Choice-based Pricing for User-Provided Connectivity*

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ABSTRACT

User-provided connectivity (UPC) leverages the network connectivity of its users to build a service offering that goes beyond their individual connectivity option, *i.e.,* allows them to roam. Because the service's overall value typically grows as a function of its coverage, it is important to devise pricing policies that make it attractive to all users, even those who derive little value from roaming. This paper builds on earlier work that explored the value of a UPC service, and proposes a new pricing policy, the *Price choice policy,* that seeks to realize an effective compromise between pricing complexity and the policy's ability to maximize system value and extract profit. The paper illustrates the benefits of the proposed policy by demonstrating why and how it outperforms several previously proposed policies.

1. INTRODUCTION

With the growing popularity of mobile connectivity, traditional infrastructure-based connectivity services have become increasingly strained [9]. A direct solution involves expanding the infrastructure capacity to keep pace with the increasing demand. The WiFi offload solutions that *User-provided connectivity* (UPC) services embody offer a more scalable and less expensive alternative. In a UPC service, a user agrees to make a fraction of her home base broadband access bandwidth available to other UPC users, in exchange for reciprocation while herself roaming. FON (`http://www.fon.com`) was arguably the first to demonstrate and popularize the benefits of such an approach, with a number of similar offerings following suit, *e.g.,* AnyFi (`anyfinetworks.com`) and more recently Comcast [8].

The "sharing" nature of UPC means that its coverage and the capacity it contributes, two key metrics of its value, increase as more users join the service, *i.e.,* the service exhibits a positive externality [4, p.71]. Conversely, as its user population grows so do the odds of having to share one's home access, *i.e.,* a negative externality. This combination of positive and negative externalities can make predicting the viability of a UPC offering and how to best price it challenging [5, 1].

We explored this question in [1] for a number of common pricing policies, including a fixed price policy similar to that of FON, and a usage-based policy that while more complex displayed a number of desirable properties (the ability to maximize both welfare and profit). The investigation of [1] revealed that while a UPC service could generate significant value, fully realizing it called for relatively complex pricing. Furthermore, successful outcomes often required the ability to subsidize a subset of users, which could in turn render pricing even more complex. In this work, we build on these earlier findings and focus on evaluating a new pricing policy, termed *Price choice policy*, which seeks to remedy some of the problems identified in [1].

The rest of the paper is structured as follow. Section 2 briefly introduces the model used to investigate UPC service adoption in [1]. Section 3 derives conditions under which successful service deployment calls for subsidies. Section 4 first reviews the fixed-price and usage-based price policies and their respective properties, which then motivates the introduction of the price choice policy. The benefits of this policy are then evaluated in Section 5, where it is compared to the fixed-price and usage-based policies.

2. SERVICE ADOPTION

In this Section we briefly introduce the basic UPC model of [1]. Section 2.1 presents the users' utility and service adoption criteria, and Section 2.2 gives the total value a UPC service generates.

2.1 User Utility

Users are assumed to adopt the UPC service if they derive positive utility from it [3]. Eq. (1) offers a simple yet representative expression[1] for the utility of a UPC service.

$$U(P,\,\Theta,\,\theta) = \gamma\,(1-\theta) + \theta\,r\,x - c\,m - p(\Theta,\,\theta)\,. \qquad (1)$$

The main feature of $U(P,\Theta,\theta)$ is in differentiating users based on their propensity to "roam." Specifically, each user is assigned a private value $\theta \in [0,1]$ that identifies how often she can be expected to benefit from using another user's broadband access. A sedentary user that never roams has a value of $\theta = 0$, while a user with a roaming value of $\theta = 1$ always seeks to connect through other users.

$U(P,\Theta,\theta)$ has two major parts. The first,

$$U_i(\Theta,\,\theta) = \gamma\,(1-\theta) + \theta\,r\,x - c\,m\,,$$

is the value or *intrinsic* utility the user derives from the service. The first term in $U_i(\Theta,\theta)$, $\gamma(1-\theta)$, corresponds to

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[1]A number of generalizations were tested in [1], and found to exhibit qualitatively similar behaviors.

the value of home connectivity, γ, while the user is at home; a fraction of time $(1-\theta)$. The second term, $r\theta x$, is the value of connectivity while the user is roaming, where $r \geq \gamma$ denotes the value of roaming connectivity, θ is the fraction of time the user spends roaming, and x corresponds to the odds of roaming within the UPC's service coverage area[2]. Finally, the last term $-cm$ reflects the (negative) impact of the roaming traffic, m.

The second part, $p(\Theta, \theta)$, in $U(P, \Theta, \theta)$ denotes the price a user with roaming value θ pays for the service under pricing policy P and given a set of adopters $\Theta \subseteq [0,1]$. A user adopts if her intrinsic utility $U_i(\Theta, \theta)$ exceeds the price $p(\Theta, \theta)$ she is being charged for the service. We discuss the impact of pricing in Section 4.

2.2 Overall System Value

The overall value created by deploying a UPC service can be obtained by summing the intrinsic utilities of all users, and subtracting the deployment cost e of the service, *i.e.,*

$$V(\Theta) = \int_{\theta\in\Theta} \big(U_i(\Theta,\, \theta) - e\big)\, d\theta, \tag{2}$$

where e is the per-user service implementation cost.

We showed in [1] that under the adoption model of Eq. (1), the total system value is generally[3] maximized at full adoption, *i.e.,* when all users adopt the service. An important goal is, therefore, to devise pricing policies that can achieve full adoption, while also enabling the service provider to recoup most of the service value as profit. As we shall see in Section 3, this may require subsidizing a subset of users to achieve full adoption, and therefore maximum service value.

3. THE NEED FOR SUBSIDIES

Eq. (1) readily shows that full adoption, *i.e.,* $\Theta = [0,1]$, implies[4] $p([0,1], \theta) < U_i([0,1], \theta), \forall \theta \in [0,1]$. An immediate consequence of this observation is that subsidies are needed, *i.e.,* $p([0,1], \theta) < 0$, whenever $U_i([0,1],\, \theta) \leq 0$ for some θ value. The next proposition characterizes when this arises and the users to which it applies.

PROPOSITION 1. *Users requiring subsidies at full adoption have roaming characteristics that satisfy*

$$\theta < \theta^* \triangleq \frac{cm^* - \gamma}{r - \gamma}\,, \tag{3}$$

where m^* is the volume of roaming traffic m at full adoption.

PROOF. The proof is immediate from Eq. (1) with $x = 1$ (full adoption) and $m = m^*$ (note that $r - \gamma > 0$). □

We note that from Eq. (3), the set of users requiring subsidies, $\Theta_S = [0, \theta^*]$, is empty only if $\theta^* < 0 \Leftrightarrow cm^* < \gamma$. This is intuitive since $cm^* < \gamma$ implies that the impact of roaming traffic at full adoption is less than the utility of basic home connectivity. However, when this condition is not satisfied, subsidies are required to offset the impact of roaming traffic for users that derive little or no benefits from the service's roaming feature, *i.e.,* users with low θ values.

We also note that the distribution of θ can affect the number of users requiring subsidies, $|\Theta_S|$, in subtle ways. In particular, increasing (decreasing) the roaming propensity of some users, *e.g.,* by creating a mode near $\theta = 1$ ($\theta = 0$), will on one hand decrease (increase) the number of users below θ^*, but on the other hand increase (decrease) the roaming traffic[5] m^* and in the process increase (decrease) θ^*.

[2]Service coverage is assumed to grow with adoption, and the analysis of [1] further assumed $x = |\Theta|$.

[3]That is, when the per-user service implementation cost, e, is *not* too high. See [1] for details.

[4]Note that this is a *necessary* condition for full adoption, and may not be *sufficient*. In particular, it does not guarantee that the system *reaches* full adoption, which depends on adoption dynamics (see [1] for details).

4. SERVICE PRICING

Pricing for the service is specified through a pricing policy, P, that determines the service cost, $p(\Theta, \theta)$, for each user. The pricing policy P is a control knob a service provider can use to affect users' adoption decisions and control profit. A sophisticated pricing policy may allow the provider to both maximize overall service value and recoup more of that value as profit. Such a sophisticated policy is, however, likely to be complex, which may translate into a higher cost and in the process affect profit.

To better assess the impact of a pricing policy cost, it is useful to split the service implementation cost e of Eq. (2) in two components $e = \widehat{e} + \widetilde{e}$, where $\widehat{e}$ is the *basic* deployment cost of the service, *e.g.,* network equipment and operation, while $\widetilde{e}$ is dedicated to the *billing* costs that depend on the pricing policy P. Different pricing policies will have different $\widetilde{e}$ values.

4.1 Fixed price policy

The simplest form of pricing is a fixed price policy, where all users are charged the same fixed price p irrespective of their usage. This is the pricing policy used by FON.

As shown in [1], when used to maximize profit, the fixed price policy often falls short of realizing full adoption, in addition to yielding a much lower profit than feasible (because it does not differentiate between users). On the other hand, its predictability of price may lead to higher user satisfaction [7]. Moreover, its billing cost $\widetilde{e}_f$ is low, which contributes to increasing profit.

4.2 Usage-based policy

A usage-based pricing policy was also investigated in [1]. It is of the form

$$p(\Theta,\, \theta) = p_h(1-\theta) + p_r\, \theta x - a, \tag{4}$$

where p_h and p_r are unit prices for home and roaming usage, respectively, with $1-\theta$ the typical home usage and θx the typical roaming usage for user θ. The term a denotes a usage allowance that can be used to tailor the policy to specific goals. The usage-based pricing policy allows a form of discriminatory pricing [2], *i.e.,* charging different users differently, and was shown in [1] to be able to jointly maximize adoption and profit. In particular, setting $p_h = \gamma$, $p_r = r$, and $a = cm^* + \epsilon$, $\epsilon > 0$, results in all users having the same utility $\epsilon > 0$. This ensures full adoption and allows all the service value to be realized as profit (for $\epsilon \approx 0$).

This usage-based pricing policy has, however, two major disadvantages. The first is its complexity that contributes to a high billing cost, *i.e.,* $\widetilde{e_u} \gg \widetilde{e_f}$, which could lower overall profit in spite of the policy's ability to extract all ser-

[5]$m^* = \int_{\theta=0}^{1} \theta\, f(\theta)\, d\theta$, where $f(\theta)$, $\theta \in [0,1]$ denotes the probability density of θ.

vice value as profit. The second important disadvantage is that when Eq. (3) yields a non-empty set of users requiring subsidies, the usage-based pricing calls for "cashback" payments to those users, *i.e.,* those whose allowance a exceeds their (home & roaming) usage. Having to handle such cases further increases the service's billing complexity and cost, and could also negatively affect users' usage patterns, *i.e.,* by creating an incentive to consume less. Hence, although some service providers have recently announced such a service option [6], it remains highly unusual. The more common version of the usage-based policy is, therefore, one with "no cashbacks," which will however be unable to realize full service adoption when subsidies are needed.

4.3 Price choice policy

In this section we introduce a pricing policy, the *price choice* policy, aimed at the shortcomings of the fixed price and usage-based policies. The policy seeks to, on one hand, improve on the fixed price policy when it comes to realizing profit. On the other hand, it targets a lower billing complexity than the usage-based policy, as well as a subsidy mechanism that avoids direct cashbacks and instead gives low θ users the ability to offset the impact of roaming traffic.

The price choice policy gives all users the option to choose between two pricing schemes. The two options for pricing under this policy are:

$$P: \begin{cases} p_1(\Theta, \theta) = p_h, & \text{(option 1)} \\ p_2(\Theta, \theta) = p_h + p_r \cdot \theta x - bm, & \text{(option 2)} \end{cases} \tag{5}$$

where, as before, the adoption level x and volume of roaming traffic m are functions of adoption set Θ. Users pick the option that yields the lowest price for them.

In option 1 roaming usage is free (as with FON), while in option 2 roaming usage (as measured by θx) is charged at a unit price of p_r. In return, option 2 compensates users for the roaming traffic (m) their home access carries[6], at a rate of b per unit of traffic. We assume that users have knowledge of what their own roaming is, and also see the roaming traffic at their home bases and use that to reevaluate their decisions about which price option to choose.

Option 2 offers a mechanism to subsidize users that see little value in the roaming feature of the service, and are negatively affected by having to share their home access with other (roaming) users. In addition, the subsidy is directly related to the "service" rendered by those users (the amount of roaming traffic they carry) rather than in the form of a cashback for unused usage.

Both price options boast a flat price p_h for home usage. This, along with the availability of options, may improve users' satisfaction with the service [7]. The price choice policy is also expected to reduce billing costs $\widetilde{e}_p$ compared to the usage-based policy, as the provider no longer needs to track users' home usage. Roaming usage still needs to be tracked, but only for users who choose pricing option 2.

The lower complexity and deployment cost of the price choice policy, however, means that this policy cannot convert all of the system value to profit (it suffers from the same limitation as the fixed price policy for users that choose pricing option 1). This happens mostly when γ is small, as the provider is then forced to use a small home price p_h to attract users. The next proposition offers a lower bound on the amount of system value the price choice policy is *unable* to convert to profit.

[6]As in [1], for simplicity roaming traffic is assumed uniformly distributed across users.

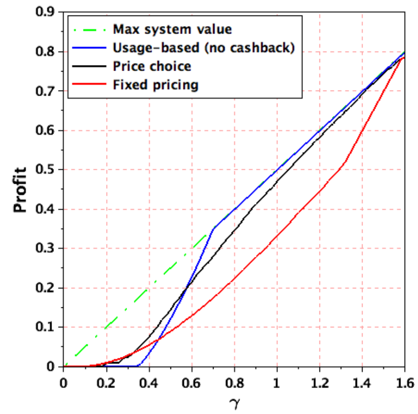


Figure 1: Maximum feasible *basic profit* under different pricing policies compared to maximum system value. Basic cost is $\widehat{e} = 0.1$, but billing costs $\widetilde{e}$ are not yet considered.

PROPOSITION 2. *Assume γ is strictly less than $r - cm^*$, such that $r - cm^* - \gamma = \kappa > 0$, where, as before, m^* denotes the volume of roaming traffic at full adoption. Then the amount $\widetilde{v}$ of system value that the price choice policy cannot convert to profit is lower-bounded by*

$$\widetilde{v} > \frac{\kappa^2}{2(r-\gamma)},$$

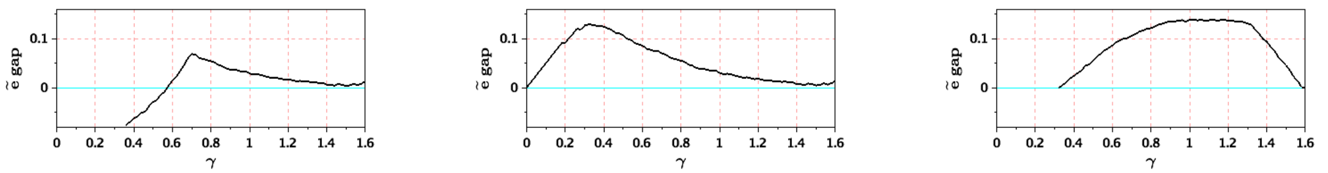
PROOF. Consider the service when adoption is zero, *i.e.,* $x = 0$ and $m = 0$. Users' utility under either pricing option is given by $U(P, \{\}, \theta) = \gamma(1-\theta) - p_h$. In order for anyone to adopt the service, it is then necessary that $p_h < \gamma$. Using this condition in the utility and price functions at full adoption, gives the result after some algebraic manipulation. □

The inevitable profit drop of Proposition 2 notwithstanding, the price choice policy may still outperform (profit-wise) the usage-based pricing policy because of its lower cost. As we shall see, it may also strictly outperform the usage-based policy, irrespective of cost, when that policy operates in the "no-cashback" mode. Finally, it typically also outperforms the fixed-price policy because of its ability to achieve a higher service value. We illustrate those claims in the next section using a number of service configurations.

5. COMPARISONS AND VALIDATIONS

This section numerically compares the maximum profit under the three different pricing policies of Section 4, which provides insight into the regimes where each policy is more profitable than others and validates the results of the previous sections. In the figures, we take the system parameters to be $r = 1.6$ and $\widehat{e} = 0.1$ (as γ varies), and assuming that θ has a uniform distribution, we obtain $m^* = 0.5$. These are representative values and perturbing them does not change the overall nature of the results. Also in order to highlight the effect of subsidies as described in Proposition 1, we use a relatively large c, taken to be $c = 1.4$.

Fig. 1 plots, as a function of γ, the maximum feasible *basic profit* (provider's profit after subtracting the basic costs $\widehat{e}$



(a) Between usage-based *without* subsidies and price choice policies ($\widetilde{e}_{u2} - \widetilde{e}_p$)

(b) Between usage-based *with* subsidies and price choice policies ($\widetilde{e}_{u1} - \widetilde{e}_p$)

(c) Between price choice and fixed price policies ($\widetilde{e}_p - \widetilde{e}_f$)

Figure 2: The threshold $\widetilde{e}$ gap between a high-cost and a low-cost pricing policy, Parameters are $r = 1.6$, $c = 1.4$, and $\widehat{e} = 0.1$.

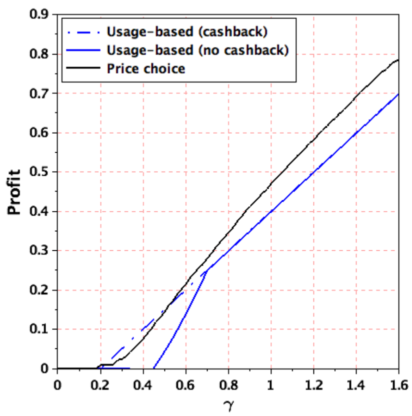


Figure 3: Maximum feasible *net profit* under price choice and usage-based policies when $\widehat{e} = 0.1$, and $\widetilde{e}_u - \widetilde{e}_p = 0.1$

but before subtracting the billing costs $\widetilde{e}$) under the three different pricing policies of Section 4, and compares them to the maximum system value (maximum system value is an upper-bound on the feasible basic profit under any policy). We showed in [1] that the usage-based pricing policy with subsidies (not plotted) is able to achieve a basic profit equal to the system value across all γ values. However, as Fig. 1 shows, the usage-based pricing policy without subsidies fails to achieve such a high basic profit when γ is small (more precisely, when $|\Theta_S|$ as described by Eq. (3) is non-empty.)

For small values of γ, the fixed price policy yields a basic profit higher than that of the usage-based policy without subsidies. The reason is, in this regime, a fixed-price policy only attracts a limited number of sedentary users who generate minimal roaming traffic, hence reducing the need for subsidies. However, the fixed price policy yields inferior basic profits for a wide range of larger γ values as it cannot differentiate between users to recoup all of the system value.

On the other hand, the price choice policy improves the basic profit in comparison to the usage-based policy without subsidies for small γ values, and also remains competitive for larger γ. This is because as γ increases towards $r = 1.6$, the provider may increase the flat home usage price p_h of the price choice policy without hindering adoption.

The basic profits plotted in Fig. 1, however, do not account for the differences in each policy's billing cost. As discussed before, different pricing policies have different billing costs $\widetilde{e}$ and this will impact how the optimal *net profit* under different policies compare to each other. For instance, assume that the billing cost $\widetilde{e}_u$ for the usage-based policy is larger than the billing cost $\widetilde{e}_p$ for the price choice policy, by a margin of 0.1, *i.e.,* $\widetilde{e}_u - \widetilde{e}_p = 0.1$. Then the corresponding plots for their maximum net profit is as shown in Fig. 3. We see that with this value of "$\widetilde{e}$ *gap*", the price choice policy always outperforms the usage-based policy without subsidies, and even outperforms the usage-based policy with subsidies for most γ values.

We can quantify the threshold value of $\widetilde{e}$ gap between any two pricing policies. The threshold gives the maximum difference between the billing costs of a high-$\widetilde{e}$ policy and a low-$\widetilde{e}$ policy, such that the high-$\widetilde{e}$ policy still generates more net profit. Fig. 2 shows this threshold value for three different pairs of policies. For instance. Fig. 2b shows the threshold $\widetilde{e}$ gap between the usage-based policy with subsidies, and the price choice policy, *i.e.,* the threshold for $\widetilde{e}_{u1} - \widetilde{e}_p$. From the figure we see that the threshold gap is larger than 0.1 only for $0.2 < \gamma < 0.5$, and that is also the interval in Fig. 3 where the usage-based policy with subsidies outperforms the price choice policy (recall that in Fig. 3 we have $\widetilde{e}_{u1} - \widetilde{e}_p = 0.1$).

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