Due 4/23:
All (turn in): page 187, 2, 6, 14
Present (me): 10

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Exercise 2

Prove that A_n is normal in S_n.

Exercise 6

Let H = { $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$: a, b, d $\in \mathbb{R}$ and ad $\neq 0$ }.
Is H a normal subgroup of GL(2, $\mathbb{R}$)?

Exercise 10

Let H = {(1), (12)(34)} in A_4.

   a. Show that H is not normal in A_4.

      Well, recall that a subgroup H of G is normal iff gH = Hg $\forall$ g $\in$ G.

      So all we need to do is find a g $\in A_4$ such that gH $\neq$ Hg.

      Notice: (23) $\in A_4$.

      (23)H = {(23)(1), (23)(12)(34)} = {(23), (1342)}

      H(23) = {(1)(23), (12)(34)(23)} = {(23), (1243)}

      {(23), (1342)} $\neq$ {(23), (1243)}

      Thus, H is not normal in A_4

   b. Referring to the multiplication table for A_4 in Table 5.1 on page 105, show that, although $\alpha_6 H = \alpha_7 H$ and $\alpha_9 H = \alpha_{11} H$, it is not true that $\alpha_6 \alpha_9 H = \alpha_7 \alpha_{11} H$.

      α_6 = (243), α_7 = (142), α_9 = (132), and α_{11} = (234)

      So, let's look at both:

      $\alpha_6 \alpha_9 H \longleftarrow ? \longrightarrow \alpha_7 \alpha_{11} H$

      (243)(132)H $\longleftarrow ? \longrightarrow$ (142)(234)H

      {(243)(132)(1), (243)(132)(12)(34)} $\longleftarrow ? \longrightarrow$ {(142)(234)(1), (142)(234)(12)(34)}

      {(12)(34), (1)} $\longleftarrow ? \longrightarrow$ {(14)(23),(13)(24)}

      Nope! Those sets are not equal, so it's not true that $\alpha_6 \alpha_9 H = \alpha_7 \alpha_{11} H$.

   c. Explain why this proves that the left cosets of H do not form a group under coset multiplication.

Exercise 14

What is the order of the element 14 + $\langle 8 \rangle$ in the factor group $\mathbb{Z}_{24}/\langle 8 \rangle$?