HW 11: page 220 - 221, #1, 2, 5 and page 226-227, # 1 - 3, 4(a)(b), 5, 11

Exercise 1 (pages 220 - 221)

Mark each statement True or False. Justify each answer.

a. Let D be a compact subset of $\mathbb{R}$ and suppose that f : D $\longrightarrow \mathbb{R}$ is continuous. Then f(D) is compact.

b. Suppose that f : D $\longrightarrow$ R is continuous. Then, there exists a point x_1 in D st $f(x_1) \geq f(x)\ \forall\ x \in D$

c. Let D be a bounded subset of $\mathbb{R}$ and assume that f : D $\longrightarrow \mathbb{R}$ is continuous. Then f(D) is bounded.

Exercise 2 (pages 220 - 221)

Mark each statement True or False. Justify each answer.

a. Let f : [a,b] $\longrightarrow \mathbb{R}$ be continuous and assume $f(a) < 0 < f(b)$. Then there exists a point $c \in (a, b)$ st $f(c) = 0$.

b. Let f : [a,b] $\longrightarrow \mathbb{R}$ be continuous and assume $f(a) \leq k \leq f(b)$. Then there exists a point $c \in [a,b]$ st $f(c) = k$.

c. If f : D $\longrightarrow \mathbb{R}$ is continuous and bounded on D, then f assumes maximum and minimum values on D.

Exercise 5 (pages 220 - 221)

Show that the equation $5^x = x^4$ has at least one real solution.

Exercise 1 (pages 226 - 227)

Let f : D $\longrightarrow \mathbb{R}$. Mark each statement True or False. Justify each answer.

a. f is uniformly continuous on D iff for every $\epsilon > 0$ there exists a $\delta > 0$ st $|f(x) - f(y)| < \delta$ whenever $|x - y| < \epsilon$ and x, y $\in$ D.

b. If D = {x}, then f is uniformly continuous at x.

c. If f is continuous and D is compact, then f is uniformly continuous on D.

Exercise 2 (pages 226 - 227)

Let f : D $\longrightarrow \mathbb{R}$. Mark each statement True or False. Justify each answer.

a.

(a) In the definition of uniform continuity, the positive depends only on the function f and the given ¿ 0. (b) If f is continuous and (xn) is a Cauchy sequence in D, then (f(xn)) is a Cauchy sequence. (c) If f:(a,b)Rcanbeextendedtoafunctionthatiscontinuouson[a,b], then f is uniformly continuous on (a,b).

Exercise 3 (pages 226 - 227)

3. Determine which of the following continuous functions are uniformly continuous on the given set. Justify your answers.
(a) f(x)= x on[2,5] (b) f(x)= x on(0,2) (c) f(x)=x2+2x7on[0,5] (d) f(x)=x2+2x7on(1,4) (e) f(x)= 12 on(0,1) x (f) f(x)=12on(0,) x (g) f(x)=x24on(2,4) x2 (h) f(x)=xsin1/x on(0,1)

Exercise 4(a)(b) (pages 226 - 227)

4. Prove that each function is uniformly continuous on the given set by directly verifyingthe propertyinDefinition4.1.
(a) f(x)=x3 on[0,2] (b) f(x)=1on[2,)

Exercise 5 (pages 226 - 227)

5. Prove that f (x) = x is uniformly continuous on [0,).

Exercise 11 (pages 226 - 227)

11. Let f : D R be uniformly continuous on the bounded set D. Prove that f is bounded on D.