Lecture 7

§1 Center of mass

1、Center of mass (质心) (COM)

1° 性质

① 所有质量集中于这一点

② 所有外力作用于这一点

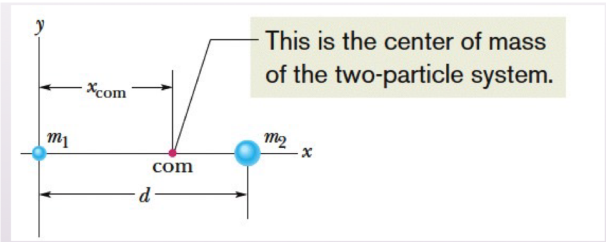
2° 作用

将一个系统的复杂运动简化为一个点的运动

3° 对于两个点：

$$m_1(-x_{com}) + m_2(d - x_{com}) = 0$$

$$x_{com} = \frac{m_2}{m_1+m_2} d$$



2、COM of the system

1° General situation (共线)

$$m_1(x_1 - x_{com}) + m_2(x_2 - x_{com}) = 0$$

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{m_1x_1+m_2x_2}{M}$$

与坐标系的选取无关

2° Many particles (共线)

$$x_{com} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i$$

3° Three dimensions

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_iy_i$$

$$z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_iz_i$$

$$\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + z_{com}\hat{k}$$

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

4° Solid bodies (无穷个点)

$$x_{com} = \frac{1}{M}\int x\,dm$$

$$y_{com} = \frac{1}{M}\int y\,dm$$

$$z_{com} = \frac{1}{M}\int z\,dm$$

5° For uniform objects

$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

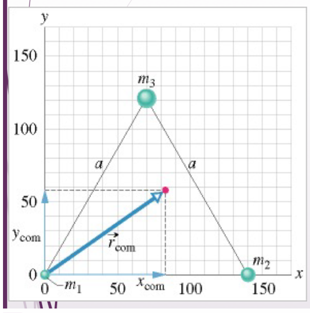
$$x_{com} = \frac{1}{V}\int x\,dV$$

$$y_{com} = \frac{1}{V}\int y\,dV$$

$$z_{com} = \frac{1}{V}\int z\,dV$$

例:

Sample Problem



Three particles of masses m_1 = 1.2 kg, m_2 = 2.5 kg, and m_3 = 3.4 kg form an equilateral triangle of edge length a = 140 cm. Where is the center of mass of this three-particle system?

SOLUTION:

Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	121

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$

$$= 83\text{ cm}$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(121\text{ cm})}{7.1\text{ kg}}$$

$$= 58\text{ cm}$$

例:

Sample Problem

Uniform metal plate P of radius 2R from which a disk of radius R has been stamped out as shown on right. Find the COM of plate P.

Solution:

- Plate S: COM at the center
- Plate C: COM at the center

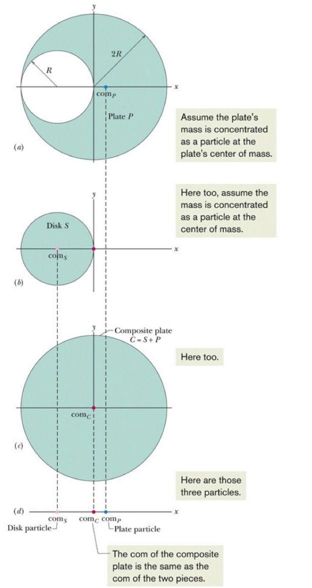


Plate	Center of Mass	Location of com	Mass
P	com$_P$	$x_P = ?$	m_P
S	com$_S$	$x_S = -R$	m_S
C	com$_C$	$x_C = 0$	$m_C = m_S + m_P$

$$x_c = \frac{m_s x_s + m_p x_p}{m_s + m_p}$$

$$x_p = -x_s \frac{m_s}{m_p}$$

$$\frac{m_s}{m_p} = \frac{\rho V_s}{\rho V_p} = \frac{A_s}{A_p} = \frac{\pi R^2}{\pi(4R^2 - R^2)} = \frac{1}{3}$$

$$x_p = \frac{1}{3}\text{R}$$

3. Newton's Second Law for a system

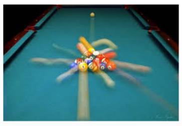
$$\vec{F}_{net} = M \cdot \vec{a}_{com}$$

$$\vec{F}_{net,x} = M\, \vec{a}_{com,x}$$

$$\vec{F}_{net,y} = M\, \vec{a}_{com,y}$$

$$\vec{F}_{net,z} = M\, \vec{a}_{com,z}$$

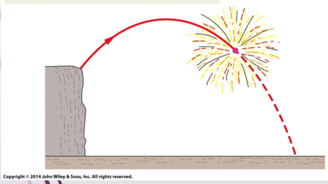
Example



Using the center of mass motion equation:

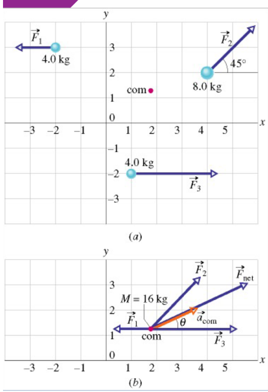
- Billiard collision: forces are only internal, $F = 0$ so $a = 0$
- Baseball bat: $a = g$, so com follows gravitational trajectory

The internal forces of the explosion cannot change the path of the com.



- Exploding rocket: explosion forces are internal, so only the gravitational force acts on the system, and the com follows a gravitational trajectory as long as air resistance can be ignored for the fragments.

Sample Problem



The three particles in the figure are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move? **The com is shown as the red dot.**

SOLUTION:

We can apply Newton's Second Law to the center of mass and obtain its acceleration.

$$\vec{F}_{net} = M\vec{a}_{com}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = M\vec{a}_{com}$$

$$\vec{a}_{com} = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3}{M}$$

$$\vec{a}_{com,x} = \frac{\vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x}}{M} = \frac{-6.0\,N + (12\,N)\cos 45^\circ + 14\,N}{16\,kg} = 1.03\,m/s^2$$

$$\vec{a}_{com,y} = \frac{\vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y}}{M} = \frac{0 + (12\text{ N})\sin 45 + 0}{16\text{ kg}} = 0.530\text{ m/s}^2$$

$$a_{com} = \sqrt{(a_{com,x})^2 + (a_{com,y})^2} = 1.16\text{ m/s}^2 \approx 1.2\text{ m/s}^2$$

$$\theta = \tan^{-1}\frac{a_{com,y}}{a_{com,x}} = 27^\circ$$

§2 Linear momentum

1. Linear momentum (线动量)

1° Linear momentum of a particle: $\vec{p} = m\vec{v}$

2° 方向与速度方向相同

3° 只能被外力改变

4° SI Unit: kg·m/s

5° $\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$

当m随v变化时失效

6° 对系统：

$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n$

$= m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n$

$= M\vec{v}_{com}$

$\vec{F}_{net} = \frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{com}}{dt} = M\vec{a}_{com}$

2、Collision (碰撞) and impulse (冲量)

1° $d\vec{p} = \vec{F}(t)\,dt$

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\,dt$

Collision and Impulse

What causes the change of the linear momentum?

The ball experiences a great force to change the velocity → change in linear momentum

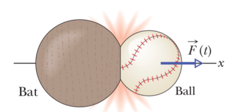


Figure 9-8 Force $\vec{F}(t)$ acts on a ball as the ball and a bat collide.

$$d\vec{p} = \vec{F}(t)dt$$

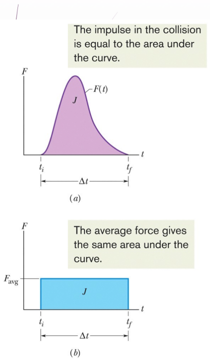
$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

2° Impulse $\vec{J}$: a measure of both the magnitude and the duration of the collision force

$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt$

3° $\Delta\vec{p} = \vec{J}$

Collision and Impulse



- Impulse $\vec{J}$: a measure of both the magnitude and the duration of the collision force.

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

- The change in momentum is equal to the impulse on the object

$$\Delta\vec{p} = \vec{J}$$

- Linear momentum-impulse theorem

$$\Delta p_x = J_x = \int_{t_i}^{t_f} F_x dt$$

$$J = F_{avg}\Delta t$$

例：

Sample Problem

- A pitched 140 g baseball, in horizontal flight with a speed v_i of 39.0 m/s, is struck by a bat. After leaving the bat, the ball travels in the opposite direction with speed v_f, also 39.0 m/s.

(a) What impulse J acts on the ball while it is in contact with the bat during the collision?

$$J = p_f - p_i = mv_f - mv_i$$

$$= (0.140\ \text{kg})(39.0\ \text{m/s}) - (0.140\ \text{kg})(-39.0\ \text{m/s})$$

$$= 10.9\ \text{kg}\cdot\text{m/s}$$

Note that J is the impulse on the ball. The final direction of the ball is positive.

(b) The impact time Δt for the baseball-bat collision is 1.20 ms. What average force acts on the baseball?

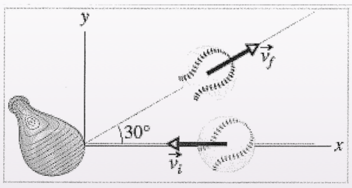
$$F_{avg} = \frac{J}{\Delta t} = \frac{10.9\ \text{kg}\cdot\text{m/s}}{0.00120\ \text{s}}$$

$$= 9080\ \text{N}$$

Note that this average force is from the bat to the ball. The positive direction of the force is in the final velocity of the ball.

Longer impact time reduces average force

(c) Now suppose the collision is not head-on, and the ball leaves the bat with a speed v_f of 45.0 m/s at an upward angle of 30.0°. What now is the impulse on the ball?



SOLUTION:

The impulse on the ball is :

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i$$

$$J_x = p_{fx} - p_{ix} = m(v_{fx} - v_{ix})$$
$$= (0.140\ \text{kg})\,[(45.0\ \text{m/s})(\cos 30.0^\circ) - (-39.0\ \text{m/s})]$$
$$= 10.92\ \text{kg}\cdot\text{m/s}$$

$$J_y = p_{fy} - p_{iy} = m(v_{fy} - v_{iy})$$
$$= (0.140\ \text{kg})\,[(45.0\ \text{m/s})(\sin 30.0^\circ) - 0]$$
$$= 3.150\ \text{kg}\cdot\text{m/s}$$

$$\vec{J} = (10.9\,\hat{i} + 3.15\,\hat{j})\ \text{kg}\cdot\text{m/s}$$

$$J = \sqrt{J_x^{\,2} + J_y^{\,2}} = 11.4\ \text{kg}\cdot\text{m/s}$$

$$\theta = \tan^{-1}\frac{J_y}{J_x} = 16^\circ$$

Note that the impulse is on the ball.

3、Conservation of linear momentum (动量守恒)

无外力作用下，$\vec{P}_i = \vec{P}_f$

Conservation of Linear Momentum

- No external force acting on a closed system ($\vec{F}_{net} = 0$)

  $\vec{P} = constant$ (closed, isolated system)
- The total linear momentum $\vec{P}$ of the system cannot change
- **Law of Conservation of the Linear Momentum**

  $\vec{P}_i = \vec{P}_f$
- Internal forces can change momenta of parts of the system, but cannot change the linear momentum of the entire system
- If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

例：*Two-dimensional explosion:* A firecracker placed inside a coconut of mass M, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. An overhead view is shown in Fig. 9-13*a*. Piece C, with mass $0.30M$, has final speed $v_{fC} = 5.0$ m/s.

(a) What is the speed of piece B, with mass $0.20M$?

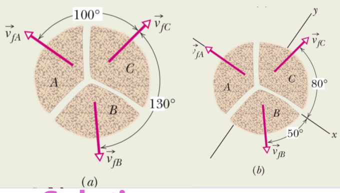


Figure 9-13 Three pieces of an exploded coconut move off in three directions along a frictionless floor. (*a*) An overhead view of the event. (*b*) The same with a two-dimensional axis system imposed.

Solution:

$$P_{iy} = P_{fy}.$$

$$p_{fA,y} = 0,$$
$$p_{fB,y} = -0.20Mv_{fB,y} = -0.20Mv_{fB}\sin 50^\circ,$$
$$p_{fC,y} = 0.30Mv_{fC,y} = 0.30Mv_{fC}\sin 80^\circ.$$

$$v_{fB} = 9.64\ \text{m/s} \approx 9.6\ \text{m/s}.$$

(b) What is the speed of piece A?

$$p_{fA,x} = -0.50Mv_{fA},$$
$$p_{fB,x} = 0.20Mv_{fB,x} = 0.20Mv_{fB}\cos 50^\circ,$$
$$p_{fC,x} = 0.30Mv_{fC,x} = 0.30Mv_{fC}\cos 80^\circ.$$

$$v_{fA} = 3.0\ \text{m/s}.$$

§3 Momentum and kinetic energy in collision

1. Elastic collisions (弹性碰撞)

1° 总动能不变

2° 现实情况的近似

2. Inelastic collisions (非弹性碰撞)

1° K.E. 不守恒

2° completely inelastic collisions (完全非弹性碰撞)

① 物体粘在一起

② 动能损失最大

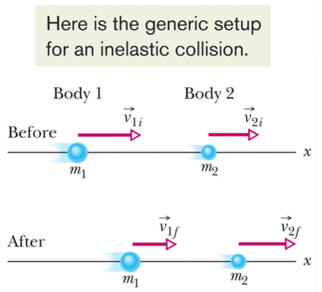
3° 动量守恒

3. 一维非弹性碰撞

1° 满足动量守恒

$$\vec{P}_{1i}+\vec{P}_{2i}=\vec{P}_{1f}+\vec{P}_{2f}$$

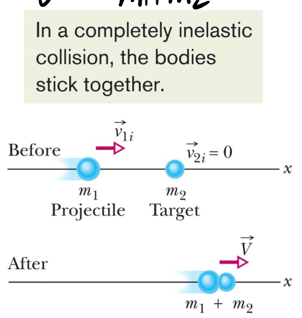
$$m_1\vec{V}_{1i}+m_2\vec{V}_{2i}=m_1\vec{V}_{1f}+m_2\vec{V}_{2f}$$



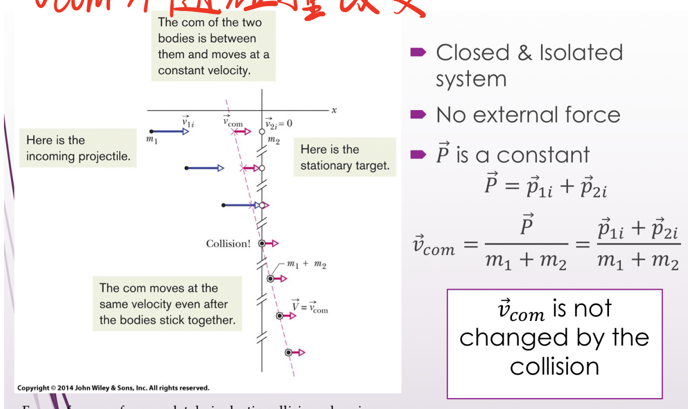
2° 若为完全非弹性碰撞

$$m_1V_{1i}=(m_1+m_2)V$$

$$V=\frac{m_1}{m_1+m_2}V_{1i}$$



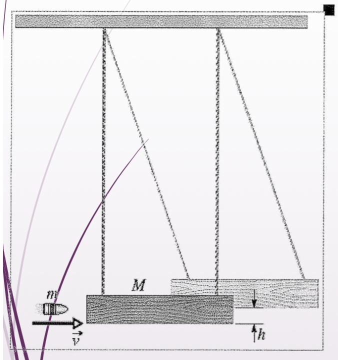
3° $\vec{V}_{com}$ 不随碰撞改变



Freeze frames of a completely inelastic collision, showing center of mass velocity.

4° 系统损失的动能被用于改变物体内在结构(bonding energy);产生热量.

例 **Sample Problem**



(冲击摆)

- The *ballistic pendulum* was used to measure the speeds of bullets before electronic timing devices were developed. The version shown in Fig. 10-11 consists of a large block of wood of mass $M = 5.4$ kg, hanging from two long cords. A bullet of mass $m = 9.5$ g is fired into the block, coming quickly to rest. The *block + bullet* then swing upward, their center of mass rising a vertical distance $h = 6.3$ cm before the pendulum comes momentarily to rest at the end of its arc. What is the speed of the bullet just prior to the collision?

SOLUTION:

Energy is not conserved before and after the collision.

$$V = \frac{m}{m+M} v$$

Energy is conserved during the swing of the pendulum after the collision.

$$\frac{1}{2}(m+M)V^2 = (m+M)gh$$

$$v = \frac{m+M}{m}\sqrt{2gh}$$

$$= \left(\frac{0.0095\text{ kg} + 5.4\text{ kg}}{0.0095\text{ kg}}\right)\sqrt{(2)(9.8\text{ m/s}^2)(0.063\text{ m})}$$

$= 630$ m/s This is the velocity of the bullet.

4. 一维弹性碰撞

1° 满足动量守恒, 系统能量守恒

$K_i = K_f$, $P_i = P_f$

对于两点系统:

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

$$\frac{1}{2}m_1 V_{1i}^2 + \frac{1}{2}m_2 V_{2i}^2 = \frac{1}{2}m_1 V_{1f}^2 + \frac{1}{2}m_2 V_{2f}^2$$

2° 对于 stationary target:

$$m_1 V_{1i} = m_1 V_{1f} + m_2 V_{2f}$$

$$\frac{1}{2}m_1 V_{1i}^2 = \frac{1}{2}m_1 V_{1f}^2 + \frac{1}{2}m_2 V_{2f}^2$$

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i}$$

① $m_1 = m_2$ 时, 交换速度

$V_{1f} = 0$, $V_{2f} = V_{1i}$

② Massive target, $m_2 \gg m_1$ 时, 原速反弹

$V_{1f} \approx -V_{1i}$, $V_{2f} \approx 0$

③ Massive projectile, $m_1 \gg m_2$ 时, 原速前进

$V_{1f} \approx V_{1i}$, $V_{2f} \approx 2V_{1i}$

3° 对于 moving target：

$$\begin{cases} m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_2 v_{2i}^2 \end{cases}$$

$$\begin{cases} m_1 (v_{1i} - v_{1f}) = -m_2 (v_{2i} - v_{2f}) \\ m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f}) \end{cases}$$

$$\begin{cases} v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \\ v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i} + \frac{2m_1}{m_1 + m_2} v_{1i} \end{cases}$$

例 Sample Problem

In Fig. 9-20*a*, block 1 approaches a line of two stationary blocks with a velocity of $v_{1i} = 10$ m/s. It collides with block 2, which then collides with block 3, which has mass $m_3 = 6.0$ kg. After the second collision, block 2 is again stationary and block 3 has velocity $v_{3f} = 5.0$ m/s (Fig. 9-20*b*). Assume that the collisions are elastic. What are the masses of blocks 1 and 2? What is the final velocity v_{1f} of block 1?

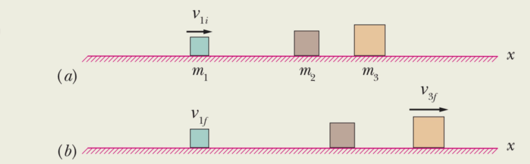


Figure 9-20 Block 1 collides with stationary block 2, which then collides with stationary block 3.

SOLUTION:

- After second collision, $v_{2f} = 0$

$$v_{2f} = \frac{m_2 - m_3}{m_2 + m_3} v_{2i},$$

$m_2 = m_3 = 6.00$ kg. $\quad v_{2i} = v_{3f} = 5.0$ m/s.

- After first collision, $v_{2f} = v_{2i} = 5.0m/s$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i},$$

$$5.0 \text{ m/s} = \frac{2m_1}{m_1 + m_2} (10 \text{ m/s}),$$

$$m_1 = \tfrac{1}{3} m_2 = \tfrac{1}{3}(6.0 \text{ kg}) = 2.0 \text{ kg}.$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i},$$

$$= \frac{\frac{1}{3} m_2 - m_2}{\frac{1}{3} m_2 + m_2} (10 \text{ m/s}) = -5.0 \text{ m/s}.$$

5. 二维碰撞

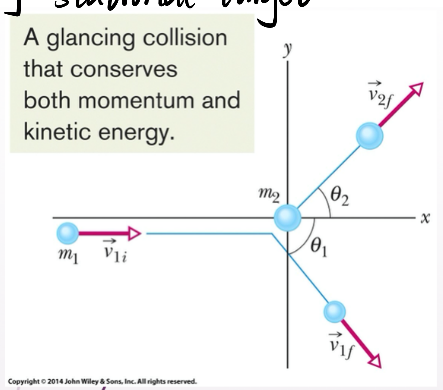
1° 各个方向上的 momentum 守恒

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

2° 若为 elastic collision

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

3° 对于 stational target



① 沿 x-axis：$m_1 v_{1i} = m_1 v_{1f}\cos\theta_1 + m_2 v_{2f}\cdot\cos\theta$

② 沿 y-axis：$0 = -m_1$

③ K.E.：$\frac{1}{2}m v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$

b. System with varying mass：火箭模型

1° 火箭与喷出物形成一个 isolated system

2° · 动量守恒：

$$Mv = -dM\cdot U + (M+dM)\cdot(v+dv) \quad ①$$

U：喷出物相对于参照系(地面)的速度

dM：$-ve$

· 利用 relative speed (v_{rel})：火箭与喷出物的相对速度

$$v + dv = v_{rel} + U$$

$$\Rightarrow U = v + dv - v_{rel}$$

代入 ①：$-dM\cdot v_{rel} = M\,dv$ ②

* 理解：假定参照系速度也为 v

$$-dM\cdot(v_{rel} - dv) = (M+dM)\cdot dv$$

$$\Rightarrow -dM\cdot v_{rel} = M\cdot dv$$

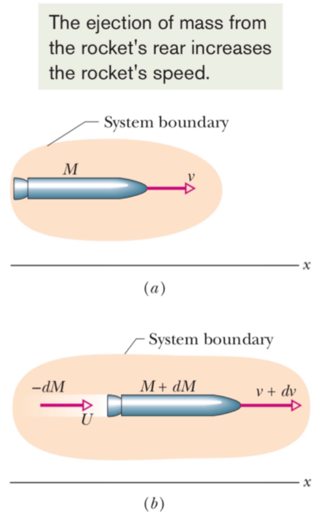


Figure 9-22 (*a*) An accelerating rocket of mass M at time t, as seen from an inertial reference frame. (*b*) The same but at time $t + dt$. The exhaust products released during interval dt are shown.

· 将②式两边同除以 dt

$$-\frac{dM}{dt}\cdot v_{rel} = M\cdot\frac{dv}{dt}$$

发现 $\frac{dM}{dt} = R$ (Mass rate of fuel consumption) 且 $\frac{dv}{dt} = a$

得 $R\cdot v_{rel} = Ma$

由此得 First Rocket Equation：

$$T = R\cdot v_{rel}$$

T 为 the Thrust (驱动力) of the rocket engine

· 将②式整理：

$$dv = -v_{rel}\frac{dM}{M}$$

对两边求积分：

$$\int_{v_i}^{v_f} dv = -v_{rel}\cdot\int_{M_i}^{M_f}\frac{dM}{M}$$

由此得 Second Rocket Equation：

$$v_f - v_i = v_{rel}\ln\frac{M_i}{M_f}$$

(可利用速度变化量推算燃料使用情况)

例 In all previous examples in this chapter, the mass of a system is constant (fixed as a certain number). Here is an example of a system (a rocket) that is losing mass. A rocket whose initial mass M_i is 850 kg consumes fuel at the rate $R = 2.3$ kg/s. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s. What thrust does the rocket engine provide?

Calculation: Here we find

$$T = Rv_{rel} = (2.3\ \text{kg/s})(2800\ \text{m/s}) = 6440\ \text{N} \approx 6400\ \text{N}.$$

(b) What is the initial acceleration of the rocket?

Calculation: We find

$$a = \frac{T}{M_i} = \frac{6440\ \text{N}}{850\ \text{kg}} = 7.6\ \text{m/s}^2. \qquad \text{(Answer)}$$

To be launched from Earth's surface, a rocket must have an initial acceleration greater than $g = 9.8\ \text{m/s}^2$. That is, it must be greater than the gravitational acceleration at the surface. Put another way, the thrust T of the rocket engine must exceed the initial gravitational force on the rocket, which here has the magnitude $M_i g$, which gives us

$$(850\ \text{kg})(9.8\ \text{m/s}^2) = 8330\ \text{N}.$$

Because the acceleration or thrust requirement is not met (here $T = 6400$ N), our rocket could not be launched from Earth's surface by itself; it would require another, more powerful, rocket.

Summary

- Center of Mass

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i,\ y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i,\ z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

- Newton's Law for a System of Particles

$$\vec{F}_{net} = M\vec{a}_{com}$$

- Linear Momentum and Newton's Second Law

$$\vec{P} = M\vec{v}_{com} \text{ and } \vec{F}_{net} = \frac{d\vec{P}}{dt}$$

- Collision and Impulse

$$\Delta\vec{p} = \vec{p_f} - \vec{p_i} = \vec{J} \text{ and } F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v$$

Summary

- Conservation of Linear Momentum

$\vec{P} = constant$ and $\vec{P}_i = \vec{P}_f$ (closed, isolated system)

- Inelastic Collision in One Dimension

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- Motion of COM for an isolated and closed system: Unchanged
- Elastic Collision in One Dimension

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$

Summary

- Collision in Two Dimensions

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

  - If the collision is elastic collision

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

- Variable-Mass Systems

$$T = Rv_{rel} = Ma$$

$$v_f - v_i = v_{rel} \ln\frac{M_i}{M_f}$$