Due 4/9:
G1 (present): page 150: 1, 7, 8
G2 (present): page 150: 3, 6, 9, 12, 14 (me: 3, 14)
All (turn in): page 150: 17, 19, 29, 36 (me)
Due 4/11:
Present: page 167: 20
All (turn in): page 167: 1, 22

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Exercise 3

Let H = {0, ±3, ±6, ±9...}. Rewrite the condition $a^{-1}b \in H$ given in property 6 of the lemma on page 139 in additive notation. Assume that the group is Abelian. Use this to decide whether or not the following cosets of H are the same.

Property 6: aH = bH iff $a^{-1}b \in H$
Rewritten: a + H = b + H iff $a^{-1} + b \in H$

a. **11 + H and 17 + H**: $-11 + 17 = 6 \in H$, so yes.

b. **−1 + H and 5 + H**: 1 + 5 = 6 mem H, so yes.

c. **7 + H and 23 + H**: $-7 + 23 = 16 \notin H$, so no.

Exercise 14

Let C^* be the group of nonzero complex numbers under multiplication and let H = $\{a + bi \in C^* : a^2 + b^2 = 1\}$. Give a geometric description of the cosets (3 + 4i)H and (c + di)H.

Exercise 17

Let G be a group with |G| = pq: p, q are prime. Prove that every proper subgroup of G is cyclic.

Exercise 19

Compute 5^{15} mod 7 and 7^{13} mod 11.

Exercise 29

Let |G| = 33. What are the possible orders for the elements of G? Show that G must have an element of order 3.

Exercise 36

Let G be a group and |G| = 21. If $g \in G$ and $g^{14} = e$, what are the possibilities for |g|?

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Exercise 1

Prove that the external direct product of any finite number of groups is a group.

Exercise 20

Find a subgroup of Z_{12} (+) Z_{18} that is isomorphic to Z_9 (+) Z_4.

Exercise 22

Determine the number of elements of order 15 and the number of cyclic subgroups of order 15 in Z_{30} (+) Z_{20}.