HW 11: page 220 - 221, #1, 2, 5 and page 226-227, # 1 - 3, 4(a)(b), 5, 11

Exercise 1 (pages 220 - 221)

Mark each statement True or False. Justify each answer.

a. Let D be a compact subset of $\mathbb{R}$ and suppose that f : D $\longrightarrow \mathbb{R}$ is continuous. Then f(D) is compact.

b. Suppose that f : D $\longrightarrow$ R is continuous. Then, there exists a point x_1 in D st $f(x_1) \geq f(x)\ \forall\ x \in D$

c. Let D be a bounded subset of $\mathbb{R}$ and assume that f : D $\longrightarrow \mathbb{R}$ is continuous. Then f(D) is bounded.

Exercise 2 (pages 220 - 221)

Mark each statement True or False. Justify each answer.

a. Let f : [a,b] $\longrightarrow \mathbb{R}$ be continuous and assume $f(a) < 0 < f(b)$. Then there exists a point $c \in (a, b)$ st $f(c) = 0$.

b. Let f : [a,b] $\longrightarrow \mathbb{R}$ be continuous and assume $f(a) \leq k \leq f(b)$. Then there exists a point $c \in [a,b]$ st $f(c) = k$.

c. If f : D $\longrightarrow \mathbb{R}$ is continuous and bounded on D, then f assumes maximum and minimum values on D.

Exercise 5 (pages 220 - 221)

Show that the equation $5^x = x^4$ has at least one real solution.

Exercise 1 (pages 226 - 227)

Let f : D $\longrightarrow \mathbb{R}$. Mark each statement True or False. Justify each answer.

a. f is uniformly continuous on D iff for every $\epsilon > 0$ there exists a $\delta > 0$ st $|f(x) - f(y)| < \delta$ whenever $|x - y| < \epsilon$ and x, y $\in$ D.

b. If D = {x}, then f is uniformly continuous at x.

c. If f is continuous and D is compact, then f is uniformly continuous on D.

Exercise 2 (pages 226 - 227)

Let f : D $\longrightarrow \mathbb{R}$. Mark each statement True or False. Justify each answer.

a. In the definition of uniform continuity, the positive δ depends only on the function f and the given $\epsilon > 0$.

b. If f is continuous and (x_n) is a Cauchy sequence in D, then $(f(x_n))$ is a Cauchy sequence.

c. If f : (a, b) $\longrightarrow \mathbb{R}$ can be extended to a function that is continuous on [a, b], then f is uniformly continuous on (a,b).

Exercise 3 (pages 226 - 227)

Determine which of the following continuous functions are uniformly continuous on the given set. Justify your answers.

a. f(x) = x on [2, 5]

b. f(x) = x on (0, 2)

c. f(x) = x^2 + 2x − 7 on [0, 5]

d. f(x) = x^2 + 2x − 7 on (1, 4)

e. f(x) = $\frac{1}{x^2}$ on (0, 1)

f. f(x) = $\frac{1}{x^2}$ on (0, ∞)

g. f(x) = $\frac{x^2-4}{x-2}$ on (2, 4)

h. f(x) = x sin($\frac{1}{x}$) on (0, 1)

Exercise 4(a)(b) (pages 226 - 227)

Prove that each function is uniformly continuous on the given set by directly verifying the ϵ - δ property in Definition 4.1.

a. f(x) = x^3 on [0, 2]

b. f(x) = $\frac{1}{x}$ on [2, ∞)

Exercise 5 (pages 226 - 227)

Prove that f(x) = $\sqrt{x}$ is uniformly continuous on [0, ∞).

Exercise 11 (pages 226 - 227)

Let f : D $\longrightarrow \mathbb{R}$ be uniformly continuous on the bounded set D. Prove that f is bounded on D.