Due 4/9:
G1 (present): page 150: 1, 7, 8
G2 (present): page 150: 3, 6, 9, 12, 14 (me: 3, 14)
All (turn in): page 150: 17, 19, 29, 36 (me)
Due 4/11:
Present: page 167: 20
All (turn in): page 167: 1, 22

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Exercise 3

Let $H = \{0, \pm 3, \pm 6, \pm 9...\}$. Rewrite the condition $a^{-1}b \in H$ given in property 6 of the lemma on page 139 in additive notation. Assume that the group is Abelian. Use this to decide whether or not the following cosets of H are the same.

a. $11 + H$ and $17 + H$

b. $-1 + H$ and $5 + H$

c. $7 + H$ and $23 + H$

Exercise 14

Let C^* be the group of nonzero complex numbers under multiplication and let $H = \{a + bi \in C^* : a^2 + b^2 = 1\}$. Give a geometric description of the cosets $(3 + 4i)H$ and $(c + di)H$.

Exercise 17

Let G be a group with $|G| = pq$: p, q are prime. Prove that every proper subgroup of G is cyclic.

Exercise 19

Compute $5^{15} \bmod 7$ and $7^{13} \bmod 11$.

Exercise 29

Let $|G| = 33$. What are the possible orders for the elements of G? Show that G must have an element of order 3.

Exercise 36

Let G be a group and $|G| = 21$. If $g \in G$ and $g^{14} = e$, what are the possibilities for $|g|$?

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Exercise 1

Prove that the external direct product of any finite number of groups is a group.

Exercise 20

Find a subgroup of Z_{12} (+) Z_{18} that is isomorphic to Z_9 (+) Z_4.

Exercise 22

Determine the number of elements of order 15 and the number of cyclic subgroups of order 15 in Z_{30} (+) Z_{20}.