Lecture 8

§1 Rotational motion

1. translation (平动) and rotation (转动)

1° translation: move without spinning (刚体中各质点运动轨迹完全相同)

2° rotation: spin about an axis (刚体中各质点都绕某一直线做圆周运动)

2. rotational variables

1° rigid body (刚体)

任何情况下形状和大小都不改变的物体

2° a fixed axis (定轴)

被称为 the axis of rotation (or rotation axis)

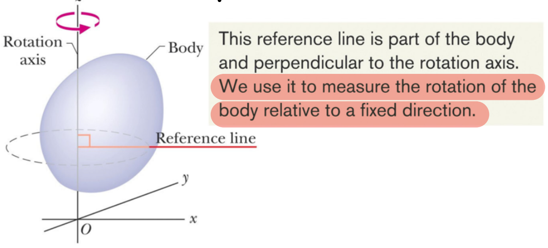
3° pure rotation

① 各质点都绕轴做圆周运动

② 各质点的矢径在相同时间内转过的角度相同

4° reference line

旋转体的一部分，与 rotation axis 垂直



§2 Angular variables

1. Angular position (角位置)

1° Angular position of the reference line

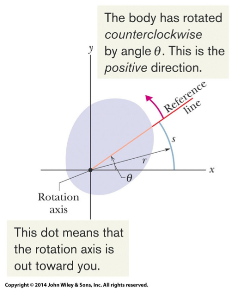
$\theta = \frac{s}{r}$

s: the circular arc, r: the radius of the circle

2° SI-unit

radian (弧度), pure number

$1\ rad = \frac{360^\circ}{2\pi} = 57.3^\circ = 0.159\ rev$



- Angular Position of the reference line: The angle (θ) of this line relative to a fixed direction, which we take as the zero angular position.

$$\theta = \frac{s}{r}$$

- s: the circular arc, r: the radius of the circle
- SI-unit: radian, pure number
- For a circle,

$$1\ rev = 360^\circ = \frac{2\pi r}{r} = 2\pi\ rad$$

And thus

$$1\ rad = 57.3^\circ = 0.159\ rev$$

2. angular displacement（角位移）($\Delta\theta$)

1° reference line 的角位置改变量

$$\Delta\theta = \theta_2 - \theta_1$$

2° 正方向：counterclockwise

负方向：clockwise

3° 对于刚体中的任意一点，都适用

例：

Checkpoint 1

A disk can rotate about its central axis like a merry-go-round. Which of the following pairs of values for its initial and final angular positions, respectively, give a negative angular displacement: (a) −3 rad, +5 rad, (b) −3 rad, −7 rad, (c) 7 rad, −3 rad?

B and C

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3. angular velocity（角速度）(ω)

1° 单位时间内的角位移量

2° average angular velocity：

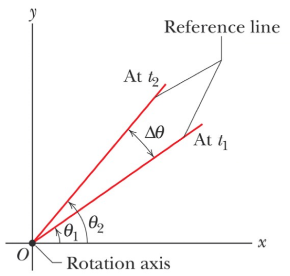
$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

3° (Instantaneous) angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

4° SI-unit：rad/s

* Angular speed（角速率）：标量



- Angular Velocity (ω): Angular Displacement during a time interval
- Average Angular Velocity

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

- (Instantaneous) Angular Velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Angular Speed: Magnitude
- SI-unit: rad/s

4. angular acceleration（角加速度）(α)

1° 单位时间内的角速度变化量

2° average angular acceleration：

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

3° (Instantaneous) angular acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

4° SI-unit：rad/s^2

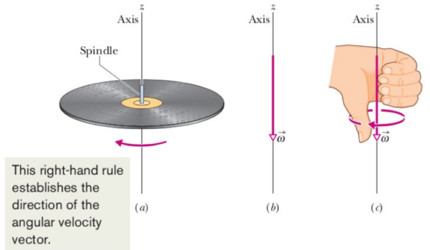
5° 对刚体上的每一点均适用

5. ω 与 α 的矢量性

与转向成右手螺旋关系

$$\vec{v} = \vec{\omega} \times \vec{r}$$

- ω *and* α: Treat them as vectors, but not 'real' vectors
- Right-Hand Rule: Rotate around the Vector, not move along the direction of the vector



This right-hand rule establishes the direction of the angular velocity vector.

For fixed rotational axis, only two ways of rotation:

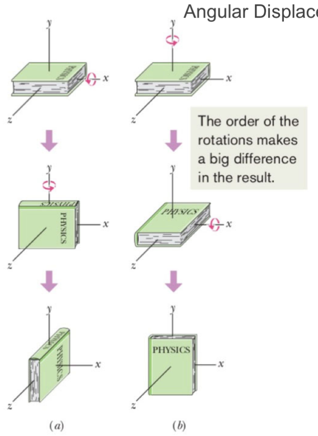
- Counterclockwise
- Clockwise
- Indicated by + and - sign

Clockwise or not depends on whether looking up or down

Angular displacement (θ) 不能被用作矢量

Special Case

Angular Displacement (θ): Treated as Vectors? **No**



The order of the rotations makes a big difference in the result.

- For vectors, the addition :

$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

- However, the addition

$$\vec{\theta}_1+\vec{\theta}_2 \neq \vec{\theta}_2+\vec{\theta}_1$$

if the rotation axes for $\vec{\theta}_1$ and $\vec{\theta}_2$ are different.

For rotations about a fixed axis, then

$$\theta_1+\theta_2=\theta_2+\theta_1$$

Indicated by + (Counterclockwise) and – sign (Clockwise)

6、定轴转动与一维直线运动的相似性

- Rotation around a fixed axis → one dimensional linear motion

Linear	Rotational
Position (x)	Angular Position (θ)
Displacement (Δx)	Angular Displacement ($\Delta\theta$)
Velocity (v)	Angular Velocity (ω)
Acceleration (a)	Angular Acceleration (α)

7、匀加速转动

1° $\omega=\omega_0+\alpha t$

2° $\theta-\theta_0=\omega_0 t+\frac{1}{2}\alpha t^2$

3° $\omega^2=\omega_0^2+2\alpha(\theta-\theta_0)$

4° $\theta-\theta_0=\frac{1}{2}(\omega_0+\omega)t$

5° $\theta-\theta_0=\omega t-\frac{1}{2}\alpha t^2$

Linear Equation	Angular Equation
$v=v_0+at$	$\omega=\omega_0+\alpha t$
$x-x_0=v_0t+\frac{1}{2}at^2$	$\theta-\theta_0=\omega_0t+\frac{1}{2}\alpha t^2$
$v^2=v_0^2+2a(x-x_0)$	$\omega^2=\omega_0^2+2\alpha(\theta-\theta_0)$
$x-x_0=\frac{1}{2}(v_0+v)t$	$\theta-\theta_0=\frac{1}{2}(\omega_0+\omega)t$
$x-x_0=vt-\frac{1}{2}at^2$	$\theta-\theta_0=\omega t-\frac{1}{2}\alpha t^2$

例:

Checkpoint 2

In four situations, a rotating body has angular position $\theta(t)$ given by (a) $\theta = 3t - 4$, (b) $\theta = -5t^3 + 4t^2 + 6$, (c) $\theta = 2/t^2 - 4/t$, and (d) $\theta = 5t^2 - 3$. To which situations do the angular equations of Table 10-1 apply?

例:

While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decrease the angular velocity of the cylinder from 3.40 rad/s to 2.00 rad/s in 20.0 rev, at constant angular acceleration.

(a) What is the constant angular acceleration during this decrease in angular speed?

SOLUTION:

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{(2.00 \text{ rad/s})^2 - (3.40 \text{ rad/s})^2}{2(125.7 \text{ rad})}$$

$$= -0.0301 \text{ rad/s}^2. \qquad \text{(Answer)}$$

§3 Linear variables

1、linear variables 与 angular variables 的联系

$s = \theta \cdot r$ (radian measure)

- s: the length of the point moves along an arc
- r: perpendicular distance from the rotational axis
- θ: for all the linear-angular unit, must use the Radian relations.

2、speed

1、速率

$$\frac{ds}{dt} = \frac{d\theta}{dt} \cdot r$$

$\Rightarrow v = \omega \cdot r$ (radian measure)

- v: speed
- ω: angular speed

2、tangential velocity (切向速度)

方向与运动轨迹相切

3、acceleration

1° acceleration 永远有一个 radial (径向) (centripetal (向心的)) component，并可能有一个 tangential component

2° radial acceleration (径向加速度) (a_r)

① 对于恒定的 speed / angular speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

$$a_r = \frac{v^2}{r} = \omega^2 r \text{ (inward)}$$

② 方向：指向圆心

③ 只要物体在转动，无论角加速度是否存在，径向加速度均不为 0.

3° tangential acceleration (切向加速度) (a_t)

① 对于角加速度 α:

$a_t = \frac{dv}{dt} = \frac{d\omega}{dt} \cdot r = \alpha \cdot r$ (radian measure)

② 方向: 与旋转路径相切

例:

- Figure 11-10 shows a centrifuge used to accustom astronaut trainees to high accelerations. The radius *r* of the circle traveled by an astronaut is 15 m.

(a) At what constant angular speed must the centrifuge rotate if the astronaut is to have a linear acceleration of magnitude 11*g*?

$$\omega = \sqrt{\frac{a_r}{r}} = \sqrt{\frac{(11)(9.8\ m/s^2)}{15\ m}}$$

$$= 2.68\ rad/s \approx 26\ rev/min$$

(b) What is the tangential acceleration of the astronaut if the centrifuge accelerates at a constant rate from rest to the angular speed of (a) in 120 s?

$$a_t = \alpha r = \frac{\omega - \omega_0}{t} r$$

$$= \frac{2.68\ rad/s - 0}{120\ s}(15\ m) = 0.34\ m/s^2$$

$$= 0.034\ g$$

例:

We are given the job of designing a large horizontal ring that will rotate around a vertical axis and that will have a radius of $r = 33.1$ m (matching that of Beijing's The Great Observation Wheel, the largest Ferris wheel in the world). Passengers will enter through a door in the outer wall of the ring and then stand next to that wall (Fig. 10-10*a*). We decide that for the time interval $t = 0$ to $t = 2.30$ s, the angular position $\theta(t)$ of a reference line on the ring will be given by

$$\theta = ct^3, \quad (10\text{-}24)$$

with $c = 6.39 \times 10^{-2}$ rad/s^3. After $t = 2.30$ s, the angular speed will be held constant until the end of the ride. Once the ring begins to rotate, the floor of the ring will drop away from the riders but the riders will not fall—indeed, they feel as though they are pinned to the wall. For the time $t = 2.20$ s, let's determine a rider's angular speed ω, linear speed v, angular acceleration α, tangential acceleration a_t, radial acceleration a_r, and acceleration $\vec{a}$.

Calculations: Let's go through the steps. We first find the angular velocity by taking the time derivative of the given angular position function and then substituting the given time of $t = 2.20$ s:

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(ct^3) = 3ct^2 \quad (10\text{-}25)$$

$$= 3(6.39 \times 10^{-2}\ rad/s^3)(2.20\ s)^2$$

$$= 0.928\ rad/s. \quad \text{(Answer)}$$

From Eq. 10-18, the linear speed just then is

$$v = \omega r = 3ct^2 r \quad (10\text{-}26)$$

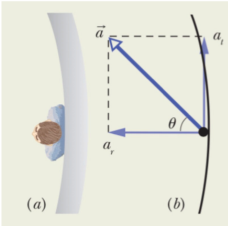
$$= 3(6.39 \times 10^{-2}\ rad/s^3)(2.20\ s)^2(33.1\ m)$$

$$= 30.7\ m/s. \quad \text{(Answer)}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(3ct^2) = 6ct$$
$$= 6(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s}) = 0.843\ \text{rad/s}^2.$$

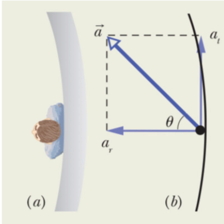
$$a_t = \alpha r = 6ctr$$
$$= 6(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s})(33.1\ \text{m})$$
$$= 27.91\ \text{m/s}^2 \approx 27.9\ \text{m/s}^2,$$

$$a_r = (3ct^2)^2 r = 9c^2t^4 r$$
$$= 9(6.39 \times 10^{-2}\ \text{rad/s}^3)^2(2.20\ \text{s})^4(33.1\ \text{m})$$
$$= 28.49\ \text{m/s}^2 \approx 28.5\ \text{m/s}^2,$$



$$a = \sqrt{a_r^2 + a_t^2}$$
$$= \sqrt{(28.49\ \text{m/s}^2)^2 + (27.91\ \text{m/s}^2)^2}$$
$$\approx 39.9\ \text{m/s}^2,$$

$$\theta = \tan^{-1}\frac{2}{3(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s})^3} = 44.4°.$$



$a_t = \alpha r = 6ctr$ $\quad$ $a_r = (3ct^2)^2 r = 9c^2t^4 r$ $\quad$ $\tan(\theta) = \frac{a_t}{a_r} = \frac{2}{3ct^3}$

Summary

Radian Measure

- Agular Position

$$\theta = \frac{s}{r}$$
$$1\ rev = 360° = \frac{2\pi r}{r} = 2\pi\ rad$$

- Angular Displacement

$$\Delta\theta = \theta_2 - \theta_1$$

- Angular Velocity and Speed

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \text{ and } \omega = \lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Angular Acceleration

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \text{ and } \alpha = \lim_{\Delta t \to 0}\frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Summary

Radian Measure

- Comparison between Linear and Rotational Motion

Linear Equation	Angular Equation
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Summary

Radian Measure

- Linear and Angular Variables Related

$$v = \omega r$$
$$a_t = \alpha r$$

- For uniform circular motion

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \text{ and } a_r = \frac{v^2}{r} = \omega^2 r$$