

Assigned: Page 22, prove Properties of Functions Theorem parts 2, 3, and 4

Given functions $\alpha : A \rightarrow B$, $\beta : B \rightarrow C$, and $\gamma : C \rightarrow D$, then

1. $\gamma (\beta \alpha) = (\gamma \beta) \alpha$ (associativity).

Let $a \in A$. Then $(\gamma (\beta \alpha))(a) = \gamma ((\beta \alpha)(a)) = \gamma (\beta (\alpha (a)))$.

On the other hand, $((\gamma \beta) \alpha)(a) = (\gamma \beta)(\alpha (a)) = \gamma (\beta (\alpha (a)))$.

So, $\gamma (\beta \alpha) = (\gamma \beta) \alpha$.

2. If α and β are one-to-one, then $\beta \alpha$ is one-to-one.
3. If α and β are onto, then $\beta \alpha$ is onto.
4. If α is one-to-one and onto, then there is a function α^{-1} from B onto A such that $(\alpha^{-1} \alpha)(a) = a$, $\forall a \in A$ and $(\alpha \alpha^{-1})(b) = b$, $\forall b \in B$.