Possibility Distribution Semantics for Probabilistic Programs with Nondeterminism

Joshua Peignier

15th May 2017 - 11th August 2017

Plan

Probabilistic programs

What are probabilistic programs ?

Probabilistic programs

What are probabilistic programs ?

- Programs including probabilistic choice

Probabilistic programs

What are probabilistic programs ?

- Programs including probabilistic choice
- For each input, returns a distribution of outputs

Probabilistic programs

What are probabilistic programs ?

- Programs including probabilistic choice
- For each input, returns a distribution of outputs
- Widely used to solve problems with a better average complexity

Probabilistic programs

What are probabilistic programs ?

- Programs including probabilistic choice
- For each input, returns a distribution of outputs
- Widely used to solve problems with a better average complexity

Example

$$\{x := -1\}[\frac{1}{3}]\{x := 1\}$$

Simulates the flipping of a biased coin.

Nondeterministic programs

What are nondeterministic programs ?

Nondeterministic programs

What are nondeterministic programs ?

- Programs including nondeterministic choice

Nondeterministic programs

What are nondeterministic programs ?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes

Nondeterministic programs

What are nondeterministic programs ?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes
- Represents the unpredictable behavior of a program.

Nondeterministic programs

What are nondeterministic programs ?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes
- Represents the unpredictable behavior of a program.

Example

$$\{x := -1\} \square \{x := 1\}$$

Nondeterministic programs

What are nondeterministic programs ?

- Programs including nondeterministic choice
- Not a concept meant to be executed: only exists for model checking purposes
- Represents the unpredictable behavior of a program.

Example

$$\{x := -1\} \square \{x := 1\}$$

Not the same as probabilistic choice: no probabilities here.

Problem

Problem

Problem

Problem

Defining proper semantics for programs containing both

Problem

Problem

Defining proper semantics for programs containing both
$\rightarrow$ Complicated

Problem

Problem

Defining proper semantics for programs containing both
$\rightarrow$ Complicated

Example

Problem

Problem

Defining proper semantics for programs containing both
$\rightarrow$ Complicated

Example

$$P_0\colon\ \{\{x := 2\} \square \{x := 5\}\}[p]\{x := 7\}$$

Problem

Problem

Defining proper semantics for programs containing both
$\rightarrow$ Complicated

Example

$$P_0\colon\ \{\{x := 2\}\square\{x := 5\}\}[p]\{x := 7\}$$

How can we semantically describe this program ?

Plan

Semantics for nondeterministic programs

- For non-deterministic programs: weakest precondition calculus.

Semantics for nondeterministic programs

- For non-deterministic programs: weakest precondition calculus.
- Dijkstra: "programs are viewed as predicate transformers"

Semantics for nondeterministic programs

- For non-deterministic programs: weakest precondition calculus.
- Dijkstra: "programs are viewed as predicate transformers"
- φ postcondition, P program

Semantics for nondeterministic programs

- For non-deterministic programs: weakest precondition calculus.
- Dijkstra: "programs are viewed as predicate transformers"
- φ postcondition, P program
  $\rightarrow$ $wp[P](\varphi)$: *weakest precondition*

Semantics for nondeterministic programs

Example

$$P_1\colon\ x := -y; x := x + 1$$

Semantics for nondeterministic programs

Example

$$P_1\colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

Semantics for nondeterministic programs

Example

$$P_1 \colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

$$wp[P_1](\varphi)$$

Semantics for nondeterministic programs

Example

$$P_1\colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

$$wp[P_1](\varphi) = wp[x := -y; x := x + 1]([x \geq 5])$$

Semantics for nondeterministic programs

Example

$$P_1\colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

$$\begin{aligned} wp[P_1](\varphi) &= wp[x := -y; x := x + 1]([x \geq 5]) \\ &= wp[x := -y]([x + 1 \geq 5]) \end{aligned}$$

Semantics for nondeterministic programs

Example

$$P_1 \colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

$$\begin{aligned} wp[P_1](\varphi) &= wp[x := -y; x := x + 1]([x \geq 5]) \\ &= wp[x := -y]([x + 1 \geq 5]) \\ &= wp[x := -y]([x \geq 4]) \end{aligned}$$

Semantics for nondeterministic programs

Example

$$P_1 \colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

$$\begin{aligned} wp[P_1](\varphi) &= wp[x := -y; x := x + 1]([x \geq 5]) \\ &= wp[x := -y]([x + 1 \geq 5]) \\ &= wp[x := -y]([x \geq 4]) \\ &= [-y \geq 4] \end{aligned}$$

Semantics for nondeterministic programs

Example

$$P_1 \colon\ x := -y; x := x + 1$$

$$\varphi = [x \geq 5]$$

$$\begin{aligned} wp[P_1](\varphi) &= wp[x := -y; x := x + 1]([x \geq 5]) \\ &= wp[x := -y]([x + 1 \geq 5]) \\ &= wp[x := -y]([x \geq 4]) \\ &= [-y \geq 4] \\ &= [y \leq -4] \end{aligned}$$

test

test

test

test

test

test