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Exercise 2

Suppose that $\langle a \rangle$, $\langle b \rangle$, and $\langle c \rangle$ are cyclic groups of orders 6, 8, and 20, respectively. Find all generators of $\langle a \rangle$, $\langle b \rangle$, and $\langle c \rangle$.

$\langle a \rangle$	a^1, a^5
$\langle b \rangle$	b^1, b^3, b^5, b^7
$\langle c \rangle$	c^1, c^3, c^7, c^9, c^{11}, c^{13}, c^{17}, c^{19}

Exercise 7

Find an example of a noncyclic group, all of whose proper subgroups are cyclic.
U(8) = {1, 3, 5, 7} works.

$$\langle 1 \rangle = \{1\}, \langle 3 \rangle = \{3, 1\}, \langle 5 \rangle = \{5, 1\}, \langle 7 \rangle = \{7, 1\}$$

Exercise 9

How many subgroups does Z_{20} have? List a generator for each of these subgroups. Suppose that $G = \langle a \rangle$ and $|a| = 20$. How many subgroups does G have? List a generator for each of these subgroups.
Six.

Z_{20}	19
Z_{10}	9
Z_5	4
Z_4	3
Z_2	1
Z_1	0

Exercise 13

In Z_{24}, find a generator for $\langle 21 \rangle \cap \langle 10 \rangle$. Suppose that $|a| = 24$. Find a generator for $\langle a^{21} \rangle \cap \langle a^{10} \rangle$. In general, what is a generator for the subgroup $\langle a^m \rangle \cap \langle a^n \rangle$?
$\boldsymbol{\langle 21 \rangle = \{0, 21, 18, 15, 12, 9, 6, 3\}}$
$\boldsymbol{\langle 10 \rangle = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22\}}$
$\boldsymbol{\langle 21 \rangle \cap \langle 10 \rangle = \{0, 6, 12, 18\}}$
Generator: 6
$\boldsymbol{\langle a^{21} \rangle = \{a^0, a^{21}, a^{18}, a^{15}, a^{12}, a^9, a^6, a^3\}}$
$\boldsymbol{\langle a^{10} \rangle = \{a^0, a^{10}, a^{20}, a^6, a^{16}, a^2, a^{12}, a^{22}, a^8, a^{18}, a^4, a^{14}\}}$
$\boldsymbol{\langle a^{21} \rangle \cap \langle a^{10} \rangle = \{a^0, a^{18}, a^{12}, a^6\}}$
Generator for $\langle a^m \rangle \cap \langle a^n \rangle$ in Z_{24}: $a^{lcm(m,n)}$

Exercise 16

Complete the statement: $|a| = |a^2|$ if and only if $|a|$ = **1 or** ∞

Exercise 32

Determine the subgroup lattice for Z_{12}. Generalize to Z_{p^2q}, where p and q are distinct primes.