Homework Due 10/12/17: (13 problems) Section 4.2 pages 177 - 178; 1, 2, 4, 5(a)(c)(e)(g)(i)(k), 9, 10, 17, 18 (for 5(i) define tn to be 1 over sm, and then show that 1 over sm goes to 0)

Problem 1

Mark each statement True or False. Justify each answer.

a. If (s_n) and (t_n) are convergent sequences with $s_n \longrightarrow s$ and $t_n \longrightarrow t$, then $\lim (s_n + t_n) = s + t$ and $\lim (s_n t_n) = st$.

b. If (s_n) converges to s and $s_n > 0 \; \forall \; n \in \mathbb{N}$, then $s > 0$.

c. The sequence (s_n) converges to s iff $\lim s_n = s$.

d. $\lim s_n = +\infty$ iff $\lim (\frac{1}{s_n}) = 0$.

Problem 2

Mark each statement True or False. Justify each answer.

a. If $s_n = s$ and $\lim t_n = t$, then $\lim (s_n t_n) = st$.

b. If $\lim s_n = +\infty$, then (s_n) is said to converge to $+\infty$.

   False. You can only converge to a finite number.

c. Given sequences (s_n) and (t_n) with $s_n \leq t_n \; \forall \; n \in \mathbb{N}$, if $\lim s_n = +\infty$, then $\lim t_n = +\infty$.

d. Suppose (s_n) is a sequence st the sequence of ratios $(\frac{s_{n+1}}{s_n})$ converges to L. If $L < 1$, then $\lim s_n = 0$.

Problem 4

a. Prove Theorem 4.2.1(b):

   Suppose that (s_n) and (t_n) are convergent sequences with $\lim s_n = s$ and $\lim t_n = t$. Then

   (b) $\lim (ks_n) = ks$ and $\lim (k + s_n) = k + s$, for any $k \in \mathbb{R}$

b. Prove Corollary 4.2.5

   Theorem 4.2.4:

   Suppose that (s_n) and (t_n) are convergent sequences with $\lim s_n = s$ and $\lim t_n = t$. If $s_n \leq t_n \; \forall \; n \in \mathbb{N}$, then $s \leq t$.

   Corollary 4.2.5:

   If (t_n) converges to t and $t_n \geq 0 \; \forall \; n \in \mathbb{N}$, then $t \geq 0$.

Problem 5

For s_n given by the following formulas, determine the convergence or divergence of the sequence (s_n). Find any limits that exist.

a. $s_n = \frac{3-2n}{1+n} \longrightarrow \frac{1}{2}$

b. $s_n = \frac{(-1)^n}{n+3} \longrightarrow 0$

c. $s_n = \frac{(-1)^n}{2n-1} \longrightarrow 0$

d. $s_n = \frac{2^{3n}}{3^{2n}} \longrightarrow 0?$

e. $s_n = \frac{n^2-2}{n+1} \longrightarrow \infty$

f. $s_n = \frac{3+n-n^2}{1+2n} \longrightarrow -\infty$

g. $s_n = \frac{1-n}{2^n} \longrightarrow 0$

h. $s_n = \frac{3^n}{n^3+5} \longrightarrow \infty$

i. $s_n = \frac{n!}{2^n} \longrightarrow \infty$

j. $s_n = \frac{n!}{n^n} \longrightarrow 0?$

k. $s_n = \frac{n^2}{2^n} \longrightarrow 0$

l. $s_n = \frac{n^2}{n!} \longrightarrow 0$

Problem 9

Prove Theorem 4.2.12:
Suppose that (s_n) and (t_n) are sequences st $s_n \leq t_n \ \forall \ n \in \mathbb{N}$

a. If $\lim s_n = +\infty$ then $\lim t_n = +\infty$

b. If $\lim s_n = -\infty$ then $\lim t_n = -\infty$

Problem 10

Prove the converse part of Theorem 4.2.13:
Let (s_n) be a sequence of positive numbers. Then, $\lim s_n = +\infty$ iff $\lim (\frac{1}{s_n}) = 0$.
$\longrightarrow$
Assume: $\lim s_n = +\infty$
Given any $\epsilon > 0$, let $M = \frac{1}{\epsilon}$. Then there exists a natural number N st $n \geq N$ implies that $s_n > M = \frac{1}{\epsilon}$.
Since each s_n is positive, we have:
$|\frac{1}{s_n} - 0| < \epsilon$, whenever $n \geq N$
Thus, $\lim (\frac{1}{s_n}) = 0$.
$\longleftarrow$

Problem 17

a. Show that $\lim\limits_{n\to\infty} \frac{k^n}{n!} = 0 \ \forall \ k \in \mathbb{R}$

b. What can be said about $\lim\limits_{n\to\infty} \frac{n!}{k^n}$?

Problem 18

Assume that (s_n) is a convergent sequence with $a \geq s_n \geq b \ \forall \ n \in \mathbb{N}$.
Prove that $a \leq \lim s_n \leq b$.