Natural Generalizations of Graphs

Rusnak works on:
Combinatorial Matrix Theory
Incidence Hypergraph Theory
At it's core, it's Graph Theory for Hypergraphs
You can bootstrap graph theory to hypergraph theory if you're careful through signed graph theory.
Somewhere in between there is incidence theory for graphs.
A directed graph consists of disjoint sets V and E, and a pair of functions (sigma, tau) from E $\longrightarrow$ V
An orientation of a graph is a graph with a preferred vertex for each edge.
An incidence hypergraph consists of 3 disjoint sets V, E, and I, and a function backwardsj: I $\longrightarrow$ V x E
A set system is a collection of labeled subsets of Powerset(V).
Incidence Matrix: H_G
Degree Matrix: D_G
Adjacency Matrix: A_G
Laplacian Matrix: L_G:= $\mathrm{D}_G - \mathrm{A}_G = \mathrm{H}_G{\mathrm{H}_G}^T$
Matrix-tree Theorem:
Let G be a graph, T(G) be the number of spanning trees of G, and L_{ij} be the ij-minor of the Laplacian, then
det(L_{ij} = something
Tutte's Transpedance Theorem