

Exercise 1

Let $A = \{0, 1, 2, 3, 4\}$ and $B = \{0, 1, 2, 3\}$. For each of the relations R from A to B listed below list all pairs $(a, b) \in \mathbb{R}$ and write the corresponding $\{0, 1\}$ -indicator-matrix.

a. $a = b : (0, 0), (1, 1), (2, 2), (3, 3)$

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
0	0	0	0

b. $a + b = 4 : (1, 3), (2, 2), (3, 1), (4, 0)$

0	0	0	0
0	0	0	1
0	0	1	0
0	1	0	1
1	0	0	0

c. $a > b : (1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3)$

0	0	0	0
1	1	0	0
1	0	0	0
1	1	1	0
1	1	1	1

d. $a \text{ divides } b : (1, 0), (2, 0), (3, 0), (4, 0), (1, 1), (1, 2), (2, 2), (1, 3)$

0	0	0	0
1	1	1	1
1	0	1	0
1	0	0	0
1	0	0	0

Exercise 2

For each of these relations on the set $\{1, 2, 3, 4\}$ decide whether or not it is reflexive, symmetric, antisymmetric, and transitive.

- $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- $\{(2, 4), (4, 2)\}$
- $\{(1, 2), (2, 3), (3, 4)\}$
- $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
- $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

<i>Relation</i>	<i>R</i>	<i>S</i>	<i>A</i>	<i>T</i>
<i>a</i>	0	0	0	1
<i>b</i>	1	1	0	1
<i>c</i>	0	1	0	1
<i>d</i>	0	0	1	0
<i>e</i>	1	1	1	1
<i>f</i>	0	0	0	1

Exercise 3

Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$, and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$ on the set $A = \{1, 2, 3, 4\}$

- Find $R \cup S$
- Find $R \cap S$
- Find $R \circ S$

Exercise 4

Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$ on the set $A = \{1, 2, 3, 4\}$.

- Find the reflexive closure of R .
- Find the symmetric closure of R .
- Find the transitive closure of R .

Exercise 5

Prove the following:

- A relation R is reflexive iff R^{-1} is reflexive (where R^{-1} is the inverse relation that just reverses the order).
- A relation R is symmetric iff $R = R^{-1}$.
- A relation R is anti-symmetric iff $R \cap R^{-1} \subset \Delta : \Delta = \{(a, a) : a \in A\}$

Exercise 6

Let R be the relation represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find the matrices for the relations:

a. R^2

b. R^3

c. R^4