Due 4/25 (Wednesday):
All (turn in): Chapter 10, page 206, 14-18, 20, 24

Chapter 10

Recall:
A homomorphism ϕ from a group G to a group G$'$ is a mapping from G into G$'$ that preserves the group operation; that is, $\phi(ab) = \phi(a)\phi(b)$ for a, b $\in$ G.
The kernel of a homomorphism ϕ from a group G to a group G$'$ with identity e$'$ is the set $\{x \in G : \phi(x) = e' \}$.

Exercise 14

Explain why the correspondence x $\longrightarrow$ 3x from Z_{12} to Z_{10} is not a homomorphism.

Because ϕ is not OP:
$\phi(3 * 4) = \phi(12) = \phi(0) = (3 * (0 \bmod 12)) \bmod 10 = e$, and
$\phi(3)\phi(4) = (3 * (3 \bmod 12) * 3 * (4 \bmod 12)) \bmod 10 = (9 * 3 * 4) \bmod 10 = (108) \bmod 10 = 8$

Exercise 15

Suppose that ϕ is a homomorphism from Z_{30} to Z_{30} and Ker $\phi = \{0, 10, 20\}$.
If $\phi(23) = 9$, determine all elements that map to 9.

$\phi(ab \bmod 30) = \phi(a \bmod 30)\phi(b \bmod 30)$
$\phi(23) = 9$.
$\phi(0) = \phi(10) = \phi(20) = 0$
It looks like it's $\phi(x) = 3x$:
$\phi(23) = 3 * 23 \bmod 30 = 69 \bmod 30 = 9$.
Thus,
$\phi(3)$, $\phi(13)$, and $\phi(23)$ all map to 9.

Exercise 16

Prove that there is no homomorphism from $Z_8 \bigoplus Z_2$ onto $Z_4 \bigoplus Z_4$.

Exercise 17

Prove that there is no homomorphism from $Z_{16} \bigoplus Z_2$ onto $Z_4 \bigoplus Z_4$.

Exercise 18

Can there be a homomorphism from $Z_4 \bigoplus Z_4$ onto Z_8? Can there be a homomorphism from Z_{16} onto $Z_2 \bigoplus Z_2$? Explain your answers.

Exercise 20

How many homomorphisms are there from Z_{20} onto Z_8? How many are there to Z_8?

Exercise 24

Suppose that $\phi\colon \mathbb{Z}_{50} \longrightarrow \mathbb{Z}_{15}$ is a group homomorphism with $\phi(7) = 6$.

    a. Determine $\phi(\mathrm{x})$.

    b. Determine the image of ϕ.

    c. Determine the kernel of ϕ.

    d. Determine $\phi^{-1}(3)$. That is, determine the set of all elements that map to 3.