HW 9: page 203 - 205, #1, 2, 3(a)(c)(e)(g), 7(c), 13, 16, 18, 19

# Exercise 1

**Let:** $f : D \longrightarrow \mathbb{R}$ and $c \in D'$

Mark each statement True or False. Justify each answer.

a. $\lim\limits_{x\to c} f(x) = L$ iff $\forall\ \epsilon > 0, \exists$ a $\delta > 0$ st $|f(x) - L| < \epsilon$ whenever $x \in D$ and $|x - c| < \delta$

**False. Can't figure out why exactly, but I know there's a case where x = c (and thus, 0 < $\delta$ ) that makes it not true.**

b. $\lim\limits_{x\to c} f(x) = L$ iff for every deleted neighborhood U of c, there exists a neighborhood V of L st $f(U \cap D) \subset V$

**True, by Theorem 5.1.2 (since it's iff).**

c. $\lim\limits_{x\to c} f(x) = L$ iff for every sequence $\{s_n\}$ in D that converges to c with $s_n \neq c\ \forall$ n, the sequence $\{f(s_n)\}$ converges to L.

d. If f does not have a limit at c, then $\exists$ a sequence $\{s_n\}$ in D with each $s_n \neq c$ st $\{s_n\}$ converges to c, but $\{f(s_n)\}$ is divergent.

# Exercise 2

**Let:** $f : D \longrightarrow \mathbb{R}$ and $c \in D'$

Mark each statement True or False. Justify each answer.

a. For any polynomial P and any $c \in \mathbb{R}$ , $\lim\limits_{x\to c} P(x) = P(c)$

b. For any polynomials P and Q, and any $c \in \mathbb{R}$ ,

$$\lim_{x\to c}\frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}$$

c. In evaluating $\lim\limits_{x\to a-} f(x)$ we only consider points that are greater than a.

d. If f is defined in a deleted neighborhood of c, then $\lim\limits_{x\to c} f(x) = L$ iff $\lim\limits_{x\to c+} f(x) = \lim\limits_{x\to c-} f(x) = L$

# Exercise 3(a)(c)(e)(g)

Determine the following limits:

a. $\lim\limits_{x\to 1} \frac{3x^2+5}{x^3+1}$

b. $\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

c. $\lim\limits_{x\to 0} \frac{x^2+5x}{x^2-2}$

d. $\lim\limits_{x\to 0-} \frac{4x}{|x|}$

## Exercise 7(c)

Find the following limit and prove your answer.
$\lim_{x \to c} \sqrt{x}$, where c $\geq 0$

## Exercise 13

Let f, g, and h be functions from D into $\mathbb{R}$ , and let c $\in$ D$'$
**Assume:** f(x) $\leq$ g(x) $\leq$ h(x) $\forall$ x $\in$ D with x $\neq$ c **Assume:** $\lim_{x \to c}$ f(x) = $\lim_{x \to c}$ h(x) = L
Prove that $\lim_{x \to c}$ g(x) = L

## Exercise 16

**Let:** f : D $\longrightarrow \mathbb{R}$ and c $\in$ D$'$
**Assume:** $\lim_{x \to c}$ f(x) $> 0$
Prove that $\exists$ a deleted neighborhood U of c st f(x) $> 0$ $\forall$ x $\in$ (U $\cap$ D)

## Exercise 18

**Let:** f : D $\longrightarrow \mathbb{R}$ and c $\in$ D$'$
**Assume:** f has a limit at c Prove that f is bounded on a neighborhood of c. That is, prove that $\exists$ a neighborhood U of c and a real number M st |f(x)| $\leq$ M $\forall$ x $\in$ (U $\cap$ D)

## Exercise 19

**Assume:** f : $\mathbb{R} \longrightarrow \mathbb{R}$ is a function st f(x + y) = f(x) + f(y) $\forall$ x, y $\in \mathbb{R}$
Prove that f has a limit at 0 iff f has a limit at every point c $\in \mathbb{R}$