Lecture 5

§1 Kinetic Energy

1、Energy

1° scalar

2° 与一个或多个物体组成的系统有关

2、Principle of Energy Conservation (能量守恒)

能量可由一种形式变为另一种，或从一个物体转移到另一个物体，但总量保持不变。

3、Kinetic energy (动能)

1° 符号：K

2° 与物体的运动状态和质量有关 ($v \ll c$)

3° $K = \frac{1}{2}mv^2$

$1 \text{ joule} = 1\,J = 1\,kg \cdot m^2/s^2$

例：

Sample Problem 7.01 **Kinetic energy, train crash**

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant $0.26\ m/s^2$, what was the total kinetic energy of the two locomotives just before the collision?

SOLUTION:

For one of the locomotives

$$x = \frac{1}{2}at^2$$

$$3.2\times10^3 = \frac{1}{2}\times0.26\times t^2$$

$$t = 156.9\ s$$

$$v = at = 40.8\ m/s$$

Total Kinetic energy of two locomotives:

$$K = 2\times\frac{1}{2}mv^2 = \frac{1.2\times10^6}{9.8}\times40.8^2\ J = 2.0\times10^8\ J$$

- The collision was like a exploding bomb
- Sending debris flying into the air to rain down upon the crowd

§2 Work and Kinetic Energy

1、Work (功)

1° 符号：W

2° 通过作用于一个物体的力来转移能量

能量转移至物体：+ve work

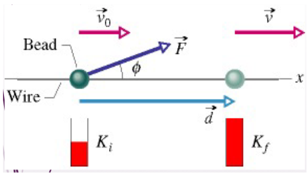
能量转移自物体：−ve work

3° Doing work: the act of trasferring energy

4° SI Unit: Joule (J)

$1\,J = 1\,kg \cdot m^2/s^2 = 1\,N \cdot m = 0.738\,ft \cdot lb$

2、外力做功

1° 

$F_x = m \cdot a_x$

$v^2 = v_0^2 + 2a_x \cdot d$

$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$

$W = F_x \cdot d = F d \cos\theta$ （恒力做功）

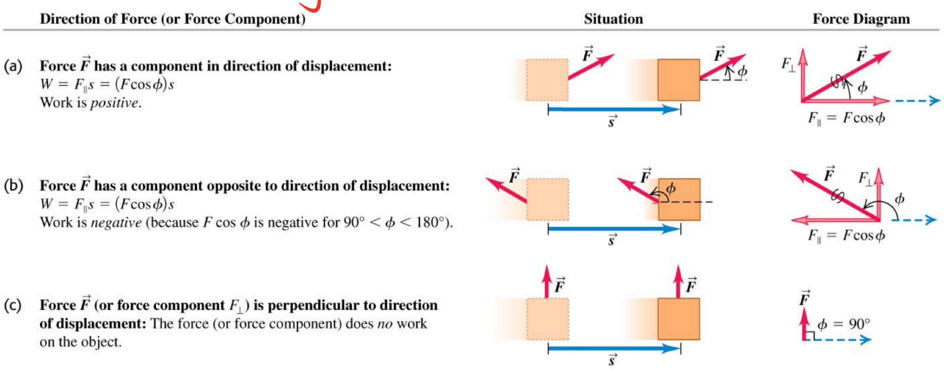
2° 仅有与物体位移方向一致的力的分量起作用

与位移方向垂直的力的分量不做功

3° $0° \le \phi < 90°$: positive work

$\phi = 90°$: zero work

$90° < \phi \le 180°$: negative work



3. Net work

the sum of the works done by the individual force

= the work done by the net force

4. Work-Kinetic Energy Theorem（动能定理）

1° 对于一个质点，合外力做功等于动能变化

$\Delta K = K_f - K_i = W$

$K_f = K_i + W$

2° +ve work : 动能增加

−ve work : 动能减少

§3 Work Done by the Gravitational Force

1. Work done by the gravitational force

1° 重力做功 : $W_g = mgd\cos\theta$

2° 上升阶段 ($\theta = 180°$) : $W_g = -mgd$

下降阶段 ($\theta = 0°$) : $W_g = mgd$

2. Work done by lifting or lowering an object

- If applying Force to the particle

$$\Delta K = K_f - K_i = W_a + W_g$$

- If the particle is stationary/moves at constant velocity before and after a lift

$$\Delta K = K_f - K_i = W_a + W_g = 0$$

$$W_a = -W_g = -\mathrm{mgd}\cos\emptyset$$

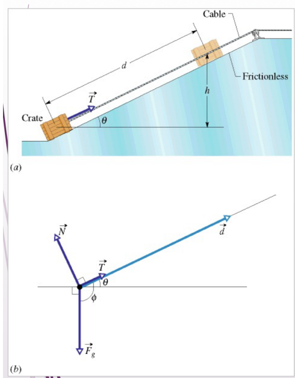
- $\emptyset$ is the angle between $\vec{F_g}$ and displacement $\vec{d}$

- Upward ($\emptyset = 180°$), $W_a = mgd$
- The work done by the applied force in lifting the object is positive.
- It equals the increase in the gravitational potential energy of the object. The applied force puts energy into the object.

- Downward ($\emptyset = 0°$), $W_a = -mgd$
- The work done by the applied force in lowering the object is negative.
- The work done by the applied force equals the decrease in the gravitational potential energy of the object. The applied force takes energy out of the object.

e.g.

Problem



An initially stationary 15.0 kg crate of cheese is pulled, via a cable, a distance d = 5.70 m up a frictionless ramp, to a height h of 2.50 m, where it stops.

(a) How much work W_g is done on the crate by the gravitational force during the lift?

(b) How much work done by the cable?

(c) How much work done by the normal force?

SOLUTION:

(a) $W_g = -mgsin\theta \cdot d = -mgh = -368\ \mathrm{J}$

(b) $W_T = mgsin\theta \cdot d = 368\ \mathrm{J}$

(c) $W_N = 0\ \mathrm{J}$

$$W_g + W_T + W_N = \Delta K = 0$$

§4 Work done by a spring force

1. Spring force (弹簧力)

variable force, as a function of d.

2. Hooke's Law (胡克定律)

1° $\vec{F_s} = -k\vec{d}$

2° k: spring constant (unit: N/m)

3° 方向与弹簧自由端位移方向相反

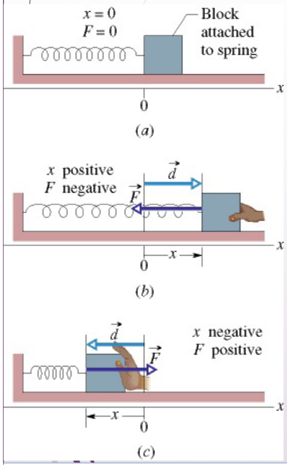
4° 假定弹簧

① massless

② 是 ideal spring, 满足 Hooke's Law

则 $F_x = F(x) = -kx$

Work Done by a Spring Force



- Spring Force: The force from Spring, variable force, as a function of d.
- (a) Relaxed State: neither compressed nor extended
- (b) Restoring Force (or Spring Force) towards left
- (c) Restoring Force (or Spring Force) towards Right

$$\vec{F_s} = -k\vec{d}$$

- Hooke's Law
- k: spring constant (unit: N/m)
- Always opposite to the direction of the displacement of the spring's free end
- Hooke's law is named after the English scientist Robert Hooke of the late 1600s.

3. Work done by a spring force

若 x 表示 the extension from zero of the spring，物块由extension x_i (initial position) 移动至 x_f (final position). 则弹簧对物块做功为

$$W_s = \int_{x_i}^{x_f} -F_x dx = \int_{x_i}^{x_f} -kx\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

若 $x_i=0$，则 $W_s = -\frac{1}{2}kx_f^2$

4. Work done by an applied force

- Applying external force $\vec{F_a}$ to the block

$$\Delta K = K_f - K_i = W_a + W_s$$

- If the block is stationary before and after the displacement,
- $\Delta K = 0 = W_a + W_s \rightarrow W_a = -W_s$
- If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

例:

Problem

A package of spicy Cajun pickles lies on a frictionless floor, attached to the free end of a spring. An applied force of magnitude F_a = 4.9 N would be needed to hold the package stationary at x_1 = 12 mm.

a) How much work does the spring force do on the package if the package is pulled rightward from x_0 = 0 to x_2 = 17 mm?
b) Next, the package is moved leftward to x_3 = -12 mm. How much work does the spring force do on the package during this displacement? Explain the sign of this work.

SOLUTION:

(a) $F_s = -kx \rightarrow k = -\frac{F_s}{x} = \frac{F_a}{x_1} = 408\ N/m$

$W_s = -\frac{1}{2}kx_f^2 = -\frac{1}{2}kx_2^2 = -0.059\ J$

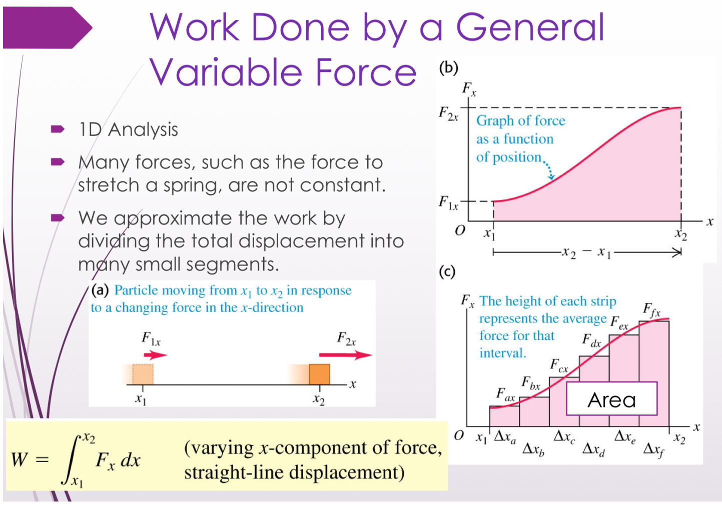
(b) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_3^2 = 0.030\ J$

The spring force does positive work as the block moves from +17mm to its relaxed position and negative work as the block moves from the relaxed position to -12mm. The former work is larger resulting in W_s being positive.

§5. Work done by a general variable force

1. 1 D analysis

$$W = \int_{x_1}^{x_2} F_x\,dx$$



2. 3-D analysis

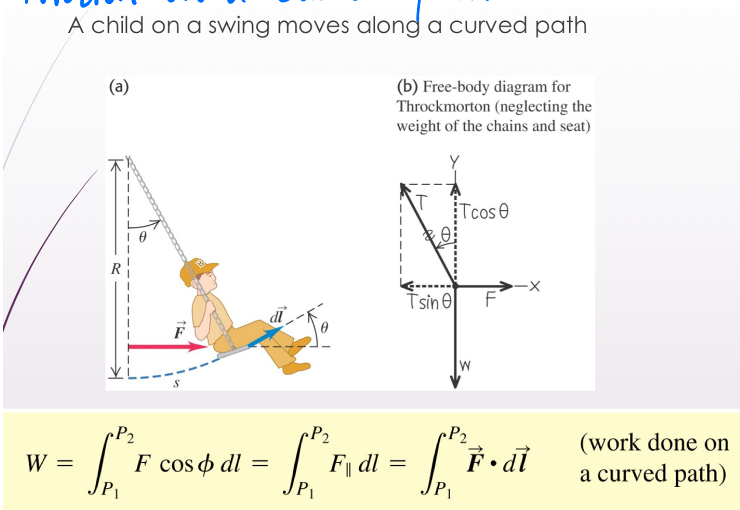
$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

3. Work-Kinetic Energy Theorem

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\cdot\frac{dx}{dt} = v_x\cdot\frac{dv_x}{dx}$$

$$W_{tot} = \int_{x_1}^{x_2} F_x\,dx = \int_{x_1}^{x_2} m a_x\,dx = \int_{v_1}^{v_2} m v_x\,dv_x = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

4. Motion on a curved path



§6 Power (功率)

1. Power

The time rate at which work is done by a force

2. Average power

$$P_{avg} = \frac{W}{\Delta t}$$

3. Instantaneous power (scalar)

$$P = \frac{dW}{dt} = \frac{F\cos\phi\,dx}{dt} = F\cos\phi\,\frac{dx}{dt} = F v\cos\phi = \vec{F}\cdot\vec{v}$$

ϕ 为力与速度的夹角

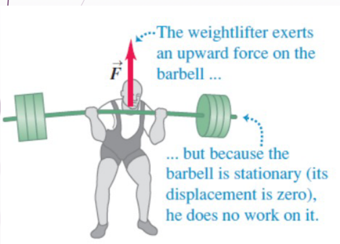
4. SI Unit

1° watt (W)

1 watt = 1 W = 1 J/s

2° kilowatt-hr (kWh)

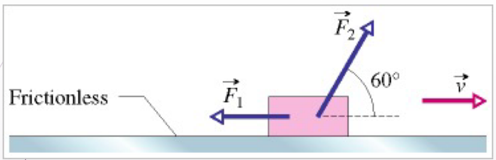
Cases in Reality



There are many situations in which forces act but do zero work. You might think it's "hard work" to hold a barbell motionless in the air for 5 minutes. But in fact, you aren't doing any work at all on the barbell because there is no displacement.

例:

Problem



The figure shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60°to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s.

(a) What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

SOLUTION:

$$P_1 = F_1 v \cos 180^\circ = (2.0\ \text{N})(3.0\ \text{m/s}) \cos 180^\circ$$
$$= -6.0\ \text{W}$$
$$P_2 = F_2 v \cos 60^\circ = (4.0\ \text{N})(3.0\ \text{m/s}) \cos 60^\circ$$
$$= 6.0\ \text{W}$$
$$P_{net} = P_1 + P_2$$
$$= 0$$

The kinetic energy of the box is not changing. The speed of the box remains at 3 m/s. The net power does not change.

(b) If the magnitude of $\vec{F}_2$ is, instead, 6.0 N, what now is the net power, and is it changing?

SOLUTION:

$$P_2 = F_2 v \cos 60^\circ = (6.0\ \text{N})(3.0\ \text{m/s}) \cos 60^\circ$$
$$= 9.0\ \text{W}$$
$$P_{net} = P_1 + P_2 = -6.0\ \text{W} + 9.0\ \text{W}$$
$$= 3.0\ \text{W}$$

There is a net rate of transfer of energy to the box. The kinetic energy of the box increases. The net power also increases. The speed of the box will increase.

Summary

- Kinetic Energy
  - $\boldsymbol{K = \frac{1}{2}mv^2}$
- Work: Energy Transfer
- Work Done by a Constant Force
  - $\boldsymbol{W = \vec{F} \cdot \vec{d} = Fd\cos\emptyset}$
  - Net Work is the sum of individual work
- Work and Kinetic Energy
  - $\Delta K = K_f - K_i = W$ & $K_f = K_i + W$
- Work done by Gravitational Force
  - $W_g = mgd\cos\emptyset$
- Work done in Lifting and Lowering an object
  - $\Delta K = K_f - K_i = W_a + W_g$
  - If $K_f = K_i$, $W_a = -W_g$

Summary

- Spring Force
  - $\vec{F_s} = -k\vec{d}$ or $F_x = F(x) = -kx$
  - Hooke's Law
- Work Done by a Spring Force
  - $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$
  - If $x_i = 0$, $W_s = -\frac{1}{2}kx_f^2$
- Work Done by a Variable Force
  - 3D, $W = \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy + \int_{z_i}^{z_f} F_z\, dz$
  - 1D, $W = \int_{x_i}^{x_f} F_x\, dx$
- Power
  - $P_{avg} = \frac{W}{\Delta t}$
  - $P = \frac{dW}{dt} = Fv\cos\emptyset = \vec{F} \cdot \vec{v}$

Some useful integrals I

1. $$\int_{x_1}^{x_2} x\,dx = \left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \frac{x_2{}^2}{2} - \frac{x_1{}^2}{2}$$

2. $$\int_{\theta_1}^{\theta_2} \sin\theta\,d\theta = \left[-\cos\theta\right]_{\theta_1}^{\theta_2} = -\cos\theta_2 + \cos\theta_1$$
$$= \cos\theta_1 - \cos\theta_2$$

3. $$\int_{\theta_1}^{\theta_2} \cos\theta\,d\theta = \left[\sin\theta\right]_{\theta_1}^{\theta_2} = \sin\theta_2 - \sin\theta_1$$

Some useful integrals II

4. $$\int_{\theta_1}^{\theta_2} \sin^2\theta\,d\theta = \int_{\theta_1}^{\theta_2} \frac{1-\cos 2\theta}{2}\,d\theta =$$

$$\left[\frac{\theta}{2}\right]_{\theta_1}^{\theta_2} - \left[\frac{\sin 2\theta}{4}\right]_{\theta_1}^{\theta_2} = \frac{\theta_2 - \theta_1}{2} - \frac{\sin 2\theta_2 - \sin 2\theta_1}{4}$$

$$\text{if } \theta_1 = 0,\ \theta_2 = 2\pi: \quad \sin 4\pi = \sin 0 = 0$$

$$\text{so:} \quad \int_{\theta_1=0}^{\theta_2=2\pi} \sin^2\theta\,d\theta = \pi$$

$$\text{or:} \quad \frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta\,d\theta = \frac{1}{2}$$