Lecture 2

§1 Motion in two or three dimensions

1、向量表示

用一个 position vector $\vec{r}$ 表示一个质点的位置

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

1° 向量由一个参照点(通常是原点)出发

2° x, y, z: 标量分量 (scalar components)

2、质点的 displacement

$\vec{r}$ 的改变量 $\Delta\vec{r}$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

3. Average velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

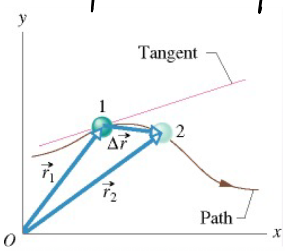
方向与 $\Delta\vec{r}$ 一致

4. Instantaneous velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

方向与 particle's path 相切于 particle's position.



例: Problem

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \quad (4\text{-}5)$$

and

$$y = 0.22t^2 - 9.1t + 30. \quad (4\text{-}6)$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

For the rabbit in the preceding sample problem, find the velocity $\vec{v}$ at time $t = 15$ s.

Check the angle calculated through $\tan^{-1}$

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

At t=15s,

$$x = 66\ m$$
$$y = -57\ m$$
$$\vec{r} = (66\ m)\hat{\imath} - (57\ m)\hat{\jmath}$$

In magnitude-angle notation,

$$r = \sqrt{x^2 + y^2} = 87\ m$$
$$\theta = \tan^{-1}\frac{y}{x} = -41°$$

Clockwise from +ve x-axis

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28) = -0.62t + 7.2$$
$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30) = 0.44t - 9.1$$

At t=15s,

$$v_x = -2.1\ m/s$$
$$v_y = -2.5\ m/s$$

In magnitude-angle notation,

$$v = \sqrt{v_x^2 + v_y^2} = 3.3\ m/s$$
$$\theta = \tan^{-1}\frac{v_y}{v_x} = -130°$$

Clockwise from +ve x-axis

5. Average acceleration

$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

方向与 $\Delta\vec{v}$ 一致

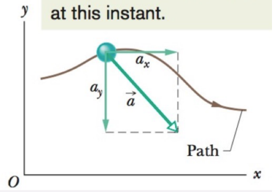
6. Instantaneous acceleration / acceleration

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$

$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

方向由 $\vec{a}$ 决定

These are the x and y components of the vector at this instant.



例: For the rabbit in the preceding two sample problems, find the acceleration $\vec{a}$ at time $t = 15$ s.

We can find $\vec{a}$ by taking derivatives of the rabbit's velocity components.

Calculations: Applying the a_x part of Eq. 4-18 to Eq. 4-13, we find the x component of $\vec{a}$ to be

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2.$$

Similarly, applying the a_y part of Eq. 4-18 to Eq. 4-14 yields the y component as

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

We see that the acceleration does not vary with time (it is a constant) because the time variable t does not appear in the expression for either acceleration component. Equation 4-17 then yields

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{i} + (0.44 \text{ m/s}^2)\hat{j}, \quad \text{(Answer)}$$

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\left(\frac{0.44 \text{ m/s}^2}{-0.62 \text{ m/s}^2}\right) = -35°. \quad -35° + 180° = 145°.$$

§2 Projectile motion

1. 抛体运动 (projectile motion)

初速度为 $\vec{v}_0$，加速度为 $a = -g$

2. 分析

水平与竖直方向的运动互不干扰

$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$

$v_{0x} = v_0 \cdot \cos\theta_0$

$v_{0y} = v_0 \cdot \sin\theta_0$

$\vec{r}$ 与 $\vec{v}$ 持续变化，$\vec{a}$ 恒定竖直向下

3. 规律

1° Y方向：

位移：$y - y_0 = (v_0\sin\theta)t - \frac{1}{2}gt^2 = (v_{\text{末}}\sin\theta)t + \frac{1}{2}gt^2$

速度：$v_y = v_0\sin\theta - gt$

速度位移公式：$v_y^2 = (v_0\sin\theta_0)^2 - 2g(y-y_0)$

2° X方向：

$x - x_0 = (v_0\cos\theta_0)t$

3° x与y的关系：

$t = \frac{x}{v_0\cos\theta_0}$

$y = v_0\sin\theta_0 t - \frac{1}{2}gt^2$

$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$

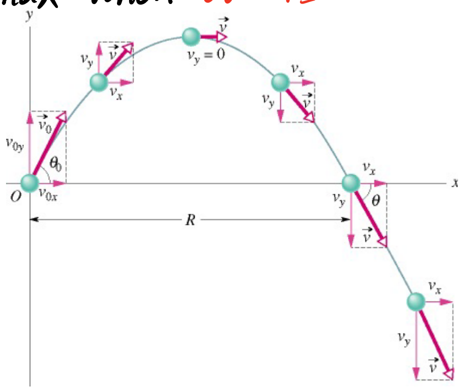
4. 水平射程 (horizontal range) R

$t = 2\cdot\frac{v_0\sin\theta_0}{g}$

horizontal range $R = v_0\cos\theta_0 t$

$= \frac{v_0^2}{g}\sin 2\theta_0$

R_{max} when $\theta_0 = 45°$



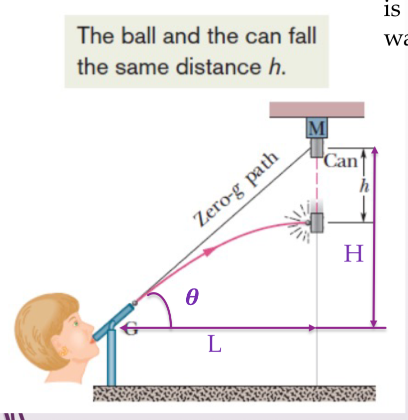
5. 最大高度

$H_{max} = \frac{(v_0\sin\theta)^2}{2g}$

$R = \frac{2v_0^2}{g}\sin\theta\cos\theta$

$\frac{H_{max}}{R} = \frac{\tan\theta_0}{4}$

例： Problem



A blowgun G using a ball as projectile is aimed at a can. Just as the ball leaves the blowgun, the can is released. How to aim it, if you want the ball to catch the can?

SOLUTION:

Assume aims at angle θ, The distance is L, and the height is H as shown left.

$$h = \frac{1}{2}gt^2$$

$$H - h = v_0 sin\theta t - \frac{1}{2}gt^2$$

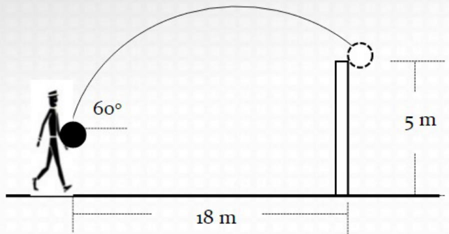
$$H = v_0 sin\theta t$$

$$L = v_0 cos\theta t$$

$$\tan\theta = \frac{H}{L}$$

Aim directly at the can

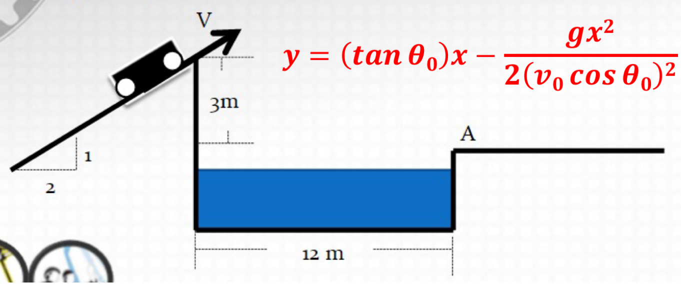
例： A ball is thrown so that it just clears a 5 meters fence 18 meters away. If it left the hand 1.5 meters above the ground and at an angle of 60° with the horizontal, what was the initial velocity of the ball?



$$\begin{cases} y = v_0 \sin\theta t - \frac{1}{2} g t^2 = 3.5 \\ x = v_0 \cos\theta t = 18 \end{cases}$$

$\Rightarrow v_0 = 15.1\ m/s$

例： A stunt man is to drive a car across the water-filled gap shown. Determine the car's minimum take velocity, landing velocity and the angle θ of the landing ramp.



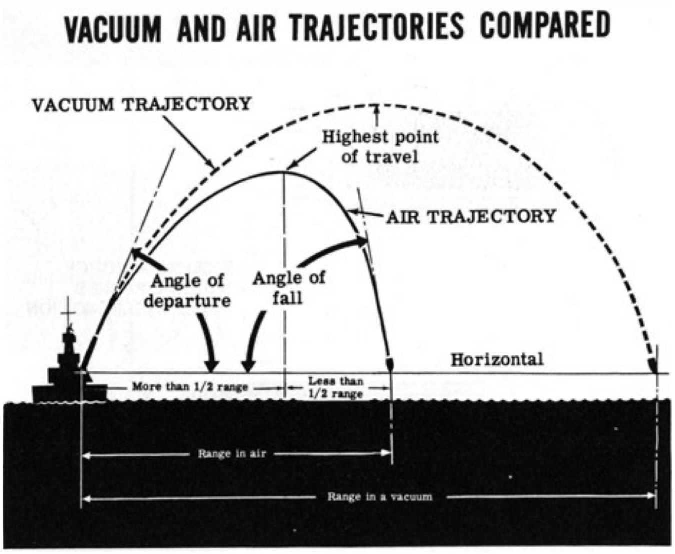
$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

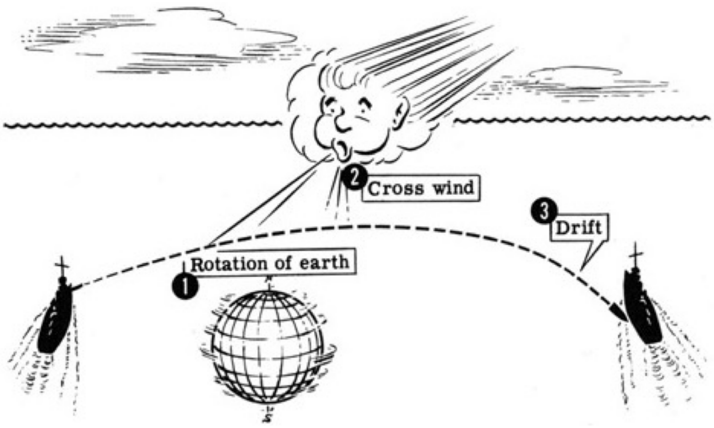
$$\begin{cases} y = v_y t - \frac{1}{2} g t^2 = -3 \\ x = v_x t = 12 \\ v_x = 2 v_y \end{cases}$$

$\Rightarrow \begin{cases} v_x = 8.85\ m/s \\ v_y = 4.42\ m/s \end{cases}$

$\Rightarrow \begin{cases} v_{land} = 12.5\ m/s \\ \theta = -45^\circ \end{cases}$

6. 考虑阻力





§3 Uniform circular motion

1. 匀速圆周运动 (uniform circular motion)

1° $\vec{v}$ 大小恒定，方向与路径相切

2° 匀速圆周运动的 $\vec{a}$ 被称为向心加速度 (centripetal acceleration)

Accelerating all the time!

方向：指向圆心

大小：$a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2}{T^2} r = v\omega = \frac{2\pi v}{T}$

3° 周期 (period (of revolution))

$T = \frac{2\pi r}{v}$

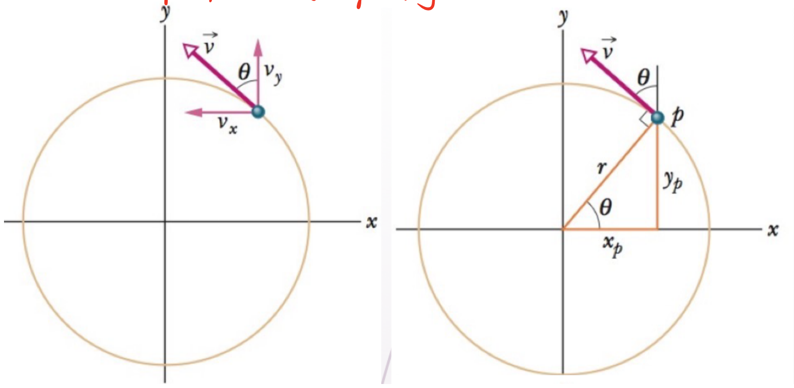
2、速度

1° 角度：$\theta = \theta$

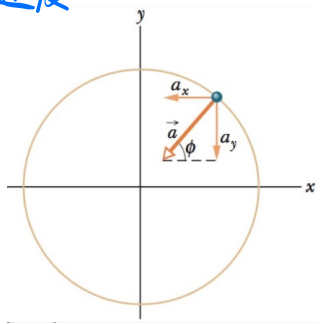
2° 速度

$$\vec{v} = V_x\hat{i} + V_y\hat{j} = (-v\sin\theta)\hat{i} + (v\cos\theta)\hat{j}$$

$$= \left(-\frac{v y_p}{r}\right)\hat{i} + \left(\frac{v x_p}{r}\right)\hat{j}$$



3、加速度



$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{i} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{j}$$

$$= \left(-\frac{v^2}{r}\cos\theta\right)\hat{i} + \left(\frac{v^2}{r}\sin\theta\right)\hat{j}$$

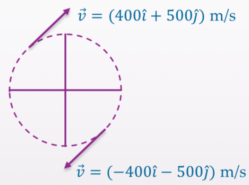
$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}$$

$$\tan\phi = \frac{a_y}{a_x} = \tan\theta$$

例：

Problem

- What is the magnitude of acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v} = (400\hat{i} + 500\hat{j})$ m/s and 24.0 s later, leaves the turn with a velocity of $\vec{v} = (-400\hat{i} - 500\hat{j})$ m/s?



SOLUTION:

$$a = \frac{v^2}{r}$$

$$T = \frac{2\pi r}{v}$$

$$a = \frac{2\pi v}{T} = \frac{2\pi \times \sqrt{(400)^2 + (500)^2}}{24 \times 2} m/s = 83.81\ m/s2$$

$$a \approx 8.6g$$

- "Top gun" pilots have long worried about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.
- There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot's vision switches to black and white and narrows to "tunnel vision." If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as **g-LOC** for "*g*-induced loss of consciousness."
- If a pilot caught in a dogfight puts the aircraft into such a tight turn, the pilot goes into *g*-LOC almost immediately, with no warning signs to signal the danger.

§4 Relative motion

1. 一维相对运动

1° 参照系 (reference frame)：一个实物（并附上坐标系）

2° $x_{PA} = x_{PB} + x_{BA}$

The coordinate x_{PA} of P as measured by A *is equal to* the coordinate x_{PB} of P as measured by B *plus* the coordinate x_{BA} of B as measured by A. Note that x is a vector in one dimension.

3° $v_{PA} = v_{PB} + v_{BA}$

The velocity v_{PA} of P as measured by A *is equal to* the velocity v_{PB} of P as measured by B *plus* the velocity v_{BA} of B as measured by A. The term v_{BA} is the velocity of frame B relative to frame A.

4° 若 v_{BA} 为常数，$a_{PA} = a_{PB} + a_{BA} = a_{PB}$

The acceleration a_{PA} of P as measured by A *is equal to* the acceleration a_{PB} of P as measured by B *plus* the acceleration a_{BA} of B as measured by A.

* 惯性系 (inertia frame)

Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle. (Inertia Frames)

例：

Problem

B's velocity relative to A is a constant v_{BA} = 52 km/h and car P is moving in the negative direction of the x axis.

(a) If A measures a constant velocity v_{PA} = -78 km/h for car P, what velocity v_{PB} will B measure?

SOLUTION:

$$v_{PA} = v_{PB} + v_{BA}$$
$$-78 = v_{PB} + 52$$
$$v_{PB} = -130\ km/h$$

(b) If car P brakes to a stop relative to A in time t = 10 s at constant acceleration, what is its acceleration a_{PA} relative to A?

SOLUTION:

$$a_{PA} = \frac{v_{PA\_f} - v_{PA}}{t} = \frac{0-(-78)\ km/h}{10\ s} \times \frac{1\ m/s}{3.6\ km/h} = 2.2\ m/s^2$$

(c) What is the acceleration a_{PB} of car P relative to B during the braking?

SOLUTION:

$$v_{PB\_f} = v_{PA\_f} - v_{BA} = 0 - 52 = -52\ km/h$$

$$a_{PB} = \frac{v_{PB\_f} - v_{PB}}{t} = \frac{-52-(-130)\ km/h}{10\ s} \times \frac{1\ m/s}{3.6\ km/h} = 2.2\ m/s^2$$

or

Since v_{BA} is constant,

$$a_{PA} = a_{PB} = 2.2\ m/s^2$$

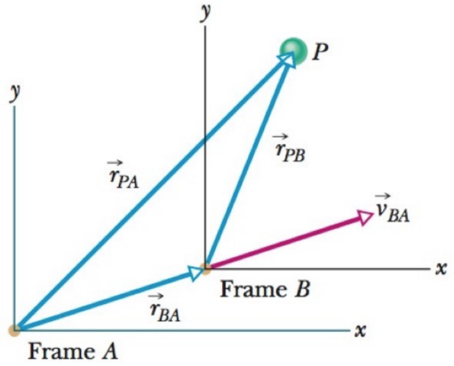
2、二维相对运动

两观察者A、B观察P，A、B相对速度恒定

则 $\vec{r}_{PA}=\vec{r}_{PB}+\vec{r}_{BA}$

$\vec{V}_{PA}=\vec{V}_{PB}+\vec{V}_{BA}$

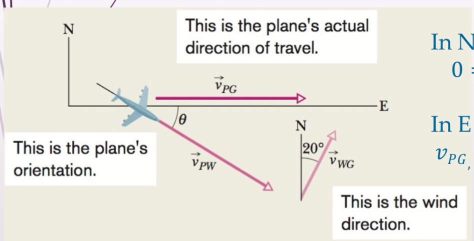
$\vec{a}_{PA}=\vec{a}_{PB}$



例：

Problem

A plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has a velocity $\vec{v}_{PW}$ relative to the wind, with a air speed of **215 km/h**, directed to angle θ south of east. The wind has velocity $\vec{v}_{WG}$ relative to the ground with speed **65.0 km/h**, directed **20.0°** east of north. What is the magnitude of the velocity $\vec{v}_{PG}$ of the plane relative to the ground, and what is θ?



$$\vec{v}_{PG}=\vec{v}_{PW}+\vec{v}_{WG}$$

In N direction,

$$0=v_{PW}\times\sin\theta+v_{WG}\times cos20.0°$$

$$\theta=-16.5°$$

In E direction,

$$v_{PG,E}=v_{PW}cos16.5°+v_{WG}sin20.0°$$

$$=228\ km/h$$

$$\vec{v}_{PG}=(228\,\hat{\imath}+0\hat{\jmath})\ \text{km/h}$$

$$v_{PG}=228\ km/h$$

Summary

- Projectile Motion

$$y-y_0=(v_0\sin\theta_0)t-\frac{1}{2}gt^2$$

$$v_y=v_0\sin\theta_0-gt$$

$$v_y{}^2=(v_0\sin\theta_0)^2-2g(y-y_0)$$

$$x-x_0=(v_0\cos\theta_0)\,t$$

$$y=(\tan\theta_0)x-\frac{gx^2}{2(v_0\cos\theta_0)^2}$$

Summary

- Position and Displacement $\Delta\vec{r}=\Delta x\hat{\imath}+\Delta y\hat{\jmath}+\Delta z\hat{k}$
- Average Velocity and Instantaneous Velocity $\vec{v}=v_x\hat{\imath}+v_y\hat{\jmath}+v_z\hat{k}$
- Average Acceleration and Instantaneous Acceleration $\vec{a}=a_x\hat{\imath}+a_y\hat{\jmath}+a_z\hat{k}$
- Projectile Motion $y=(\tan\theta_0)x-\frac{gx^2}{2(v_0\cos\theta_0)^2}$
- Uniform Circular Motion $a=\frac{v^2}{r}$ $T=\frac{2\pi r}{v}$
- Relative Motion in One Dimension
- Relative Motion in Two Dimension $\vec{r}_{PA}=\vec{r}_{PB}+\vec{r}_{BA}$, $\vec{v}_{PA}=\vec{v}_{PB}+\vec{v}_{BA}$, $\vec{a}_{PA}=\vec{a}_{PB}$