Possibility Distribution Semantics for Probabilistic Programs with Nondeterminism

Joshua Peignier

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Plan

- 1 Introduction
- 2 Problem
- 3 The previous semantics
- 4 Our semantics
- 5 Conclusion

What are probabilistic programs?

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Programs including probabilistic choice

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Example

$${x := -1}[\frac{1}{3}]{x := 1}$$

Simulates the flipping of a biased coin.



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$${x := -1} \square {x := 1}$$

Not the same as probabilistic choice: no probabilities here.



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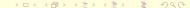
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How can we semantically describe this program ?



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- Dijkstra: "programs are viewed as predicate transformers"
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 - \rightarrow wp[P](φ): weakest precondition

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$$= [y < -4]$$

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 - → Conventions must be chosen

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Possibility-based semantics

■ Idea: rather use possibility based semantics



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- Possibility measure: $\Pi(U \cup V) = \text{Max}(\Pi(U), \Pi(V))$

test

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- Future work: find a complete partial order → Then, our semantics will be well-defined

