell; MATH 5374; Exercises 11.3, 12.2

11.3

Take m = 50, n = 12. Using MATLAB's linspace, define t to be the m vector corresponding to linearly spaced grid points from 0 to 1. Using MATLAB's vander and fliplr, define A to be the m by n matrix associated with least squares fitting on this grid by a polynomial of degree n − 1. Take b to be the function cos(4t) evaluated on the grid. Now, calculate and print (to sixteen digit precision) the least squares coefficient vector x by 6 methods:

1. Formulation and solution of the normal equations, using MATLAB's backslash.
2. QR factorization computed by mgs (Ex 8.2)
3. QR factorization computed by house (Ex 10.2)
4. QR factorization computed by MATLAB's qr
5. x = A / b in MATLAB.
6. SVD, using MATLAB's svd

The calculations above will produce six lists of twelve coefficients. In each list, shade with red pen the digits that appear to be wrong (affected by rounding error). Comment on what differences you observe. Do the normal equations exhibit instability? You don't have to explain your observations.

normal	mgs	house
0.999999997567501	0.999999997983937	1.0000000009966
1.19953127407744e-07	4.78462577347366e-07	-4.22742792398997e-07
-7.99999434000841	-8.00001529261281	-7.99998123569055
-0.000206053460520519	0.000185380629590156	-0.000318763183438443
10.6690658679798	10.6655568413165	10.6694307956079
-0.0140486066062536	0.0037165176931723	-0.0138202868619189
-5.64190754803297	-5.69692814231864	-5.64707563027385
-0.091917808094205	0.0161887718253664	-0.0753160192413515
1.72057294151753	1.58527555554907	1.69360695765875
-0.0185769442051395	0.085914578547369	0.00603211297775285
-0.362218729231468	-0.407608493574646	-0.374241705194851
0.0855874800273986	0.094070189115174	0.088040576386011

qr_built_in	qr_normal	svd
1.00000000099661	1.00000000099661	1.00000000099661
-4.22743079283837e-07	-4.22743318060613e-07	-4.22743078905345e-07
-7.99998123568522	-7.99998123567744	-7.99998123568494
-0.00031876323121668	-0.00031876333071431	-0.000318763235895195
10.6694307958579	10.6694307965368	10.6694307958938
-0.0138202876981328	-0.0138202904904775	-0.0138202878532729
-5.64707562840455	-5.64707562106602	-5.64707562799499
-0.0753160220795574	-0.0753160346954749	-0.0753160227650345
1.69360696055882	1.69360697468771	1.69360696128871
0.0060321110639337	0.00603210113142263	0.00603211058541004
-0.374241704456949	-0.374241700477198	-0.374241704281134
0.088040576259666	0.088040575566367	0.0880405762320284

For the most part, house, SVD, x = A / b and MATLAB's built in QR line up pretty well. It's the normal equations and mgs that are noticably wrong (as compared to the other 4). When comparing mgs and normal equations to the other 4, mgs and normal equations are very often wrong on the first significant decimal place, and when they're right on the first, they're wrong by the 3rd or 4th decimal place at the latest.

The normal equations definitely exhibit instability (as indicated by MATLAB and this error):

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Warning: Matrix is close to singular or badly scaled. Results may be inaccurate.
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12.2

In example 11.1 we remarked that polynomial interpolation in equispaced points is ill-conditioned. To illustrate this phenomenon, let x_1, x_2, ..., x_n and y_1, y_2, ..., y_m be n and m equispaced points from -1 to 1, respectively.

1. Derive a formula for the m x n matrix A that maps an n-vector of data at $\{x_j\}$ to an m-vector of sampled values $\{p(y_j)\}$, where p is the degree $n - 1$ polynomial interpolant of the data (see Example 1.1).

2. Write a program to calculate A and plot $||A||_\infty$ on a semilog scale for $n = 1, 2, ..., 30$, and $m = 2n - 1$.

   In the continuous limit $m \longrightarrow \infty$, the numbers $||A||_\infty$ are known as Lebesgue constants for equispaced interpolation, which are asymptotic to $2^n/(e(n - 1) \log n)$ as $n \longrightarrow \infty$.

3. For $n = 1, 2, ..., 30$ and $m = 2n - 1$, what is the ∞ norm condition number k of hte problem of interpolating the constant function 1? (Use 12.6).

4. How close is your result for $n = 11$ to the bound implicit in Figure 11.1?