HW 8: pages 193, #1, 2, 3, 5, 9, 10, 17
For 2(c), see Theorem 1 and Example 9 from Lecture 15
Make sure when you do these problems, justify the answer by either writing down the theorem name or providing a counter example.

Exercise 1

Mark each statement True or False. Justify each answer.

a. A sequence (s_n) converges to s iff every subsequence of (s_n) converges to s.

   True.

b. Every bounded sequence is convergent.

   False.

   Counter example: $(s_n) = (-1)^n$

c. Let (s_n) be a bounded sequence. If (s_n) oscillates, then the set S of subsequential limits of (s_n) contains at least two points.

d. Let (s_n) be a bounded sequence and let $m = \lim\sup s_n$. Then, $\forall\ \epsilon > 0$, $\exists$ $N \in \mathbb{N}$ st $N \geq n$ implies $s_n > m - \epsilon$

e. If (s_n) is unbounded above, then (s_n) contains a subsequence that has ∞ as a limit.

Exercise 2

Mark each statement True or False. Justify each answer.

a. Every sequence has a convergent subsequence.

b. The set of subsequential limits of a bounded sequence is always nonempty.

c. (s_n) converges to s iff $\lim\inf s_n = \lim\sup s_n = s$

d. Let (s_n) be a bounded sequence and let $m = \lim\sup s_n$. Then, $\forall\ \epsilon > 0$, there are infinitely many terms in the sequence greater than $m - \epsilon$.

e. If (s_n) is unbounded above, then $\lim\inf s_n = \lim\sup s_n = \infty$

Exercise 3

For each sequence, find the set S of subsequential limits, the limit inferior, and the limit inferior.

a. $s_n = 1 + (-1)^n$

b. $t_n = (0, \frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{4}{5}, \frac{1}{6}, \frac{6}{7})$

c. $u_n = n^2(-1 + (-1)^n)$

d. $v_n = n \sin \frac{n\pi}{2}$

Exercise 5

Use exercise 4.3.14 to find the limit of each sequence:

a. $s_n = (1 + \frac{1}{2n})^{2n}$

b. $s_n = (1 + \frac{1}{n})^{2n}$

c. $s_n = (1 + \frac{1}{n})^{n-1}$

d. $s_n = (\frac{n}{n+1})^{n}$

e. $s_n = (1 + \frac{1}{2n})^{n}$

f. $s_n = (\frac{n+2}{n+1})^{n+3}$

Exercise 9

Let (s_n) be a bounded sequence.
Assume: $\liminf s_n = \limsup s_n = s$
Prove that (s_n) is convergent and that $\lim s_n = s$

Exercise 10

Assume: $x > 1$
Prove that $\lim x^{\frac{1}{n}} = 1$

Exercise 17

Prove that if $\limsup s_n = \infty$ and $k > 0$, then $\limsup (ks_n) = \infty$