

Homework Due 10/12/17: (13 problems)

Section 4.2 pages 177 - 178; 1, 2, 4, 5(a)(c)(e)(g)(i)(k), 9, 10, 17, 18

Test up to 4.2.4 on 10/5/17

(theorem 4.2.1 is a possibility on test)

Since 4.2.1 is a possibility, I'm going to redo it here:

## Theorem 4.2.1

Assume that  $(s_n)$  and  $(t_n)$  are convergent sequences with  $\lim s_n = s$  and  $\lim t_n = t$ . If  $s_n \leq t_n \forall n \in \mathbb{N}$ , then  $s \leq t$ .

*Proof.*

Suppose, instead, that  $s > t$ .

**Let:**  $\epsilon = \frac{(s-t)}{2} > 0$

Now, we have  $2\epsilon = s - t$

$$t = s - 2\epsilon$$

$$s = t + 2\epsilon$$

$$t + \epsilon = s - \epsilon$$

By the definition of convergent sequences,

$$\exists N_1 \in \mathbb{N} \text{ st } n \geq N_1 \text{ implies } s - \epsilon > s_n > s + \epsilon$$

and, similarly

$$\exists N_2 \in \mathbb{N} \text{ st } n \geq N_2 \text{ implies } t - \epsilon > t_n > t + \epsilon$$

**Let:**  $N = \max \{N_1, N_2\}$

Then, for  $n \geq N$ , we have

$$t_n > t + \epsilon = s - \epsilon > s_n$$

Which contradicts the assumption that  $s_n \leq t_n \forall n \in \mathbb{N}$

Thus, we conclude that  $s \leq t$

□