Lecture 3

§1 Newtonian Mechanics

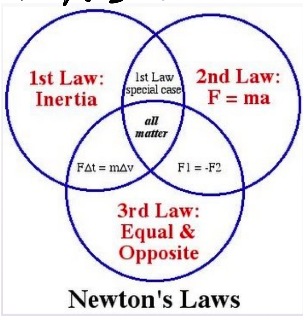
1. Isaac Newton

于 later 1600s 第一个理解 forces.

2. Newton's laws of motion

1° 适用于经典物理学中的运动物体

2° 由大量实验验证 (vertify)



3. Newtonian mechanics (牛顿力学)

1° 对 force 与 acceleration 的研究

2° 当速度过大 (接近光速) 时，被 Einstein's special theory of relativity (爱因斯坦狭义相对论)

3° 当物体过小 (在原子 $10^{-10}m$ 或纳米 $10^{-9}m$ 尺度) 时，被 quantum mechanics (量子力学).

§2 Newton's First Law

1. Force

1° 作用于物体，改变 velocity，或产生 acceleration.

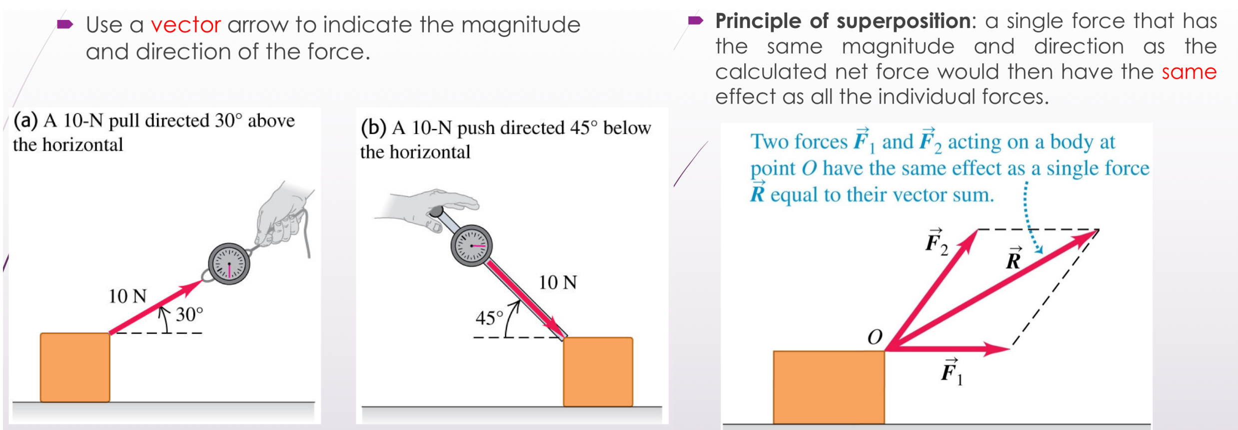
2° 单位：Newton (N)

$$1\ N = 1\ kg \times 1 m/s^2$$

3° vector：(1) 与加速度方向相同

(2) Net force：将单独的 force vectors 相加

(3) 遵从 principle of superposition (叠加原理)



2. Newton's first law

Newton's First Law: If no force acts on a body, the body's velocity cannot change; the body cannot accelerate

Newton's First Law: If no Net force $\vec{F}$ ($\vec{F} = 0$) acts on a body, the body's velocity cannot change; the body cannot accelerate

If an object has no acceleration, can you conclude there are no forces acting on it? Explain.

No, there may be several forces acting on an object but the net force, i.e., the vector sum of the forces, is zero.

If a body's velocity is constant, we can immediately say that the Net force $\vec{F}$ is zero ($\vec{F} = 0$)

1° 惯性参考系 (inertial reference frame)

静止或以恒定速度运动的参照系为惯性参照系

- An inertial reference frame is one in which Newton's law hold
- Assume ground is the inertia reference frame
  - Neglect Earth's astronomical motion
  - Curling Sports refer to the ground ✓
  - An accelerating elevator is a non-inertial frame.
- A reference frame at rest or with constant velocity is an inertial frame.
  - e.g. earth/solar system/FK_4 system

2° 惯性定理 (Law of inertia)

moving body → keep moving
at rest → at rest
} unless it is acted on by a net force

More Examples

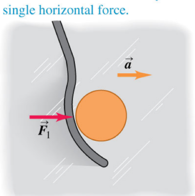
- Figure a: Net force acts, causing acceleration
- Figure b: No net force acts, resulting in no acceleration

Equilibrium (平衡状态)

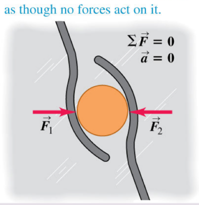
Law of inertia
- A body at rest, or moving with constant velocity, will remain at rest, or moving with constant velocity, unless it is acted on by a net force
- Tendency
  - Moving body → keep moving
  - At rest → at rest

$\vec{F} = 0$ ($\sum Fx = 0$ and $\sum Fy = 0$.) → EQUILIBRIUM

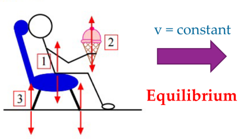
(a) A puck on a frictionless surface accelerates when acted on by a single horizontal force.



(b) An object acted on by forces whose vector sum is zero behaves as though no forces act on it.



A "simple" example ....

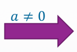


Equilibrium

[1] The force on you due to the Earth, i.e., your weight, (*downward*) is balanced by the force of the seat of the chair on you (*upward*). No net force.

[2] The force on the ice cream cone due to the Earth (*downward*) is balanced by the force of your hand on the cone (*upward*). No net force.

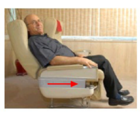
[3] The total weight of you, the ice cream cone and the chair (*downward*) is balanced by the force of the ground on the chair legs (*upward*). No net force.

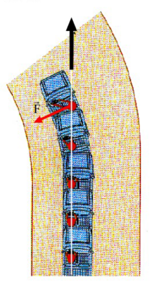


Acceleration

Did you apply "throwing" force to the seat?

Are you "thrown" backwards when a car accelerates from rest? *NO!! There is no "throwing" force; you tend to remain at rest as the seat moves forward!*



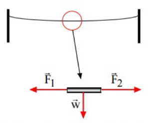


Are you "thrown" outwards when a car turns a corner? *NO!! There is no "throwing" force; as the car turns, you tend to continue to move in a straight line; so the side of the car exerts an inwards force on you!*

Law of inertia
A body at rest, or moving with constant velocity, will remain at rest, or moving with constant velocity, unless it is acted on by a net force

3° Newton's Law 无法被证明

Why does a wire stretched between two posts always sag in the middle no matter how tightly it is stretched?



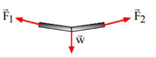
Assume wire is horizontal and consider a small section of wire in the middle:

As the wire is in equilibrium, i.e., no motion, then there is no net force, i.e.,

$|\vec{F}_1| = |\vec{F}_2|$

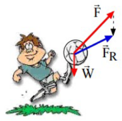
... but there are no vertical components of $\vec{F}_1$ a... compensate $\vec{w}$ and produce no net vertical force. Therefore, our assumption that the wire is horizontal is *incorrect*; clearly, to compensate for $\vec{w}$, $\vec{F}_1$ and $\vec{F}_2$ must have vertical components. Therefore, they must "slope upward", i.e., the wire must sag in the middle, viz:

$\vec{F} = 0$ ($\sum Fx = 0$ and $\sum Fy = 0$.)
→ EQUILIBRIUM

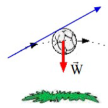


Equilibrium

Two forces: the weight of the ball downward (due to the attraction of the Earth) is *balanced* by the "normal" force of the ground on the ball.
No net force ⇒ no motion



Two forces: the weight of the ball downward and the force of the foot on the ball.
Net resultant force ⇒ change in motion
acceleration



One force: the weight of the ball downward.
Net force ⇒ change in motion
acceleration

Important concept ...

... we cannot neglect the effect of the Earth ... it always produces a non-contact force!

Can Newton's Law be proven?
The answer is no, because it's impossible to be sure that your reference frame is without any accelerations.

Do we believe in this?
Yes, we do.
We believe in it since it is consistent within the uncertainty of the measurements with all experiments that have been done.

§3 Newton's Second Law

1. Mass and weight

1° mass (质量)

The characteristic that relates a force on the body to the resulting acceleration

Intrinsic characteristic (内在特性)

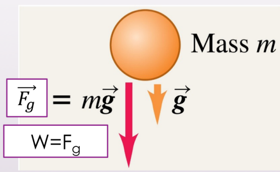
2° weight (重力)

The magnitude of the net force required to prevent the body from falling freely.

Mass

- Common sense: the object with large mass accelerates less
- What is mass?
  - Size
  - Density
  - Weight (always confusing)
  - The amount of matter → how to define it?
  - The characteristic that relates a force on the body to the resulting acceleration. Intrinsic Characteristic

The weight of a body is the magnitude of the net force required to prevent the body from falling freely, as measured from the reference frame of the ground.



On Moon, the weight of an object changes, since g is different.
While, the mass of the object doesn't change with locations.

2. Newton's second law

The net force on a body is equal to the product of the body's mass and its acceleration.

1° 每条坐标轴上的 acceleration component 仅由同一轴上的 force components 的和决定，与其他轴上的力无关

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net,x} = m\vec{a}_x,\ \vec{F}_{net,y} = m\vec{a}_y,\ \vec{F}_{net,z} = m\vec{a}_z$$

2° Units in Newton's Second Law

$\vec{F}_{net} = 0$ ($\sum Fx = 0$ and $\sum Fy = 0$.)
→ EQUILIBRIUM

$\vec{F}_{net} = 0 \rightarrow \vec{a}_{net} = 0$

$$1\,\mathrm{N} = (1\,\mathrm{kg})(1\,\mathrm{m/s^2}) = 1\,\mathrm{kg \cdot m/s^2}$$

TABLE 5-1 Units in Newton's Second Law

System	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
CGS[a]	dyne	gram (g)	cm/s^2
British[b]	pound (lb)	slug	ft/s^2

[a] 1 dyne = 1 $g \cdot cm/s^2$.
[b] 1 lb = 1 $slug \cdot ft/s^2$.

3. Free-body diagram (受力分析图)

1° 将 body 视为 dot

2° 多个 bodies 的组合被称为系统 (system)

3° 系统外作用于系统内物体的力为外力 (external force)

4° net force $\vec{F}$ 是所有外力的向量和.

Free-body Diagram

To solve the problem with Newton's Second Law

Free-body diagram

- In a coordinate system, we represent the body as a dot, each force on the body is a vector arrow with its tail on the body.
- A collection of two or more bodies is called a system, and any force on the bodies inside the system caused by external agents outside the system is called an external force. If the bodies are rigidly connected, then we can treat the system as one composite body, and the net force $\vec{F}$ on it is the vector sum of all external forces.

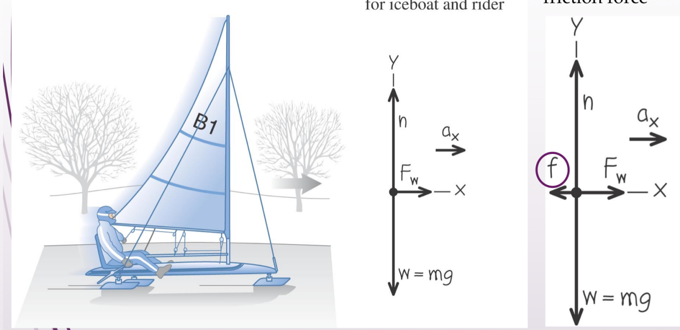
Examples

The wind exerts a constant horizontal force on the boat.

(a) Iceboat and rider on frictionless ice

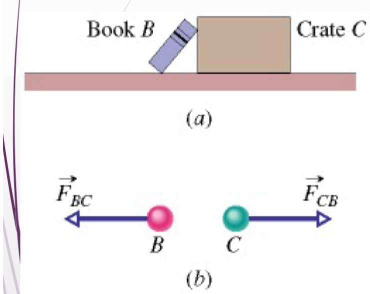
(b) Free-body diagram for iceboat and rider

With horizontal friction force



§4 Newton's Third Law

Newton's Third Law

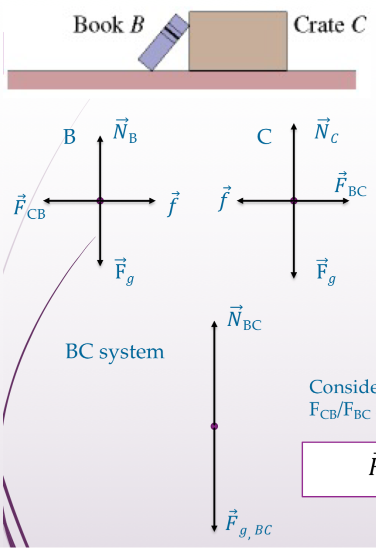


$$\vec{F}_{CB} = -\vec{F}_{BC}$$

- Interact: Push or Pull each other
- When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.
- *Third-law Force Pair*

Every action has an equal and opposite reaction.

Be careful how you treat others.



Consider B and C separately:
- External forces
- $\vec{F}_{CB}$ is the force from crate on book and $\vec{F}_{BC}$
- is the force from book on crate.

Consider B and C as a system: F_{CB}/F_{BC} are internal forces

$$\vec{F}_{CB} = -\vec{F}_{BC}$$

§5. Some particular force

1、种类

- Gravitational force (引力)
- Normal force (法向力)
- Friction (摩擦力)
- Tension (张力)

or

- contact forces (接触力): push, pull, friction, etc. ⟶ direct force
- non-contact (or field) forces (非接触力): gravity, magnetic, electric, etc.

⟶ "action" at a distance and long range.

2、引力

Gravitational Force

- A force that pulls on a body directly toward the center of Earth (Non-contact Force)

$$\vec{F}_g = m\vec{a}$$

- Free-Fall, the acceleration

$$a = -g$$

$$-F_g = m\times(-g) \rightarrow F_g = mg$$

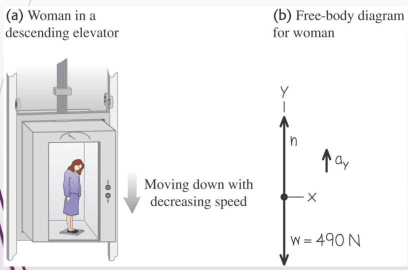
- Weight: the magnitude of the net force required to prevent the body from falling freely. (Scalar)

$$W - F_g = 0 \rightarrow W = F_g = mg$$

- Weighing
  - Balance/Spring Scale (秤)
  - Must measure in the reference frame without acceleration

Weighing

A woman inside the elevator is standing on a scale. How will the acceleration of the elevator affect the scale reading?



$\vec{a_y}$: +ve Y-axis

$$W - mg = ma_y$$

$$W = m(g + a_y)$$

$\vec{a_y}$: -ve Y-axis

$$W - mg = -ma_y$$

$$W = m(g - a_y)$$

Problems

Free-fall is motion in which gravity is the only force acting on an object.

(a) Is a skydiver who has reached terminal speed in free-fall? ... explain why or why not.

No, the skydiver is not in free-fall because in addition to the force due to gravity there is a resisting force due to the air.

When the terminal velocity is reached, the two forces are equal in magnitude but in opposite directions, so there is no net force and the diver continues at constant velocity.

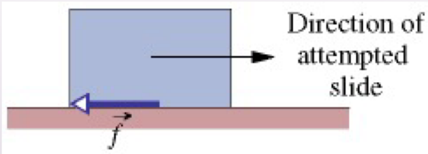
(b) Is a satellite circling the Earth in space in free-fall? ... explain why or why not.

Yes, a satellite is in free-fall because there is no air resistance. Therefore, the only force acting on the satellite is centripetal and it is the gravitational force.

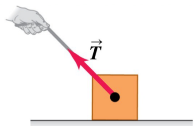
3. 摩擦力

Friction

- If we slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. The resistance is considered to be a single force $\vec{f}$, called the frictional force, or simply friction.
- This force is directed along the surface, opposite the direction of the intended motion. Sometimes, to simplify a situation, friction is assumed to be negligible (the surface is frictionless). (光滑的)

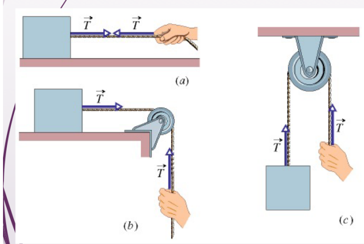


4. 张力



Tension

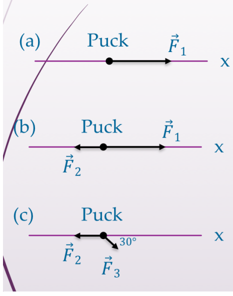
- Tension: A pulling force exerted on an object by a rope or cord. This is a contact force.
- Assumptions: Cord (绳) → Massless; Pulley (滑轮) → Frictionless
- A string has a single tension force (magnitude). The direction depends on the body on which this force acts upon. The tension forces on two sides of a frictionless pulley are the same in magnitude.



- Net Force on pulley: Vector sum of Tensions
- E.g. (c) the net force on the pulley from the cord: $-2\vec{T}$
- How about (b)?

5. Applying Newton's Law

Forces act on the puck that moves over frictionless ice along an x-axis. The puck mass is 0.20 kg. Forces are directed as shown in the fig below, $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ have magnitudes of 4.0, 2.0, and 1.0 N, respectively. In each situation, what is the acceleration of the puck?



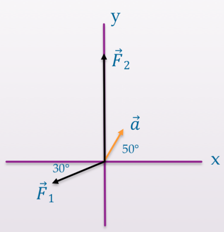
a) $a_x = \frac{F_1}{m} = \frac{4.0}{0.20} = 20\ m/s^2$
Accelerating in +ve x-axis direction

b) $a_x = \frac{F_1 - F_2}{m} = \frac{4.0-2.0}{0.20} = 10\ m/s^2$
Accelerating in +ve x-axis direction

c) $a_x = \frac{F_3 \times \cos 30° - F_2}{m} = -\frac{2.0 - 1.0\cos 30°}{0.20} = -5.7\ m/s^2$
in –ve x-axis direction

More Problems

Here we find a missing force by using the acceleration. In the overhead view of Fig., a 2.0 kg cookie tin is accelerated at 3.0 m/s² in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. what is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?



SOLUTION

$$\vec{F}_{net} = m\vec{a}$$
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}$$

x-axis:

$$F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$
$$= 2.0 \times 3.0 \times \cos 50° - (-10 \times \cos 30°) - 0 = 12.5\ N$$

y-axis:

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$
$$= 2.0 \times 3.0 \times \sin 50° - (-10 \times \sin 30°) - 20 = -10.4\ N$$

In unit-vector notation

$$\vec{F}_3 = (12.5\ N)\hat{\imath} - (10.4\ N)\hat{\jmath}$$

In magnitude-angle notation

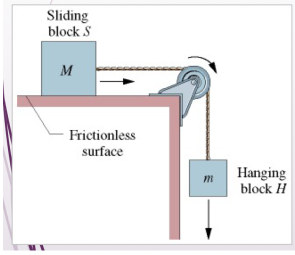
$$F_3 = \sqrt[2]{F_{3,x}^2 + F_{3,y}^2} = 16.3\ N$$

$$\theta = \tan^{-1}\frac{F_{3,y}}{F_{3,x}} = -40°$$

PAY ATTENTION TO THE DIRECTION!

(滑块)

The figure shows a block S (the *sliding block*) with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface such as an air table. We assume that the table is an inertia frame. This first block is connected by a cord that wraps over a frictionless pulley to a second block H (the hanging block), with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks (they are "massless"). The hanging block H falls as the sliding block S accelerates to the right.



Find (a) the acceleration of the sliding block S,
(b) the acceleration of the hanging block H, and
(c) the tension in the cord.

SOLUTION

For S, $\vec{T} = M\vec{a}_S$

For H, $\vec{F}_g + \vec{T} = m\vec{a}_H$

(Vector sum of the forces acting on m)

$-mg + Ma_S = -ma_H$

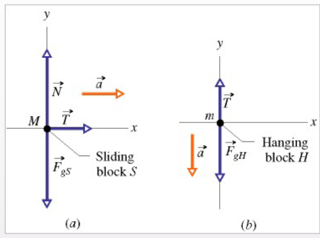
The +ve direction is upward

$a_S = a_H = a$

The two objects moving at the same speed and magnitude of acceleration

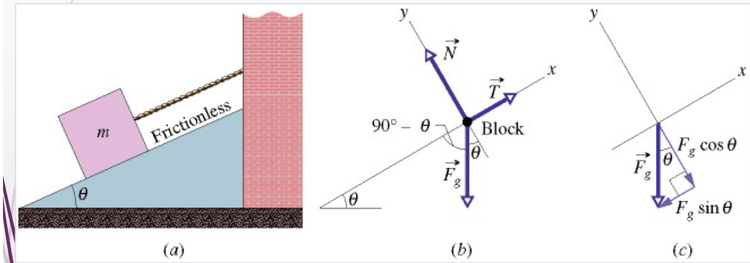
$a = \dfrac{mg}{M+m} = 3.8\ m/s^2$

$T = Ma = 3.3\times 3.8 = 13\ N$



More Problems

In the figure, a cord holds a stationary block of mass $m = 15$ kg, on a frictionless plane that is inclined at angle = 27°. (a) What are the magnitudes of the force $\vec{T}$ on the block from the cord and the normal force $\vec{N}$ on the block from the plane?



SOLUTION:

$$\vec{T} + \vec{N} + \vec{F}_g = 0$$

$$T + 0 - mg\sin\theta = 0$$

$$T = mg\sin\theta$$

$$= (15\ \text{kg})(9.8\ \text{m/s}^2)(\sin 27^\circ)$$

$$= 67\ \text{N}$$

$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$$= (15\ \text{kg})(9.8\ \text{m/s}^2)(\cos 27^\circ)$$

$$= 131\ \text{N}$$

If We now cut the cord. As the block then slides down the inclined plane, does it accelerate? If so, what is its acceleration?

SOLUTION:

$$mg\sin\theta = ma$$

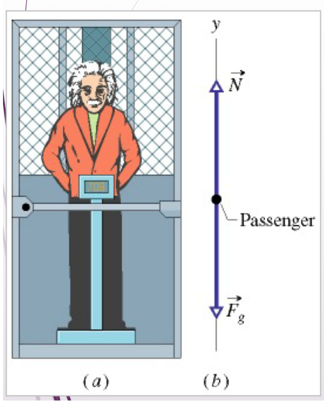
Positive direction is down the inclined plane

$$a = g\sin\theta$$

$$a = (9.8\ \text{m/s}^2)(\sin 27^\circ) = 4.4\ \text{m/s}^2$$

Direction: Downwards along the plane surface

In the figure, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary, and when it is moving up or down. (a) Find a general solution for the scale reading, whatever the vertical motion of the cab.



SOLUTION:

$N - mg = ma$

$N = m(g + a)$

With +ve direction upward

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

SOLUTION:

Constant Velocity, no net force acting on the body

$N = m(g + a) = mg = 707.6\ \text{N}$

(c) What does the scale read if the cab accelerates upward at 3.20 m/s² and downward at 3.20 m/s²?

SOLUTION:

Moving upward

$a = +3.2\ m/s^2$

$N = m(g + a) = 72.7\times(9.8 + 3.2)\ \text{N} = 938.6\ \text{N}$

Moving downward

$a = -3.2\ m/s^2$

$N = m(g + a) = 72.2\times(9.8 - 3.2)\ \text{N} = 476.5\ \text{N}$

(d) During the upward acceleration in part (c), what is the magnitude F_{net} of the net force on the passenger?

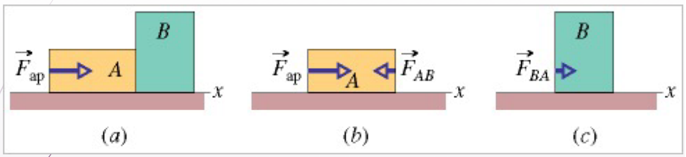
SOLUTION:

Moving upward/downward

$\vec{a} = \pm 3.2\ m/s^2$

$F_{net} = ma = 72.2\times 3.2\ N = 231.0\ N$

In the figure, a constant horizontal force $\vec{F}_{ap}$ of magnitude 20 N is applied to block A of mass m_A = 4.0 kg, which pushes against block B of mass m_B = 6.0 kg. The blocks slide over a frictionless surface, along an x axis. (a) What is the acceleration of the blocks?



SOLUTION:

$$F_{net} = ma$$

$20\ N = 4.0\ kg \times a$ → ~~a=5.0 m/s²~~

1. Identify A and B as one system

$$20\ N = (4.0 + 6.0)\ kg \times a \rightarrow a = 2.0\ m/s^2$$

2. Analyze each block separately

A: $F_{ap} - F_{AB} = m_A \times a_A$

B: $F_{BA} = m_B \times a_B$

$F_{AB} = F_{BA}$ & $a_A = a_B = a$

$F_{ap} - m_B \times a = m_A \times a$

$20\ N - 6.0\ kg \times a = 4.0\ kg \times a \rightarrow a = 2.0\ m/s^2$

+ve direction along x-axis

(b) What is the horizontal force $\vec{F}_{BA}$ on block B from block A in Fig(c)?

$$F_{BA} = m_B \times a_B = 6.0 \times 2.0\ N = 12.0\ N$$

+ve direction along x-axis

- Remember to indicate the direction for vectors.

  Velocity, Acceleration, Force, and etc.

Summary

- Newtonian Mechanics
- Force
- Newton's First Law
- Inertial Reference System
- Mass
- Newton's Second Law
- Newton's Third Law
  - Free-body Diagram
  - Some Particular Forces: Gravitational Force, weight, normal forces, Friction and Tension

Summary

If no *Net force* $\vec{F}$ ($\vec{F} = 0$) acts on a body, the body's velocity cannot change; the body cannot accelerate

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$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net,x} = m\vec{a}_x,\ \vec{F}_{net,y} = m\vec{a}_y,\ \vec{F}_{net,z} = m\vec{a}_z$$

When the two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

$$\vec{F}_{CB} = -\vec{F}_{BC}$$