# Lecture 4

## §1 Friction

### 1. Friction

方向：parallel to the surface and directed so as to oppose the sliding.

- When a force $\vec{F}$ tends to slide a body along a surface, a frictional force $\vec{f}$ from the surface acts on the body. The frictional force is parallel to the surface and directed so as to oppose the sliding. It is due to the bonding between the body and the surface

Disadvantages
- Counteract the frictional force to make the vehicle moving

Advantages
- Walking/Climbing
- Holding things

### 2. 静摩擦力 (static friction force)

1° $\vec{f_s}$ 指向与 applied force 相反的方向

2° 无运动

3° 随着 applied force 增大，static friction force $\vec{f_s}$ 大小也增大，但 block 保持静止.

4° 只有当滑块刚开始滑动前 static friction 有 maximum value $\vec{f}_{s\,max}$

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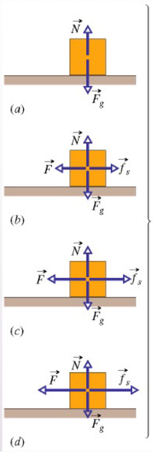
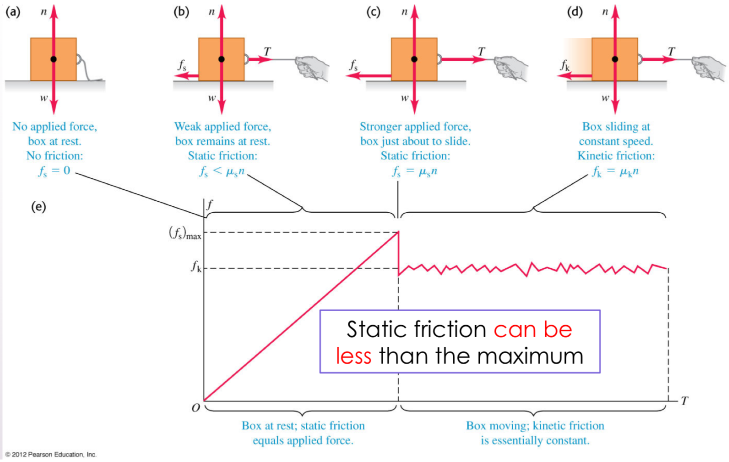
- In response to an applied force, a frictional force $\vec{f_s}$ is directed in the opposite direction, exactly balancing the applied force.
- No motion
- If you increase the magnitude of your applied force, the magnitude of the static frictional force $\vec{f_s}$ also increases and the block remains at rest.
- Static friction only has its maximum value $\vec{f}_{s\,max}$ just before the box "breaks loose" and starts to slide.




### 3. 滑动摩擦力 (kinetic friction force)

1° 阻碍运动

2° 通常 $f_k < f_{s\,max}$

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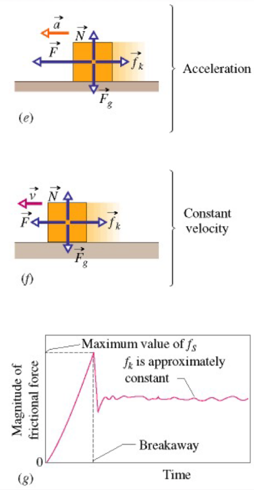
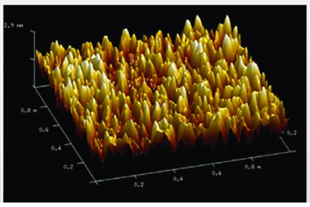
- As shown in the right, as increasing magnitude of the applied force $\vec{F}$, the block "breaks away" from its intimate contact with the table top and accelerates leftward.
- The frictional force that then opposes the motion is called the Kinetic frictional force $\vec{f_k}$
- Usually, $f_k < f_{s,max}$

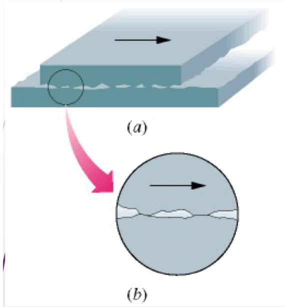


# 4. 摩擦力的微观分析

## Cold-Weld

- Assume two smooth and clean metal surfaces brought together in vacuum
- Atomic interactions occur → one piece of metal → cold-weld
- However, it is not possible to have that much atom-to-atom contact
- 3D Surface profile of a-well polished metal by AFM
- Far from being flat in atomic scale
- Covered with metal oxide layer






In reality

- When two ordinary surfaces are placed together, only the high points touch each other.
- The actual *microscopic* area of contact is much less than the apparent *macroscopic* contact area, perhaps by a factor of $10^4$.
- Many contact points do **cold-weld** together.
- These welds produce static friction $\vec{f_s}$ when an applied force attempts to slide the surfaces relative to each other.

- If the applied force is great enough to pull one surface across the other, there is first a tearing of welds (at breakaway) and then a continuous re-forming and tearing apart of welds as movement occurs and chance contacts are made. The kinetic frictional force $\vec{f_k}$ that opposes the motion is the vector sum of the forces at those many chance contacts.
- If the two surfaces are pressed together harder, many more points cold-weld. Then, getting the surfaces to slide relative to each other requires a greater applied force: The static frictional force $\vec{f_s}$ has a greater maximum value.
- Over jerky surface: Producing sounds, eg. Violin

# 5. 摩擦力的性质

1° Static friction force

$f_{s\,max} = \mu_s F_N$

$\mu_s$: coefficient of static friction ; $F_N$: Normal force

2° Kinetic friction force

$f_k = \mu_k F_N$

$\mu_k$: coefficient of kinetic friction ; $F_N$: Normal force

3° $\mu_k / \mu_k$

(1) dimensionless (无量纲/无单位的)

(2) 必须由实验测定

(3) 大小取决于 body 和 surface

(4) $\mu_k$ 与速度无关

A force $\vec{F}$ (or vector sum of several applied forces) attempts to slide the body along the surface:

- If the body does not move, then the static frictional force $\vec{f_s}$ and the component of $\vec{F}$ that is parallel to the surface balance each other. They are equal in magnitude, and $\vec{f_s}$ is directed opposite that component of $\vec{F}$.
- Static Frictional Force

$$f_{s,max} = \mu_s F_N$$

$\mu_s$: Coefficient of static friction; $F_N$: Normal Force

- Kinetic Frictional Force

$$f_k = \mu_k F_N$$

$\mu_k$: Coefficient of kinetic friction; $F_N$: Normal Force

- $F_N$: How firmly the body presses against the surface.

$$F_{CB} = -F_{BC}$$

  - The harder you press, the larger the Normal Force
  - $\vec{F_N} \perp \vec{f_s}/\vec{f_k}$
- $\mu_s/\mu_k$: Dimensionless, must be determined experimentally
  - Depends on the body and surface
  - e.g. The value of $\mu_s$ between an egg and a Teflon-coated skillet is 0.04, but that between rock-climbing shoes and rock is as much as 1.2
  - $\mu_k$: Assume the value doesn't depends on the speed at which the body slides along the surface

# Friction in Horizontal Motion

Before the crate moves, static friction acts on it.
After it starts to move, kinetic friction acts.

(a) Pulling a crate
(b) Free-body diagram for crate just before it starts to move
(c) Free-body diagram for crate moving at constant speed

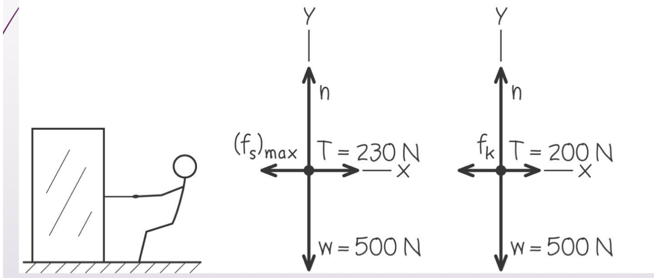


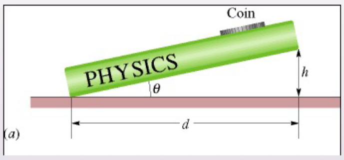

**Table 5.1 Approximate Coefficients of Friction**

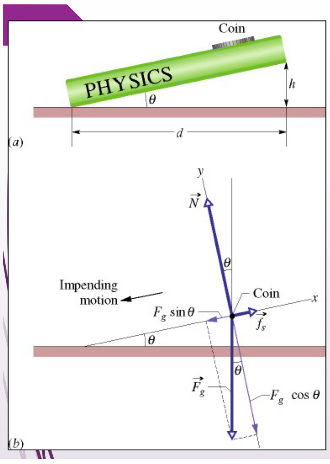
| Materials | Coefficient of Static Friction, $\mu_s$ | Coefficient of Kinetic Friction, $\mu_k$ |
|---|---|---|
| Steel on steel | 0.74 | 0.57 |
| Aluminum on steel | 0.61 | 0.47 |
| Copper on steel | 0.53 | 0.36 |
| Brass on steel | 0.51 | 0.44 |
| Zinc on cast iron | 0.85 | 0.21 |
| Copper on cast iron | 1.05 | 0.29 |
| Glass on glass | 0.94 | 0.40 |
| Copper on glass | 0.68 | 0.53 |
| Teflon on Teflon | 0.04 | 0.04 |
| Teflon on steel | 0.04 | 0.04 |
| Rubber on concrete (dry) | 1.0 | 0.8 |
| Rubber on concrete (wet) | 0.30 | 0.25 |

例:

## Problem

The figure shows a coin of mass *m* at rest on a book that has been tilted at an angle with the horizontal. By experimenting, you find that when $\theta$ is increased to 13°, the coin is on the *verge* of sliding down the book, which means that even a slight increase beyond 13° produces sliding. What is the coefficient of static friction $\mu_s$ between the coin and the book?




**SOLUTION:**
The coin stays at rest, the net force is zero along x and y axis.

In x-axis

$$f_{s,max} - F_g \sin\theta = 0$$

$$f_{s,max} = F_g \sin\theta$$

In y-axis

$$N - F_g \cos\theta = 0$$

$$N = F_g \cos\theta$$

$$\mu_s = \frac{f_{s,max}}{N} = \frac{F_g \sin\theta}{F_g \cos\theta} = \tan\theta = 0.23$$

Bigger $\mu_s$ → Bigger $\theta$

## §2 The drag force

1. Fluid (流体)

任何可流动的物体，无论气态或液态

2. Drag force $\vec{D}$ (阻力)

当物体与流体存在相互运动时，会受到阻力

1° 与相对运动方向相反

2° Blunt body (钝体) 在空气中快速相对运动:

$$D = \frac{1}{2} C \rho A v^2$$

C: drag coefficience (风阻系数)　$\rho$: 空气密度　A: effective cross-sectional area (垂直于速度)　v: 相对速度　*: $C \in (0.4, 1.0)$

3. Terminal speed

一个 blunt body 在空气中由静止释放

$$D - F_g = ma$$

$$\frac{1}{2} C \rho A v_t^2 - F_g = 0$$

terminal speed $= v_t = \sqrt{\frac{2F_g}{C\rho A}}$

## Problem

A raindrop with radius $R = 1.5$ mm falls from a cloud that is at height $h = 1200$ m above the ground. The drag coefficient $C$ for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water $\rho_w$ is 1000 kg/m$^3$, and the density of air $\rho_a$ is 1.2 kg/m$^3$.

(a) As Table 6-1 indicates, the raindrop reaches terminal speed after falling just a few meters. What is the terminal speed?

## SOLUTION

- Spherical drop feels gravitational force $F = mg$:
  - Express in terms of density of water

$$F_g = V\rho_w g = \tfrac{4}{3}\pi R^3 \rho_w g.$$

- So plug in to the terminal velocity equation using the values provided in the text:
  - Use $A = \pi R^2$ for the cross-sectional area

$$v_t = \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{(8)(1.5\times10^{-3}\text{ m})(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)}{(3)(0.60)(1.2\text{ kg/m}^3)}}$$

$$= 7.4\text{ m/s} \approx 27\text{ km/h.} \qquad \text{(Answer)}$$

With no drag force to reduce the drop's speed during the fall, the drop would fall with the constant free-fall acceleration $g$, so the constant-acceleration equations of Table 2-1 apply.

*Calculation:* Because we know the acceleration is $g$, the initial velocity $v_0$ is 0, and the displacement $x - x_0$ is $-h$, we use Eq. 2-16 to find $v$:

$$v = \sqrt{2gh} = \sqrt{(2)(9.8\text{ m/s}^2)(1200\text{ m})}$$

$$= 153\text{ m/s} \approx 550\text{ km/h.} \qquad \text{(Answer)}$$

Had he known this, Shakespeare would scarcely have written, "it droppeth as the gentle rain from heaven, upon the place beneath." In fact, the speed is close to that of a bullet from a large-caliber handgun!

# §3 Uniform Circular Motions

## 1、Uniform circular motion

1° Acceleration

$a = \frac{v^2}{r}$

2° The centripetal force for the object

$F = m\frac{v^2}{r}$

方向：指向圆心

## Uniform Circular Motions

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

- Acceleration:

$$a = \frac{v^2}{r} \qquad \text{Magnitude of the acceleration}$$

- The centripetal force (magnitude) for an object in uniform circular motion is

$$F = m\frac{v^2}{r}$$

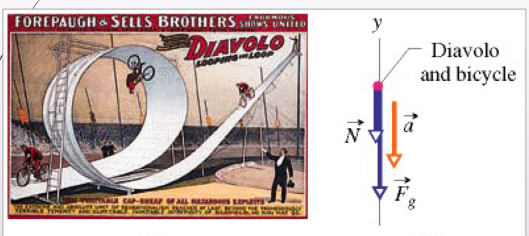
- The direction?

Pointing to the center of the circle all the time

例：

## Problem

In a 1901 circus performance, Allo "Dare Devil" Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (see figure). Assuming that the loop is a circle with radius $R = 2.7$ m, what is the least speed $v$ Diavolo could have at the top of the loop to remain in contact with it there?




(a) (b)

## SOLUTION:

Define the positive direction vertically downward

In the circular motion:

$$N + mg = m\left(\frac{v^2}{R}\right)$$

Positive direction towards center

If N=0, then

$$v = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(2.7\text{ m})}$$

$$= 5.1\text{ m/s}$$

Therefore, he must maintain at least 5.1 m/s at the top of the loop. Otherwise, he'll fall off the track.

例:

## Pilot in a Loop

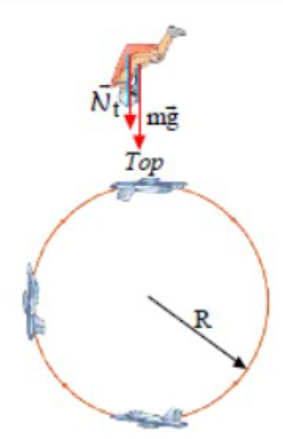
At the top (inside the loop)

$$F_{top} = N_t + mg = \frac{mv^2}{R}.$$

$$N_t = \frac{mv^2}{R} - mg.$$

If $\frac{mv^2}{R} \geq mg$, i.e., $v \geq \sqrt{gR}$, then $N_t > 0$

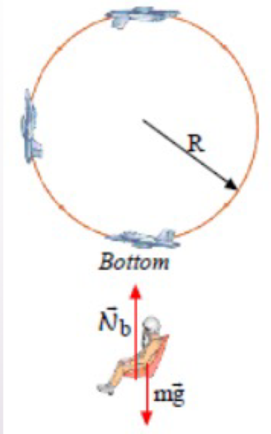
If $\frac{mv^2}{R} < mg$, i.e., $v < \sqrt{gR}$, then $N_t < 0$




At the bottom (inside the loop):

$$\frac{mv^2}{R} = N_b - mg.$$

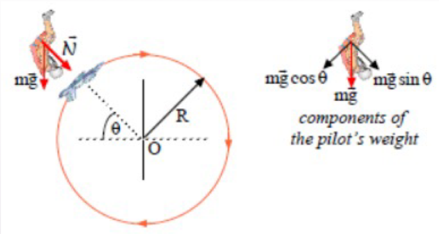
$$N_b = mg + \frac{mv^2}{r}$$




At some angle (inside the loop):

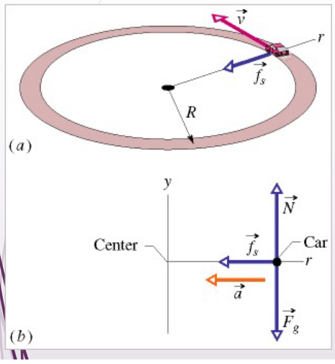
$$\Sigma F_R = \frac{mv^2}{R} = N + mg\sin\theta,$$

$$N = m\left(\frac{v^2}{R} - g\sin\theta\right).$$




例:

## Circular Turn




The figure represents a stock car of mass $m$ = 1600 kg traveling at a constant speed $v$ = 20 m/s around a flat, circular track of radius $R$ = 190 m. For what value of $\mu_s$ between the track and the tires of the car will the car be on the verge of sliding off the track?

**SOLUTION:**

$$f_s = m\frac{v^2}{R}$$ Direction towards the center

$$= \mu_s mg$$

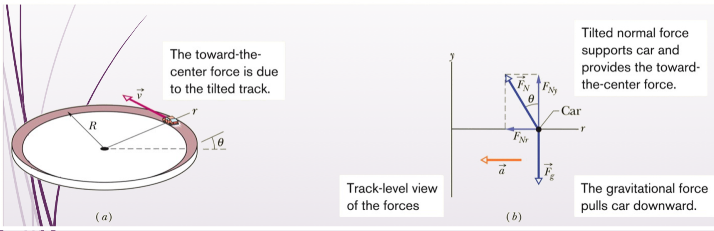
$$\mu_s = \frac{mv^2}{mgR} = \frac{v^2}{gR}$$

$$= \frac{(20\ m/s)^2}{(9.8\ m/s^2)(190\ m)}$$

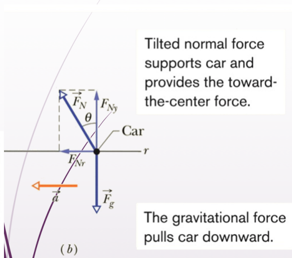
$$= 0.21$$

例:

The figure below represents a car of mass m as it moves at a constant speed v of 20 m/s around a banked circular track of radius R=190 m. If the frictional force from the track is negligible, what bank angle $\theta$ prevents sliding?




**SOLUTION**




The component of normal force Provide the centripetal force

In x-axis:

$$-F_N \sin\theta = -m\frac{v^2}{R}$$

In y-axis:

$$F_N \cos\theta - F_g = 0$$

$$F_N \cos\theta = mg$$

Thus,

$$\tan\theta = \frac{v^2}{Rg} = 12.1°$$

# Summary

- Friction
  - Static Frictional Force
    $$f_{s,max} = \mu_s F_N$$
  - Kinetic Frictional Force
    $$f_k = \mu_k F_N$$
  - Understand the formation of frictional forces in microscopic view
- Drag Force and Terminal Speed
  $$D = \frac{1}{2} C\rho A v^2 \qquad v_t = \sqrt{\frac{2F_g}{C\rho A}}$$
- Circular Motion
  $$F = m\frac{v^2}{r}$$