Lecture 1

§1 Measurement

1. Why need measurement

Physics and engineering are based on the precise measurement of physical quantities.

$\Rightarrow$ Standard and Units for measurement

$\Rightarrow$ Accessible & Invariable

2. Base quantities (基本量)

1° 7 fundamental quantities

2° assigned standards with SI units

Table 1. SI base units

Base quantity	SI base unit Name	SI base unit Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

3° Define other physical quantities

3. Length

1 meter $\stackrel{def}{=}$ the length of the path traveled by light in vacuum (真空) during a time interval of 1 / 299792458 of a second

$c = 299792458\,m/s$

(b) Measuring the meter

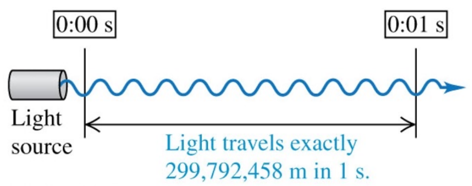


Table 1-3 Some Approximate Lengths

Measurement	Length in Meters
Distance to the first galaxies formed	2×10^{26}
Distance to the Andromeda galaxy	2×10^{22}
Distance to the nearby star Proxima Centauri	4×10^{16}
Distance to Pluto	6×10^{12}
Radius of Earth	6×10^{6}
Height of Mt. Everest	9×10^{3}
Thickness of this page	1×10^{-4}
Length of a typical virus	1×10^{-8}
Radius of a hydrogen atom	5×10^{-11}
Radius of a proton	1×10^{-15}

4. Time

1 second $\stackrel{def}{=}$ the time taken by 9192631770 oscillations of light emitted at the transition between the two hyperfine levels of the ground state of a cesium-133 atom.

Table 1-4 Some Approximate Time Intervals

Measurement	Time Interval in Seconds	Measurement	Time Interval in Seconds
Lifetime of the proton (predicted)	3×10^{40}	Time between human heartbeats	8×10^{-1}
Age of the universe	5×10^{17}	Lifetime of the muon	2×10^{-6}
Age of the pyramid of Cheops	1×10^{11}	Shortest lab light pulse	1×10^{-16}
Human life expectancy	2×10^{9}	Lifetime of the most unstable particle	1×10^{-23}
Length of a day	9×10^{4}	The Planck time[a]	1×10^{-43}

[a]This is the earliest time after the big bang at which the laws of physics as we know them can be applied.

5. Mass

The carbon-12 atom has been assigned a mass of 12 atomic mass units (u).

$1\ u = 1.6605402 \times 10^{-27}$ kg

Table 1-5 Some Approximate Masses

Object	Mass in Kilograms
Known universe	1×10^{53}
Our galaxy	2×10^{41}
Sun	2×10^{30}
Moon	7×10^{22}
Asteroid Eros	5×10^{15}
Small mountain	1×10^{12}
Ocean liner	7×10^{7}
Elephant	5×10^{3}
Grape	3×10^{-3}
Speck of dust	7×10^{-10}
Penicillin molecule	5×10^{-17}
Uranium atom	4×10^{-25}
Proton	2×10^{-27}
Electron	9×10^{-31}

6. Units prefixes

Table 1.1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and living cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for sound to travel 0.35 m)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^{3} m (a 10-minute walk)		

7. Scientific notation

employs powers of 10 to write large or small numbers

$$3\,560\,000\,000 \text{ m} = 3.56 \times 10^{9} \text{ m}$$

$$0.000\,000\,492 \text{ s} = 4.92 \times 10^{-7} \text{ s}.$$

8. Conversion factor

1° is a ratio of units that is equal to 1

2° is used to convert between units

$$2 \text{ min} = (2 \text{ min})(1) = (2\ \cancel{\text{min}})\left(\frac{60 \text{ s}}{1\ \cancel{\text{min}}}\right) = 120 \text{ s}.$$

3° units obey the same algebraic rules as variables and numbers

9. Significant figures

- Generally, round to the least number of significant figures of the given data
  - Round up for 5+ (13.5 → 14, but 13.4 → 13)
- Significant figures are not decimal places
  - 0.00356 has 5 decimal places, 3 significant figures

- In general, tailing zeros are not significant

    In other words, 3000 may have 4 significant figures

    but usually 3000 will have only 1 significant figure!

  When in doubt, use scientific notation 3.000×10^3 （4） or 3×10^3 （1）

Another example

The world's largest ball of string is about 2 m in radius.

Estimate the total length of the string. Diameters of the string is around 0.004 m.

$$V = \frac{4\pi}{3}R^3 = \pi\left(\frac{d}{2}\right)^2 L$$

$$L = \frac{16R^3}{3d^2} \approx 2.6\times10^6\ m$$

$$\approx 3\times10^6\ m$$

(1 significant figure)

10. Density

Density $\stackrel{def}{=\!=\!=}$ Mass per unit volume

$$\rho = \frac{m}{V}$$

Summary

Measurement

- Defined by relationships to base quantities
- Each defined by a standard, and given a unit

Changing Units

- Use chain-link conversions
- Write conversion factors as unity
- Manipulate units as algebraic quantities

SI Units

- International System of Units
- Each base unit has an accessible standard of measurement

Length

- Meter is defined by the distance traveled by light in a vacuum in a specified time interval

Summary

Time

- Second is defined in terms of oscillations of light emitted by a cesium-133 source
- Atomic clocks are used as the time standard

Mass

- Kilogram is defined in terms of a platinum-iridium standard mass
- Atomic-scale masses are measured in u, defined as mass of a carbon-12 atom

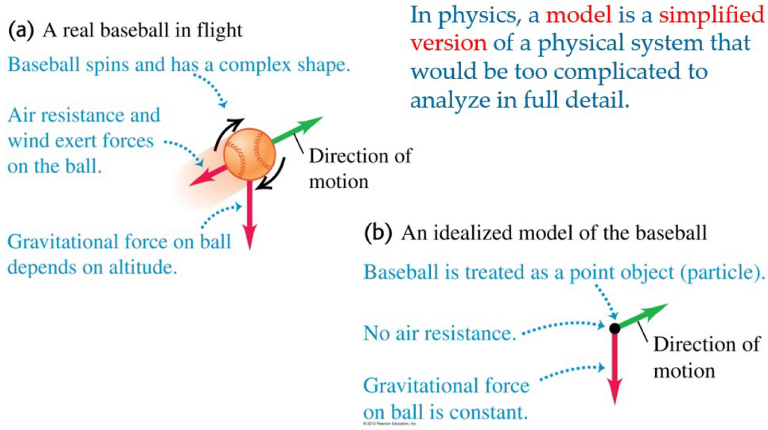
Density

- Mass/volume

$$\rho = \frac{m}{V}$$

§2 Motion along a straight line

1. Particle (质点): an idealized model



2. Position and displacement

1° Origin (or zero point) and direction

- **Origin (or Zero point)**: Reference point to locate the object
- **Positive (+ve) direction** of the axis is in the direction of increasing numbers (coordinates); the opposite is the **Negative (-ve) direction**

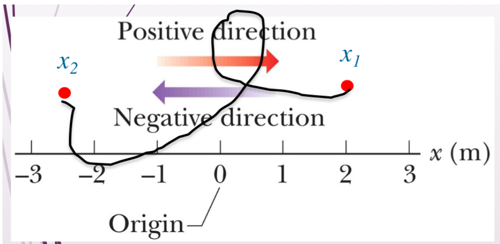
2° Displacement (位移)

x_1: Initial position

x_2: Final position

displacement: A vector (矢量) with direction and magnitude (数值)

$\Delta x = x_2 - x_1$

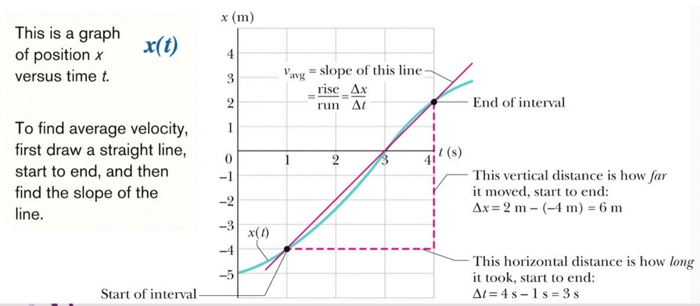


3. Average velocity (平均速度)

v_{avg}: The ratio of the displacement Δx that occurs during a particular time interval Δt

v_{avg}: Slope of the straight line of $x(t)$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$



4. 平均速度 (average speed)

S_{avg}：一段时间 Δt 内的总路程 (total distance)

$$S_{avg} = \frac{\text{total distance}}{\Delta t}$$ 标量 (scalar)

$$|V_{avg}| \leq S_{avg}$$

5. 瞬时速度 (Instantaneous velocity)

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$x(t)$ 曲线的切线

6. 瞬时速率 (Instantaneous speed)

瞬时速度的大小 (magnitude)

7. 加速度 (acceleration)

1° 平均加速度 (average acceleration)

$$a_{avg} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

单位：m/s^2

2° (瞬时) 加速度 ((instantaneous) acceleration)

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

是一个有大小与方向的向量

方向取决于 dv

加速 (acceleration)：$+ve$

减速 (deceleration)：$-ve$

8. 匀变速 (constant acceleration) 运动

1° $V = V_0 + at$

2° $V_{avg} = \frac{1}{2}(V_0 + V)$

3° $x - x_0 = V_0 t + \frac{1}{2}at^2$

4° $x - x_0 = V_{avg} t$

$\quad\quad = \frac{1}{2}(V_0 + V)t$

5° $V^2 = V_0^2 + 2a(x - x_0)$

6° $x - x_0 = Vt - \frac{1}{2}at^2$

9. 自由落体 (free-fall) 运动

1° 在竖直 y 方向的运动，以竖直向上为 y 轴正方向 (+y vertically up)

2° g 是自由落体运动的加速度大小

$a = -g = -9.8\ m/s^2$ (忽略纬度(latitude)与海拔(elevation)影响)

3° 与质量、密度、形状等无关

4° 不计空气影响

例:

Sample Problem

In the figure, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

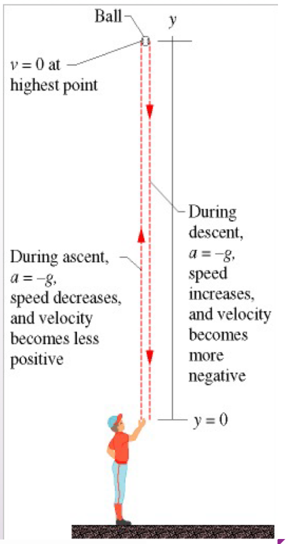
a) What is the acceleration at the highest point?

b) How long does it take to reach the maximum height?

c) What is the maximum height above its release point?

d) How long does the ball take to reach a point 5.0 m above its release point?

e) Describe the motion



a) What is the acceleration at the highest point?

SOLUTION: Once the ball leaves the pitcher and before it returns to his hand, its acceleration is always constant.

As +y is vertically up

$$a = -g = -9.8\ m/s^2$$

b) How long does it take to reach the maximum height?

SOLUTION:

- The velocity v at the maximum height is 0.
- Initial velocity $\mathbf{v_0}$ pointing up, +ve

$$t = \frac{v - v_0}{a} = \frac{0 - 12\ m/s}{-9.8\ m/s^2} = 1.2\ s$$

c) What is the ball's maximum height above its release point?

SOLUTION:

- Take the ball's release point to be $y_0 = 0$
- $y - y_0 = y$ and $v = 0$ (at the maximum height)

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\ m/s)^2}{2(-9.8\ m/s^2)} = 7.3\ m$$

or

$$\mathbf{y = v_0 t + \tfrac{1}{2}at^2}$$

$$= 12\ m/s \times 1.2\ s + \tfrac{1}{2} \times (-9.8\ m/s^2) \times (1.2\ s)^2$$

$$= 7.3\ m$$

d) How long does the ball take to reach a point 5.0 m above its release point?

SOLUTION: The ball passes the point ***twice***, once on the way up and once on the way down

$$\mathbf{y = v_0 t + \tfrac{1}{2}at^2}$$

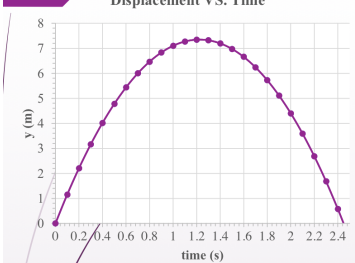
$$5.0\ m = 12\ m/s \times t + \tfrac{1}{2} \times (-9.8\ m/s^2) \times t^2$$

$$t = 0.53\ s \text{ or } 1.9\ s$$

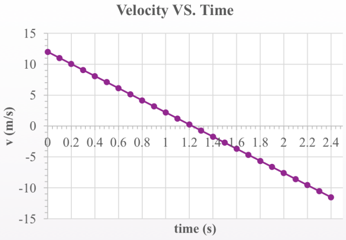
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graphical Analysis

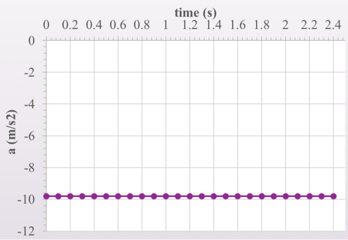
Displacement VS. Time



Velocity VS. Time



Acceleration VS. Time



- Area in v vs. t curve: Changing in Position
- Area in a vs. t curve: Changing in Velocity

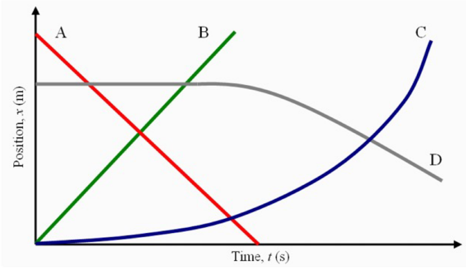
e) Describe the motion

SOLUTION:

- t = 0 s, the particle at $y_0 = 0$, with initial v_0 = 12 m/s
- 0 < t < 1.2 s, the particle has a +ve velocity, with constant a = –g. Decelerating
- t = 1.2 s, v = 0, the particle reaches its highest point y = 7.3 m
- 1.2 s < t < 2.4 s, -ve velocity with a = -g. Accelerating
- t = 2.4 s, y = 0, the ball returns back to the pitcher with v = -12 m/s

例:

2.7.5. Consider the graph the position versus time graph shown. Which curve on the graph best represents a constantly accelerating car?



a) A

b) B

c) C

d) D

e) None of the curves represent a constantly accelerating car.

例:

- A car moves along a straight line. Which one is accelerating with negative $\vec{a}$?
  A. Moving in the positive direction with increasing speed
  B. Moving in the positive direction with decreasing speed
  C. Moving in the negative direction with increasing speed
  D. Moving in the negative direction with decreasing speed

§3 矢量 (vectors)

1. 矢量/向量

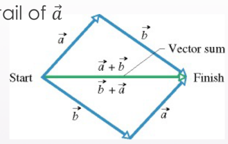
1° 与具体路径无关

2° 相同方向与长度/大小的向量可替换

2. 向量加法

Adding Vectors Geometrically

- Vector Sum, Head-to-tail, from the tail of $\vec{a}$ to the head of $\vec{b}$

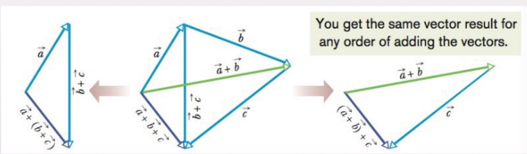


$$\vec{s} = \vec{a} + \vec{b}$$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

- More than 2 vectors, group at any orders

$$\vec{a} + \vec{b} + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

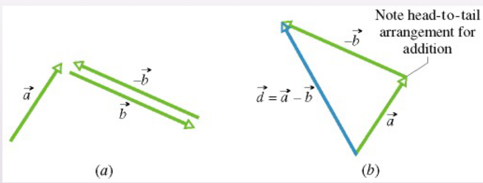


Adding Vectors Geometrically

- Adding $-\vec{b}$ has the same effect of subtracting $\vec{b}$

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

$$\vec{d} + \vec{b} = \vec{a} \text{ or } \vec{a} = \vec{d} + \vec{b}$$



Important Properties

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$	Commutative Law
$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$	Associative Law
$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$	Vector Subtraction

Checkpoint 1

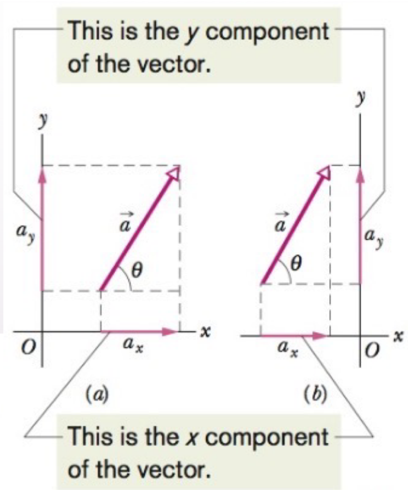
The magnitudes of displacements $\vec{a}$ and $\vec{b}$ are 3 m and 4 m, respectively, and $\vec{c} = \vec{a} + \vec{b}$. Considering various orientations of $\vec{a}$ and $\vec{b}$, what are (a) the maximum possible magnitude for $\vec{c}$ and (b) the minimum possible magnitude?

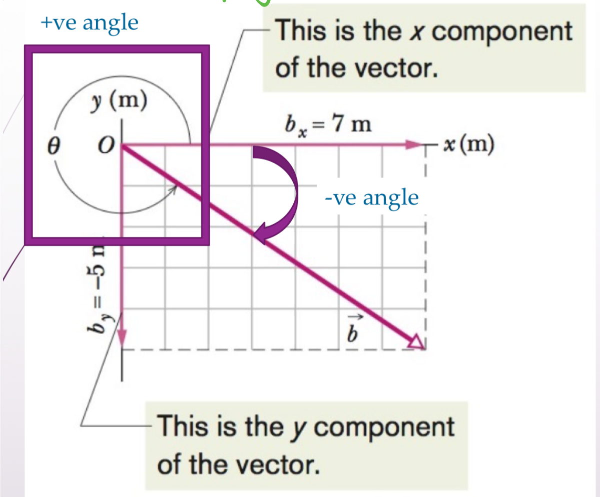
7 or 1 m

3. 分向量 (components of vectors)

将向量置于平面直角坐标系 (2-dimentional rectangular coordinate system)

分量：向量在坐标轴 (axes) 上的投影 (projection)





$a_x = |\vec{a}| \cdot \cos\theta$

$a_y = |\vec{a}| \cdot \sin\theta$

4. 单位向量 (unit vectors)

1° 单位向量长度为1，方向任意

2° x, y, z轴方向的单位向量如下图 ($\hat{i}$, $\hat{j}$, $\hat{k}$)

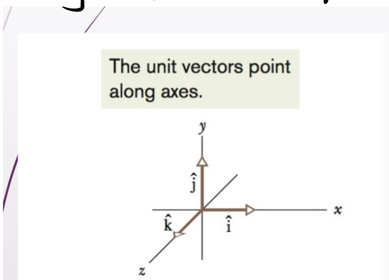
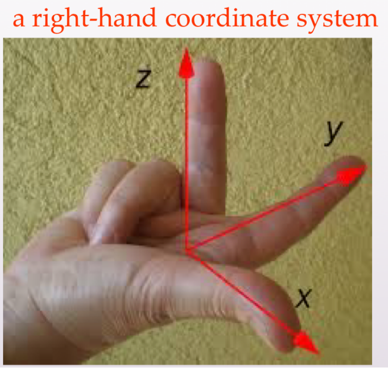
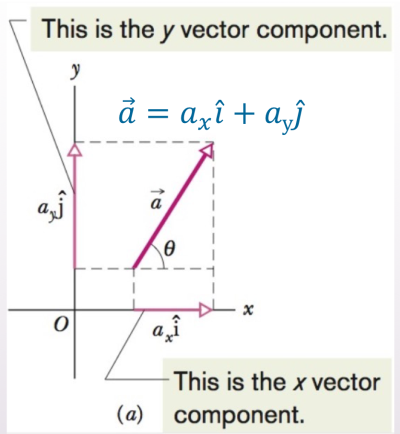


Figure 3.13 Unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ define the directions of a right-handed coordinate system.





5. 用分向量表示向量加减

$\vec{r} = \vec{a} + \vec{b}$

$$\begin{cases} r_x = a_x + b_x \\ r_y = a_y + b_y \\ r_z = a_z + b_z \end{cases}$$

Problem

- Three Vectors $\vec{a}, \vec{b},$ *and* $\vec{c}$, what is the vector sum $\vec{r}$?

$$\vec{a} = (4.2\,\text{m})\,\hat{i} - (1.5\,\text{m})\hat{j},$$
$$\vec{b} = (-1.6\,\text{m})\,\hat{i} + (2.9\,\text{m})\hat{j},$$
$$\vec{c} = (-3.7\,\text{m})\hat{j}.$$

SOLUTION:

$$r_x = a_x + b_x + c_x \qquad r_y = a_y + b_y + c_y$$
$$= 4.2\,\text{m} - 1.6\,\text{m} + 0 = 2.6\,\text{m} \qquad = -1.5\,\text{m} + 2.9\,\text{m} - 3.7\,\text{m} = -2.3\,\text{m}$$

$$\vec{r} = (2.6\,\text{m})\,\hat{i} - (2.3\,\text{m})\,\hat{j}, \qquad r = \sqrt{(2.6\,\text{m})^2 + (-2.3\,\text{m})^2} \approx 3.5\text{m}$$

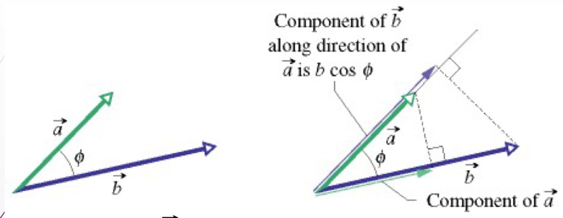
$$\theta = \tan^{-1}\left(\frac{-2.3\,\text{m}}{2.6\,\text{m}}\right) = -41^\circ$$

6. 向量点乘 (scale / dot product)

结果为标量

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$

Scalar or Dot Product



$$\vec{a} \cdot \vec{b} = a\,b\cos\phi$$

cos∅ = cos(360°-∅)

$$\vec{a} \cdot \vec{b} = (a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}) \cdot (b_x\,\hat{i} + b_y\,\hat{j} + b_z\,\hat{k}),$$
$$\vec{a} \cdot \vec{b} = a_x\,b_x + a_y\,b_y + a_z\,b_z,$$
$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

7. 向量叉乘 (vector / cross product)

$\vec{c} = \vec{a} \times \vec{b}$

1° 大小: $|\vec{c}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\theta$

θ: $\vec{a}$ 与 $\vec{b}$ 夹角 $(0 \le \theta \le 180^\circ)$

2° 方向: 右手定则

四指由 $\vec{a}$ 绕向 $\vec{b}$，大拇指所指方向即为 $\vec{c}$ 方向

- Magnitude

$$\vec{c} = \vec{a} \times \vec{b}$$
$$c = ab\sin\emptyset$$

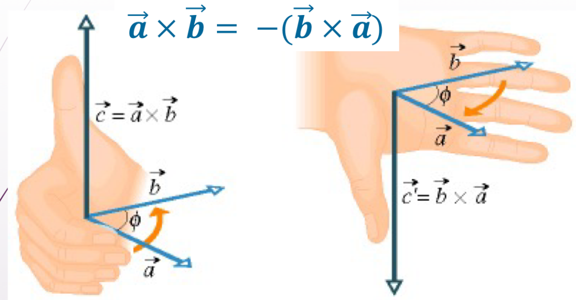
∅: the smaller of the two angles between the vectors, since $\sin\emptyset \neq \sin(360^\circ - \emptyset)$

- If ∅ = 90°, c ➔ Max
- If ∅ = 0° or 180°, c ➔ 0

$$\vec{c} = \vec{a} \times \vec{b} = 0$$

Parallel or antiparallel

- Direction
  - Right-Hand Rule



3° 计算: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

则 $\vec{c} = (a_yb_z - b_ya_z)\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$

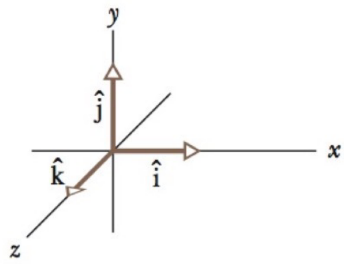
法一:

$$\vec{a}\times\vec{b} = (a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k})\times(b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k})$$

$$= \cancel{a_xb_x\hat{\imath}\times\hat{\imath}} + a_xb_y\hat{\imath}\times\hat{\jmath} + a_xb_z\hat{\imath}\times\hat{k} + a_yb_x\hat{\jmath}\times\hat{\imath} + \cancel{a_yb_y\hat{\jmath}\times\hat{\jmath}} + a_yb_z\hat{\jmath}\times\hat{k} + a_zb_x\hat{k}\times\hat{\imath} + a_zb_y\hat{k}\times\hat{\jmath} + \cancel{a_zb_z\hat{k}\times\hat{k}}$$

$$= a_xb_y\hat{k} - a_xb_z\hat{\jmath} - a_yb_x\hat{k} + a_yb_z\hat{\imath} + a_zb_x\hat{\jmath} - a_zb_y\hat{\imath}$$

$$= (a_yb_z - a_zb_y)\hat{\imath} + (a_zb_x - a_xb_z)\hat{\jmath} + (a_xb_y - a_yb_x)\hat{k}$$



法二:

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_yb_z - b_ya_z)\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$$

- Tips
  - Arrange vectors tail-to-tail: you must shift the vectors in the tail-to-tail arrangement (or move them to the same origin in *x-y-z* coordinate system)
  - Twist your right hand to apply the Right-Hand Rule

* 标量&矢量三重积

Products of Vectors

- Triple Products
  - Given three vectors expressed in terms of the Cartesian unit base vectors, i.e.

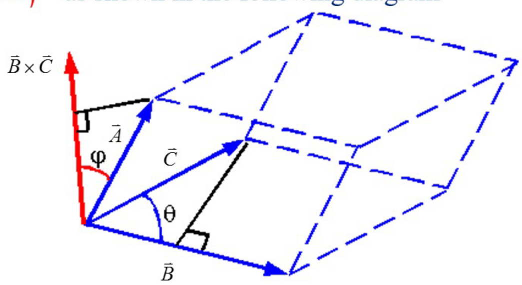
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k},\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \text{ and } \vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

the ***Triple Scalar Product*** can be expressed as:

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

Products of Vectors

- Triple Products
  - The volume of the parallelepiped formed by vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ is determined by their ***triple scalar product*** $\vec{A}\cdot(\vec{B}\times\vec{C})$ as shown in the following diagram



Products of Vectors

- As the volume of the parallelepiped does not depend on the order of the vectors, we expect
- Proof: $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} B_x & B_y & B_z \\ C_x & C_y & C_z \\ A_x & A_y & A_z \end{vmatrix} = \vec{B}\cdot(\vec{C}\times\vec{A})$$

Products of Vectors

- Triple Products
  - For three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$; the ***Triple Vector Product*** can be expressed as:

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$

  - It is easily to show that

$$\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$$

in general.

Problem

Two vectors lie with their tails at the same point. When the angle between them is increased by 20° their scalar product has the same magnitude but changes from positive to negative. The original angle between them was:

A) 0°
B) 60°
C) 70°
D) 80°
E) 90°

Problem

If $\vec{a} = 3\hat{\imath} - 4\hat{\jmath}$ and $\vec{b} = -2\hat{\imath} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

SOLUTION:

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

$$\vec{c} = (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k})$$
$$= -12\hat{i} - 9\hat{j} - 8\hat{k}$$

$\hat{c} \perp \hat{a}$ **and** $\hat{c} \perp \hat{b}$;

A cross product gives a perpendicular vector;
Or a vector perpendicular to the plane formed by
$\vec{a}$ and $\vec{b}$

- Vectors $\vec{A}$ and $\vec{B}$ have the magnitudes of 3 and 4, respectively. What is the angel between the directions of $\vec{A}$ and $\vec{B}$ if $\vec{A} \cdot \vec{B}$ equals

1. Zero: 90 deg
2. 12 : 0 deg
3. -12 : 180 deg

- If $\vec{A} \times \vec{B}$ equals

1. Zero : 0 deg or 180 deg
2. 12 : 90 deg

Summary

- Position
- Displacement
- Average Velocity and Speed
- Instantaneous Velocity and Speed
- Average Acceleration
- Instantaneous Acceleration
- Constant Acceleration
- Free-Fall Acceleration

Summary

- Scalar and Vectors
- Adding Vectors Geometrically or in Component Form
- Product of a Scalar and a Vector
- The Scalar Product
- The Vector Product