Due 4/23:
All (turn in): page 187, 2, 6, 14
Present (me): 10

Page 187

Exercise 2

Prove that A_n is normal in S_n.

Exercise 6

Let H = { $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$: a, b, d $\in \mathbb{R}$ and ad $\neq 0$ }.
Is H a normal subgroup of GL(2, $\mathbb{R}$)?

Exercise 10

Let H = {(1), (12)(34)} in A_4.

a. Show that H is not normal in A_4.

   Well, recall that a subgroup H of G is normal iff gH = Hg $\forall$ g $\in$ G.

   So all we need to do is find a g $\in A_4$ such that gH $\neq$ Hg.

   Notice: (23) $\in A_4$.

   (23)H = {(23)(1), (23)(12)(34)} = {(23), (1342)}

   H(23) = {(1)(23), (12)(34)(23)} = {(23), (1243)}

   {(23), (1342)} $\neq$ {(23), (1243)}

   Thus, H is not normal in A_4

b. Referring to the multiplication table for A_4 in Table 5.1 on page 105, show that, although $\alpha_6 H = \alpha_7 H$ and $\alpha_9 H = \alpha_{11} H$, it is not true that $\alpha_6 \alpha_9 H = \alpha_7 \alpha_{11} H$.

   α_6 = (243), α_7 = (142), α_9 = (132), and α_{11} = (234)

   So, let's look at both:

   $\alpha_6 \alpha_9 H \longleftarrow ? \longrightarrow \alpha_7 \alpha_{11} H$

   (243)(132)H $\longleftarrow ? \longrightarrow$ (142)(234)H

   {(243)(132)(1), (243)(132)(12)(34)} $\longleftarrow ? \longrightarrow$ {(142)(234)(1), (142)(234)(12)(34)}

   {(12)(34), (1)} $\longleftarrow ? \longrightarrow$ {(14)(23),(13)(24)}

   Nope! Those sets are not equal, so it's not true that $\alpha_6 \alpha_9 H = \alpha_7 \alpha_{11} H$.

c. Explain why this proves that the left cosets of H do not form a group under coset multiplication.

   Because the order of the permutations results in different output permutation, which means that coset multiplication isn't associative. Therefore, it can't form a group.

Exercise 14

What is the order of the element 14 + $\langle 8 \rangle$ in the factor group $Z_{24}/\langle 8 \rangle$?