PHY2048 Formulas

Constants and General Relations

$g = 9.8$ m/s^2 1 mile = 1600 m Area circle = πr^2

Vectors

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ Magnitudes: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ $|\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$

Scalar Product: $\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z = |\vec{a}||\vec{b}|\cos\theta$ (θ = angle between $\vec{a}$ and $\vec{b}$)

Vector Product: $\vec{a}\times\vec{b} = (a_yb_z - a_zb_y)\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$

Magnitude: $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$ (θ = angle between $\vec{a}$ and $\vec{b}$)

Component of $\vec{a}$ in Direction of $\vec{b} = a_{\parallel} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Motion

Displacement: $\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$

Average Velocity: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(t_2)-\vec{r}(t_1)}{t_2-t_1}$

Average Speed: s_{avg} = (total distance)/Δt

Instantaneous Velocity: $\vec{v} = \frac{d\vec{r}(t)}{dt}$

Relative Velocity: $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$

Average Acceleration: $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t_2)-\vec{v}(t_1)}{t_2-t_1}$

Instantaneous Acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

Equations of Motion for Constant Acceleration

General

$\vec{v} = \vec{v}_0 + \vec{a}t$

$\vec{r} - \vec{r}_0 = \vec{v}_0t + \frac{1}{2}\vec{a}t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$ (in each of 3 dim)

$\Delta\vec{r} = \frac{1}{2}(\vec{v}_0 + \vec{v})\Delta t$

Projectile Motion

$x = x_0 + v_{x0}t$

$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

$v_x = v_{x0}$

$v_y = v_{y0} - gt$

Newton's Laws

$\vec{F}_{net} = 0 \Leftrightarrow \vec{v}$ is a constant (Newton's First Law)

$\vec{F}_{net} = m\vec{a}$ (Newton's Second Law)

"Action = Reaction" (Newton's Third Law)

Friction & Drag

Static: $f_s \le \mu_s F_N$

Kinetic: $f_k = \mu_k F_N$

Drag: $D = \frac{1}{2}C\rho A v^2$

Spring Force

$\vec{F}_{spring} = -k\Delta\vec{x}$

Uniform Circular Motion (Radius R, Tangential Speed $v = R\omega$, Angular Velocity ω)

Centripetal Acceleration: $a = \frac{v^2}{R} = R\omega^2$

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$

Work (W) & Kinetic Energy (K)

Kinetic Energy: $K = \frac{1}{2}mv^2$ Work: $W = \int_{\vec{r}_1}^{\vec{r}_2}\vec{F}\cdot d\vec{r}$ (when force is constant $W = \vec{F}\cdot\vec{d}$)

Power: $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$ Work-Energy Theorem: $W_{\text{external}} = \Delta K$ $W_{gravity} = mg(h_i - h_f)$ $W_{spring} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Work (W), Mechanical Energy (E, Kinetic Energy (K), Potential Energy (U))

Potential Energy: $\Delta U = -W = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$ $\quad U_{gravity} = mgh \quad U_{spring} = \frac{1}{2}kx^2$

Mechanical Energy: $E_{\text{mech}} = K + U \quad E_{\text{mech}} =$ constant for an isolated system with conservative forces

Work-Energy: $W_{ext} = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int} \qquad \Delta E_{th} = f_k d$

Momentum

Center of Mass: $\vec{r}_{\text{com}} = \frac{1}{M_{\text{tot}}} \sum_{i=1}^{N} m_i \vec{r}_i \quad M_{\text{tot}} = \sum_{i=1}^{N} m_i$

Linear Momentum: $\vec{p} = m\vec{v}$ $\quad$ Impulse: $\vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt = \vec{F}_{\text{av}}\Delta t$

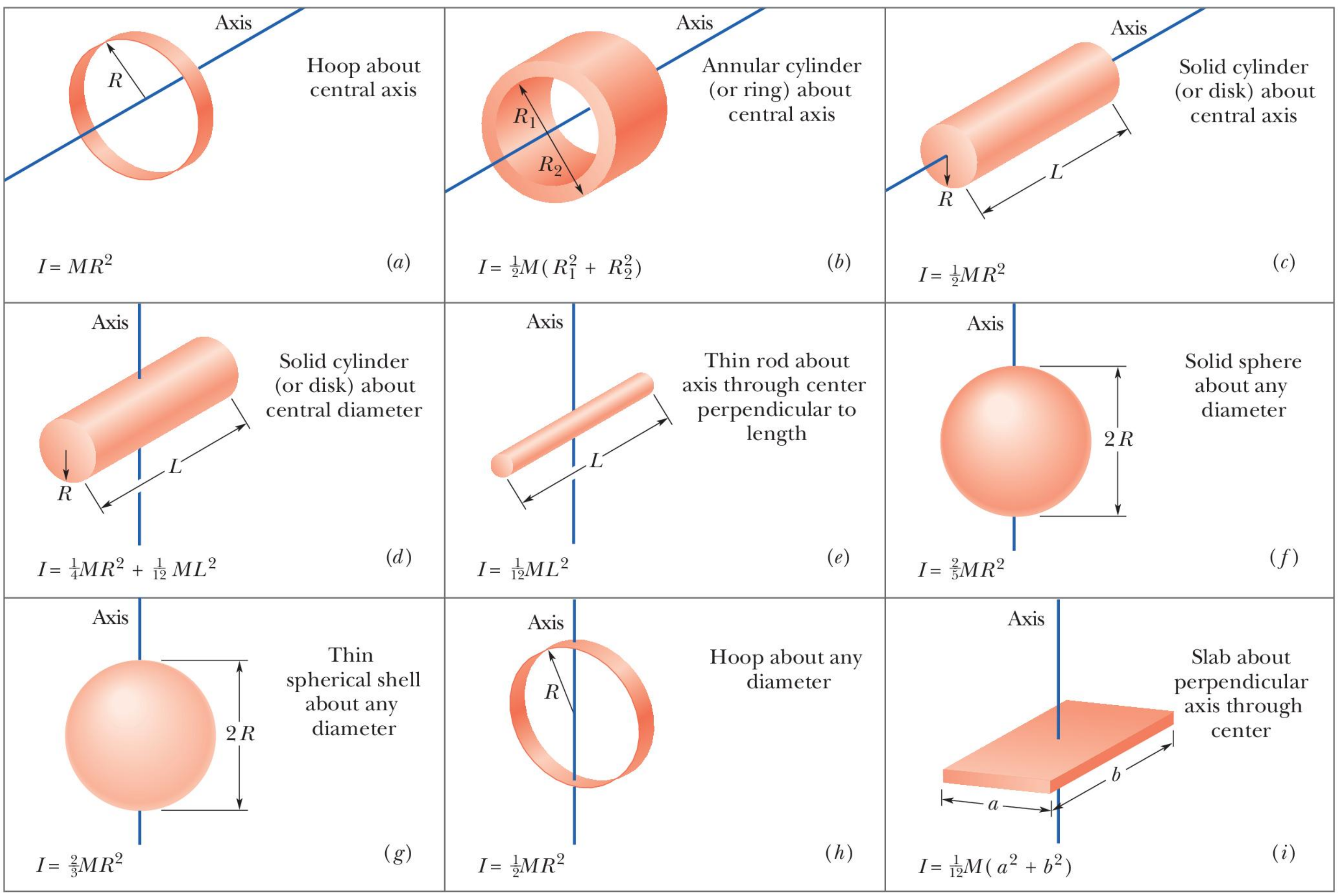
$\vec{F} = \frac{d\vec{p}}{dt}$ $\quad$ If $\vec{F} = \frac{d\vec{p}}{dt} = 0$ then $\vec{p} =$ constant and $\vec{p}_f = \vec{p}_i$

$\vec{P}_{\text{tot}} = M_{\text{tot}}\,\vec{v}_{\text{com}} = \sum_{i=1}^{N} \vec{p}_i \qquad \vec{F}_{\text{net}} = \frac{d\vec{P}_{\text{tot}}}{dt} = M_{\text{tot}}\,\vec{a}_{\text{com}}$

Elastic Collisions of Two Bodies, 1D

$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$

Moments of Inertia



Rotational Variables

angular position: $\theta(t)$ angular velocity: $\omega(t) = \frac{d\theta(t)}{dt}$

angular acceleration: $\alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2}$

Motion with constant angular acceleration α:

$\omega = \omega_0 + \alpha t$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\theta = \theta_0 + \frac{1}{2}(\omega + \omega_0)t$

Angular to linear relationships for circular motion

arc length: $s = r\theta$ velocity: $v = r\omega$

tangential acceleration: $a_{\mathrm{T}} = r\alpha$ centripetal acceleration: $a_{\mathrm{c}} = r\omega^2$

Rotational Inertia: $I = \sum_{i=1}^{N} m_i r_i^2$ (discrete) $I = \int r^2\, dm$ (continuous)

Parallel Axis: $I = I_{\mathrm{com}} + M_{\mathrm{tot}} d^2$ (d is displacement from c.o.m.)

Rotational, Rolling Kinetic Energy: $K_{\mathrm{rot}} = \frac{1}{2}I\omega^2$ $K_{\mathrm{roll}} = \frac{1}{2}Mv_{\mathrm{com}}^2 + \frac{1}{2}I_{\mathrm{com}}\omega^2$

Rolling without slipping: $x_{\mathrm{com}} = R\theta$ $v_{\mathrm{com}} = R\omega$ $a_{\mathrm{com}} = R\alpha$

Torque and Angular Momentum

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$ $\tau = rF\ \sin$ (angle between $\vec{r}$ and $\vec{F}$) $= rF_{\perp}$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$ $\vec{\tau} = \frac{d\vec{L}}{dt}$ $L = rp\ \sin$ (angle between $\vec{r}$ and $\vec{p}$) $= rp_{\perp}$ $L = I\omega$

If $\vec{\tau}_{\mathrm{net}} = \frac{d\vec{L}}{dt} = 0$ then $\vec{L} =$ constant and $\vec{L}_f = \vec{L}_i$

Work done by a constant torque: $W = \tau\Delta\theta = \Delta K_{\mathrm{rot}} = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$; General: $W = \int \tau d\theta$

Power done by a constant torque: $P = \tau\omega$

For torque acting on a body with rotational inertia I: $\vec{\tau} = I\vec{\alpha}$

Elasticity

Stress $= \frac{F}{A}$ and Strain $= \frac{\Delta L}{L}$ (Y = Young's modulus, G = shear modulus)

Tensile: $\frac{F_{\perp}}{A} = Y\frac{\Delta L}{L}$ Shear: $\frac{F_{\parallel}}{A} = G\frac{\Delta x}{L}$

Gravitation

Newton's law of gravitation $\vec{F} = \frac{Gm_1m_2}{r^2}\hat{r}$ where $G = 6.67408 \times 10^{-11}$ Nm^2/kg^2 $U_g = -\frac{GMm}{r}$

Kepler's Area Law: $\frac{dA}{dt} = constant$ Kepler's Law of periods: $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

Escape speed from Earth: $v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$ with $M_E = 5.97 \times 10^{24}$ kg and $R_E = 6.38 \times 10^6$ m

Fluids

Fluid density: $\rho = \frac{m}{V}$ Pressure: $P = \frac{F}{A}$ $P_2 = P_1 + \rho g(y_1 - y_2)$

Archimedes: $F_b = m_f g$ Flow of fluid: $Av = \text{constant}$ Pascal's principle: $F_i/A_i = F_o/A_o$

Bernoulli's Eqn: $P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$

Waves and Oscillators

Simple harmonic motion $\frac{d^2x}{dt^2} = -\omega^2 x$ $x = x_m \cos(\omega t + \phi)$ Energy (undamped) $E = \frac{1}{2}kx_m^2 = \frac{1}{2}m\omega^2 x_m^2$

Frequency (Hz) and period (s): $f = \frac{1}{T}$ Angular freq. $\omega = 2\pi f$ $v_m = \omega x_m$ $a_m = \omega^2 x_m$

Simple pendulum: $\omega = \sqrt{\frac{g}{L}}$ Physical pendulum: $\omega = \sqrt{\frac{mgh}{I}}$

Linear oscillator: $\omega = \sqrt{\frac{k}{m}}$

Linear oscillator with damping force $-bv$: $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ $x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)$

Sinusoidal waves: $y(x,t) = y_m \sin(kx \mp \omega t)$ $k = \frac{2\pi}{\lambda}$ $\omega = \frac{2\pi}{T}$ $v = \frac{\omega}{k} = f\lambda$

Wave equation: $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$ Wave speed on string (linear mass density μ) $v = \sqrt{F_T/\mu}$

n^{th} harmonic resonance on string of length L or an open-open tube of length L: $f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}, n = 1, 2, 3, ...$

n^{th} harmonic resonance in an open-closed tube of length L: $f_n = \frac{v}{\lambda_n} = n\frac{v}{4L}, n = 1, 3, 5, ...$

Path difference and phase $\phi = \frac{2\pi \Delta L}{\lambda}$ Constructive: $\phi = 2m\pi$, Destructive: $\phi = (2m+1)\pi$

Sound intensity $I = \frac{P}{A} = \frac{1}{2}\rho v \omega^2 s_m^2$ and sound level: $\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_0}\right)$ (where $I_0 = 10^{-12}$ W/m^2)

Doppler effect for source moving at v_S and detector moving at v_D: $f_D = f_S\left(\frac{v_{snd} \pm v_D}{v_{snd} \mp v_S}\right)$
top signs: $v_D(v_S)$ moves towards $v_S(v_D)$
bottom signs: $v_D(v_S)$ moves away from $v_S(v_D)$