Joshua Mitchell; MATH 5374; Exercises 10.1a, 10.3

10.1a

Determine the eigenvalues of a Householder reflector. Give a geometric as well as an algebraic proof.

The eigenvalues of a Householder reflector F are all 1's.

Algebraic Proof:

A house holder reflector F takes the form:

$$F = I - 2\frac{vv^*}{v^*v}$$

Notice:

$$Fx = ||x||e_k$$

For all x $\in \mathbb{C}^{m-k+1}$
Since eigenvalues take the form:

$$\begin{aligned} Fx &= \lambda x \\ &= ||x||e_k \end{aligned}$$

That means that e_k must be an eigenvector of F. Since e_k has length 1, that means 1 is an eigenvalue.
Since F is unitary, that implies that there are no eigenvalues that aren't 1.

Geometric proof:

A Householder reflector always reflects across H to align with e_k. The only way it wouldn't change x's orientation is if x were already a multiple of e_k (which would make $e_k \in$ H). Since F is unitary, it doesn't stretch vectors, which makes 1 its only possible eigenvalue, and e_k its only possible eigenvector.

10.3

Let Z be the matrix:

$$Z = \underset{5\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix}}$$

Compute 3 reduced QR factorizations of Z in MATLAB: by the Gram-Schmidt routine mgs of Exercise 8.2, by the Householder routines house and formQ of Exercise 10.2, and by MATLAB's built-in command [Q, R] = qr(Z, 0). Compare these three and comment on any differences you see.

They actually seem identical, save a few negative signs. Sometimes the Q column is negative and the corresponding R entries are positive, but sometimes the opposite is true.

There are also a few columns in mgs Q that are the negative of the other Q's (i.e. column 2).

```
mgs_Q =

    0.1010    0.3162    0.5420
    0.4041    0.3534    0.5162
    0.7071    0.3906   -0.5248
    0.4041   -0.5580    0.3871
    0.4041   -0.5580   -0.1204


mgs_R =

    9.8995    9.4954    9.6975
         0    3.2919    3.0129
         0         0    1.9701

house_Q =
```

```
   -0.1010   -0.3162    0.5420
   -0.4041   -0.3534    0.5162
   -0.7071   -0.3906   -0.5248
   -0.4041    0.5580    0.3871
   -0.4041    0.5580   -0.1204

house_R =

   -9.8995   -9.4954   -9.6975
         0   -3.2919   -3.0129
         0         0    1.9701
         0         0   -0.0000
         0         0    0.0000

builtin_Q =

   -0.1010   -0.3162    0.5420
   -0.4041   -0.3534    0.5162
   -0.7071   -0.3906   -0.5248
   -0.4041    0.5580    0.3871
   -0.4041    0.5580   -0.1204


builtin_R =

   -9.8995   -9.4954   -9.6975
         0   -3.2919   -3.0129
         0         0    1.9701
```