Homework Due 10/5/17 (7 problems):
Section 4.1 pages 169 - 170; 1, 6(b), 7(f), 9(a), 11, 12, 15

#1

Mark each statement True or False. Justify each answer.

a. If (s_n) is a sequence and $s_i = s_j$ then i = j.

   False.

   Let: $(s_n) = \{1^n\}$

b. If $s_n \longrightarrow s$, then, for every $\epsilon > 0$, $\exists$ N $\in \mathbb{N}$ st n $\geq$ N implies $|s_n - s| < \epsilon$.

   A sequence $\{S_n\}$ is said to **converge** to s $\in \mathbb{R}$ provided that $\forall\ \epsilon > 0$

   $\exists$ N $\in \mathbb{N} \leq$ n st

   $|S_n - S| < \epsilon$

c. If $s_n \longrightarrow k$ and $t_n \longrightarrow k$, then $s_n = t_n\ \forall$ n $\in \mathbb{N}$.

   False.

   Let: $s_n = \sum_{i=0}^{\infty} \frac{1}{2^i}$, $t_n = 2 - \sum_{i=0}^{\infty} \frac{1}{2^i}$

d. Every convergent sequence is bounded.

   By Theorem 4.1.13, this is true.

6(b)

Definition 4.1.2
A sequence $\{s_n\}$ is said to **converge** to s $\in \mathbb{R}$ provided that $\forall\ \epsilon > 0$
$\exists$ N $\in \mathbb{N} \leq$ n st
$|s_n - s| < \epsilon$

Using only definition 4.1.2, prove the following:
For k > 0, k $\in \mathbb{R}$, $\lim\limits_{n\to\infty} (\frac{1}{n^k}) = 0$
Let: $\frac{1}{n^k} = \{s_n\}$

7(f)

Using any of the results in this section (4.1), prove the following:
If $|x| < 1$, then $\lim\limits_{n\to\infty} x^n = 0$

9(a)

For each of the following, prove or give a counter example:
If (s_n) converges to s, then $(|s_n|)$ converges to $|s|$.

11

Given the sequence (s_n), $k \in \mathbb{N}$, let (t_n) be the sequence defined by $t_n = s_{n+k}$. That is, the terms in (t_n) are the same as that of the terms in (s_n) after the first k terms have been skipped. Prove that (t_n) converges iff (s_n) converges, and if they converge, show that $\lim t_n = \lim s_n$. Thus, the convergence of a sequence is not affected by omitting (or changing) a finite number of terms.

12

a. Assume that $\lim s_n = 0$. If (t_n) is a bounded sequence, prove that $\lim(s_n t_n) = 0$.

b. Show by example that the boundedness of (t_n) is a necessary condition in part (a).

15

a. Prove that x is an accumulation point of a set S iff $\exists$ a sequence (s_n) of points in $S \setminus \{x\}$ st (s_n) converges to x.

b. Prove that a set S is closed iff, whenever (s_n) is a convergent sequence of points in S, it follows that $\lim s_n$ is in S.