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Exercise 1

Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$
Compute each of the following:

a. α^{-1}

$$\begin{bmatrix} 2 & 1 & 3 & 5 & 4 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

b. $\beta\ \alpha$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{bmatrix}$$

c. $\alpha\ \beta$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{bmatrix}$$

Exercise 3

Write each of the following permutations as a product of disjoint cycles:

a. (1235)(413)

$1 \longrightarrow 3 \longrightarrow 5 \longrightarrow 1$

(15)

$2 \longrightarrow 2 \longrightarrow 3$

$3 \longrightarrow 4 \longrightarrow 4$

$4 \longrightarrow 1 \longrightarrow 2$

(234)

(15)(234)

b. (13256)(23)(46512)

$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 2$

$2 \longrightarrow 4 \longrightarrow 4 \longrightarrow 4$

$3 \longrightarrow 3 \longrightarrow 2 \longrightarrow 5$

$4 \longrightarrow 6 \longrightarrow 6 \longrightarrow 1$

$5 \longrightarrow 1 \longrightarrow 1 \longrightarrow 3$

$6 \longrightarrow 5 \longrightarrow 5 \longrightarrow 6$

(124)(35)(6)

c. (12)(13)(23)(142)

$1 \longrightarrow 4 \longrightarrow 4 \longrightarrow 4 \longrightarrow 4$

$2 \longrightarrow 1 \longrightarrow 1 \longrightarrow 3 \longrightarrow 3$

$3 \longrightarrow 3 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$

$4 \longrightarrow 2 \longrightarrow 3 \longrightarrow 1 \longrightarrow 2$

(1423)

Exercise 39

In S_4, find a cyclic subgroup of order 4 and a noncyclic subgroup of order 4.

Exercise 40

In S_3, find elements α and β such that $|\ \alpha\ | = 2$, $|\ \beta\ | = 2$, and $|\ \alpha\ \beta\ | = 3$.