Possibility Distribution Semantics for Probabilistic Programs with Nondeterminism

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Plan

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2 Problem

3 The previous semantics

4 Our semantics

5 Conclusion

Probabilistic programs

What are probabilistic programs ?

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Example

$$\{x := -1\}[\frac{1}{3}]\{x := 1\}$$

Simulates the flipping of a biased coin.

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$$\{x := -1\}\Box\{x := 1\}$$

Not the same as probabilistic choice: no probabilities here.

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$$P_0\text{: } \{\{x := 2\}\square\{x := 5\}\}[p]\{x := 7\}$$

How can we semantically describe this program ?

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Semantics for nondeterministic programs

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- Dijkstra: "programs are viewed as predicate transformers"
- φ postcondition, P program
  $\rightarrow$ $wp[P](\varphi)$: *weakest precondition*

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Example

$$P_1\colon\ x := -y; x := x + 1$$

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$$wp[P_1](\varphi) = wp[x := -y; x := x + 1]([x \geq 5])$$

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$$P_1\colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

$$\begin{aligned} wp[P_1](\varphi) &= wp[x := -y; x := x + 1]([x \geq 5]) \\ &= wp[x := -y]([x + 1 \geq 5]) \end{aligned}$$

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- Problem: not possible to assign probabilities to nondeterministic choices
  $\rightarrow$ Conventions must be chosen

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wp[P_0]([x = 7]) &= 1 - p\\
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wp[P_0]([x = 5]) &= 0\\
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\end{aligned}$$

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Possibility-based semantics

- Idea: rather use possibility based semantics

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- Possibility measure: $\Pi(U \cup V) = \mathrm{Max}(\Pi(U), \Pi(V))$

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Conclusion

- Original problem: existing semantics for probabilistic nondeterministic programs
  → Imprecise, miss information
- Our idea: correct this lack of information with possibility theorey
- New rules, our semantics describes the behavior of some programs better than the existing semantics
- Main problem: not proven to be well-defined for program with rules
- Future work: find a complete partial order → Then, our semantics will be well-defined