Practice: (2.2) 3, 4, (2.3) 2, 3

Hand In: (2.2) 7, 10, (2.3) 16, 28

Hint for 2.2.7: By symmetry, each edge of K_n appears in the same number of spanning trees of K_n.
Hint for 2.2.10: Only compute $\tau(K_{2,m})$

2.2.7

(!) Use Cayley's Formula to prove that the graph obtained from K_n by deleting an edge has (n - 2)n^{n-3} spanning trees.

Given n vertices, there are $2^{\binom{n}{2}}$ possible simple graphs. Using this, we may determine that there are n^{n-2} possible spanning trees (Cayley's Formula).
Since K_n contains the maximum possible amount of edges, K_n also contains the maximum possible amount of spanning trees for n vertices: n^{n-2}

2.2.10

Compute $\tau(K_{2,m})$.

$\tau(K_{2,m})$ = the number of spanning trees in G.

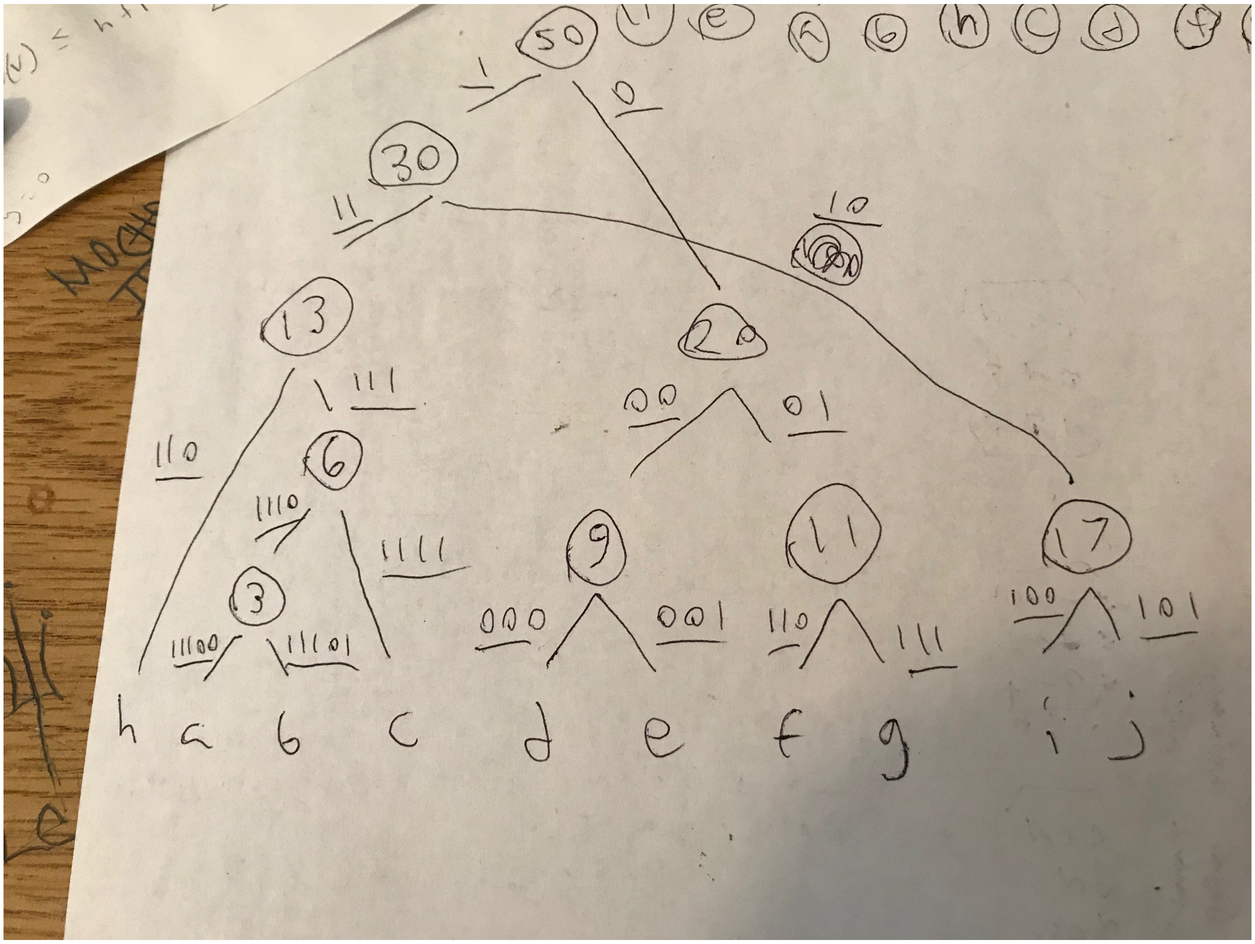
2.3.16

Four people must cross a canyon at night on a fragile bridge. At most two people can be on the bridge at once. Crossing requires carrying a flashlight, and there is only one flashlight (which can cross only by being carried). Alone, the four people cross in 10, 5, 2, 1 minutes, respectively. When two cross together, they move at the speed of the slower person. In 18 minutes, a flash flood coming down the canyon will wash away the bridge. Can four people get across in time? Prove your answer without using graph theory and describe how the answer can be found using graph theory.

Yes, it is possible. Just send the person who takes 2 minutes with the person who takes 1 minute (let's call these people by the number of minutes they take to cross the bridge), and then have 1 return with the flashlight. At this point, 3 minutes have elapsed.
Then, send 5 and 10 with the flashlight and have 2 return with it. At this point, 3 + 10 + 2 = 15 minutes have elapsed.
Finally, have 1 and 2 cross the bridge. At this point, 17 minutes have elapsed. They have crossed the bridge in time.
In terms of graph theory, if we think of the amount of time it takes to cross the bridge as a weight for the edge between two vertices, then we just have to come up with all possible combinations of states this scenario could have, and represent them as vertices.
Then, if it's possible to get from one state to another, we draw an edge between each state's vertex and put a weight corresponding to the amount of time between each state change.
Finally, we're trying to get from some initial state to the ending state in the shortest time possible, so it makes sense to use a shortest path algorithm (like Dijkstra's) to find the shortest path.

2.3.28

Compute a code with minimum expected length for a set of ten messages whose relative frequencies are 1, 2, 3, 4, 5, 5, 6, 7, 8, 9. What is the expected length of a message in this optimal code?



item	frequency	code	length
a	1	11100	5
b	2	11101	5
c	3	1111	4
d	4	000	3
e	5	001	3
f	5	110	3
g	6	111	3
h	7	110	3
i	8	100	3
j	9	101	3

$$1 + 2 + 3 + 4 + 5 + 5 + 6 + 7 + 8 + 9 = 50 \text{ occurrences.}$$

$$5 + 5 + 4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 35 \text{ total length of all codes.}$$

$$35/10 = 3.5 \text{ expected value of a code or message}$$

5