# Possibility Distribution Semantics for Probabilistic Programs with Nondeterminism

Joshua Peignier

15th May 2017 - 11th August 2017



### Plan

- 1 Introduction
- 2 Problem
- 3 The previous semantics
- 4 Our semantics
- 5 Conclusion

What are probabilistic programs?

What are probabilistic programs?

Programs including probabilistic choice

What are probabilistic programs?

- Programs including probabilistic choice
- For each input, returns a distribution of outputs

What are probabilistic programs?

- Programs including probabilistic choice
- For each input, returns a distribution of outputs
- Widely used to solve problems with a better average complexity

What are probabilistic programs?

- Programs including probabilistic choice
- For each input, returns a distribution of outputs
- Widely used to solve problems with a better average complexity

#### Example

$${x := -1}[\frac{1}{3}]{x := 1}$$

Simulates the flipping of a biased coin.



What are nondeterministic programs?

What are nondeterministic programs?

■ Programs including nondeterministic choice

What are nondeterministic programs?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes

What are nondeterministic programs?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes
- Represents the unpredictable behavior of a program.

What are nondeterministic programs?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes
- Represents the unpredictable behavior of a program.

$${x := -1} \square {x := 1}$$



What are nondeterministic programs?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes
- Represents the unpredictable behavior of a program.

#### Example

$${x := -1} \square {x := 1}$$

Not the same as probabilistic choice: no probabilities here.



Problem

#### Problem

Defining proper semantics for programs containing both

#### Problem

Defining proper semantics for programs containing both

 $\rightarrow \, \mathsf{Complicated}$ 

#### Problem

Defining proper semantics for programs containing both

 $\rightarrow \, \mathsf{Complicated}$ 

#### Problem

Defining proper semantics for programs containing both

 $\rightarrow \, \mathsf{Complicated}$ 

$$P_0$$
:  $\{\{x := 2\} | \{x := 5\}\} [p] \{x := 7\}$ 

#### Problem

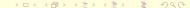
Defining proper semantics for programs containing both

 $\rightarrow \ \mathsf{Complicated}$ 

#### Example

$$P_0$$
: {{ $x := 2$ } $\square$ { $x := 5$ }}[ $p$ ]{ $x := 7$ }

How can we semantically describe this program ?



### Plan

- 1 Introduction
- 2 Problem
- 3 The previous semantics
- 4 Our semantics
- 5 Conclusion

■ For non-deterministic programs: weakest precondition calculus.

- For non-deterministic programs: weakest precondition calculus.
- Dijkstra: "programs are viewed as predicate transformers"

- For non-deterministic programs: weakest precondition calculus.
- Dijkstra: "programs are viewed as predicate transformers"
- lacksquare  $\varphi$  postcondition, P program

- For non-deterministic programs: weakest precondition calculus.
- Dijkstra: "programs are viewed as predicate transformers"
- lacksquare  $\varphi$  postcondition, P program
  - $\rightarrow$  wp[P]( $\varphi$ ): weakest precondition

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$\varphi = [x \ge 5]$$

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$\varphi = [x \ge 5]$$

$$wp[P_1](\varphi)$$

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$\varphi = [x \ge 5]$$

$$wp[P_1](\varphi) = wp[x := -y; x := x + 1]([x \ge 5])$$

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$\varphi = [x \ge 5]$$

$$wp[P_1](\varphi) = wp[x := -y; x := x + 1]([x \ge 5])$$
  
=  $wp[x := -y]([x + 1 \ge 5])$ 

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$\varphi = [x \ge 5]$$

$$wp[P_1](\varphi) = wp[x := -y; x := x + 1]([x \ge 5])$$

$$= wp[x := -y]([x + 1 \ge 5])$$

$$= wp[x := -y]([x \ge 4])$$

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$\varphi = [x \ge 5]$$

$$wp[P_1](\varphi) = wp[x := -y; x := x + 1]([x \ge 5])$$

$$= wp[x := -y]([x + 1 \ge 5])$$

$$= wp[x := -y]([x \ge 4])$$

$$= [-y \ge 4]$$

$$P_1$$
:  $x := -y$ ;  $x := x + 1$ 

$$\varphi = [x \ge 5]$$

$$wp[P_1](\varphi) = wp[x := -y; x := x + 1]([x \ge 5])$$

$$= wp[x := -y]([x + 1 \ge 5])$$

$$= wp[x := -y]([x \ge 4])$$

$$= [-y \ge 4]$$

$$= [y < -4]$$

### test

test



### test

test



#### test

test

