Optimal Pricing for Local and Global WiFi Markets

Lingjie Duan, Jianwei Huang, and Biying Shou

Abstract—**This paper analyzes two pricing schemes commonly used in WiFi markets: flat-rate pricing and usage-based pricing. The flat-free pricing encourages users to achieve the maximum WiFi usage and targets at users with high valuations in mobile Internet access, whereas the usage-based pricing is flexible to attract more users – even those with low valuations. First, we show that for a local provider, the flat-rate pricing provides more revenue than the usage-based pricing, which is consistent with the common practice in today's local markets. Second, we study how Skype may work with many local WiFi providers to provide a global WiFi service. We formulate the interactions between Skype, local providers, and users as a two-stage dynamic game. In Stage I, Skype bargains with each local provider to determine the global Skype WiFi service price and revenue sharing agreement; in Stage II, local users and travelers decide whether and how to use local or Skype WiFi service. Our analysis discovers two key insights behind Skype's current choice of usage-based pricing for its global WiFi service: to avoid severe competition with local providers and attract travelers to the service. We further show that at the equilibrium, Skype needs to share the majority of his revenue with a local provider to compensate the local provider's revenue loss due to competition. When there are more travelers or fewer local users, the competition between Skype and a local provider becomes less severe, and Skype can give away less revenue and reduce its usage-based price to attract more users.**

I. Introduction

The 802.11 standard based wireless local area network technology, also known as WiFi, is one of the most well-known successful stories in modern wireless communications [1]. Operating in the unlicensed 2.4GHz and 5GHz spectrum band, WiFi networks do not require exclusive spectrum licenses as the cellular technologies, and can provide high-speed wireless access to mobile users within tens to hundreds of meters of WiFi access points (APs) [16]. Furthermore, APs in WiFi networks are inexpensive and can be easily deployed and maintained [18]. The annual revenue in the WiFi industry has exceeded $4 billion in 2007 and has been growing rapidly in recent years ([4], [17]).

In order to provide close to seamless high performance mobile communication experiences, many WiFi providers (e.g., AT&T in US, BT Openzone in UK, and PCCW in Hong Kong) are deploying a large number of WiFi APs in their local markets. In the Hong Kong market alone, PCCW has rolled out nearly ten thousand APs covering almost all popular places (e.g., convenient stores and shopping malls, coffee shops and hotels, train stations, and education institutes). For these local providers, we often observe them charging local users (subscribers) a monthly flat fee ([3], [23], [24]), where a user pays a fixed amount per month independent of the actual usage. This motivates us to ask the first key question in this paper: *Why does a local WiFi provider prefer to charge its local users a flat fee instead of based on usage?*

Notice that a WiFi AP can serve not only local users, but also travelers who visit a particular city/country for a short period of time. But paying a monthly flat fee is often not a good choice for a traveler. To cater the needs of travelers, Skype [27] has pioneered in providing a global WiFi service under the band name of *Skype WiFi*, through collaborating with many local WiFi providers who own a total of more than 1 million WiFi APs worldwide. Once a user subscribes to the Skype WiFi service, he can use any of the associated WiFi AP with his Skype account, and pays according to usage with his Skype Credit (i.e., "only pays for the time you are online", as Skype puts it). Such flexible Skype WiFi service provides great convenience for travelers, but also introduces competition with local WiFi providers among local users. In order to promote such cooperation, Skype needs to share part of the revenue with the local WiFi provides who provide the WiFi APs. Thus the local WiFi providers will only have incentives to collaborate with Skype and share their infrastructure if they can also gain from this new service. This motivates us to ask the second key question in this paper: *Why does Skype choose usage-based pricing for its global WiFi service, and how will he share the benefits with the local WiFi providers?*

To answer the first key question, we focus on a local market with a monopoly local WiFi provider and a group of local users. We model their interactions as a two-stage Stackelberg game: the local provider (leader) determines pricing scheme (flat-fee or usage-based) in Stage I, and local users (followers) decide whether they will subscribe (and how much to use in the usage-based pricing) in Stage II. We show that the flat-fee pricing can offer a higher revenue than the usage-based pricing for such a monopoly local provider. To answer the second key question, we study how Skype may provide a global WiFi service by cooperating with local providers, given the local providers' optimal flat-fee based pricing. We formulate the problem as a two-stage dynamic game. In Stage I, Skype negotiates with each local provider about the Skype WiFi price and the revenue sharing portion based on Nash bargaining. In

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Stage II, local users choose between Skype WiFi and the local provider's service, and travelers choose their usage levels in the Skype WiFi service.

Our key results and contributions are as follows:

- *Flat-fee pricing dominates the local WiFi markets*: In Sections II, III and IV, we analyze a local provider's optimal choice of pricing scheme by comparing the usage-based pricing and the flat-rate pricing. We show that the flat-rate pricing allows maximum usage and is effective in attracting the high valuation users, while the usage-based pricing provides incentives for the users to optimize their usage and is attractive to the low valuation users. However, given the abundant WiFi capacity in today's market, the flat-rate pricing brings a larger revenue source to the local provider than the usage-based pricing.[1]
- *Win-win situation with Skype's usage-based pricing:* In Section V, we show that by choosing the usage-based pricing for global WiFi service, Skype avoids severe competition with local providers' own flat-fee based local services. Moreover, the global Skype WiFi service attracts travelers with all valuations as well as local users with low valuations, and will provide enough revenue that can cover the user loss of the local providers due to competition. When such revenue is shared properly, Skype and each local provider achieve a win-win situation.
- *Nash bargaining on Skype WiFi price and revenue sharing:* In Section VI, we show that Skype always needs to share more than half of its revenue with the local providers, to incentivize them to cooperate and share WiFi APs. As the local user population decreases or the traveler population increases, the competition between Skype and a local provider decreases. This enables Skype to share less revenue and lower its price (and can be as low as the monopoly price).

A. Related Work

We can divide the related work on WiFi pricing into three categories. The first category of results studied how a local provider optimizes the price to maximize his revenue ([14], [22]). Focusing on some generic network, these results did not consider WiFi's limited coverage and the users' movements across different WiFi markets. Moreover, they studied either flat-rate pricing or usage-based pricing, without analytical comparison between the two schemes. Results in the second category focused on the perspective of an individual WiFi AP owner, who charges visitors for using his AP's resources ([11], [12], [21]). The key design challenge here is the asymmetric information, i.e., visitors know more about their utility functions than the AP owner. Results in the third category studied wireless social community networks, where WiFi owners form a community to share their APs with each other, so that one AP owner can use other APs to access the Internet during travel (e.g., [2], [18]). In this line of literature, the main design objective is to encourage as many AP owners to join the community as possible. The revenue maximization becomes a secondary concern.

In this paper, we study the optimal pricing schemes in both local and global WiFi markets. We consider several key and practical features of WiFi networks (e.g., WiFi's limited coverage and users' movement across different WiFi markets) in our pricing design, and compare the pros and cons of flat-fee and usage-based pricing. Furthermore, we are the first to study how a global WiFi service provider (such as Skype) may cooperate with local providers, negotiate pricing and revenue sharing schemes, and achieve win-win situation.

B. Some Key Concepts

The following concepts will be useful throughout this paper.

- *Local provider:* A WiFi provider who deploys APs to provide service to a single dedicated region. For example, PCCW serves the Hong Kong market only, and AT&T serves the USA market only.
- *Global provider*: A WiFi provider who serves multiple local markets, by using the network infrastructure (APs) of the corresponding local providers. For example, the Skype Wifi service covers many countries with collaborations with local provides, but Skype does not own any physical WiFi APs.
- *Local market:* A market that is served by a single local provider (and possibility by a global provider). There are a set $\mathcal{I} = \{1, 2, ..., I\}$ of disjoint local markets, and hence I local providers.[2]
- *Local user:* a user who lives in a particular local market as a long-term resident. There are N_i local users in each local market $i \in \mathcal{I}$.
- *Traveler:* when a user travels to a market other than his own local market, he becomes a traveler. We use the parameter α_j^i to denote the percentage of users in local market j who are willing to pay short-term visits to local market i, and thus the total travelers from market j to i is $\alpha_j^i N_j$.

II. Usage-based Pricing for Local WiFi Service

We will first study how a local provider will optimize the price to maximize the revenue, assuming that he will choose the usage-based pricing. In Section III, we will derive the optimal pricing term should the local provider choose to use the flat-fee pricing. In Section IV, we will compare these two cases, and show that flat-fee pricing always brings more revenue than the usage-based pricing in the local WiFi service.

We consider a two-stage Stackelberg game between a local provider i and a group of N_i local users. In Stage I, the

[1] In today's market, the latest WiFi technology 802.11n [15] can offer a much larger throughput per user than the cellar network, and it does not incur network congestion in most of places and time. For analysis simplicity, we assume that the WiFi network has an unlimited capacity.

[2] The assumption of one local provider per local market is for analysis convenience. The focus of this paper is not on competition between the local providers, but on the competition and cooperation between local providers and the global provider. When there are multiple local providers in the same market, then the competition between local markets might make the entry of Skype easier. We will address the interaction between these two different types of competitions in a future work.

provider i announces the price p_i (per unit of usage time) to maximize its revenue. In Stage II, users decides whether and how much to use the service to maximize their payoffs. At a Subgame Perfect Equilibrium (SPE, or simply *equilibrium*) of the game, the provider and users will not have incentives to change their pricing and usage choices.

Next we will analyze the equilibrium of the game using backward induction [13]. We will first study the users' decisions in Stage II for any given price, and then look at how the provider should determine the price in Stage I by taking the users' decisions into consideration.

A. Stage II: Users' Usage Choices

Due to the limited number of APs, a local WiFi provider typically cannot provide a complete coverage in a region. Let us denote the local provider's WiFi's coverage as $G_i(M_i) \in (0,1)$, where M_i is the total number of deployed APs. In this paper, we will assume that M_i is fixed, and thus will simply write $G_i(M_i)$ as G_i. As today's WiFi technology can provide very high speed (e.g., a maximum of 54Mbps for 802.11g and 150Mbps for 802.11n) within its coverage, we assume that the resource constraint for users is not the network capacity but the usage time. This is actually the fact in many today's WiFi networks, where the ISM band is enough to support a large number of wireless users with tolerable congestion and many commercial APs remain under-utilized most of the time.

When a local user in market i is in the WiFi coverage, we denote his usage level as $d^i \in [0,1]$, which represents the percentage of Internet connection time over the whole time in WiFi coverage. Note that users may demand different usage levels as they have different valuations towards Internet connection. When $d^i = 1$, then the user always stays online whenever WiFi is available. Different users have different valuations of the Internet access time, which is characterized by a type parameter θ. Unlike d^i, the parameter θ is not a decision variable. A larger θ implies the user's higher valuation of the Internet access time. For analysis tractability, we assume that θ follows a uniform distribution in $[0,1]$.[3] We further assume that a type-θ user's utility $u(\theta, d^i)$ is linearly increasing in θ and concavely increasing in d^i. The latter is to represent his diminishing return in Internet access time. One commonly used utility function satisfying our requirement is[4]

$$u(\theta, d^i) = \theta \ln(1 + kd^i), \tag{1}$$

where the parameter $k > 0$ represents the elasticity of demand, i.e., the ratio between the percent change of demand and the percent change of price [19]. A larger k leads to a higher valuation of the same usage level.

When using the service, the user needs to pay linearly proportionally to his usage level or time and the unit price p_i. This is motivated by the fact that many providers charge based on connection time instead of data volume. As the user's usage and payment are only meaningful when he is within the WiFi coverage, his overall payoff v^i is linear in coverage G_i, i.e.,

$$v^i(\theta, p_i, d^i) = G_i(\theta \ln(1 + kd^i) - p_i d^i). \tag{2}$$

Maximizing payoff v^i over d^i leads to the optimal usage level

$$d^{i*}(\theta, p_i) = \min\left(\max\left(\frac{\theta}{p_i} - \frac{1}{k}, 0\right), 1\right), \tag{3}$$

which is increasing in the user's type θ and the elasticity parameter k, and is decreasing in price p_i. Furthermore, only users with $\theta \geq p_i/k$ will have a positive usage (i.e., subscribe to the service).

Next we exploit how users' optimal usage levels change with the price p_i. Depending on whether the price is low or high, one of the following two scenarios is possible: (i) some users will choose the maximum usage level $d^{i*} = 1$, or (ii) no users will choose the maximum usage level. By assuming that the two terms in the min operation in (3) are equal at $\theta = 1$, we can derive the price threshold that separate the two scenarios:

$$p_i^{th} = \frac{k}{k+1}. \tag{4}$$

Depending on whether the price p_i is larger or smaller than the threshold, the provider will face two different types of market demands. This will be analyzed next in Stage I.

B. Stage I: The Local Provider's Pricing Choice

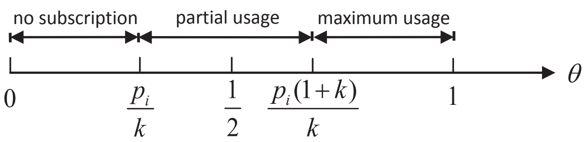

Fig. 1. Users' WiFi usage choices in the low price regime

1) Low price regime: $p_i < k/(k+1)$: Figure 1 summarizes users' optimal usage levels in this case. There are three categories of users based on the type parameter θ: (i) a user with a small type $\theta \in [0, p_i/k)$ will not subscribe to the WiFi service, (ii) a user with a medium type $\theta \in [p_i/k, p_i(1+k)/k]$ will subscribe with a partial usage level (i.e., $d^{i*}(\theta, p_i) = \theta/p_i - 1/k < 1$), and (iii) a user with a high type $\theta \in (p_i(1+k)/k, 1]$ will have the maximum usage (i.e., $d^{i*}(\theta, p_i) = 1$). The provider's total revenue collected from the latter two categories of users is

$$\begin{aligned}\pi_i(p_i) &= N_i G_i p_i \left(\int_{\frac{p_i}{k}}^{\frac{p_i(k+1)}{k}} \left(\frac{\theta}{p_i} - \frac{1}{k}\right) d\theta + \int_{\frac{p_i(k+1)}{k}}^{1} 1 d\theta\right) \\ &= N_i G_i \left(p_i - p_i^2\left(\frac{1}{2} + \frac{1}{k}\right)\right). \end{aligned} \tag{5}$$

By checking the first and second order derivatives of $\pi_i(p_i)$, we can show that $\pi_i(p_i)$ is concave in p_i. Thus the optimal price that maximizes the revenue in the low price regime is

$$p_i^L = \frac{k}{k+2}. \tag{6}$$

[3]The assumption of uniform distribution can facilitate our analysis. Many work also made such assumption and the relaxation to more general distributions often do not change the main engineering insights (e.g., [20], [26]).

[4]The logarithmic utility is widely used in the networking literature to model elastic applications (*e.g.,* [5], [7], [9], [10]).

The provider's maximum revenue in the low price regime is

$$\pi_i(p_i^L) = N_i G_i \frac{k}{2(k+2)}. \tag{7}$$

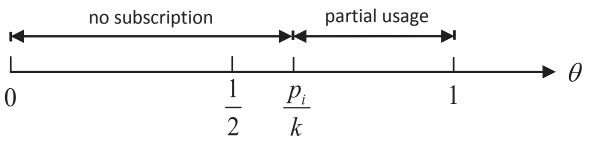

Fig. 2. Users' WiFi usage choices in the high price regime

2) High price regime: $p_i \geq k/(k+1)$: Figure 2 summarizes users' optimal usage in this case. There are two categories of users based on the type parameter θ: (i) a user with a low type $\theta \in [0, p_i/k)$ will not subscribe to the WiFi service, and (ii) a user with a high type $\theta \in [p_i/k, 1]$ choose to subscribe the WiFi service with a partial usage level (i.e., $d^{i*}(\theta, p_i) = \theta/p_i - 1/k < 1$). The provider's total revenue collected from the second category users is

$$\begin{aligned}\pi(p_i) &= N_i G_i p_i \int_{\frac{p_i}{k}}^{1} \left(\frac{\theta}{p_i} - \frac{1}{k}\right) d\theta \\ &= N_i G_i \left(\frac{1}{2} - \frac{p_i}{k} + \frac{p_i^2}{2k^2}\right). \end{aligned} \tag{8}$$

Its first derivative over p_i is

$$\frac{d\pi_i(p_i)}{dp_i} = N_i G_i \frac{p_i - k}{k^2}. \tag{9}$$

Notice that to obtain a positive revenue, the provider should set the price such that the highest type user is willing to subscribe, i.e., $d^{i*}(1, p_i) = 1/p_i - 1/k_i > 0$. This means $p_i < k$, which implies (9) is negative. Thus the optimal price in the high price regime is

$$p_i^H = \frac{k}{k+1},$$

which is the boundary case of the low price regime.

Summarizing the results from both low and price regimes, we have the following result.

Proposition 1: The provider's equilibrium usage-based price that maximizes his revenue is

$$p_i^* = \frac{k}{k+2}, \tag{10}$$

which is increasing in the elasticity parameter of demand k and is independent of coverage G_i. The provider's maximum revenue under usage-based pricing is

$$\pi_i(p_i^*) = N_i G_i \frac{k}{2(k+2)}. \tag{11}$$

The independence of p_i^* in G_i is due to the fact that a user only pays when he uses the service in the WiFi coverage area.

Figure 3 summarizes all users' usage behaviors at the equilibrium. The flexibility of usage-based pricing attracts the majority of users to the service, since the threshold type $p_i^*/k = 1/(k+2) < 1/2$. As the elasticity parameter k

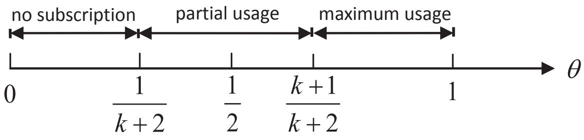

Fig. 3. Users' WiFi usage choices at the equilibrium

increases, the type threshold will decrease and more users will join the service. Users' total usage level, however, is

$$\begin{aligned} D_i(p_i^*) &= N_i G_i \left(\int_{\frac{p_i^*}{k}}^{\frac{p_i^*(1+k)}{k}} \left(\frac{\theta}{p_i^*} - \frac{1}{k}\right) d\theta + \int_{\frac{p_i^*(1+k)}{k}}^{1} 1 d\theta \right) \\ &= \frac{N_i G_i}{2}, \end{aligned} \tag{12}$$

and independent of k.

III. Flat-rate Pricing for Local WiFi Service

Similar to Section II, in this section we also consider a two-stage Stackelberg game played by the provider i and N_i users. The difference is that the provider will announce a flat-fee in Stage I, and users decide whether to subscribe the service in Stage II. Since a user's payment is independent of his usage level, he will always choose the maximum usage time $d^i = 1$ (*i.e.,* 100% of the time) whenever he subscribes. Next we derive the game equilibrium by using backward induction.

A. Stage II: Users' Subscription Choices

In Stage II, by joining the flat-rate price plan, a type-θ user's payoff is

$$v^i(\theta, P_i) = G_i u(\theta, 1) - P_i = G_i \theta \ln(1+k) - P_i.$$

Notice that the flat fee P_i is independent of usage, and thus is also independent of whether the user is in the WiFi coverage area In other words, once a user subscribes to the WiFi service, he will be charged a flat fee at the end of that month. This means that the *effective* price by considering the limited coverage is $\tilde{P}_i := P_i/G_i > P_i$.

It is clear that only users who have high valuations of mobile Internet access would subscribe to the WiFi service and obtains a positive payoff. The minimum type among these users is

$$\theta^{th}(P_i) = \frac{P_i}{G_i \ln(1+k)}. \tag{13}$$

B. Stage I: The Local Provider's Pricing Choice

Back to Stage I, the provider wants to maximize his revenue by collecting payment from users with $\theta \in [\theta^{th}(P_i), 1]$, i.e.,

$$\max_{P_i \geq 0} \pi_i(P_i) = N_i P_i \left(1 - \frac{P_i}{G_i \ln(1+k)}\right). \tag{14}$$

It is easy to verify that $\pi_i(P_i)$ is concave in P_i, and we can derive the optimal price as follows.

Proposition 2: The provider's equilibrium flat-rate price is

$$P_i^* = \frac{G_i \ln(1+k)}{2}, \tag{15}$$

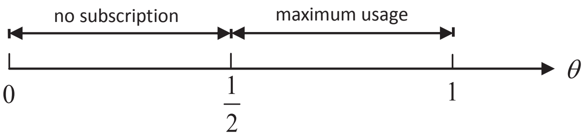

Fig. 4. Users' WiFi usage choices at the equilibrium

which is increasing in the coverage G_i and elasticity parameter k. The provider's maximum revenue with flat fee is

$$\pi_i(P_i^*) = \frac{N_i}{4} G_i \ln(1+k), \tag{16}$$

which is increasing in G_i and k.

Figure 4 summarizes users' usage behaviors at the equilibrium. The inflexibility of this pricing scheme attracts fewer users than the usage-based scheme, while the WiFi usage level for each user is the maximum.

Intuitively, a better WiFi coverage and a larger elasticity parameter of demand encourage more users to use WiFi service, and the provider can charge more. Users' total usage is

$$D_i(P_i^*) = \frac{N_i}{2} G_i, \tag{17}$$

which is the same as in the usage-based pricing case (see (12)).

IV. Flat-rate Pricing Outperforms Usage-based Pricing for Local WiFi Service

Now we are ready to compare the two pricing scheme and see which one leads to a larger provider revenue.

Let us define the ratio between the equilibrium revenues of flat-rate pricing scheme and usage-based pricing scheme as $r := \pi_i(P_i^*)/\pi_i(p_i^*)$. Based on (11) and (16), we can rewrite the ratio as a function of k, i.e.,

$$r(k) = \frac{(k+2)\ln(1+k)}{2k}. \tag{18}$$

The first order derivative of $r(k)$ over k is

$$\frac{dr(k)}{dk} = \frac{k(k+2)/(k+1) - 2\ln(1+k)}{2k^2}, \tag{19}$$

and we can show that such derivative is positive for all positive values of k. By using L'Hospital law, we can further check

$$\lim_{k\to 0} r(k) = \left. \frac{\ln(1+k) + (k+2)/(k+1)}{2} \right|_{k=0} = 1.$$

This means that $r(k) > 1$ for any $k > 0$. Thus, we have the following result.

Theorem 1: A local provider obtains a larger revenue with the flat-rate pricing than with the usage-based pricing. The revenue gap increases in the demand elasticity parameter k.

Theorem 1 is consistent with the current industry practice, where most WiFi providers offer flat-rate pricing instead of usage-based pricing in local markets (e.g., Orange in UK [23], AT&T in US [3], and PCCW in Hong Kong [24]). Another benefit of the flat-rate pricing (that is not explicitly modeled here) is that it is easy to implement with little overhead for billing, while the usage-based pricing requires the provider to record users' mobile traffic for payment calculation and collection over time [6].

V. Skype's Global WiFi Service

Now let us look at the WiFi service in a global market. If one firm wants to provide a global WiFi service, he can either densely deploy APs worldwide or cooperate with many local WiFi providers. The former approach typically requires an extremely large investment, while the latter approach is more feasible. In fact, today Skype uses the latter approach to provide a global WiFi service called "Skype WiFi", which involves more than 1 million hotspots deployed by many local providers worldwide (e.g., Best Western in US, BT Openzone in UK, and PCCW in Hong Kong). To motivate the cooperation of local providers, Skype shares some of his WiFi revenue with these cooperators [28]. Here is what each side will gain and lose during this cooperation.

- *Skype's gain:* Skype can gain extra revenue by providing WiFi service. Skype used to be just a software provider without any physical WiFi infrastructure. With the cooperation and a usage-based pricing, Skype is able to serve travelers who are not willing to sign a long-term flat-free agreement with local providers. Furthermore, Skype can attract some low valuation local users who do not subscribe the flat-fee local WiFi service, or prefer usage-based pricing to the flat-fee pricing. During this process, Skype needs to share part of the revenue with the local WiFi providers to achieve a win-win situation.
- *Local provider's benefit and loss:* When Skype starts to provide WiFi service in a local market i, the local provider i will experience new market competition and a reduced number of subscribers. However, as Skype relies on the local provider's WiFi infrastructure, the provider has the market power to negotiate with Skype on Skype WiFi's price to avoid severe competition.[5] Furthermore, he can share part of the Skype's revenue to compensate his loss and potentially increase his total revenue.

The slogan of Skype WiFi is "only pay for the time you're online" (i.e., usage-based pricing). It should be noted that Skype has the following three advantages over many local providers to implement a usage-based pricing.

- *Existing mechanism to record users' traffic:* Skype can use the same traffic recording system in Skype WiFi as in the existing *Skype Internet Call service*. Besides, existing Skype users do not need to install additional applications on laptops or smartphones to support ID authentication or traffic recording.
- *Trustworthy global billing system:* Skype has built a reputable global billing platform with its existing services. As Skype has successfully cooperated with many local telecommunication companies to provide Skype Call, it is easy for Skype to convince local WiFi providers to be new collaborators.
- *High market penetration and brand visibility:* Skype has high market penetration through the popular Skype

[5]For simplicity, we assume that a local provider i will still charge the same flat fee P_i^* in (15) after Skype's entry. In practice, a local provider may not be able to change the flat-rate price very often due to the reputation issue [5].

Call service. Skype can easily advertise Skype WiFi to many users, while many local providers have little brand visibility outside their local markets.

Even with these advantages, one may still wonder why Skype does not choose flat-fee pricing, as what the local providers are doing for local WiFi services. The key reason is that Skype needs to avoid severe competition with local providers in order to reach a win-win situation.

To make the discussion more concrete, let us first look at the users' choices. After Skype's entry, a user in his own local market can choose between the local WiFi service and Skype Wifi service. When the user travels to a different market, he will only choose Skype WiFi as he does not want to pay a monthly flat fee in a different market.

Now consider the possibility of Skype adopting the flat-rate pricing scheme for the global WiFi service. This can further include two variations: a *market-dependent* flat-rate pricing and a *market-independent* one. In the market-dependent scheme, a user needs to pay a separate flat-rate price for each market (either local or foreign) he might enter. In this way, Skype WiFi is just replicating many local services at a global scale. This leads to direct competition with local providers in each local market (e.g., all local users in a market will choose Skype WiFi if its flat-fee is lower than the local provider's charge). Furthermore, such scheme is not attractive for a user who travels in many markets, as more markets means more payment. In the market-independent scheme, a user subscribing to Skype WiFi only needs to pay a single flat fee to receive services in all markets. Then many users no longer need to use the local WiFi service. To summary, in each of the two cases, the local WiFi provider will suffer from Skype's flat-fee pricing, and will not have the incentive to cooperate. Hence, we successfully prove that why in practice Skype chooses the usage-based pricing scheme.

VI. SKYPE'S OPTIMAL USAGE-BASED PRICING SCHEME

Now we will analyze the optimal usage-based pricing scheme for Skype WiFi. We will consider the market-dependent usage-based pricing scheme (which is Skype's current practice), where a user pays different usage-based prices when he is in different markets.[6] Under such a scheme, we can model the interactions between Skype, a local provider i, and local users as well as travelers in market i as a two-stage dynamic game. In Stage I, Skype and provider i jointly decide the Skype WiFi usage-based price p_i^{Skype} and the revenue sharing portion η_i (as a compensation of using provider i's network infrastructure). In Stage II, each of the N_i local users chooses between Skype WiFi and the provider i's local service (and the usage level if choosing Skype WiFi), and travelers decide their usages of the Skype WiFi service. In the following, we use backward induction to examine Stage II first.

[6]The market-independent usage based pricing is a special case of the market-dependent one.

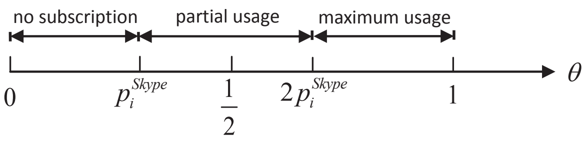

Fig. 5. Local users' service choices in market i in the low price regime

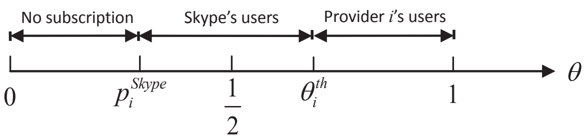

Fig. 6. Local users' service choices in market i in the medium price regime

A. Stage II: Local Users' and Travelers' choices

Consider a total of I markets in the global market. A type-θ local user in the local market i has two types of demands:

- *Demand in his local market:* he can choose Skype WiFi's usage-based price (with the optimal usage $d^{i*}(\theta, p_i^{Skype})$ as in (3)) or provider i's flat-rate price P_i^* in (15) (with the optimal maximum usage $d^{i*}(\theta, P_i^*) = 1$).
- *Demand when he travels in non-local markets:* he will only choose Skype WiFi's usage-based prices in other $I-1$ markets. The probability for this user traveling to a market j is $\alpha_i^j < 1$. By demanding a usage level $d^{j*}(\theta, p_j^{Skype})$ in market j as in (3), this user's aggregate payoff in all non-local markets is
$$\sum_{j \neq i} \alpha_i^j G_j(u(\theta, d^{j*}(\theta, p_j^{Skype})) - p_j^{Skype} d^{j*}(\theta, p_j^{Skype})).$$

Apparently, the user's usage in non-local markets (the second type of demand) does not affect his choice of service in the local market (the first type of demand). To study a local user's local service choice, we can simply compare his optimal local payoff with Skype WiFi,
$$v^{Skype} = G_i(\theta \ln(1+kd^{i*}(\theta, p_i^{Skype})) - p_i^{Skype} d^{i*}(\theta, p_i^{Skype}))$$
to the optimal payoff with local provider i,
$$v^i = G_i \theta \ln(1+k) - P_i^*.$$
In the following, we analyze the local users' equilibrium behaviors given any possible value of p_i^{Skype}.[7] To facilitate analysis in this section, we assume the elasticity parameter of demand $k = 1$ and utility $u(\theta, d) = \theta \ln(1+d)$.[8]

Proposition 3: At Stage II, local users' equilibrium decisions in market i depend on the Skype WiFi price p_i^{Skype} as follows:

- *Low price regime ($p_i^{Skype} \leq \ln(2)/2$):* no local users will choose provider i's local service. Users with types $\theta \in [p_i^{Skype}, 1]$ will choose Skype WiFi. Their equilibrium usage levels are illustrated in Fig. 5.
- *Medium price regime ($\ln(2)/2 < p_i^{Skype} < 1/2$):* both local provider i and Skype have positive numbers

[7]Note that the other decision η in Stage I does not affect users' decisions.
[8]Similar to Sections II, III and IV, our results here can also be extended to the case with any positive k value.

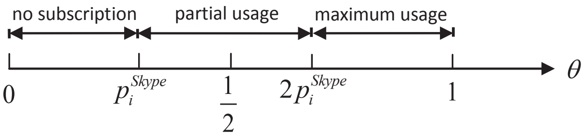

Fig. 7. Travelers' usage choices in market i in Skype WiFi in the medium price regime

  of subscribers. Figure 6 illustrates local users' service subscriptions, where there are three categories of users depending on their valuations: (i) a user with the type $\theta \in [0, p_i^{Skype})$ will not choose any service, (ii) a user users with the type $\theta \in [p_i^{Skype}, \theta_i^{th})$ will choose Skype WiFi, and (iii) a user with the type $\theta \in [\theta_i^{th}, 1]$ will choose local provider i's service. The threshold type θ_i^{th} is the unique solution to

$$\theta_i^{th} \ln\left(\frac{\theta_i^{th}}{p_i^{Skype}}\right) - \theta_i^{th} + p_i^{Skype} - \left(\theta_i^{th} - \frac{1}{2}\right)\ln(2) = 0, \quad (20)$$

  which satisfies $2p_i^{Skype} > \theta_i^{th}$ (i.e., $d^{i*}(\theta_i^{th}, p_i^{Skype}) < 1$), and θ_i^{th} is decreasing in p_i^{Skype}. Provider i's local service targets at high valuation users, whereas Skype WiFi targets at low valuation users and none of Skype WiFi subscribers request maximum usage level.
- *High price regime ($p_i^{Skype} > 1/2$):* no local users will choose Skype WiFi. Users with types $\theta \in [1/2, 1]$ choose Provider i's service as in Fig. 4.

The proof of Proposition 3 is given in our online technical report [8].

Proposition 3 shows that two services will coexist only in the medium price regime, when the two prices are comparable to each other. When p_i^{Skype} decreases in this regime, more local users will switch from the local provider i to Skype WiFi, resulting in a larger partition threshold θ_i^{th}. Moreover, θ_i^{th} only depends on p_i^{Skype} and is independent of WiFi coverage G_i, as both services are competing with each other using the same network infrastructure.

B. Stage I: Negotiation Between Skype and Local Provider

As the low and high price regimes in Theorem 3 will drive either local provider i or Skype out of the local market, they are not likely to be viable choices for the negotiation in Stage I. In fact, we can prove that the medium price regime is the only feasible choice for the equilibrium of the whole game.

Theorem 2: In Stage I, Skype and local provider i will only agree on a Skype WiFi price in the medium price regime, i.e., $\ln(2)/2 < p_i^{Skype} < 1/2$.

The proof of Theorem 2 is given in our online technical report [8]. Next we focus on the medium price regime and study the revenues for both Skype and Provider i.

First consider Skype, who can obtain revenue by serving local users and travelers in market i, but needs to share η_i portion to local provider i in using the local WiFi infrastructure.[9] By serving local users with types $\theta \in [p_i^{Skype}, \theta_i^{th})$, Skype collects a total payment

$$\Delta\pi_i^{Skype}(\eta_i, p_i^{Skype}, \theta_i^{th}) = (1-\eta_i) p_i^{Skype} N_i G_i \int_{p_i^{Skype}}^{\theta_i^{th}} d^{i*}(\theta, p_i^{Skype}) d\theta. \quad (22)$$

with $d^{i*}(\theta, p_i^{Skype}) = \theta / p_i^{Skype} - 1$. As for travelers in market i, they can be divided into three categories depending on their valuations (and independent of where they come from) as in Fig. 7: (i) travelers with types $\theta \in [0, p_i^{Skype})$ demand zero usage, (ii) travelers with types $\theta \in [p_i^{Skype}, 2p_i^{Skype})$ demand partial usage, and (iii) travelers with types $\theta \in [2p_i^{Skype}, 1]$ demand the maximum usage. Similar to (5), we can derive that the total payment collected by Skype from the travelers in market i as

$$\Delta\pi_{-i}^{Skype}(\eta_i, p_i^{Skype}, \theta_i^{th}) = (1-\eta_i) \sum_{\forall j \neq i} \alpha_j^i N_j G_i p_i^{Skype} \cdot \left(\int_{p_i^{Skype}}^{2p_i^{Skype}} \left(\frac{\theta}{p_i^{Skype}} - 1\right) d\theta + \int_{2p_i^{Skype}}^{1} 1 d\theta\right). \quad (23)$$

By summing up (22) and (23), Skype's revenue increase by cooperating with local provider i is

$$\Delta\pi^{Skype}(\eta_i, p_i^{Skype}, \theta_i^{th}) = (1-\eta_i) N_i G_i \frac{(\theta_i^{th} - p_i^{Skype})^2}{2} + (1-\eta_i) \sum_{\forall j \neq i} \alpha_j^i N_j G_i \left(p_i^{Skype} - \frac{3(p_i^{Skype})^2}{2}\right), \quad (24)$$

which is linearly decreasing in the revenue sharing portion η_i. Notice that with the practical use of coverage factor G_i for modeling market i's revenue, the revenue increment for Skype in one market can be decoupled with another.

Now we look at the revenue increase of local provider i through the cooperation. As Skype's entry will result in service competition, provider i will lose those users with types $\theta \in [1/2, \theta_i^{th}]$ to Skype WiFi. Compared with provider i's original revenue in (16) with $k = 1$, such competition reduces the revenue by

$$\Delta\pi_i^i(\eta_i, p_i^{Skype}, \theta_i^{th}) = -\frac{\ln(2)}{2} G_i N_i \left(\theta_i^{th} - \frac{1}{2}\right) < 0.$$

On the other hand, Skype will share part of his revenue with local provider i:

$$\Delta\pi_{-i}^i(\eta_i, p_i^{Skype}, \theta_i^{th}) = \frac{\eta_i}{1-\eta_i} \Delta\pi^{Skype}(\eta_i, p_i^{Skype}, \theta_i^{th}).$$

Thus local provider i's total revenue increase is

$$\Delta\pi^i(\eta_i, p_i^{Skype}, \theta_i^{th}) = -\frac{\ln(2)}{2} G_i N_i \left(\theta_i^{th} - \frac{1}{2}\right) + \eta_i G_i \cdot \left(N_i \frac{(\theta_i^{th} - p_i^{Skype})^2}{2} + \sum_{\forall j \neq i} \alpha_j^i N_j \left(p_i^{Skype} - \frac{3(p_i^{Skype})^2}{2}\right)\right), \quad (25)$$

which is linearly increasing in η_i.

[9] Note Skype WiFi's income in other markets is not related to local provider i, and can be treated as a constant normalized to 0 in the following analysis.

$$\eta_i^*(p_i^{Skype}) = \min\left(1, \frac{\frac{\ln(2)}{4}N_i\left(\theta_i^{th}(p_i^{Skype})-\frac{1}{2}\right)}{\sum_{\forall j\neq i}\alpha_j^i N_j\left(p_i^{Skype}-\frac{3(p_i^{Skype})^2}{2}\right)+N_i\frac{(\theta_i^{th}(p_i^{Skype})-p_i^{Skype})^2}{2}} + \max\left(\frac{\frac{\ln(2)}{4}N_i\left(\theta_i^{th}(p_i^{Skype})-\frac{1}{2}\right)}{\sum_{\forall j\neq i}\alpha_j^i N_j\left(p_i^{Skype}-\frac{3(p_i^{Skype})^2}{2}\right)+N_i\frac{(\theta_i^{th}(p_i^{Skype})-p_i^{Skype})^2}{2}}, \frac{1}{2}\right)\right), \tag{21}$$

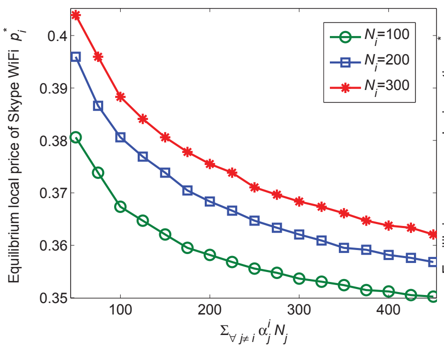

Fig. 8. Equilibrium price of Skype WiFi p_i^{Skype*} in market i

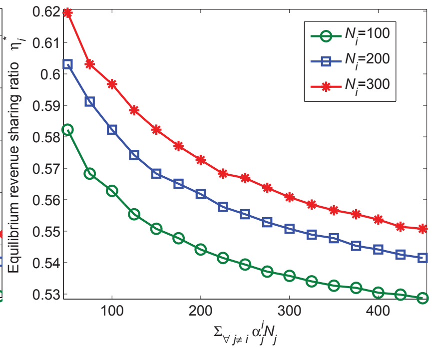

Fig. 9. Equilibrium revenue sharing portion η_i^* between Provider i and Skype in market i

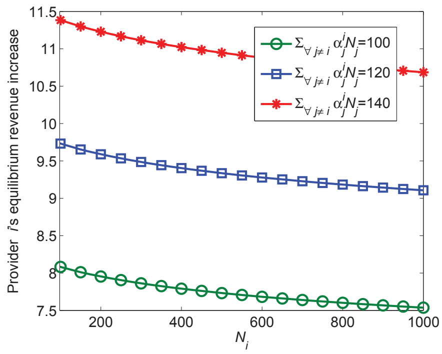

Fig. 10. Provider i's equilibrium revenue increase as a function of traveler population $\sum_{\forall j\neq i}\alpha_j^i N_j$ and local user population N_i.

Now we discuss how Skype bargains with local provider i on p_i^{Skype} and η_i based on (24) and (25). We will use the Nash bargaining framework to resolve this issue. According to [13], the Nash bargaining equilibrium is Pareto efficient, symmetric, and independent of irrelevant alternatives. It is the same as Zeuthen's solution of a general bargaining problem where two players could bargain for infinite rounds. In our problem, the Nash bargaining leads to the following joint optimization problem of the revenue increase product,[10]

$$\max_{\eta_i, p_i^{Skype}, \theta_i^{th}} \quad \Delta\pi^{Skype}(\eta_i, p_i^{Skype}, \theta_i^{th})\Delta\pi^i(\eta_i, p_i^{Skype}, \theta_i^{th})$$

$$\text{subject to,} \qquad 0 \leq \eta \leq 1,$$

$$\frac{\ln(2)}{2} \leq p_i^{Skype} \leq \frac{1}{2},$$

$$\theta_i^{th}\ln\left(\frac{\theta_i^{th}}{p_i^{Skype}}\right) - \theta_i^{th} + p_i^{Skype} - \left(\theta_i^{th}-\frac{1}{2}\right)\ln(2) = 0, \tag{26}$$

where the last constraint comes from (20). Notice that θ_i^{th} only depends on p_i^{Skype}, and thus we can express it as $\theta_i^{th}(p_i^{Skype})$. This means that we need to solve the remaining two variables η_i and p_i^{Skype} in Problem (26). We can take a sequential optimization approach: first optimize over η_i given a fixed p_i^{Skype}, and then optimize over p_i^{Skype}. We can show that the objective function of Problem (26) is strictly concave in η_i, which leads to the following result by examining the first order condition.

[10]We can add different weights to each term in the product to reflect different market powers of Skype and provider i, but this will not change the key insights of this paper. More discussions are presented in our online technical report [8].

Proposition 4: At the equilibrium, Skype will share the *majority* of his revenue in market i with local provider i, i.e., $\eta_i^* > 1/2$. More specifically, given any feasible price $\ln(2)/2 < p_i^{Skype} < 1/2$, the unique optimal $\eta_i^*(p_i^{Skype})$ is given in (21), which increases in the local user population N_i and decreases in the traveler population $\sum_{\forall j\neq i}\alpha_j^i N_j$ in market i.

Notice that Skype's entry brings in competition and reduces provider i's original revenue, but such cooperation only brings benefit to Skype. To incentivize provider i to share its network infrastructure, Skype needs to share more than half of his revenue with Provider i. As the local user population N_i increases, Skype WiFi will take away more local users from provider i, and provider i needs to share a larger portion of the revenue as compensation. As more travelers coming, the importance of local market decreases, and hence the Skype can keep a higher percentage of the revenue (but still less than half).

With (21), we can simplify Problem (26) into the following one variable optimization problem:

$$\max_{p_i^{Skype}} \Delta\pi^{Skype}\left(\eta_i^*(p_i^{Skype}), p_i^{Skype}, \theta_i^{th}(p_i^{Skype})\right)$$

$$\cdot\,\Delta\pi^i\left(\eta_i^*(p_i^{Skype}), p_i^{Skype}, \theta_i^{th}(p_i^{Skype})\right)$$

$$\text{subject to,} \qquad \frac{\ln(2)}{2} \leq p_i^{Skype} \leq \frac{1}{2}, \tag{27}$$

where $\eta_i^*(p_i^{Skype})$ is given in (21) and $\theta_i^{th}(p_i^{Skype})$ (though not in closed-form) can be derived from (20). We can check that the objective of Problem (27) may not be concave in p_i^{Skype} and Problem (27) is not a convex optimization problem. Despite this, we can still use an efficient one-dimensional exhaustive search algorithm to find the global optimal solution p_i^{Skype*} ([25]). Next we provide comprehensive numerical results by solving Problem (27).

Observation 1: At the equilibrium of market i, both the Skype WiFi price p_i^{Skype*} and revenue sharing portion η_i^* between Skype and provider i are independent of the local WiFi coverage G_i. As the local user population N_i decreases or the traveler population $\sum_{\forall j \neq i} \alpha_j^i N_j$ increases, both p_i^{Skype*} and η_i^* will decrease (see Figs. 8 and 9).

As both provider i's and Skype's revenue increases in Problem (27) are linear in coverage G_i, the solutions p_i^{Skype*} and η_i^* are independent of G_i. More interestingly, note that Skype is the monopolist in serving travelers, whereas both Skype and provider i are competing in serving local users. Compared to the monopoly usage-based price $1/3$ in (10) with $k = 1$, the price of Skype WiFi p_i^{Skype*} needs to be higher than $1/3$ to avoid severe price competition with provider i's local flat-rate pricing service. When the traveler population $\sum_{j \neq i} \alpha_j^i N_j$ increases or local user population N_i decreases, Skype is gaining a market power approaching a monopolist in serving the whole market, and it is efficient for Skype to lower the price (and eventually approach the monopoly benchmark of $1/3$). Meanwhile, local provider i's loss of revenue due to Skype's competition is smaller, and Skype only needs to share a smaller portion η_i^* with provider i.

Observation 2: The equilibrium revenue increases of both provider i and Skype are increasing in $\sum_{j \neq i} \alpha_j^i N_j$ and G_i, but are decreasing in N_i (see Fig. 10).

Intuitively, a larger coverage G_i improves the quality of both two services, and a larger $\sum_{\neq i} \alpha_j^i N_j$ provides a larger cooperation benefit between Skype and local provider i. However, a larger population N_i increases the competition between Skype and the local provider and thus reduces the cooperation benefit.

VII. Conclusion

This paper studies how local and global WiFi providers should choose and optimize their pricing schemes. We first study the business operation of a local WiFi market, where the local provider can choose from two pricing schemes: flat-rate pricing and usage-based pricing. Given the abundant WiFi capacity in today's markets, we show that flat-rate pricing can lead to higher revenue for a local provider than usage-based pricing. This well explains why flat-fee pricing is currently the mainstream in local WiFi markets. Then we further study how a global WiFi provider (e.g., Skype) cooperates with many local providers in using their WiFi infrastructures to provide a global WiFi service. We explain why Skype adopts usage-based pricing, as it wants to avoid severe competition with the local providers and also attract travelers to the service. We show that at the equilibrium, Skype needs to share the majority of his revenue to compensate local providers' revenue loss and thus realize a win-win situation.

There are some possible directions to extend the results in this paper. First, we may consider oligopolistic instead of monopolistic service provider in a local market, where different providers have different WiFi coverages in their QoS and they have incentive to lower prices for a larger market share. Intuitively, it is easier for Skype to enter such a market under competition and he may have different strategies in negotiating with these providers. Users' choice model would be more complex given more options. Moreover, we can study the sequential negotiation between Skype and several local providers who decide whether to join Skype's global WiFi network, taking into consideration the network effect.

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