

Due 4/25 (Wednesday):

All (turn in): Chapter 10, page 206, 14-18, 20, 24

Chapter 10

Recall:

A homomorphism ϕ from a group G to a group G' is a mapping from G into G' that preserves the group operation; that is, $\phi(ab) = \phi(a)\phi(b)$ for $a, b \in G$.

The kernel of a homomorphism ϕ from a group G to a group G' with identity e' is the set $\{x \in G : \phi(x) = e'\}$.

Exercise 14

Explain why the correspondence $x \mapsto 3x$ from Z_{12} to Z_{10} is not a homomorphism.

Because ϕ is not OP:

$$\phi(3 * 4) = \phi(12) = \phi(0) = (3 * (0 \bmod 12)) \bmod 10 = e, \text{ and}$$

$$\phi(3)\phi(4) = (3 * (3 \bmod 12)) * 3 * (4 \bmod 12) \bmod 10 = (9 * 3 * 4) \bmod 10 = (108) \bmod 10 = 8$$

Exercise 15

Suppose that ϕ is a homomorphism from Z_{30} to Z_{30} and $\text{Ker } \phi = \{0, 10, 20\}$. If $\phi(23) = 9$, determine all elements that map to 9.

Exercise 16

Prove that there is no homomorphism from $Z_8 \oplus Z_2$ onto $Z_4 \oplus Z_4$.

Exercise 17

Prove that there is no homomorphism from $Z_{16} \oplus Z_2$ onto $Z_4 \oplus Z_4$.

Exercise 18

Can there be a homomorphism from $Z_4 \oplus Z_4$ onto Z_8 ? Can there be a homomorphism from Z_{16} onto $Z_2 \oplus Z_2$? Explain your answers.

Exercise 20

How many homomorphisms are there from Z_{20} onto Z_8 ? How many are there to Z_8 ?

Exercise 24

Suppose that $\phi: Z_{50} \rightarrow Z_{15}$ is a group homomorphism with $\phi(7) = 6$.

- Determine $\phi(x)$.
- Determine the image of ϕ .
- Determine the kernel of ϕ .
- Determine $\phi^{-1}(3)$. That is, determine the set of all elements that map to 3.