Assignment Set: 1, 2, 3, 4, 6, 8 from pages 148 - 149

1)

Mark each statement as true or false. Justify each answer.

a. A set S is compact iff every open cover of S contains a finite subcover.

b. Every finite set is compact.

c. No infinite set is compact.

d. If a set is compact, then it has a maximum and a minimum.

e. If a set has a maximum and a minimum, then it is compact.

2)

Mark each statement as true or false. Justify each answer.

a. Some unbounded sets are compact.

b. If S $\subset \mathbb{R}$ is compact, then $\exists$ x $\in \mathbb{R}$ st s $\in$ S$'$

c. If S is compact and s $\in$ S$'$, then s $\in$ S.

d. If S is unbounded, then S has at least one accumulation point.

e. **Let:** F = $\{$A$_i$, i $\in \mathbb{N}$ $\}$. Suppose that the intersection of any finite subfamily of F is nonempty. If $\cap$ F = $\emptyset$, then, for some k $\in \mathbb{N}$, A$_k$ is not compact.

3)

Show that each subset of $\mathbb{R}$ is not compact by describing an open cover for it that has no finite subcover.

a. ‘[1, 3)

b. ‘[1,2) $\cup$ (3, 4]

c. $\mathbb{N}$

d. $\{\frac{1}{n}$: n $\in \mathbb{N}$ $\}$

e. $\{$x $\in \mathbb{Q}$: 0 $\leq$ x $\leq$ 2$\}$

4)

Prove that the intersection of any collection of compact sets is compact.

6)

Show that compactedness is necessary in Corollary 3.5.8. That is, find a family of intervals $\{A_n : n \in \mathbb{N}\}$ with $A_{n+1} \subset A_n \; \forall$ n, $\bigcup_{n=1}^{\infty} A_n = \emptyset$, and such that:

a. The sets A_n are all closed.

b. The sets A_n are all bounded.

8)

If $S \subset \mathbb{R}$ is compact and $T \subset S$ is closed, then T is compact.

a. Prove this using the definition of compactness.

b. Prove this using the Heine-Borel theorem.