Lecture 24

§1 Intensity of sound

1. Intensity of sound (声强)

1° 对于一个表面的声强：单位时间单位面积穿过该曲面的能量

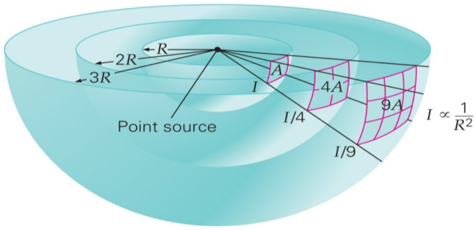
$$I = \frac{P}{A}$$

P 为 power (time rate of energy transferring)

2° 对于一个 isotropic point source with power P_s：

$$I = \frac{P_s}{4\pi r^2}$$

- Where, P is the power (time rate of energy transferring)



- For an isotropic point source with power P_s

$$I = \frac{P_s}{4\pi r^2}$$

- The intensity of sound from an isotropic point source decreases with the square of the distance r from the source.

2. 声强与振幅 S_m 的关系

$$I = \frac{1}{2}\rho v \omega^2 S_m^2$$

证：a thin slice of air of thickness dx, area A, and mass dm

- 考虑 K.E.

$$dK = \frac{1}{2} dm \times u^2$$

$$u = \frac{\partial S}{\partial t} = \omega S_m \sin(kx - \omega t)$$

$$dK = \frac{1}{2}(\rho A dx)[\omega S_m \sin(kx - \omega t)]^2$$

$$= \frac{1}{2}\rho A v dt\ \omega^2 S_m^2 \sin^2(kx - \omega t)$$

$$\frac{dK}{dt}_{avg} = \frac{1}{4}\rho A v \omega^2 S_m^2$$

- 势能与动能相等

$$P_{avg} = \left(\frac{dE}{dt}\right)_{avg} = 2 \times \left(\frac{dK}{dt}\right)_{avg} = \frac{1}{2}\rho A v \omega^2 S_m^2$$

- 因此，

$$I = \frac{P_{avg}}{A} = \frac{1}{2}\rho v \omega^2 S_m^2$$

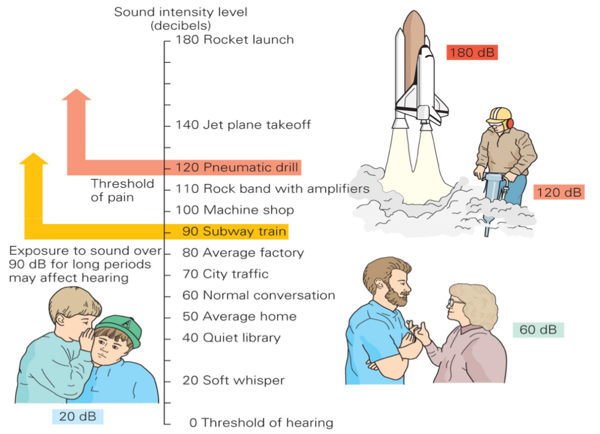
3. Sound level (声级) β

比起声强，声级用起来更方便

$$\beta = (10\,dB)\lg\frac{I}{I_0}$$

- 单位：decibel (dB)
- Standard reference intensity：$I_0 = 10^{-12}\ W/m^2$

Some Sound Levels



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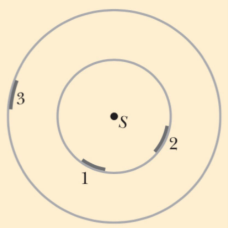
例:

Problem

Checkpoint 2

The figure indicates three small patches 1, 2, and 3 that lie on the surfaces of two imaginary spheres; the spheres are centered on an isotropic point source S of sound. The rates at which energy is transmitted through the three patches by the sound waves are equal. Rank the patches according to (a) the intensity of the sound on them and (b) their area, greatest first.



$$I = \frac{P_s}{4\pi r^2}$$

$$I = \frac{P}{A}$$

$$P = \frac{A}{4\pi r^2} P_s$$

Answer: (a) $I_1 = I_2 > I_3$
(b) $A_3 > A_2 = A_1$

例:

Problem

An electric spark jumps along a straight line of length ***L*=10 m**, emitting a pulse of sound that travels radially outward from the spark. (The spark is said to be a line source of sound.) The power of this acoustic emission is **P_s =1.6x10⁴ W**.

a) What is the intensity ***I*** of the sound when it reaches a distance ***r* =12 m** from the spark?

b) At what time rate **P_d** is sound energy intercepted by an acoustic detector of area **A_d =2.0 cm²**, aimed at the spark and located a distance ***r* =12 m** from the spark?

Solution:

a)The Area is

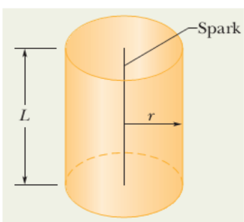
$$A = 2\pi r L$$

$$I = \frac{P_s}{A} = \frac{P_s}{2\pi r L} = 21.2\,\frac{W}{m^2} \approx 21\ W/m^2$$

b)By a detector with area A_d

$$I = \frac{P_d}{A_d}$$

$$P_d = I A_d = 4.2\ mW$$



例：

Problem

Many veteran rockers suffer from acute hearing damage because of the high sound levels they endured for years. Many rockers now wear special earplugs to protect their hearing during performances (Fig. 17-11). If an earplug decreases the sound level of the sound waves by 20 dB, what is the ratio of the final intensity I_f of the waves to their initial intensity I_i?

Solution: The difference in the sound levels is

$$\beta_f - \beta_i = (10\ \text{dB})\left(\log\frac{I_f}{I_0} - \log\frac{I_i}{I_0}\right).$$

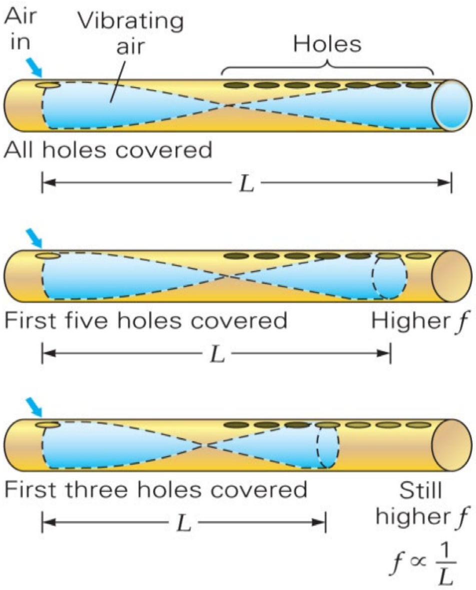
$$\beta_f - \beta_i = (10\ \text{dB})\log\frac{I_f}{I_i}.$$

$$\frac{I_f}{I_i} = \log^{-1}(-2.0) = 0.010.$$

§2 Standing waves in pipes

1. Musical sound

- 很多乐器，如木管乐器与铜管乐器，通过管内的驻波制造声音。驻波由声波在管的boundaries反射，并互相干涉而成.
- 通过调整管的boundaries，可以调整管内驻波的mode.



2. Standing waves in pipes

1° Two open ends

类似于横波的 two free boundaries:

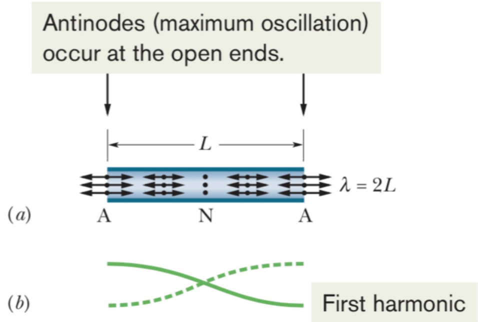
在 boundaries 处为 antinodes with maximum oscillation

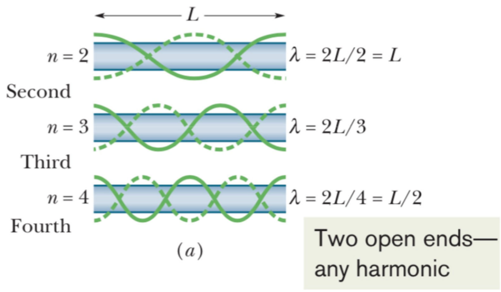
因此，

$$\lambda = \frac{2L}{n}$$

Resonant frequency 为

$$f = \frac{v}{\lambda} = \frac{nv}{2L} \quad (n = 1, 2, 3, \cdots\cdots)$$





2° One open end

类似于 one hard boundary, and one free boundary:

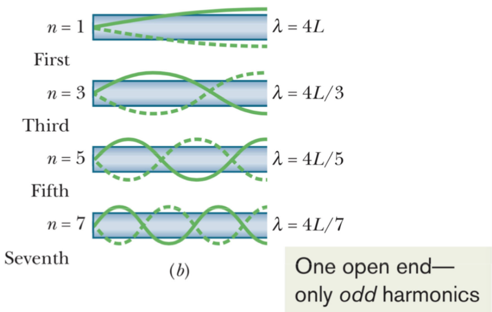
One is antinode and one is node.

因此，

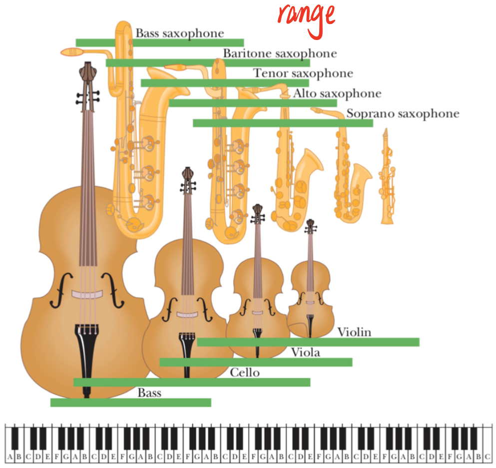
$$\lambda = \frac{4L}{n}$$

Resonant frequency 为

$$f = \frac{v}{\lambda} = \frac{nv}{4L} \quad (n = 1, 3, 5, \cdots\cdots)$$



- The length of a musical instrument reflects the ~~rang~~ range of frequencies



3. Net wave

The quality of a sound (音色) 取决于 the shape of its waveform.

The pitch of a sound (音调) 取决于 the fundamental frequency (基频)

Fundamental frequency 会被 overtones (泛音) 加强，且形成自己独特的音色.

Fundamental frequency 与 overtones 合成之后形成最后的波形.

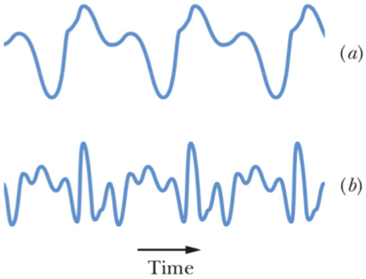
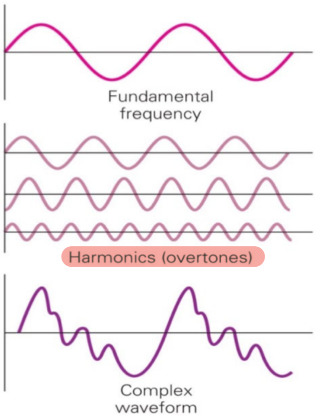


Figure 17-16 The wave forms produced by (*a*) a flute and (*b*) an oboe when played at the same note, with the same first harmonic frequency.



例:

Problem

Pipe A is open at both ends and has length $L_A = 0.343$ m. We want to place it near three other pipes in which standing waves have been set up, so that the sound can set up a standing wave in pipe A. Those other three pipes are each closed at one end and have lengths $L_B = 0.500L_A$, $L_C = 0.250L_A$, and $L_D = 2.00L_A$. For each of these three pipes, which of their harmonics can excite a harmonic in pipe A?

Solution

Pipe A: Let's first find the resonant frequencies of symmetric pipe A (with two open ends) by evaluating Eq. 17-39:

$$f_A = \frac{n_A v}{2L_A} = \frac{n_A(343 \text{ m/s})}{2(0.343 \text{ m})}$$

$$= n_A(500 \text{ Hz}) = n_A(0.50 \text{ kHz}), \quad \text{for } n_A = 1, 2, 3, \ldots.$$

The first six harmonic frequencies are shown in the top plot in Fig. 17-17.

Pipe B: Next let's find the resonant frequencies of asymmetric pipe B (with only one open end) by evaluating Eq. 17-41, being careful to use only odd integers for the harmonic numbers:

$$f_B = \frac{n_B v}{4L_B} = \frac{n_B v}{4(0.500L_A)} = \frac{n_B(343 \text{ m/s})}{2(0.343 \text{ m})}$$

$$= n_B(500 \text{ Hz}) = n_B(0.500 \text{ kHz}), \quad \text{for } n_B = 1, 3, 5, \ldots.$$

Comparing our two results, we see that we get a match for each choice of n_B:

$$f_A = f_B \quad \text{for } n_A = n_B \quad \text{with } n_B = 1, 3, 5, \ldots. \quad \text{(Answer)}$$

Pipe C: Let's continue with pipe C (with only one end) by writing Eq. 17-41 as

$$f_C = \frac{n_C v}{4L_C} = \frac{n_C v}{4(0.250L_A)} = \frac{n_C(343 \text{ m/s})}{0.343 \text{ m/s}}$$

$$= n_C(1000 \text{ Hz}) = n_C(1.00 \text{ kHz}), \quad \text{for } n_C = 1, 3, 5, \ldots.$$

From this we see that C can excite some of the harmonics of A but only those with harmonic numbers n_A that are twice an odd integer:

$$f_A = f_C \quad \text{for } n_A = 2n_C, \quad \text{with } n_C = 1, 3, 5, \ldots. \quad \text{(Answer)}$$

Pipe D: Finally, let's check D with our same procedure:

$$f_D = \frac{n_D v}{4L_D} = \frac{n_D v}{4(2L_A)} = \frac{n_D(343 \text{ m/s})}{8(0.343 \text{ m/s})}$$

$$= n_D(125 \text{ Hz}) = n_D(0.125 \text{ kHz}), \quad \text{for } n_D = 1, 3, 5, \ldots.$$

As shown in Fig. 17-17, none of these frequencies match a harmonic frequency of A. (Can you see that we would get a match if $n_D = 4n_A$? But that is impossible because $4n_A$ cannot yield an odd integer, as required of n_D.) Thus D cannot set up a standing wave in A.

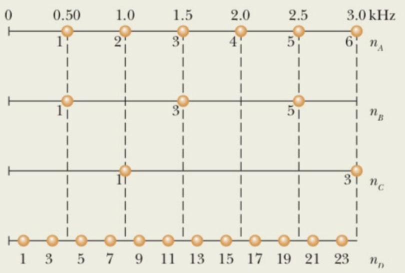


Figure 17-17 Harmonic frequencies of four pipes.

Summary

- Sound Intensity

$$I = \frac{P}{A}$$

- Sound Intensity

$$I = \frac{P_s}{4\pi r^2} \text{ and } I = \frac{1}{2}\rho v \omega^2 s_m^2$$

- Sound Level in Decibels (dB)

$$\beta = (10\ dB) log \frac{I}{I_0}$$

- Standing Wave Patterns in Pipes
  - Two Open Ends

$$f = \frac{v}{\lambda} = \frac{nv}{2L} (n = 1, 2, 3 \dots)$$

  - One Closed End and One Open End

$$f = \frac{v}{\lambda} = \frac{nv}{4L} (n = 1, 3, 5 \dots)$$