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Exercise 1

Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$

Compute each of the following:

a. α^{-1}

$$\begin{bmatrix} 2 & 1 & 3 & 5 & 4 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

b. $\beta \alpha$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{bmatrix}$$

c. $\alpha \beta$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{bmatrix}$$

Exercise 3

Write each of the following permutations as a product of disjoint cycles:

a. $(1235)(413)$

$$1 \rightarrow 3 \rightarrow 5 \rightarrow 1$$

$$(15)$$

$$2 \rightarrow 2 \rightarrow 3$$

$$3 \rightarrow 4 \rightarrow 4$$

$$4 \rightarrow 1 \rightarrow 2$$

$$(234)$$

$$(15)(234)$$

b. $(13256)(23)(46512)$

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 2$$

$$2 \rightarrow 4 \rightarrow 4 \rightarrow 4$$

$$3 \rightarrow 3 \rightarrow 2 \rightarrow 5$$

$$4 \rightarrow 6 \rightarrow 6 \rightarrow 1$$

$$5 \rightarrow 1 \rightarrow 1 \rightarrow 3$$

$$6 \rightarrow 5 \rightarrow 5 \rightarrow 6$$

$$(124)(35)(6)$$

c. $(12)(13)(23)(142)$

$$1 \rightarrow 4 \rightarrow 4 \rightarrow 4 \rightarrow 4$$

$$2 \rightarrow 1 \rightarrow 1 \rightarrow 3 \rightarrow 3$$

$$3 \rightarrow 3 \rightarrow 2 \rightarrow 2 \rightarrow 1$$

$$4 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 2$$

$$(1423)$$

Exercise 39

In S_4 , find a cyclic subgroup of order 4 and a noncyclic subgroup of order 4.

$$\langle (1234) \rangle = \{e, (1234), (1234)^2, (1234)^3\}$$

The set $\langle (1234) \rangle$ under composition is a cyclic group of order 4 and a subgroup of S_4 .

Exercise 40

In S_3 , find elements α and β such that $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = 3$.

$$S_3 = \{e, (23), (12), (132), (123), (13)\}$$

Let $\alpha = (12)$, $\beta = (23)$

Notice: $\alpha = 213$, $\alpha^2 = 123$, $\beta = 132$, $\beta^2 = 123$.

Notice also: $(\alpha\beta) = 312$, $(\alpha\beta)^2 = 231$, and $(\alpha\beta)^3 = 123$.

Hence, $\alpha = (12)$ and $\beta = (23)$ is a solution.