Lecture 20

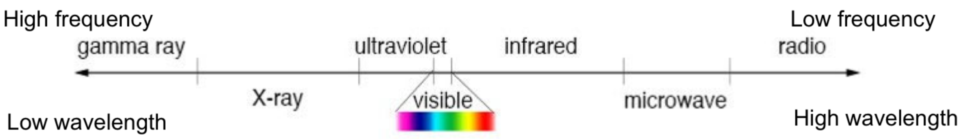
§1 Wave

1. Wave

- A wave can be any kind of oscillation that travels in space and transfers energy from one place to another place.
  - Sound Wave.
  - Radio Wave
- The process of the oscillation with time is described by the wave equation.

2. 波的种类

- **Electromagnetic Waves** (电磁波)
  - Follow Maxwell's Equation
  - Require no medium to travel and all travel at the same speed $c = 299792458\ m/s$ in vacuum
  - Visible Light, radio waves, X-ray and etc.



- **Matter Waves:** (物质波)
  - Follow Schrodinger's Equation and Dirac's Equation
  - Particles in quantum mechanics, like electrons, protons, and other fundamental particles, and even atoms and molecules.

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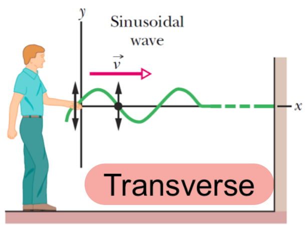
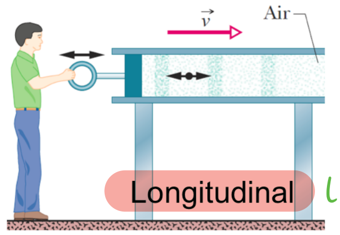
- **Mechanical Waves:** (机械波)
  - Governed by Newton's Law
  - Exist only within **material media** (介质)
  - Water Wave, Sound Wave, and Seismic Wave
  - Transverse and longitudinal waves:



The oscillation is perpendicular to the direction of the wave's travel.



(纵向的)

The oscillation is parallel to the direction of the wave's travel.

§2 Transverse waves

1. Transverse waves (横波)

振动方向与传播方向垂直

- Travelling waves: 从一点移动至另一点
- 物体并不从一点移动至另一点
- Sinusoidal wave (正弦波)

2. Wave equation（波动方程）

- 若 $t=0$ 时，$y = h(x,0) = f(x)$

  则 time t 时，$y = h(x,t) = f(x \pm vt)$

- $\dfrac{\partial h}{\partial t} = \pm v \cdot f'(x \pm vt)$, $\dfrac{\partial h}{\partial x} = f'(x \pm vt)$

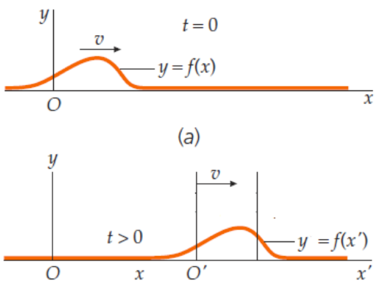
  $\dfrac{\partial^2 h}{\partial t^2} = v^2 \cdot f''(x+vt)$, $\dfrac{\partial^2 h}{\partial x^2} = f''(x+vt)$

- $\dfrac{1}{v^2} \cdot \dfrac{\partial^2 h}{\partial t^2} = \dfrac{\partial^2 h}{\partial x^2}$

- To illustrate a wave on a string, we have a function to describe the wave.
- Describe the motion of any element along its length

$$y = h(x,t)$$

- y: the transverse displacement of any string element
- x: position of the element along the string at time t.



- At t=0 s, the wave is described as:

$$y = h(x,0) = f(x)$$

- At time t, the wave is:

$$y = h(x,t) = f(x')$$

- The wave travels at velocity v

$$f(x - vt) = f(x')$$

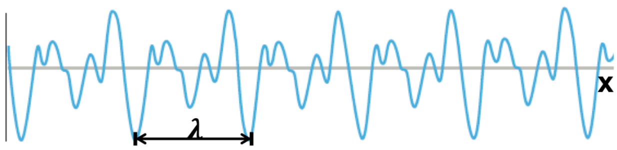
- Since the wave moves toward right.

- Therefore, $h(x,t) = f(x \pm vt)$
- Then the displacement fit the differential equation:

$$\frac{1}{v^2}\frac{\partial^2 h}{\partial t^2} = \frac{\partial^2 h}{\partial x^2}$$

3. Periodic wave（周期波）

沿着波传播的方向 spatially（空间地）重复它的形状



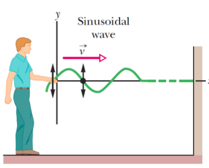
4. Sinusoidal function（波形图）

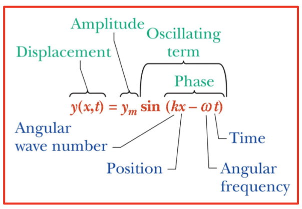
$y(x,t) = y_m \sin(kx - \omega t)$

- The elements oscillate parallel to the y axis. At time t, the displacement y of the element, located at position x, is given by

$$y(x,t) = y_m \sin(kx - \omega t)$$

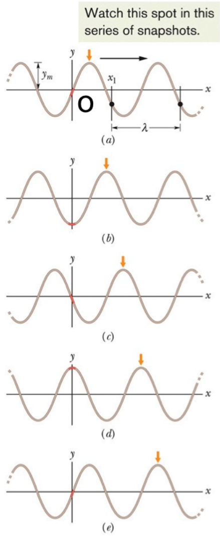
- Give the displacement of the element at position x as a function of time.
- Give the shape of the wave at any given time





- Amplitude y_m (振幅): element 相对于平衡位置的最大位移的大小
- Phase $kx-\omega t$ (相位): 在一个特定的位置 x, phase 随时间线性变化
- Wavelength: λ (波长): 沿着波传播方向, 相同两个波形间的距离.
- Angular wave number k (波数)

$$y(x,0) = y_m \sin kx$$
$$y_m \sin(kx_1) = y_m \sin k(x_1+\lambda)$$
$$\Rightarrow k\lambda = 2\pi$$
$$\Rightarrow k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$



- **Amplitude** y_m: Magnitude of the maximum displacement of the element, relative to its equilibrium position
- **Phase**: At particular position x, the phase changes linearly with time t.
- **Wavelength** λ: the distance (parallel to the direction of the wave's travel) between the repetitions of the shape of the wave.

$$y(x,0) = y_m sinkx$$
$$y_m sinkx_1 = y_m sink(x_1+\lambda)$$
$$k\lambda = 2\pi$$
$$k = \frac{2\pi}{\lambda}$$

- **Angular Wave Number** k (rad/m)

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5. Sinusoidal function (质点振动方程)

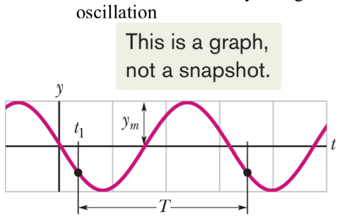
在 $x=0$ 处, 质点做 SHM:

$$y = y_m \sin(-\omega t)$$

且满足

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- **Period** T: the time any string element takes to move through one full oscillation



- ω : **Angular frequency** of the wave
- **Frequency** f: number of oscillations per second.

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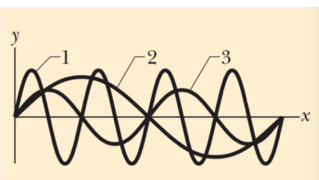
例: **Problem**

Checkpoint 1

The figure is a composite of three snapshots, each of a wave traveling along a particular string. The phases for the waves are given by (a) $2x - 4t$, (b) $4x - 8t$, and (c) $8x - 16t$. Which phase corresponds to which wave in the figure?



$$k = \frac{2\pi}{\lambda}$$

Answer:
a. 2
b. 3
c. 1

6. Phase constant（相位常量）

1° Without phase constant

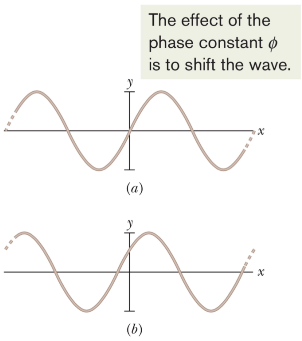
$\varnothing = 0$

2° With phase constant

$y(x,t) = y_m \sin(kx - \omega t + \varnothing)$

$\varnothing$为正：图像往x轴负方向移

$\varnothing$为负：图像往x轴正方向移



- (a) Without Phase Constant,

$$\varnothing = 0$$

- (b) With Phase Constant
  $y(x,t) = y_m \sin(kx - \omega t + \varnothing)$
- Other terms donot change
- A positive value of $\varnothing$ shifts the curve in the negative direction of the x axis
- A negative value shifts the curve in the positive direction.

7. the speed of traveling wave（波速）

- $V = \pm\frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$

若向正方向传播：$V = \frac{\omega}{k}$

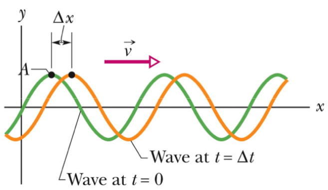
若向负方向传播：$V = -\frac{\omega}{k}$

- At a short time interval Δt, the entire wave pattern moves a distance Δx.
- Wave speed

$$v = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Each point of the wave form retains its displacement

$$kx - \omega t = constant$$



$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} = v$$

$$v = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} = \lambda f$$

T: time for one complete wave to pass

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- 波的表达式为

$h(x,t) = f(kx \pm \omega t)$

f可以表示任何函数，sine为其中之一

- 注意区分波速与 the oscillating speed $v = \frac{\partial h}{\partial t}$ of the medium particle.

$V_m = \omega \cdot f_m$（f_m为振幅）

例：

Checkpoint 2

Here are the equations of three waves:
(1) $y(x,t) = 2\sin(4x - 2t)$, (2) $y(x,t) = \sin(3x - 4t)$, (3) $y(x,t) = 2\sin(3x - 3t)$.
Rank the waves according to their (a) wave speed and (b) maximum speed perpendicular to the wave's direction of travel (the transverse speed), greatest first.

$v = \frac{\omega}{k}$

Answer: (a) 2, 3, 1; (b) 3, 1&2

$v_m = \omega f_m$

There are two velocities in a travelling wave:

1) The wave velocity **v** is constant

2) The particle velocity **u** is a function of x and t

例:

Problem

A wave traveling along a string is described by

$$y(x,t) = 0.00327\, sin(72.1x - 2.72t)$$

in which the numerical constants are SI units (0.00327 m, 72.1 rad/m, and 2.72 rad/s).

a) What is the amplitude of this wave?

b) What are the wavelength, period, and frequency of this wave?

c) What is the velocity of this wave?

d) What is the displacement of the string at x = 22.5 cm and t = 18.9s?

Solution:

$$y(x,t) = 0.00327 \;\sin(72.1x - 2.72t)$$

(a) $y_m = 0.00327\ m = 3.27\ mm$

(b) $k = 72.1 rad / m \qquad \omega = 2.72 rad / s$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{72.1} = 8.71 cm \qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{2.72} = 2.31 s$$

$$f = \frac{1}{T} = \frac{1}{2.31 s} = 0.433\ Hz$$

(c) $v = \frac{\omega}{k} = \frac{2.72}{72.1} = 0.0377\ m/s$ v is the speed of the wave

(d) $y = 0.00327 \;\sin(72.1 \times 0.225 - 2.72 \times 18.9)$

$= 0.00327 \;\sin(-35.1855\ rad) = 1.92\ mm$

例:

Problem

In the preceding sample problem, we showed that at t = 18.9 s the transverse displacement y of the string at x = 22.5 cm due to the wave of Eq.

$$y(x,t) = 0.00327\, sin(72.1x - 2.72t)$$

is 1.92 mm.

a) What is the transverse velocity u of the same element of the string at that time?

b) What is the transverse acceleration a_y of the same element at that time?

Solution:

(a) $u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$

u is the speed of a particle in the wave

$$u = (-2.72 rad / s)(3.27\ mm)\cos(-35.1855\ rad) = 7.20\ mm / s$$

(b) $a_y = \frac{\partial u}{\partial t} = -\omega^2 y_m \sin(kx - \omega t) = -\omega^2 y$

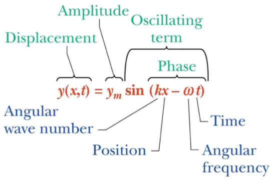
$$a_y = -(2.72 rad / s)^2 (1.92\ mm) = -14.2 mm / s^2$$

Summary

- Mechanical Waves
- Transverse and Longitudinal Waves
- Equation of a Travelling Wave

$$y(x,t) = h(x \pm vt) = h(kx \pm \omega t)$$

- Sinusoidal Waves



Summary

- Wavelength λ

$$k = \frac{2\pi}{\lambda}$$

- The Period T and Frequency f:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Wave Speed v of Traveling Waves:

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$