ELECTROMAGNETIC WAVE THEORY I

16/12/2021

Statics & Dynamics

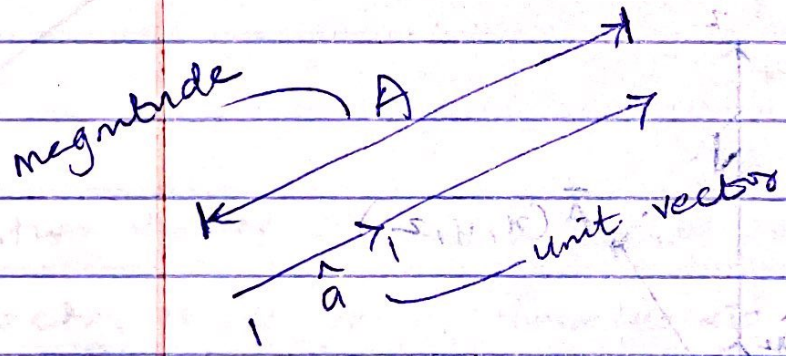
We would dwell more on Statics.

Governing law for electromagnetics is the Maxwell Equation. We would be using distribution form ~~to~~ to do it.

We would be considering charges at rest or in constant speed (Statics).

VECTOR ANALYSIS (REVIEW)

Here, we would be talking about magnitude and direction.



$$\bar{A} = |\bar{A}|\hat{a}_r = A\hat{a}_r$$

always denote your vectors this way compulsorily!!!

Unit vectors in terms of rectangular coordinates

$\hat{x}$

$\hat{i}$ ⟶ we do not use i in electrical because i is

$\hat{a}_x$ the standard for instantaneous current

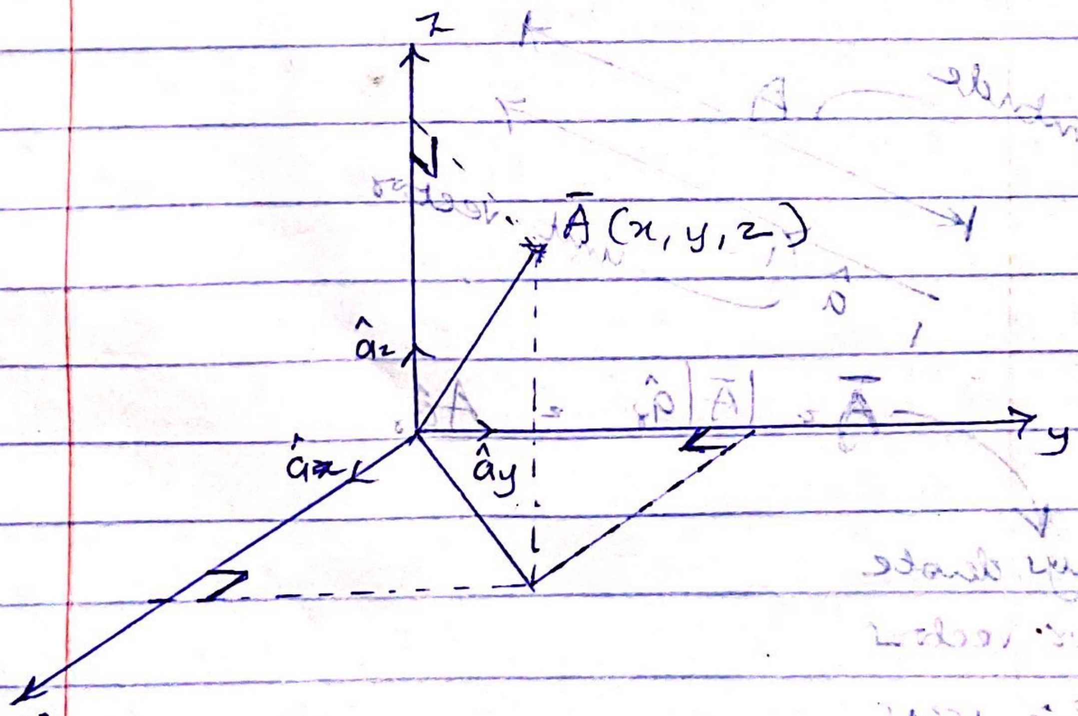
We use this unit vector.

rectangular Cartesian coordinate system (x, y, z)

Cylindrical " (r, ϕ, z)

spherical " (r, θ, ϕ)

CYLINDRICAL



The arrangement to be used is magnitude and unit vector.

$$\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

By pythagoras theorem, $A = |\bar{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$

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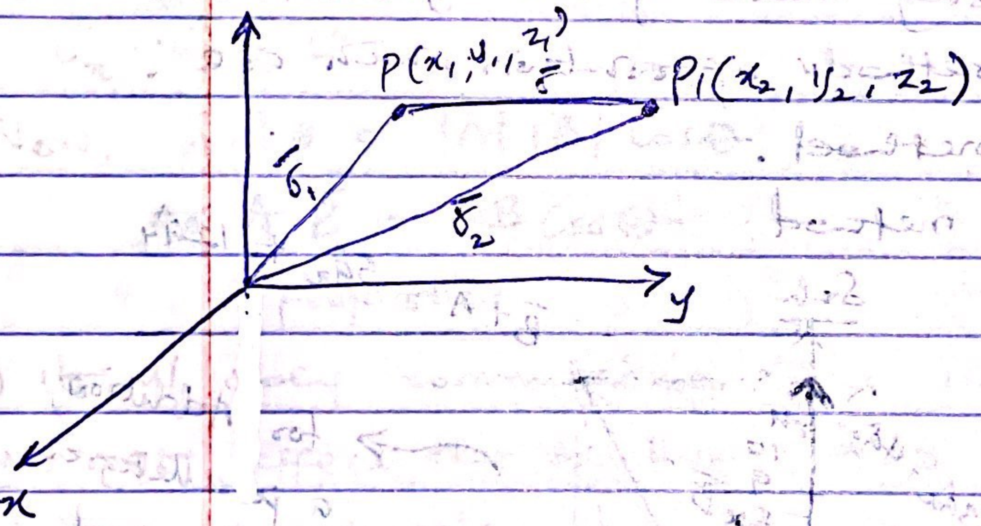
Position vector : origin is at zero and we measure to where the point is.



Position Vector : has a point in the space which gives the vector that ranges from point P_1 (???)

$$\bar{r} = \bar{r}_2 - \bar{r}_1$$

$$\bar{r}_2 = X_2 \hat{a}_x + Y_2 \hat{a}_y + Z_2 \hat{a}_z$$

$$(x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

If we are asked to look for distance, where distance

Unit vector shows the direction.

is a scalar.

To calculate the distance, we have:

$$|r| = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right]^{1/2}$$

Example: Given the vectors:

$\bar{A} = 4\hat{a}_x + 4\hat{a}_y$

$\bar{B} = \hat{a}_x + 8\hat{a}_y$

Find the vectors $\bar{B} \pm \bar{A}$ and their magnitudes.

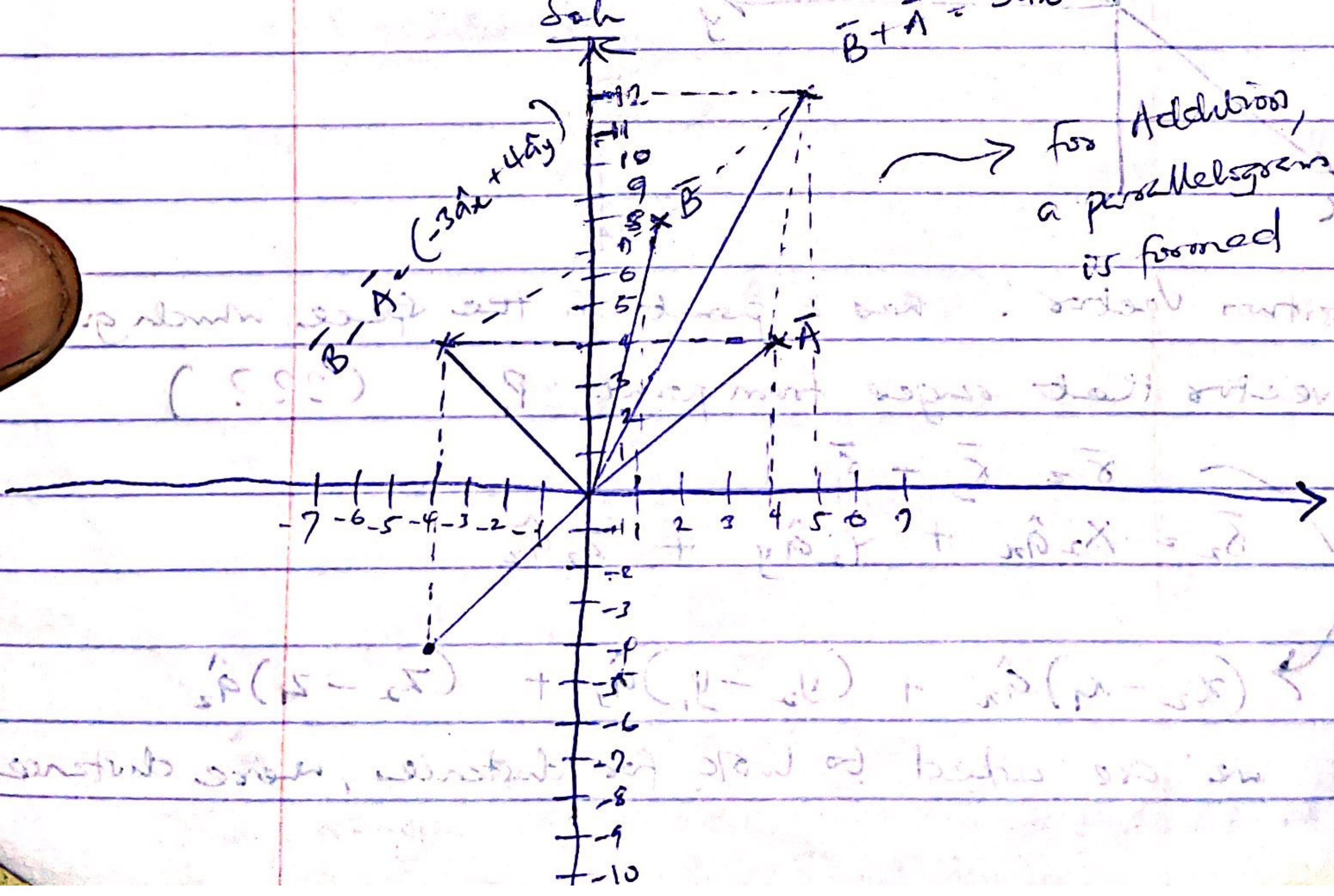
We would be using more of the geometrical solution.

there are two methods of solution which are:

→ analytical method.

→ Geometrical method.

$\bar{B} + \bar{A} = 5\hat{a}_x + 12\hat{a}_y$



→ for Addition, a parallelogram is formed

$\bar{A} \cdot B \longrightarrow$ it is not correct!!!! (Do not Forget)

Multiplication of vectors:

– Vector × scalar – This gives a vector with it's magnitude changed but direction remains the same.

Scalar multiplication of vectors (Dot product)

Dot product exists between two vectors. If you dot ~~two~~ vectors, we obtain a scalar.

$\hat{a}_x \cdot \hat{a}_x = 1$

$\hat{a}_x \cdot \hat{a}_y = 0$

$\hat{a}_x \cdot \hat{a}_z = 0$

Now, $\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}| \cos\theta$

or $\bar{A} \cdot \bar{B} = AB \cos\theta$

Properties

(i) It will obey commutative law i.e arrangement does not affect the solution ($\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$)

(ii) It is distributive

$\bar{A} \cdot (\bar{B} + \bar{C}) = \bar{A} \cdot \bar{B} + \bar{A} \cdot \bar{C}$

Now, θ_{AB} or θ_{BA} does not really matter.

Practice many questions to avoid errors.

Symbols are very important here.

CROSS PRODUCT

$$\bar{A} \times \bar{B} = AB \sin\theta \, a_n$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\hat{a}_x \times \hat{a}_x = 0 \qquad \hat{a}_x \times \hat{a}_y = \hat{a}_z \qquad \hat{a}_x \times \hat{a}_z = -\hat{a}_y$

Properties

- It is distributive → and

i.e $\bar{A} \times (\bar{B} + \bar{C}) = (\bar{A} \times \bar{B}) + (\bar{A} \times \bar{C})$

In a cartesian coordinate, vector $\bar{A}$ points from the origin to point P_1 where $P_1 = (2, 3, 3)$ and vector $\bar{B}$ is directed from P_1 to point P_2 where $P_2 = (1, -2, 2)$. find:

(a) Vector $\bar{A}$, its magnitude and unit vector

(b) The angle between vector $\bar{A}$ and the y-axis.

(c) Vector $\bar{B}$

(d) The angle between vector $\bar{A}$ and $\bar{B}$

(e) Perpendicular distance from the origin to vector $\bar{B}$

Always draw a geometric solution

Soln

(a) $\bar{A} = 2\hat{a}_x + 3\hat{a}_y + 3\hat{a}_z$

where $\bar{A} = OP_1$ i.e $P_1 - O$

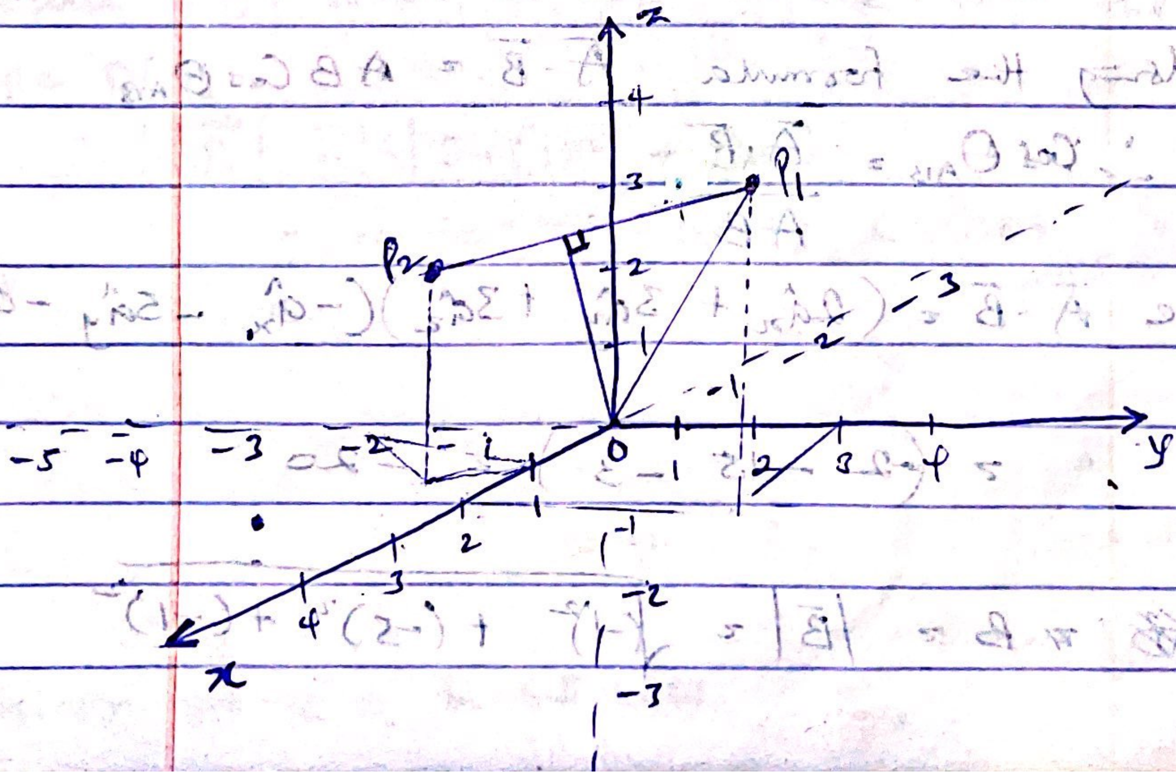
$= (2, 3, 3) - (0, 0, 0)$

(b) $|\bar{A}| = \sqrt{2^2 + 3^2 + 3^2}$

$|\bar{A}| = \sqrt{22} = \sqrt{22}$

$\hat{A} = \dfrac{\bar{A}}{|A|} = \dfrac{2\hat{a}_x + 3\hat{a}_y + 3\hat{a}_z}{\sqrt{22}}$

$\hat{A} = \dfrac{2\hat{a}_x}{\sqrt{22}} + \dfrac{3\hat{a}_y}{\sqrt{22}} + \dfrac{3\hat{a}_z}{\sqrt{22}}$



(b) To obtain the angle between vector $\bar{A}$ and the y-axis

$$\bar{A} \cdot \hat{a}_y = A \cos\theta$$

$$\cos\theta = \frac{\bar{A} \cdot \hat{a}_y}{A} = \frac{(2\hat{a}_x + 3\hat{a}_y + 3\hat{a}_z)(\hat{a}_y)}{\sqrt{22}}$$

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{22}}\right) = 50.23^\circ$$

(c) $\bar{B} = P_2 - P_1$

$$= (1-2)\hat{a}_x + (-2-3)\hat{a}_y + (2-3)\hat{a}_z$$

$$\bar{B} = -\hat{a}_x - 5\hat{a}_y - \hat{a}_z$$

(d) Using the formula $\bar{A} \cdot \bar{B} = AB\cos\theta_{AB}$

$$\therefore \cos\theta_{AB} = \frac{\bar{A} \cdot \bar{B}}{AB}$$

where $\bar{A} \cdot \bar{B} = (2\hat{a}_x + 3\hat{a}_y + 3\hat{a}_z)(-\hat{a}_x - 5\hat{a}_y - \hat{a}_z)$

$$= (-2 - 15 - 3) = -20$$

$\cancel{AB} \ne B = |\bar{B}| = \sqrt{(-1)^2 + (-5)^2 + (-1)^2}$

Read Scalar triple product

" vector triple products

$B = 3\sqrt{3}$

$$\cos\theta_{AB} = \frac{-20}{3\sqrt{3}\cdot\sqrt{22}}$$

$$\theta_{AB} = \cos^{-1}\left(-20/3\sqrt{66}\right)$$

$= 145.1°$

(e) I don't understand his solution

the perpendicular distance between the origin and vector B is the distance $|\overrightarrow{OP_3}|$ shown in the figure. From the right triangle OP_1P_3.

$$|\overrightarrow{OP_3}| = |A| \sin(180° - \theta_{AB})$$

$$= \sqrt{22} \sin(180° - 145.1) = 2.68$$

Answer to e = 2.68 units

Scalar triple

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

Vector triple

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B})$$

identity for the vector triple product

OR ~~Do~~ the normal curl of ($\bar{B}$ and $\bar{C}$) and then curl the result with $\bar{A}$.

Example: Given the vectors below:

$\bar{A} = \hat{a}_x - \hat{a}_y + 2\hat{a}_z$

$\bar{B} = \hat{a}_y + \hat{a}_z$

$\bar{C} = -2\hat{a}_x + 3\hat{a}_z$

Find $\bar{A} \times (\bar{B} \times \bar{C})$

Using the identity $\bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B})$

where $\bar{A} \cdot \bar{C} = 4$

$\bar{B}(\bar{A} \cdot \bar{C}) = 4\hat{a}_y + 4\hat{a}_z$

$\bar{A} \cdot \bar{B} = 1$

therefore $\bar{C}(\bar{A} \cdot \bar{B}) = -2\hat{a}_x + 3\hat{a}_z$

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Upon substituting, we have:

$4\hat{a}_y + 4\hat{a}_z - (-2\hat{a}_x + 3\hat{a}_z)$

$\Rightarrow 4\hat{a}_y + 4\hat{a}_z + 2\hat{a}_x - 3\hat{a}_z$

So we have:

$2\hat{a}_x + 4\hat{a}_y + \hat{a}_z$

Using the normal cross product

$$\bar{B} \times \bar{C} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & 1 \\ -2 & 0 & 3 \end{vmatrix} = \hat{a}_x(3-0) - \hat{a}_y(0+2) + \hat{a}_z(0+2)$$

$$\bar{B} \times \bar{C} = 3\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z$$

$$\bar{A} \times (\bar{B} \times \bar{C}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & -1 & 2 \\ 3 & -2 & 2 \end{vmatrix}$$

$$\Rightarrow \hat{a}_x(-2+4) - \hat{a}_y(2-6) + \hat{a}_z(-2+3)$$

$$= 2\hat{a}_x + 4\hat{a}_y + \hat{a}_z \quad \text{Ans}$$

Orthogonal coordinate system

Orthogonal Coordinate System

→ Cartesian

→ Cylindrical

→ Spherical

For the cartesian, we have, 1 – dimensional, 2 – dimensional and 3 – dimensional

1 – φ → δL (differential length)

2 – φ → δA (" area)

3 – φ → δV (" volume)

12/1/2022

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} 1\,dy\,dx$$

$$4\int_{0}^{a}\int_{0}^{\sqrt{a^2-x^2}} dy\,dx$$

$$\Rightarrow 4\int_{0}^{a}\left[\,y\,\Big|_{0}^{\sqrt{a^2-x^2}}\right] dx$$

$$\Rightarrow 4\int_{0}^{a} \sqrt{a^2-x^2} - 0 \; dx$$

$\frac{0}{a}$

$$4\int_0^a \sqrt{a^2-x^2}\, dx$$

$$= \frac{4}{2}\left[x\sqrt{a^2-x^2} + a^2 \sin^{-1}(x/a)\right]_0^a$$

$$2\left[a(\sqrt{a^2-a^2}) + a^2 \sin^{-1}(1) - 0\right]$$

$$2\left[a^2 \pi/2\right] = \pi a^2$$

Differential length dl

$dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

$dr\,\hat{a}_r + r\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$

$dR\,\hat{a}_r + R\,d\theta\,\hat{a}_\theta + r\sin\theta\, d\phi\,\hat{a}_\phi$

Differential surface area

$$\left.\begin{aligned} ds_x &= dy\,dz\,\hat{a}_x \\ ds_y &= dx\,dz\,\hat{a}_y \\ ds_z &= dx\,dy\,\hat{a}_z \end{aligned}\right\} \text{rectangular}$$

$$\left.\begin{aligned} ds_r &= r\,d\phi\,dz\,\hat{a}_r \\ ds_\phi &= dr\,dz\,\hat{a}_\phi \\ ds_z &= r\,dr\,d\phi\,\hat{a}_z \end{aligned}\right\} \text{cylindrical}$$

$$\left.\begin{aligned} ds_r &= r^2 \sin\theta\, d\theta\, d\phi \\ ds_\theta &= r\sin\theta\, dr\, d\phi\,\hat{a}_\theta \\ ds_\phi &= r\,dr\,d\theta\,\hat{a}_\phi \end{aligned}\right\} \text{spherical}$$

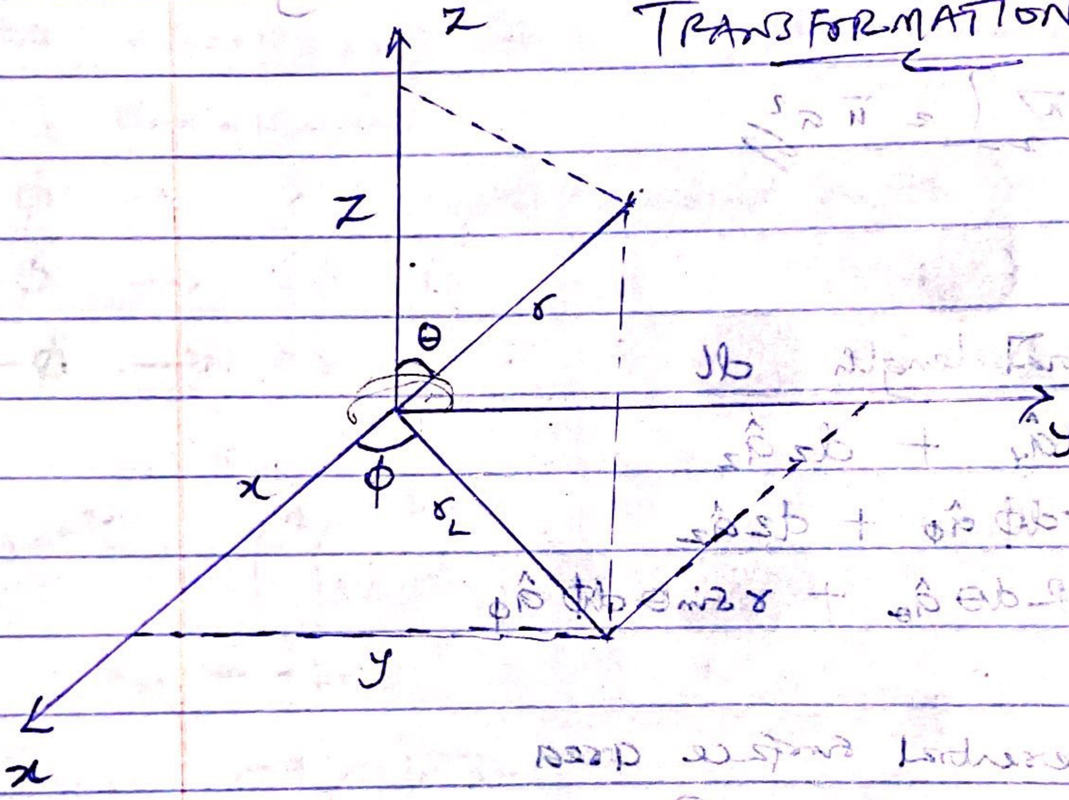
Differential volume

$dx\,dy\,dz$ — rectangular

$r\,dr\,d\phi\,dz$ — cylindrical

$r^2 \sin\theta\,dr\,d\theta\,d\phi$ — spherical.

TRANSFORMATION



Changing Cartesian to Spherical

$x = r_L \cos\phi$ $\quad$ $z = r\cos\theta$

$y = r_L \sin\phi$ $\quad$ $r_L = r\sin\theta$ → By observation

$\therefore\ x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

More importantly, put things in geometrical form.

(Quiz)

Using cylindrical coordinates

$\int dA = \iint r\,dr\,d\theta \longrightarrow$ so substitute and get your answer

$$\int_0^{2\pi}\int_0^{r} r\,dr\,d\theta = \int_0^{2\pi} d\theta\left[\frac{r^2}{2}\Big|_0^r\right]$$

$$= \int_0^{2\pi}\frac{r^2}{2}\,d\theta = \frac{\theta r^2}{2}\Big|_0^{2\pi} = \frac{\cancel{2}\pi r^2}{\cancel{2}} - 0$$

where $r = a$ since $r = a$, $A = \pi a^2$

Using the spherical coordinates (QUIZ)

$$V_s = \int r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$\int_0^{2\pi}\int_0^{\pi}\left[\frac{r^3}{3}\Big|_0^R\right]\sin\theta\,d\theta\,d\phi$$

$$\frac{R^3}{3}\int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta\,d\phi$$

$$\frac{R^3}{3}\int_0^{2\pi}(-\cos\theta)\Big|_0^{\pi}d\phi = \frac{2R^3}{3}\int_0^{2\pi}d\phi = \frac{4\pi R^3}{3}$$

Review of Vector Analysis

27/1/2022

Questions

Find the angle between the vectors shown in fig Q.1

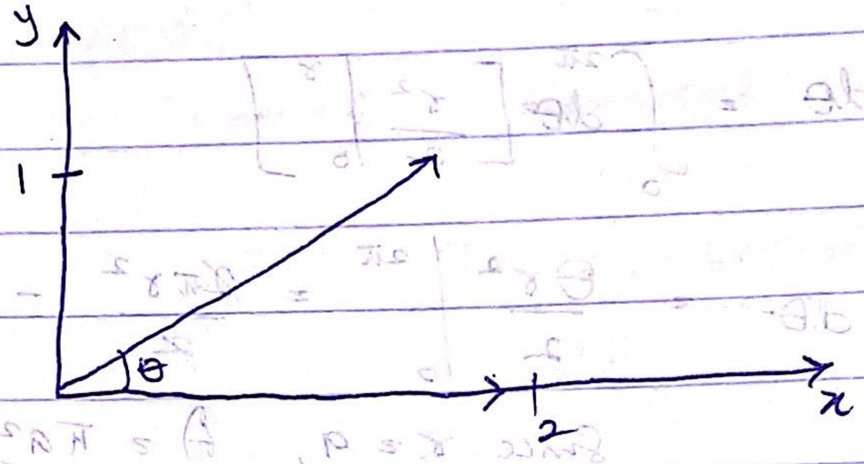


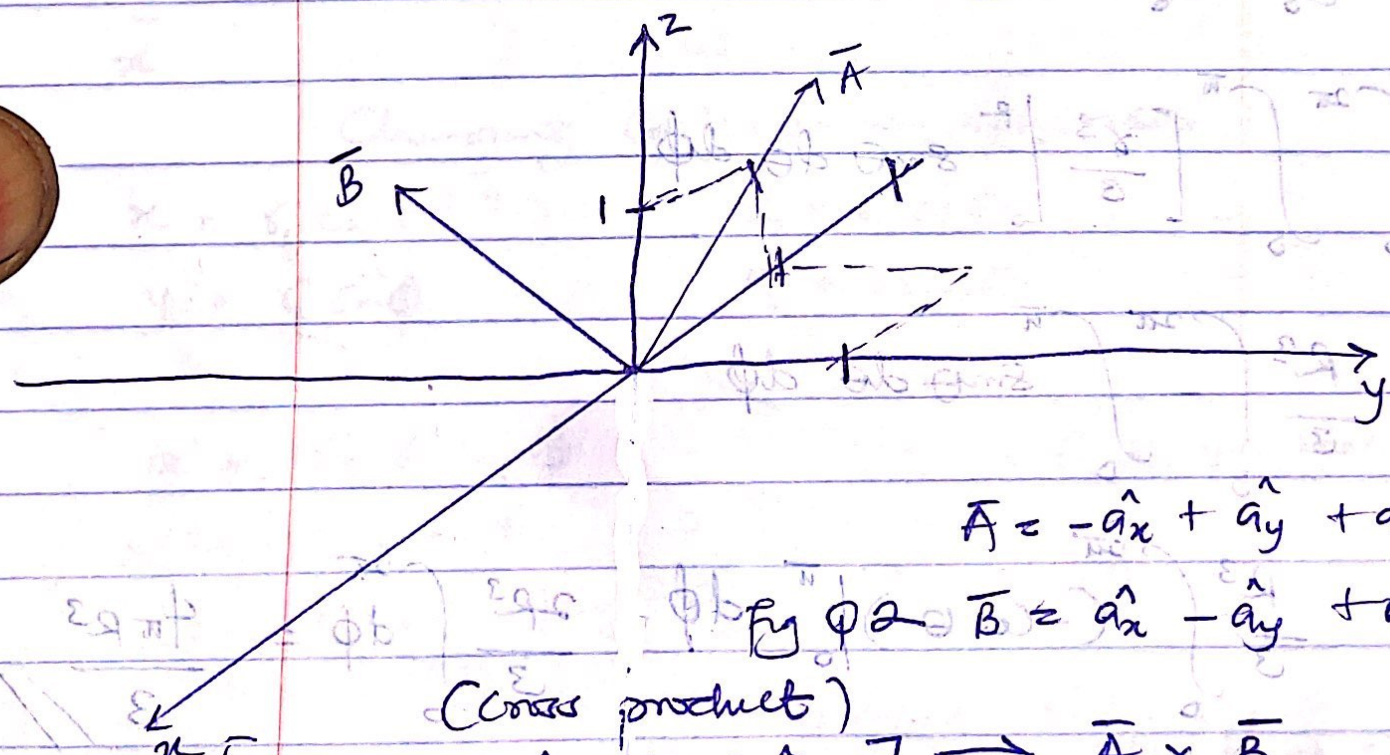
Figure Q1

$\bar{A} = \sqrt{3}\,\hat{a}_x + \hat{a}_y$

$\bar{B} = 2\hat{a}_x$

(Ans $\theta = 30°$)

(2) Find the unit vector $\hat{a}_n$ perpendicular in the right hand sense to the vectors shown in figure 2



$\bar{A} = -\hat{a}_x + \hat{a}_y + \hat{a}_z$

Fig Q2 $\bar{B} = \hat{a}_x - \hat{a}_y + \hat{a}_z$

(Cross product)

[Ans = $2\hat{a}_x + 2\hat{a}_y$] $\longrightarrow$ $\bar{A} \times \bar{B}$

Unit vector ans $\rightarrow \frac{1}{\sqrt{2}}(\hat{a}_x + \hat{a}_y)$ $\hat{a}_n = \frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|}$

Find angle between vectors $\bar{A}$ and $\bar{B}$.

* Ans $= \frac{|\bar{A} \times \bar{B}|}{|\bar{A}| \cdot |\bar{B}|}$ $\theta = 70.5°$ or $109.5°$ (taking quadrants into consideration)

(3) Find the gradient of each of the following functions where a and b are constants

(a) $f = ax^2y + by^3z$

(b) $f = ar^2\sin\phi + brz\cos$.

(c) $f = \frac{a}{r} + br\sin\theta\cos\phi$

4) For $f = x^2y$, verify that $\int_a^b df = f_b - f_a = \int_a^b \bar{\nabla}f \cdot d\bar{l}$

between the origin to the Point P at x_0, y_0.

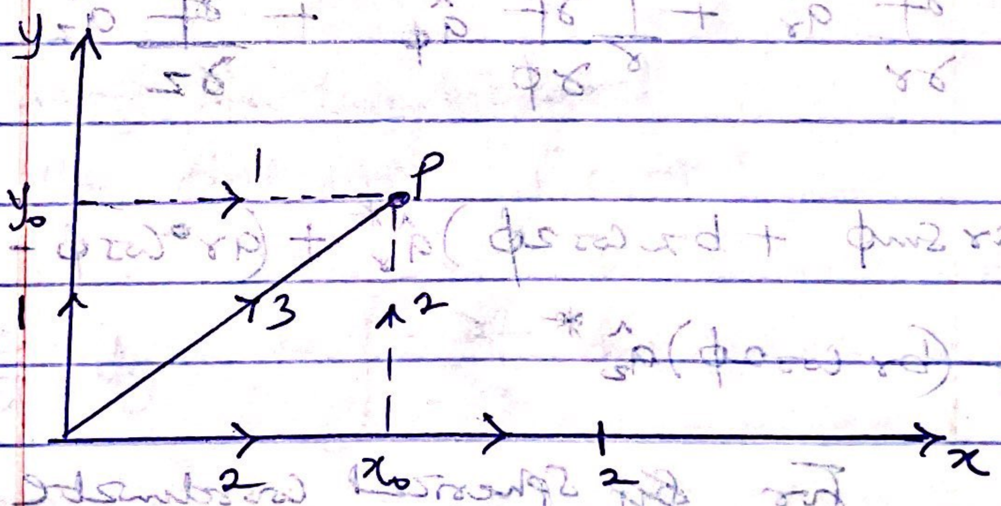


Figure Q4

try to solve using geometrical method

You can also use cross product to obtain the angles

Question 2 : If you do dot product, you will get
109.5°

Question 3 If you use ∇ operation on a scalar, you will get a vector out of it

$\bar{\nabla}$ – del operator

→ for cartesian coordinate

$$\bar{\nabla} f = \frac{\partial f}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z$$

$$= 2axy\,\hat{a}_x + (ax^2 + 3by^2z)\hat{a}_y + (by^3)\hat{a}_z$$

$\bar{\nabla}$ → varies for different coordinates

For Cylindrical coordinates

$$\bar{\nabla} f = \frac{\partial f}{\partial r}\hat{a}_r^* + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{a}_\phi^* + \frac{\partial f}{\partial z}\hat{a}_z^*$$

$$= (2ar\sin\phi + bz\cos 2\phi)\hat{a}_r^* + (ar\cos\phi - 2bz\sin 2\phi)\hat{a}_\phi^* + (br\cos 2\phi)\hat{a}_z^*$$

*

For Spherical coordinate

$$\bar{\nabla} f = \frac{\partial f}{\partial r}\hat{a}_r^* + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{a}_\theta^* + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{a}_\phi^*$$

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$dl \longrightarrow$ line integral

QUESTION 4

$$\int_a^b df = f_b - f_a \therefore \int_a^b df = x_0^2 y_0$$

$$\oint x_0^2 y_0 \longrightarrow \text{normal answer}$$

$f_b = x_0^2 y_0$ $\therefore f_b - f_a = x_0^2 y_0 - 0$

$f_a = 0$

$$\int_a^b \bar{\nabla} f \cdot \bar{dl} \longrightarrow \text{Evaluating}$$

along the line integral
for the first path $(1 \to 1$

$$\int_0^{y_0} \frac{\partial f}{\partial y}\, dy + \int_0^{x_0} \frac{\partial f}{\partial x}\, dx$$

$$\int_0^{y_0} x^2 dy + \int_0^{x_0} 2xy\, dx$$

along this path $x = ...$

$$\left[x^2 y\right]_0^{y_0} + \left[\frac{2x^2 y}{2}\right]_0^{x_0}$$

Along that path, $y = y_0$

so we have;

$$\frac{\cancel{2} x_0^2 y_0}{\cancel{2}} - 0 = x_0^2 y_0 \;//$$

Along the second path is also similar to the first path.

Along the third path

Obtaining the equation of the line

$y = mx + c$

where $c = 0$

m = gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y_2 = y_0 \quad x_2 = x_0$$
$$y_1 = 0 \quad x_1 = 0$$

$$m = \frac{y_0 - 0}{x_0 - 0} = \frac{y_0}{x_0}$$

$$\therefore y = \frac{x y_0}{x_0}$$

$$dy = \frac{y_0\, dx}{x_0}$$ so with this, we substitute

$$\int_0^P \left(\frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial x} dx \right)$$

$$\int_0^P x^2 dy + 2xy\, dx$$

$$\Rightarrow \int_0^P x^2 \frac{y_0}{x_0} dx + 2xy\, dx$$

$$\Rightarrow \int_0^P x^2 \frac{y_0}{x_0} dx + 2\frac{y_0}{x_0} x^2 dx$$

$$\Rightarrow \int_0^{x_0} \left[x^2 \frac{y_0}{x_0} + 2x^2 \frac{y_0}{x_0} \right] dx$$

$$\Rightarrow \int_0^{x_0} 3x^2 \frac{y_0}{x_0} dx \longrightarrow \frac{\cancel{3}x^3}{\cancel{3}} \cdot \frac{y_0}{x_0} \Bigg|_0^{x_0}$$

$$\Rightarrow x_0^3 \cdot \frac{y_0}{x_0} - 0$$

Ans $\Rightarrow x_0^2 y_0$ Ans

ELECTROSTATICS

We have the maxwells equations given as:

$$\left.\begin{array}{l} - \bar{\nabla} \times \bar{E} = \dfrac{\partial B}{\partial t} \\ - \nabla \times \bar{H} = \bar{J} + \dfrac{\partial D}{\partial t} \end{array}\right\} \text{two curl functions}$$

$$\left.\begin{array}{l} - \nabla \cdot \bar{D} = e \\ - \nabla \cdot \bar{B} = 0 \end{array}\right\} \text{two divergence functions}$$

$\bar{E}$ – Electric field

$\bar{B}$ – Magnetic "

$\bar{H}$ – " "

$\bar{J}$ – Current density

$\bar{D}$ – displacement current

e = Charge density

In statics, we do not talk about time. In statics the charge can either be zero or it is going with a steady current.

Since we do not talk about time, from the maxwell equations, $\dfrac{\partial \bar{B}}{\partial t} = 0$ $\dfrac{\partial \bar{D}}{\partial t} = 0$

wrt = with respect to

So we would then have Electrostatics and magnetostatics we would have a resulting decoupled equation.

(i) $\bar{\nabla} \times \bar{E} = 0$

(ii) $\nabla \times \bar{H} = \bar{J}$

(iii) $\nabla \cdot \bar{D} = \rho$

(iv) $\nabla \cdot \bar{B} = 0$

Electrostatics (i & iii)

Magnetostatics (ii & iv)

We are to look at Electrostatics and magnetostatics and we would start with electrostatics

CHARGE DISTRIBUTION: This exists wrt electron and nucleus. In macroscopic levels, charges can be distributed over a surface or space. It can be over a line, area, or volume (contd on next page)

Practical Applications of Electrostatics

(I) Some components that operate using electrostatics principle

(i) X-ray

(ii) Oscilloscope

(iii) Printers

(iv) Liquid crystal display

(II) Some medical equipments include

(i) Electrocardiograph

Movement of charge from one place to another gives current

* take time to look at the practical applications of electrostatics.

So we have: Line Charge distribution

Surface " "

Volume " "

The electric flux density can be related to the electric field. That type of relationship is called constitutive relationship

$\bar{D} = \varepsilon \bar{E}$ where ε – permittivity

where we can ~~can~~ have free space

$\bar{D} = \varepsilon_0 \bar{E}$ and relative permittivity.

SURFACE CHARGE DENSITY

$\rho_s = \frac{dq}{ds}$ (Coulomb/m^2) $= \frac{\Delta q}{\Delta s}$ → Elemental

Same thing applies to line and volume charge density

$Q = \int_s \rho_s \, ds$

* read divergence theorem & stokes theorem.

Line Charge density $= dq/dl$

Volume " $= dq/dr$

COULOMB'S LAW: This law deals with the interaction btw charges wrt the force. It states that the electrostatic force between two point charges (q_1, q_2) is directly proportional to the product of magnitude of the charges and inversely proportional to the square of the distance between them.

Mathematically

$$\bar{F} \propto \frac{q_1 q_2}{r^2} \hat{a}_r \qquad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\bar{F} = \frac{k q_1 q_2}{r^2} \hat{a}_r$$

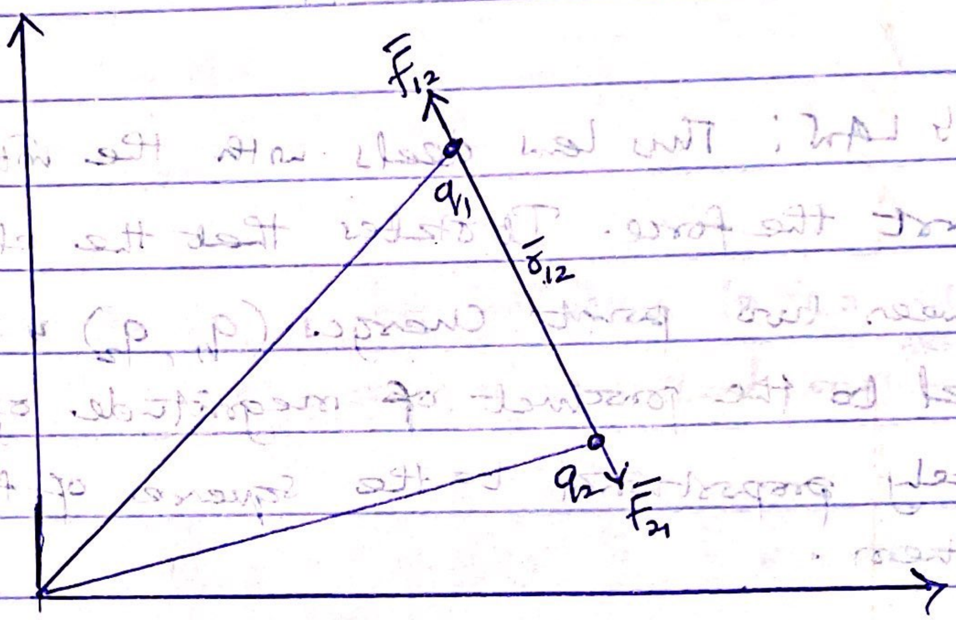
$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{a}_r$$

Keywords in the law → (the type of force being spoken about). Instead of electrostatic force, you can write the force of attraction or repulsion between them.

You can write point charges or test charges.

(Very important. Take note!!)

Now, we can have: $\bar{F}_{12} = \dfrac{k q_1 q_2 \hat{a}_{12}}{r^2}$ showing direction



$$\bar{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{a}_{12} \quad (N)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{(\bar{r}_2 - \bar{r}_1)}{|\bar{r}_2 - \bar{r}_1|^3}$$

* ~~Look at~~ superposition of Electro

Recommend Textbooks: Fundamental of applied Electromagnet

Any book on microwave engineering

3/2/2022

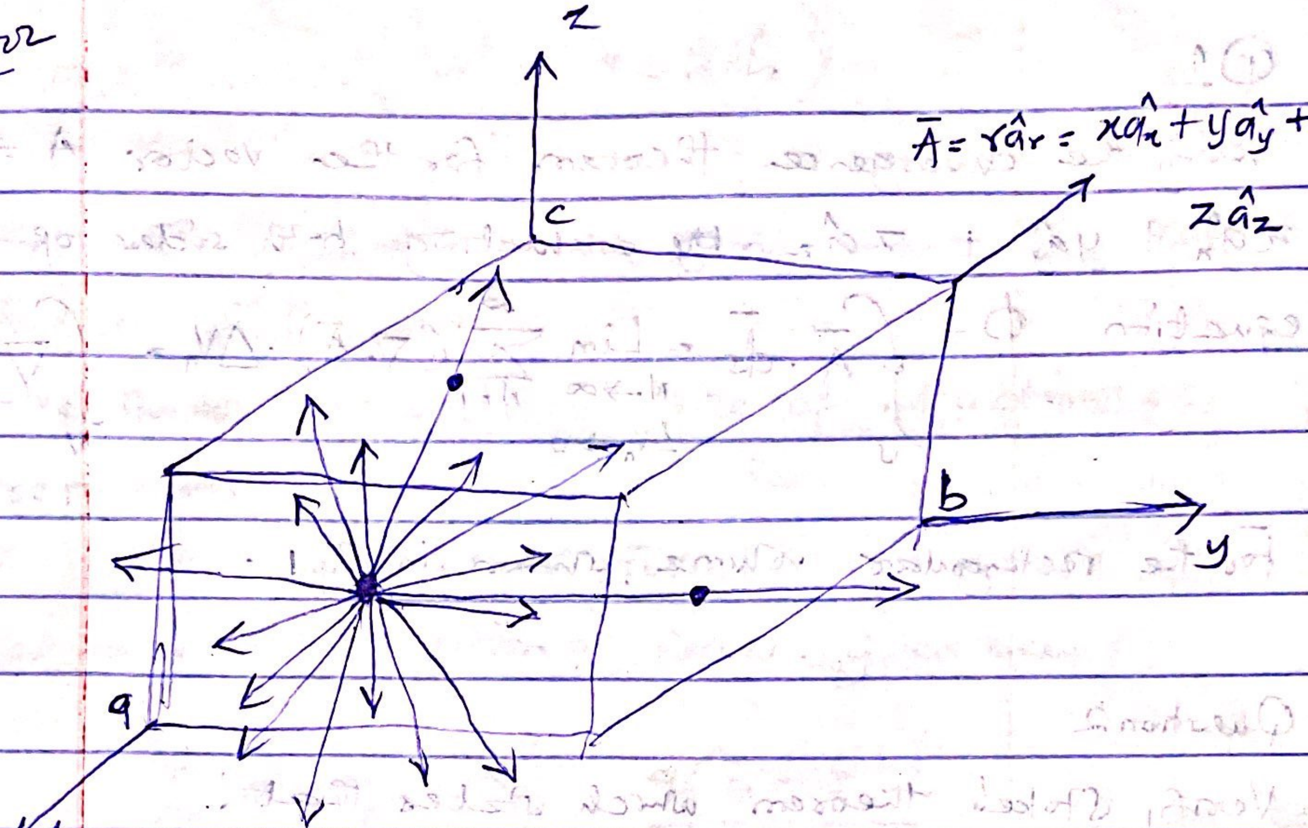


figure 1

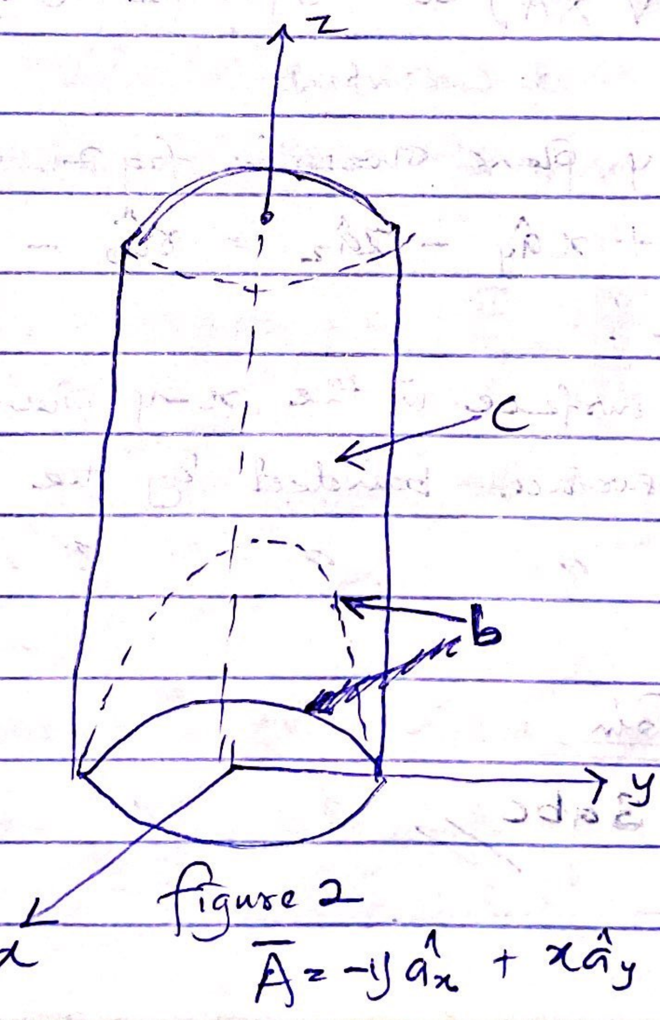


figure 2

$\bar{A} = -y\hat{a}_x + x\hat{a}_y - z\hat{a}_z = r\hat{a}_\phi - z\hat{a}_z$

Q1

Verify the divergence theorem for the vector $\bar{A} = r\hat{a}_r = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ by evaluating both sides of the equation
$$\phi = \oint_S \bar{A}\cdot\bar{ds} = \lim_{\substack{N\to\infty\\ \Delta V_n \to 0}} \sum_{N=1}^{\infty} (\nabla\cdot\bar{A})\cdot\Delta V_n = \int_V \nabla\cdot\bar{A}\,dv$$

For the rectangular volume shown in fig 1.

Question 2

Verify Stoke's theorem which states that:
$$\oint_L \bar{A}\cdot\bar{dl} = \int_S (\nabla\times\bar{A})\cdot\bar{ds}$$
for the circular bounding contour ~~for~~ in the x-y plane shown in fig 2 with a vector field $\bar{A} = -y\hat{a}_x + x\hat{a}_y - z\hat{a}_z = r\hat{a}_\phi - z\hat{a}_z$. Check the result for the:

(a) Flat circular surface in the x-y plane

(b) Hemispherical surface bounded by the contour

(c) Cylindrical " " " "

Soln

(1) Ans = ~~3a...~~ 3abc //

$$\oint_L \bar{A} \cdot d\bar{l}$$

(iv) Ans $2\pi r^2$ $\quad$ ($Curl\ \bar{A} = 2\hat{a}_z$)

From Maxwell's Equations, we can derive many laws, Ohm's law, Gauss' law and so on.

+ve or −ve shows whether it is force of attraction or force of repulsion.

Characteristics of Forces

(1) We can have superposition of forces. given below,

$$\bar{F}_N = \frac{q}{4\pi \varepsilon_0} \sum_{n=1}^{N} \frac{q_n(\bar{r}_n - \bar{r}')}{|\bar{r}_n - \bar{r}'|^3}$$

we put n because the observation will vary

Electric field: This is a disturbance or space in between the two charges that will allow the message or effect to be felt.

~~Unit~~ force / unit charge i.e $E = \frac{F}{q}$

$$\bar{E}_{21} = \frac{\bar{F}_{21}}{q_2} \quad (\text{solving for effect on } \underline{1})$$

If you substitute in the force equation, you would have,

$$\bar{E}_{21} = \frac{q_1}{4\pi \varepsilon_0} \frac{(\bar{r}_2 - \bar{r}_1)}{|\bar{r}_2 - \bar{r}_1|^3}$$

You can have a continuous charge and a discrete charge

$$d\bar{E} = \frac{dq\,(\bar{r}_n - \bar{r}')}{4\pi\varepsilon_0 \|\bar{r}_n - \bar{r}'\|^3}$$

$\bar{r}'$ – source
$\bar{r}$ – observation

Another way to solve for the electric field is when we use Gauss law $\boxed{\text{This states that}}$

Gauss law: The total electric flux through a closed surface is equal to the total charge enclosed by the surface.

$$\nabla \cdot \bar{D} = \rho_s \implies \varepsilon \nabla \cdot \bar{E} = \rho_s$$

$$\oint_s \bar{D} \cdot \bar{ds} = q_{enc} \implies \oint_s E \cdot \bar{ds} = \frac{q_{enc}}{\varepsilon_0}$$

to calculate

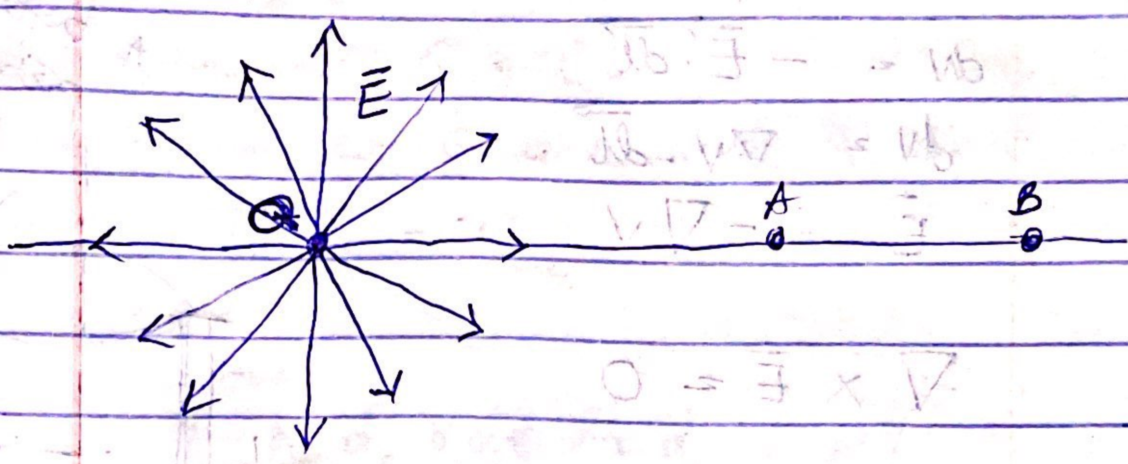
Three ways to ~~look~~ at electric field are:

- Coulomb's law
- Gauss' law
- Electric Potential

Potential from the view of workdone (either in attraction or repulsion).

It is either that the charge gains energy when moved in the direction of a force.

Potential Energy depends on position.



$$W = -\int_{r_a}^{r_b} \vec{F} \cdot \vec{dl}$$ (Workdone done is opposite to the force, that is why it is -ve)

$$V_{r_b} - V_{r_a} = \frac{W}{q} = -\int_{r_a}^{r_b} \vec{E} \cdot \vec{dl}$$

Once we substitute, we have:

$$V_{r_b} - V_{r_a} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

i.e. the point a

Assuming the point is at infinity (reference)

$$V(r) = \frac{q}{4\pi\varepsilon_0 r}$$

In terms of closed path $W = \oint_L q E \cdot \overline{dl} = 0$

$$\Rightarrow \oint \bar{E} \cdot \overline{dl} = 0$$

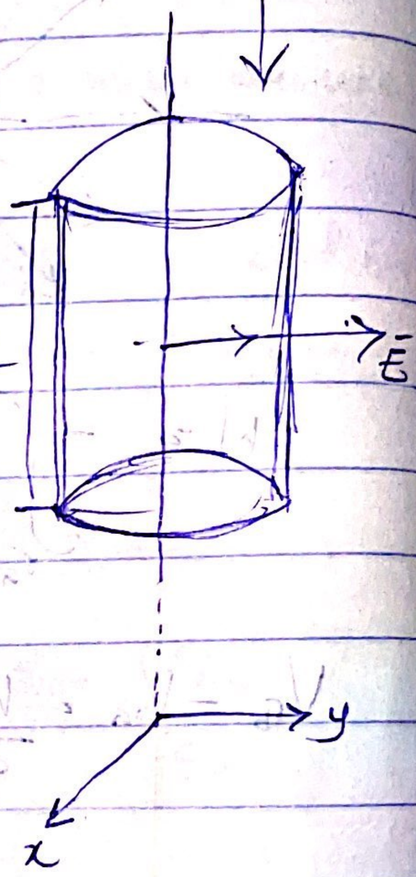
$$dV = -\vec{E} \cdot \overrightarrow{dl}$$

$$dV = \nabla V \cdot \overrightarrow{dl}$$

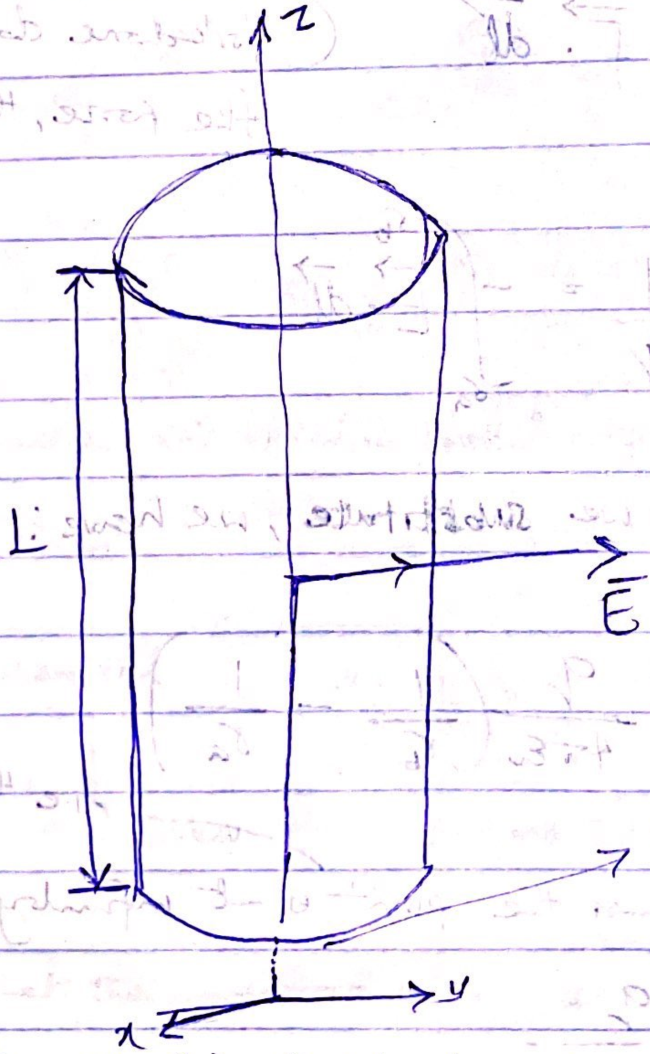
$$\bar{E} = -\nabla V$$

$$\nabla \times \bar{E} = 0$$

A better picture



Examples



dot dot mean infinite

figure 3 : Infinite Line Charge.

Determine the electric field due to an infinitely long line with uniform charge density ρ_L that is distributed along the z axis in free space as in figure 3

Soln

It was changed to r

$$\bar{D} = D_\rho \hat{a}_\rho \longrightarrow D_r \hat{a}_r$$

From Charge distribution, $Q = \rho L$

We go on to use the Gauss Law

$$\oint_s \bar{D}\, d\bar{s} = q_{enc}$$

$$\int_0^L \int_0^{2\pi} D_r \hat{a}_r \, r \, d\phi \, dz \, \hat{a}_r = \rho_L l$$

$$D_r = \frac{\rho_L}{2\pi r} \qquad \bar{E} = \frac{\rho_L}{2\pi \varepsilon_0 r} \hat{a}_r$$

$$\bar{E} = \frac{\bar{D}}{\varepsilon_0}$$

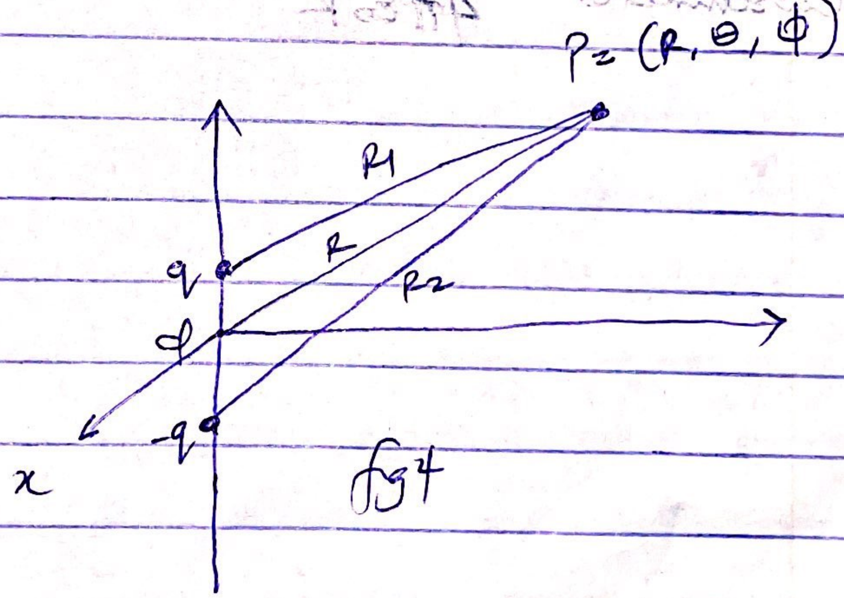


fig 4

An electric dipole consists of two point charges of equal magnitude but opposite polarity, separated by a distance d as illustrated in figure 4. Determine the electric potential and electric field at point P, given that P is at a distance $R \gg d$ from the dipole centre and the dipole resides in free space.

Soln

$$R_1 = R - \frac{d}{2}\cos\theta$$

$$R_2 = R + \frac{d}{2}\cos\theta$$

$$V = \frac{qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

$\bar{E}$

Using spherical coordinates $\rightarrow \frac{qd}{4\pi\varepsilon_0 R^3}\left(2\cos\theta\,\hat{a}_R + \sin\theta\,\hat{a}_\theta\right)$

Practise a lot of Questions.