Lecture 25

§1 Beats

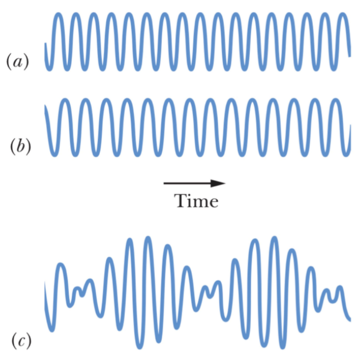
1、Beats (拍)

- 当两波频率 f_1 与 f_2 偏差很小时，它们合成之后会形成 beats
- 若两个有相同振幅，相似频率的波发生干涉时，resultant wave 会有 intensity fluctuating，频率为两波的 frequency difference —— beat phenomena.

  $f_{beat} = |f_1 - f_2|$

2、推导

Beats



- Wave (a)

$$s_1 = s_m \cos(kx + \omega_1 t)$$

- Wave (b)

$$s_2 = s_m \cos(kx + \omega_2 t)$$

- The net wave

$$s' = s_1 + s_2$$

- Since

$$\cos\alpha + \cos\beta$$
$$= 2\cos[\tfrac{1}{2}(\alpha - \beta)]\cos[\tfrac{1}{2}(\alpha + \beta)]$$

- Thus,

$$s' = 2s_m \cos\omega' t \cos(kx + \omega t)$$

- Where $\omega' = \frac{1}{2}(\omega_1 - \omega_2)$ and $\omega = \frac{1}{2}(\omega_1 + \omega_2)$

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Beats

$$s' = 2s_m \cos\omega' t \cos(kx + \omega t)$$

- The frequencies are slightly different from each other

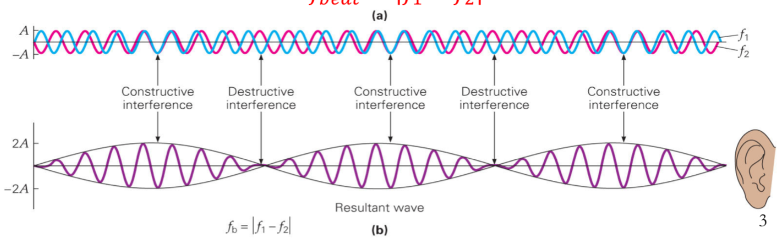
$$\omega \gg \omega'$$

- Regard the function as a cosine function with amplitude of s'_m fluctuating with time.

$$s'_m = |2s_m \cos\omega' t|$$

$$\omega_{beat} = 2\omega' = |\omega_1 - \omega_2|$$

$$f_{beat} = |f_1 - f_2|$$



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Additional Information

- The intensity of the beats: $I = \frac{1}{2}\rho v \omega^2 s_m^2$

$$s'(x,t) = 2s_m \cos\omega' t \cos(kx + \omega t)$$

$$I = \frac{1}{2}\rho v \omega^2 4 s_m^2 \cos^2(\omega' t)$$

- Since $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$,

$$I = \frac{1}{2}\rho v \omega^2 4 s_m^2 \times \frac{1}{2}[1 + \cos(2\omega' t)]$$
$$= \rho v \omega^2 s_m^2 + \rho v \omega^2 s_m^2 \cos(\Delta\omega t)$$

- The intensity fluctuates as $\cos(\Delta\omega t)$
- The maximum intensity at $t = 0$ or $t = \frac{2\pi}{\Delta\omega} = \frac{1}{f_{beat}}$

$$I_{max} = 2\rho v \omega^2 s_m^2$$

- The minimum intensity

$$I_{min} = 0$$

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例: **Problem**

Emperor penguins vocalize by using both sides of its vocal organ simultaneously. Each side sets up acoustic standing waves in the bird's throat and mouth, much like in a pipe with **two open ends**. Suppose that the frequency of the first harmonic produced by side A is f_{A1} =**432 Hz** and the frequency of the first harmonic produced by side B is f_{B1} =**371 Hz**. What is the **beat frequency** between those two first-harmonic frequencies and between the two second-harmonic frequencies?

Solution:

- First Harmonic

$$f_{beat,1} = |f_{A1} - f_{B1}| = 61 \text{ Hz}$$

- Second Harmonic for two open ends

$$f = \frac{v}{\lambda} = \frac{nv}{2L} (n = 1, 2, 3 \ldots)$$

$$f_2 = 2 \times \frac{v}{2L} = 2f_1$$

$$f_{beat,2} = |2f_{A1} - 2f_{B1}| = 2 \times 61 \text{ Hz} = 122 \text{ Hz}$$

§2 The doppler effect

1. The doppler effect (多普勒效应)

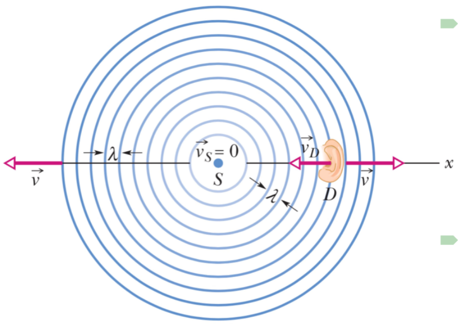
- 当 source 与 detector 有 relative motion 时，detector 接收到的频率 f' 与 source 发出的频率 f 不同.
- 同样适用于 electromagnetic waves
  - E.g. A parked police sounding its siren at 1000 Hz,
    - If you are parked as well, you hear 1000 Hz
    - If you are driving towards it (120 km/h), you will hear a higher frequency
    - If you are driving away from it (120 km/h), you will hear a lower frequency

2. General equation

$$f' = f \frac{v \pm v_D}{v \pm v_S}$$

- Where, v is the speed of sound through the air, v_D is the detector's speed relative to air, and v_s is the source's speed relative to the air.
- When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from each other, the sign on its speed must give a downward shift in frequency.

- Moving detector and Stationary source



- Moving toward each other

$$T = \frac{\lambda}{v} = \frac{1}{f}$$

$$T' = \frac{\lambda}{v + v_D} = \frac{1}{f'}$$

$$f' = \frac{1}{\lambda}(v + v_D) = \frac{f}{v}(v + v_D)$$

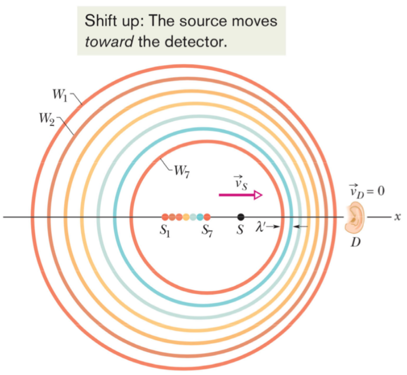
- Moving away from each other

$$f' = \frac{f}{v}(v - v_D)$$

- Generally,

$$f' = f\frac{v \pm v_D}{v}$$

- Moving Source and Stationary Detector



- Moving toward each other
- The Wavelength

$$\lambda' = vT - v_s T$$

- Thus,

$$f' = \frac{v}{\lambda'} = \frac{1}{T} \times \frac{v}{v - v_s} = f\frac{v}{v - v_s}$$

- Moving away

$$f' = f\frac{v}{v + v_s}$$

- Generally,

$$f' = f\frac{v}{v \pm v_s}$$

- The general equation:

$$f' = f\frac{v \pm v_D}{v \pm v_s}$$

- Choose the right sign

S	***D***	f'/f
●→	←●	$\frac{v + v_D}{v - v_s}$
←●	●→	$\frac{v - v_D}{v + v_s}$
●→	●→	$\frac{v - v_D}{v - v_s}$
←●	←●	$\frac{v + v_D}{v + v_s}$

例:

Problem

Checkpoint 4

The figure indicates the directions of motion of a sound source and a detector for six situations in stationary air. For each situation, is the detected frequency greater than or less than the emitted frequency, or can't we tell without more information about the actual speeds?

	Source	Detector		Source	Detector
(a)	⟶	• 0 speed	(d)	⟵	⟵
(b)	⟵	• 0 speed	(e)	⟶	⟵
(c)	⟶	⟶	(f)	⟵	⟶

Answer: (a) greater (b) less (c) cannot tell
(d) cannot tell (e)greater (f) less

例: Problem

$v_{air} = 343\ m/s$

Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than can be heard by a human. Suppose a bat emits ultrasound at frequency f_{be} =**82.52 kHz** while flying with velocity $\vec{v}_b = (9.00\ \mathrm{m/s})\ \hat{\imath}$ as it chases a moth that flies with velocity $\vec{v}_m = (8.00\ \mathrm{m/s})\ \hat{\imath}$.

a) What frequency f_{md} does the moth detect?

b) What frequency f_{bd} does the bat detect in the returning echo from the moth?

Solution:

a)The Moth is the detector while the bat is the source

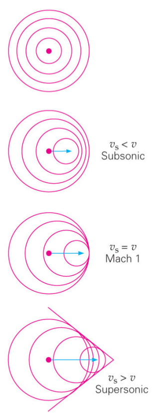
$$f_a = f\frac{v \pm v_D}{v \pm v_s} = f\frac{v - v_m}{v - v_b} = 82.8\ kHz$$

b)The Moth is the source for the echo, while the bat is the detector

$$f_b = f_a\frac{v \pm v_D}{v \pm v_s} = f_a\frac{v + v_b}{v + v_m} = 83.0\ kHz$$

§3 Supersonic speeds

1、Supersonic speeds (超声速)



- Moving Source and Stationary detector

$$f' = f\frac{v}{v - v_s}$$

- If $v = v_s$, the source keep pace with its own spherical wavefronts
- If $v < v_s$, the general equation doesn't work.
- it will outpace its sound waves, creating a sonic boom. A similar phenomenon produces the *wake* from a boat—it is going faster than the wave speed in water.

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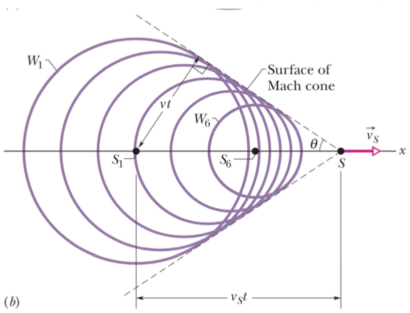
2、Mach cone (马赫锥)

- As shown in the figure below, all the wavefronts bunch along a V-shaped envelope in this 2D drawing.
- Actually, it is a 3D cone called the ***Mach cone***. A ***shock wave*** exists along the surface of this cone.
- The ***Mach cone angle*** is given by



The ***Mach number***

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$

1.

Summary

- Sound Waves

$$v = \sqrt{\frac{B}{\rho}}$$

(a) $\overbrace{s(x,t)}^{\text{Displacement}} = \underbrace{s_m}_{\text{Displacement amplitude}} \underbrace{\cos(kx - \omega t)}_{\text{Oscillating term}}$

(b) $\underbrace{\Delta p(x,t)}_{\text{Pressure variation}} = \underbrace{\Delta p_m}_{\text{Pressure amplitude}} \overbrace{\sin(kx - \omega t)}^{\text{Oscillating term}}$

- Pressure Variation Amplitude

$$\Delta p_m = (v\rho\omega)s_m$$

- Interference: two identical wave passing through one common point

$$\emptyset = 2\pi \frac{\Delta L}{\lambda}$$

- Fully Constructive

$$\frac{\Delta L}{\lambda} = 0, 1, 2 \ldots$$

- Fully Destructive

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5 \ldots$$

- Sound Intensity

$$I = \frac{P}{A}$$

  - Sound Intensity

$$I = \frac{P_s}{4\pi r^2} \text{ and } I = \frac{1}{2}\rho v \omega^2 s_m^2$$

  - Sound Level in Decibels (dB)

$$\beta = (10\ dB) log \frac{I}{I_0}$$

  - Standing Wave Patterns in Pipes
    - Two Open Ends

$$f = \frac{v}{\lambda} = \frac{nv}{2L} (n = 1, 2, 3 \ldots)$$

    - One Closed End and One Open End

$$f = \frac{v}{\lambda} = \frac{nv}{4L} (n = 1, 3, 5 \ldots)$$

- Beats

$$f_{beat} = |f_1 - f_2|$$

- The Doppler Effect: General Equation with $v > v_s$

$$f' = f \frac{v \pm v_D}{v \pm v_s}$$

- Shock Wave: When $v < v_s$, the Mach cone angle

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$