**Study Guide for Exam 1**

In preparing for our first exam you should review your notes from lecture and your homework solutions. Most of you will also try some of the old exams on our Canvas web site. This guide is a list of skills and facts that we will be testing you on. It will hopefully help you focus on those areas that you need to brush up on and review in more detail.

**Unit conversion:** Be able to convert between different units, e.g. miles and meters, by multiplying by "one", (1600 meters)/(1 mile) or (1 mile)/(1600 meters) .

**One dimensional kinematics:** Position, velocity, and acceleration are related by

$$v = \frac{dx}{dt} \quad \text{and} \quad x(t_2) - x(t_1) = \int_{t_1}^{t_2} v\,dt \tag{1}$$

$$a = \frac{dv}{dt} \quad \text{and} \quad v(t_2) - v(t_1) = \int_{t_1}^{t_2} a\,dt. \tag{2}$$

These equations are general. You should be able to take derivatives and integrals of simple functions like polynomials to go betwen $x$, $v$, and $a$. You should also be able to do this graphically using slope (derivative) and area (integral).

Speed is the absolute value of the velocity, $|v|$. For the average velocity and acceleration replace the derivative by a difference

$$v_{avg} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} \quad \text{and} \quad a_{avg} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}. \tag{3}$$

The average speed is the total distance traveled divided by the total time.

For the special case of constant acceleration these equations reduce to

$$x = x_o + v_o t + \frac{1}{2}at^2 \tag{4}$$

$$v = v_o + at. \tag{5}$$

**Vectors:**

- Know how to add, subtract, and scalar multiply vectors both using component notation and graphically.
- Know how to compute the magnitude of a vector using the Pythagorean theorem and the direction of of a vector in polar coordinates.
- Be able to go from polar coordinates to x-y coordinates using $x = r\cos(\theta)$ and $y = r\sin(\theta)$.
- Know how to take the dot product both with components and via $\vec{a} \cdot \vec{b} = ab\cos(\theta)$. Understand how the dot product is related to the component of a vector in a particular direction.
- Know how to take the cross product both with components and using the right hand rule along with $|\vec{a} \times \vec{b}| = ab\sin(\theta)$. Understand how the cross product is related to the component of a vector perpendicular to a particular direction.

**Kinematics in two and three dimensions:** In two and three dimensions position, velocity, and acceleration are vectors.

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \text{and} \quad \vec{r}(t_2) - \vec{r}(t_1) = \int_{t_1}^{t_2} \vec{v}\,dt \tag{6}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \text{and} \quad \vec{v}(t_2) - \vec{v}(t_1) = \int_{t_1}^{t_2} \vec{a}\,dt \tag{7}$$

Remember each vector equation is 2 scalar equations in two dimensions, and 3 scalar equations in three dimensions. To generalize Eqs. (3) - (5) to higher dimensions one just changes the scalar position, velocity, and accelerations into vectors.

For projectile motion one further specializes to the case that the constant acceleration is downwards at $g = 9.8m/s^2$.

$$x = x_o + v_{x,o}t \tag{8}$$

$$y = y_o + v_{y,o}t - \frac{1}{2}gt^2 \tag{9}$$

$$v_x = v_{x,o} \tag{10}$$

$$v_y = v_{y,o} - gt \tag{11}$$

In projectile motion and other problems you are expected to be able to do algebraic manipulations to solve for the unknowns. This may involve solving two simultaneous equations or the quadratic equation.

**Circular motion:** A particle undergoing circular motion with radius r at a constant speed, $|v|$, has acceleration $a = v^2/r$ towards the center of the circle. The angular velocity, $\omega$, is $2\pi/T$, where $T$ is the time or period it takes to go around the circle one time. The velocity can be written as $v = 2\pi r/T = r\omega$.

**Relative motion:** The velocity of an object is the vector sum of the velocity of the object in the reference frame plus the velocity of the reference frame. A common example of a relative motion problem is a person swimming in a river with a current.

**Newton's laws problems:** You should know Newton's three laws and be able to apply them to qualitative problems. (You will find examples in lecture and on old exams.) For quantitative Newton's law problems there are usually three basic steps:

1. Draw a picture (force diagram) showing all the forces on an object. Use one picture per object so as not to get confused by equal and opposite pairs of forces as required by Newton's third law.

2. Choose a coordinate system and write down $\vec{F} = m\vec{a}$ in that coordinate system. Note that this is a vector equation so in two dimensions there will be two scalar equations: one for the x-component and one for the y-component.

3. Solve for the unknowns. Each problem is different, but as indicated earlier you are expected to be able do algebraic manipulations to solve problems.

It is sometimes tempting to skip steps 1 and 2 above; however, that comes at a great risk. Just one sign error is enough to turn the right answer to the wrong one. For most of the problems on the exam there is very little calculation required. You should have time to read the problems carefully as well as draw pictures and set up the equations.

There are a number of special forces that you need to know:

- Gravity acts downwards with magnitude $mg$.

- The normal force, $F_N$, is perpendicular to the surface.

- Tension acts along a string or rope. Two objects connected by a rope experience forces in equal and opposite directions.

- The static frictional force occurs when an object is not moving relative to a surface. It is in the opposite direction to the way it would move without friction. The *maximum* value of the static frictional force is $\mu_s F_N$. The static frictional force is only equal to $\mu_s F_N$ when it is just starting to move.

- The kinetic frictional force occurs when an object is moving with respect to a surface. It opposes the motion along the surface and is equal in magnitude to $\mu_k F_N$.

- The force due to a spring is equal to $F = -kx$. In this equation $x = 0$ is the point where the force is zero. Springs always push an object towards $x = 0$.

**Work:** The work done by a constant force, $\vec{F}$, is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement (change in position between initial and final times). The dot product here is important. The work can be positive, negative, or zero depending on the angle between $\vec{F}$ and $\vec{d}$. For a force that varies in space in one dimension the work is given by

$$W = \int_{x_i}^{x_f} F\,dx. \tag{12}$$

Of particular interest to us is the case of the work done by a spring

$$W_{spring} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2, \tag{13}$$

and the work done by gravity

$$W_{gravity} = mg(h_i - h_f), \tag{14}$$

where here we have replaced $x$ by $h$ to indicate the object is moving up and down - not horizontally.

**Kinetic energy:** The kinetic energy of a moving mass is $\frac{1}{2}mv^2$.

**Work-energy theorem:** The net work done on an object due to all forces is equal to the change in the kinetic energy of the object. If the work done is positive, the object will speed up, and if the work done is negtive, the object will slow down.

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \tag{15}$$

**Power:** Power is the rate at which work is done.

$$P = \frac{dW}{dt} \quad \text{and} \quad W = \int_{t_i}^{t_f} P\,dt \tag{16}$$