Study Guide for Exam 2

This guide provides you with a big picture view of the topics you need to know for Exam 2. It is not a substitute for practicing solving problems, which is the bulk of what you'll be asked to do on the exam. There will be a few conceptual questions in addition to these mathematical problem-solving questions.

Note that the formula sheet for Exam 2 includes all of the formulas from the Exam 1 formula sheet as well as additional formulas that are not explicitly covered in this review. A great self-test of your foundational knowledge is to explain the formula sheet to yourself or a friend: can you say what each variable in an equation stands for, what its units are, and what the equation is for? Being able to confidently do this for every equation on the formula sheet means that you have an excellent baseline understanding of the topics covered in this class.

Finally, remember that the material in this course is cumulative, meaning that you need to know the material in the earlier part of the course to solve problems in the later part of the course. Thus, when you have a problem for this exam on a topic like statics, you will need to use what you learned in the first part of the course on solving force problems and using vectors.

Energy

The work done in going from an initial position, i, to a final position, f, does not depend on the path taken for conservative forces. The gravitational force and the force due to a spring are conservative forces. Friction is not a conservative force. For conservative forces we can define the potential energy, U:

$$W = \int_i^f F dx = -(U_f - U_i). \text{ (for conservative forces)} \tag{1}$$

Note that the potential energy is only defined up to an additive constant since only the difference appears in Eq. (1). One can go from the potential energy to the force by taking a derivative: $F = -dU/dx$. The potential energy for the gravity near the surface of the Earth and for springs is

$$U_{gravity} = mgh \text{ with h = height} \tag{2}$$

$$U_{spring} = \frac{1}{2}kx^2 \text{ with x = 0 having F = 0.} \tag{3}$$

From the previous exam we have the work-energy theorem which applies to all forces.

$$W = \int_i^f F_{net}\, dx = K_f - K_i. \text{ (net is the sum of all forces)} \tag{4}$$

If we only have conservative forces, Eq. (1) and Eq. (4) can be combined to form the law of conservation of mechanical energy:

$$E_i = K_i + U_i = K_f + U_f = E_f \text{ (only conservative forces)} \tag{5}$$

The complete law of conservation of energy includes the work done by nonconservative forces, the change in thermal energy due to kinetic friction acting in the system, and the change in the internal energy of the system:

$$W = \Delta K + \Delta U + \Delta E_{th} + \Delta E_{int} \text{ (all energy terms)} \tag{6}$$

The increase in thermal energy, ΔE_{th}, is equal to the magnitude of the force of friction f_k multiplied by the distance traveled d: $\Delta E_{th} = f_k d$. Unless we are specifically asking about the change in internal energy, ΔE_{int}, we can safely ignore this term in solving energy problems in this class.

Momentum and Center of Mass

For multi-particle systems it is useful to rewrite Newton's second law in terms of the center of mass and the momentum.

$$\vec{F}_{net} = M\frac{d^2\vec{r}_{CM}}{dt^2}, \text{ where } \vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \text{ and } M = m_1 + m_2 \tag{7}$$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}, \text{ where } \vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 \tag{8}$$

$\vec{F}_{net}$ here is the sum of all external forces on the system. To compute the center of mass or momentum for more than two particles just add the extra terms in the above equations. You should be able to compute the center of mass of a set of objects in both one and two dimensions.

If there are no external forces acting on a system, then the momentum is conserved,

$$\vec{p}_i = \vec{p}_f. \tag{9}$$

The momentum of a system is also equal to the mass times the velocity of the center of mass, $M\vec{v}_{CM}$. Thus, if there are no external forces, the velocity of the center of mass is constant as well. If there is a net external force, then the momentum changes. This is quantified with the impulse, $\vec{J}$.

$$\vec{J} = \int_i^f \vec{F}\, dt = \vec{p}_f - \vec{p}_i \approx \vec{F}_{avg}\Delta t \tag{10}$$

Because collisions take place over a very short time scale it is a good approximation to neglect any external forces compared to the internal forces of the collision. Thus, for collisions we will be taking momentum to be conserved. On the other hand, before and after collisions the external forces are important and we can not necessarily take momentum to be conserved. Thus, problems with collisions often have a "before" part where momentum is not conserved, the collision where momentum is conserved, and an "after" part where momentum is not conserved. For the "before" and "after" parts one must use other physics such as kinematics, Newton's second law, or conservation of energy.

A collision in which the kinetic energy is conserved in addition to the momentum is are an elastic collision. You are given the formulas for elastic collisions in one dimension on your formula sheet; however, keep in mind that these formulas are only valid if the kinetic energy is conserved. We are always taking momentum to be conserved in collisions.

Rotation

Kinematics

Rotational kinematics is described by the angle measured in radians, θ, the angular velocity, ω, and the angular acceleration, α.

$$\omega = \frac{d\theta}{dt} \tag{11}$$

$$\alpha = \frac{d\omega}{dt}. \tag{12}$$

These equations have the same structure as for motion in one dimension so we can use all the one dimensional kinematics equations with x, v, a replaced by θ, ω, α.

The tangential velocity and acceleration of a point is related to the angular velocity and acceleration via

$$v = r\omega \tag{13}$$

$$a_t = r\alpha. \tag{14}$$

Objects moving in a circle also have acceleration towards the center of the circle

$$a_r = \frac{v^2}{r} = \omega^2 r. \tag{15}$$

The total acceleration is the *vector* sum of the tangential and radial accelerations.

Rotational Inertia

The rotational inertia, I, plays the role that mass does for linear motion. It is given by

$$I = \sum_i m_i r_i^2, \tag{16}$$

where r_i is the distance from mass i to the axis of rotation. The rotational inertia depends on the axis of rotation. An object has different rotational inertias depending on which axis you are rotating about. One can break any solid object up into infinitesimal masses using calculus and evaluate I. The results for some standard objects rotating about their center of mass are given in the formula sheet and in a table in the book. To get the rotational inertia for an object not rotating about its center of mass use the parallel axis theorem,

$$I = I_{CM} + Mh^2, \tag{17}$$

where h is the distance from the rotational axis to the center of mass.

Torque

Torque plays the analog of force for rotational motion. It is a vector,

$$\vec{\tau} = \vec{r} \times \vec{F}, \tag{18}$$

where $\vec{r}$ is a vector which goes from the axis of rotation to the applied force. In practice we will usually compute the magnitude of the torque using

$$\tau = rF\sin(\phi) = r_\perp F = rF_\perp \tag{19}$$

and then assign a plus sign for a force that acts to produce a counterclockwise rotation and a minus sign for a force that acts to produce a clockwise rotation. The angle between r and F is ϕ.

Second Law for Rotation

The analog of Newton's second law for rotational motion is

$$\tau = I\alpha. \tag{20}$$

This equation is derivable from $F = ma$.

Energy of Rotation and Rolling

When an object is rotating in addition to having its center of mass move, there are two contributions to the kinetic energy.

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{21}$$

For the case of an object rolling without slipping the translational velocity and angular velocity are related via $v = r\omega$.

Angular Momentum

The torque can also be related to the time derivative of the angular momentum just as the force is related to the time derivative of the momentum.

$$\vec{\tau} = \frac{d\vec{L}}{dt} \tag{22}$$

We use two methods to calculate the angular momentum. For a particle moving the angular momentum is the cross product of the position and the linear momentum, while for a solid object rotating about an axis it is $I\omega$.

$$\vec{L} = \vec{r} \times \vec{p} \tag{23}$$

$$L = I\omega \tag{24}$$

If the external torque is zero, the angular momentum is constant, $L_i = L_f$, which is called conservation of angular momentum. In class we did demonstrations with the rotating chair illustrating this principle. We also did the gyroscope demonstration.

Statics

When an object is not moving, the net force and torque on it are zero.

$$\vec{F} = 0 \text{ and } \vec{\tau} = 0 \tag{25}$$

We solved many different statics problems in three steps.

- Draw a force diagram for each object, including drawing the forces where they are acting on the object, choosing a pivot point, indicating the direction of the torque produced by each force, and labeling the distances of the force vectors from the pivot. Remember that the gravitational force is applied at the center of mass of the object.
- Write down the $\vec{F}_{net} = 0$ and $\vec{\tau}_{net} = 0$ (vector) equations.
- Solve using algebra.
- Check that your results make physical sense.

Elasticity

We discussed two types of stress in class: tensile and shear. In the case of tensile stress, an object is stretched or compressed by a force acting perpendicular to the cross-sectional area of the object (typically, the force is acting along the length of the object). In the case of shear stress, the force is acting parallel to the face of the cross-sectional area of the object. The tensile stress equation is:

$$\frac{F_\perp}{A} = E\frac{\Delta L}{L} \tag{26}$$

The shear stress equation is:

$$\frac{F_\parallel}{A} = G\frac{\Delta x}{L} \tag{27}$$

Here, E is Young's modulus and G is the shear modulus, which vary based on the material making up the object under consideration.