Assigned: Page 54, Exercise 2, 4, 23, 25, 33

Exercise 2

Which of the following binary operations are associative?

a. subtraction of integers - **No**

b. division of nonzero rationals - **No**

c. function composition of polynomials with real coefficients - **Yes**

d. multiplication of 2 x 2 matrices with integer entries - **No**

e. exponentiation of integers - **Yes**

Exercise 4

Which of the following sets are closed under the given operation?

a. 0, 4, 8, 12 addition mod 16 - **Yes**

b. 0, 4, 8, 12 addition mod 15 - **No**

c. 1, 4, 7, 13 multiplication mod 15 - **Yes**

d. 1, 4, 5, 7 multiplication mod 9 - **No**

Exercise 23

(Law of Exponents for Abelian Groups)
Let a and b be elements of an Abelian group and let n be any integer.
Show that $(ab)^n = a^n b^n$.
Let a, b $\in$ G, an Abelian group.
Let n $\in \mathbb{Z}$

$$
\begin{aligned}
(ab)^n &= ab \times ab \times ab \times ... \times ab \text{ (n times)} \\
&= a \times a \times a \times ... \times a \times b \times b \times b \times ... \times b \text{ (by commutativity)} \\
&= (a)^n (b)^n
\end{aligned}
$$

Is this also true for non-Abelian groups?

Exercise 25

Prove that a group G is Abelian iff $(ab)^{-1} = a^{-1} b^{-1}$, $\forall$ a, b $\in$ G.

Exercise 33

Suppose the table below is a group table. Fill in the blank entries.

	e	a	b	c	d
e	e	-	-	-	-
a	-	b	-	-	e
b	-	c	d	e	-
c	-	d	-	a	b
d	-	-	-	-	-

$\longrightarrow$

	e	a	b	c	d
e	e	-	-	-	-
a	-	b	-	-	e
b	-	c	d	e	-
c	-	d	-	a	b
d	-	-	-	-	-