Due 4/25 (Wednesday):
All (turn in): Chapter 10, page 206, 14-18, 20, 24

Chapter 10

Recall:
A homomorphism ϕ from a group G to a group G$'$ is a mapping from G into G$'$ that preserves the group operation; that is, ϕ(ab) = ϕ(a)ϕ(b) for a, b $\in$ G.
The kernel of a homomorphism ϕ from a group G to a group G$'$ with identity e$'$ is the set {x $\in$ G : ϕ(x) = e$'$ }.

Exercise 14

Explain why the correspondence x $\longrightarrow$ 3x from Z_{12} to Z_{10} is not a homomorphism.

Because ϕ is not OP:
ϕ(3 * 4) = ϕ(12) = ϕ(0) = (3 * (0 mod 12)) mod 10 = e, and
ϕ(3)ϕ(4) = (3 * (3 mod 12) * 3 * (4 mod 12)) mod 10 = (9 * 3 * 4) mod 10 = (108) mod 10 = 8

Exercise 15

Suppose that ϕ is a homomorphism from Z_{30} to Z_{30} and Ker ϕ = {0, 10, 20}.
If ϕ(23) = 9, determine all elements that map to 9.

ϕ(ab mod 30) = ϕ(a mod 30)ϕ(b mod 30)
ϕ(23) = 9.
ϕ(0) = ϕ(10) = ϕ(20) = 0
It looks like it's ϕ(x) = 3x:
ϕ(23) = 3 * 23 mod 30 = 69 mod 30 = 9.
Thus,
ϕ(3), ϕ(13), and ϕ(23) all map to 9.

Exercise 16

Prove that there is no homomorphism from $Z_8 \bigoplus Z_2$ onto $Z_4 \bigoplus Z_4$.
Suppose $\exists$ ϕ: $Z_8 \bigoplus Z_2 \longrightarrow Z_4 \bigoplus Z_4$, such that ϕ is a homomorphism.
Then ϕ(ab) = ϕ(a)ϕ(b) for a, b $\in Z_8 \bigoplus Z_2$ and $\phi(g^n) = \phi(g)^n$ for n $\in \mathbb{Z}$.

Exercise 17

Prove that there is no homomorphism from $Z_{16} \bigoplus Z_2$ onto $Z_4 \bigoplus Z_4$.

Exercise 18

Can there be a homomorphism from $Z_4 \bigoplus Z_4$ onto Z_8? Can there be a homomorphism from Z_{16} onto $Z_2 \bigoplus Z_2$? Explain your answers.

Exercise 20

How many homomorphisms are there from Z_{20} onto Z_8? How many are there to Z_8?

Exercise 24

Suppose that ϕ: $\mathrm{Z}_{50} \longrightarrow \mathrm{Z}_{15}$ is a group homomorphism with $\phi(7) = 6$.

a. Determine ϕ(x).

b. Determine the image of ϕ.

c. Determine the kernel of ϕ.

d. Determine $\phi^{-1}(3)$. That is, determine the set of all elements that map to 3.