§1 Fluid

1. Fluid

1° 可以flow

2° 形状由盛装的容器决定

因为无法维持一个平行于表面的力（无法承受 shearing stress）

3° 可以维持一个垂直于表面的力

2. Density

1° Density : $\rho = \frac{\Delta m}{\Delta V}$

2° Uniform density : $\rho = \frac{m}{V}$

3° 标量

4° SI-Unit : kg/m^3

Table 14-1 Some Densities

Material or Object	Density (kg/m^3)
Interstellar space	10^{-20}
Best laboratory vacuum	10^{-17}
Air: 20°C and 1 atm pressure	1.21
20°C and 50 atm	60.5
Styrofoam	1×10^2
Ice	0.917×10^3
Water: 20°C and 1 atm	0.998×10^3
20°C and 50 atm	1.000×10^3
Seawater: 20°C and 1 atm	1.024×10^3
Whole blood	1.060×10^3
Iron	7.9×10^3
Mercury (the metal, not the planet)	13.6×10^3
Earth: average	5.5×10^3
core	9.5×10^3
crust	2.8×10^3
Sun: average	1.4×10^3
core	1.6×10^5
White dwarf star (core)	10^{10}
Uranium nucleus	3×10^{17}
Neutron star (core)	10^{18}

3. Pressure（压强）

1° Pressure : $p = \frac{\Delta F}{\Delta A}$

2° 标量

3° SI-Unit : Pa (pascal)

$1\ N/m^2 = 1\ Pa$

$1\ atm = 1.01 \times 10^5\ Pa$

Problem

- A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m. (a) What does the air in the room weigh when the air pressure is 1.0 atm?

SOLUTION:

$mg = (\rho V) g$ (Use ρ of air from Table 14-1)

$= (1.21\ kg/m^3)(3.5\ m \times 4.2\ m \times 2.4\ m)(9.8\ m/s^2)$

$= 418\ N \approx 420\ N$

(b) What is the magnitude of the atmosphere's force on the floor of the room?

SOLUTION:

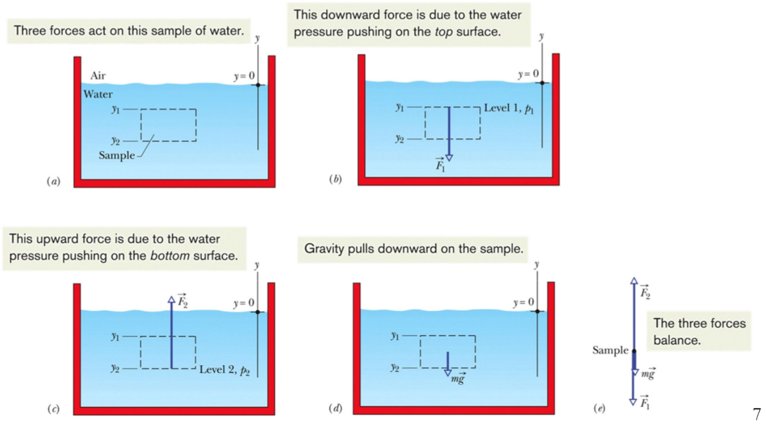
$$F = p\,A = (1.0\ atm)\left(\frac{1.01 \times 10^5\ N/m^2}{1.0\ atm}\right)(3.5\ m)(4.2\ m)$$

$$= 1.5 \times 10^6\ N$$

§2 Fluid at rest

1、hydrostatic pressure (静水压强)

1° 由静止的流体产生



2° ·
$$\begin{cases} F_2 = F_1 + mg \\ F_1 = p_1 A \\ F_2 = p_2 A \end{cases}$$

· 因此，$p_2 A = p_1 A + \rho A g (y_1 - y_2)$

$$p_2 = p_1 + \rho g (y_1 - y_2)$$

3° Absolute pressure (绝对压强)

深度为h的压强：

$$p = p_0 + \rho g h$$

Absolute pressure (p) 包含两部分：

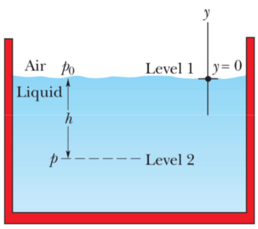
① p_0：大气压强

② $\rho g h$：The gauge pressure (表压)：绝对压强和大气压强的差值

4° Atmospheric pressure (大气压强)

$$p = p_0 - \rho_{air} g d$$

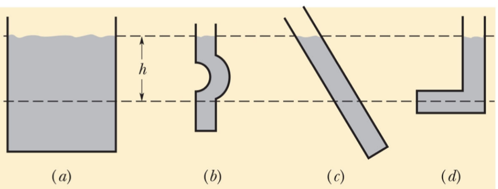
d：到I平面的距离



例：

Checkpoint 1

The figure shows four containers of olive oil. Rank them according to the pressure at depth h, greatest first.



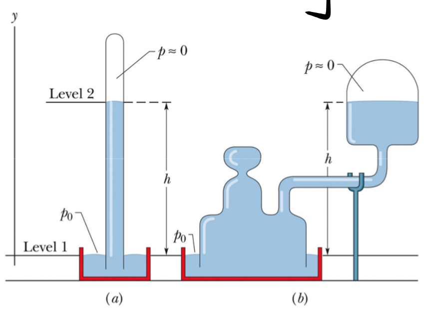
Answer: All the pressures will be the same. All that matters is the distance h, from the surface to the location of interest, and h is the same in all cases.

2. Measuring pressure

1° Mercury Barometer (水银气压计)

$p_0 = \rho g h$

- ρ: 水银密度
- h: air-mercury interface 与 mercury level 间的距离



- Independent of the cross-sectional area

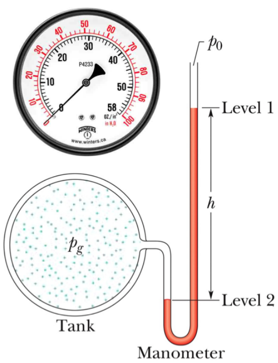
2° The open-tube manometer (开管压力计)

- 测量的是气体的 gauge pressure p_g:

$p_g = p - p_0 = \rho g h$

ρ: 液体密度

- p_g 与 h 成正比



- p_g is directly proportional to h
- The gauge pressure p_g can be positive or negative, depending on whether $p > p_0$ or $p < p_0$
- Eg. Suck on straw to pull the liquid up: the absolute pressure in your lung is less than atmospheric pressure

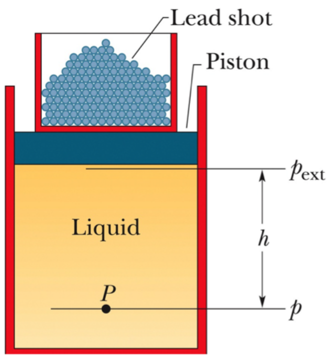
3. Pascal's principle (帕斯卡原理)

1° governs the transmission of pressure through an incompressible fluid.

2° 不可压缩静止流体中任一点受外力产生压强增值后，此压强增值瞬时间传至静止流体各点.

$p = p_{ext} + \rho g h$

$\Delta p = p - \rho g h = p_{ext}$

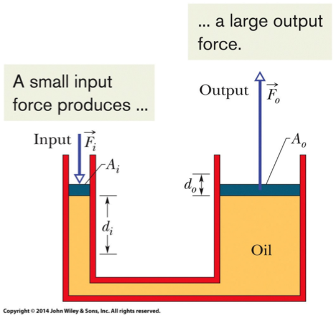


- The pressure at any point in the liquid is:

$$p = p_{ext} + \rho g h$$
$$\Delta p = p - \rho g h = p_{ext}$$

3° 应用: Hydraulic lever (液压杆)

· 力增大, 功不变



$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o}$$

$$F_o = F_i \frac{A_o}{A_i}$$

$$V = A_i d_i = A_o d_o$$

$$d_o = d_i \frac{A_i}{A_o}$$

$$W = F_o d_o = F_i \frac{A_o}{A_i} \times d_i \frac{A_i}{A_o} = F_i d_i$$

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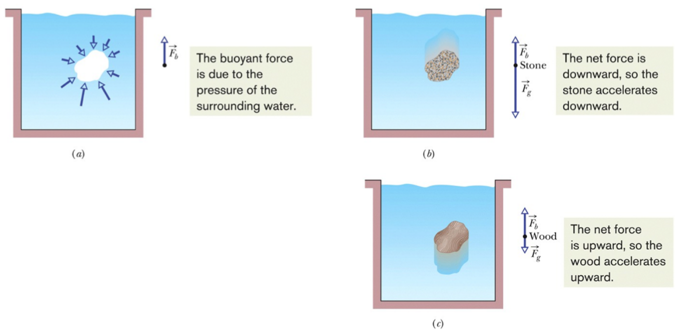
4. Archimedes' principle (阿基米德原理)

1° The buoyant force (浮力) is the net upward force on a submerged object by the fluid in which it is submerged

- This force opposes the weight of the object, and comes from the increase in pressure with depth



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2° $F_b = m_f g$

m_f 为物体排开水的质量

- When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body.
- The buoyant force on a body

$$F_b = m_f g$$

- Where m_f is the mass of the fluid that is displaced by the body

3° 若物体 floats in the fluid, 则

$$F_b = F_g = m_f g$$

- When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude of the gravitational force F_g on the body.

$$F_b = F_g$$

- When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f g$ of the fluid that has been displaced by the body.

$$F_g = m_f g$$

4° Apparent weight in a fluid

weight app = weight − F_b

- Apparent Weight is related to the actual weight of a body and the buoyant force on the body by

(apparent weight) = (actual weight) – (buoyant force)

Or

$$weight_{app} = weight - F_b$$

例:

Checkpoint 2

A penguin floats first in a fluid of density ρ_0, then in a fluid of density $0.95\rho_0$, and then in a fluid of density $1.1\rho_0$. (a) Rank the densities according to the magnitude of the buoyant force on the penguin, greatest first. (b) Rank the densities according to the amount of fluid displaced by the penguin, greatest first.

Answer: (a) all the same (b) $0.95\rho_0$, $1\rho_0$, $1.1\rho_0$

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§3 Ideal fluids in motion

1. 关于 ideal fluid 的四个假设

1° Steady flow: 流动液中任意一点的速度不随时间改变

2° Incompressible flow: ideal fluid is incompressible, 它的密度均匀不变

3° Nonviscous (无粘性的) flow: 无 viscous drag force: 没有由 viscosity 导致的 resistive force.

4° Irrotational flow: test body 不会绕着 COM 旋转

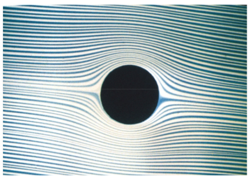
2. streamline (流线)

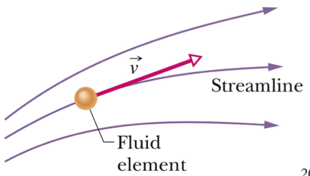
1° the path that a tiny element of the fluid would take as the fluid flows

2° 分子速度方向与流线相切

3° 两条流线不会相交

若两条流线相交,交点处的分子会同时有两个方向上的速度,这是不可能的





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3. The equation of continuity (连续性方程)

- 流速取决于流过的截面 A
- 由 incompressible flow: 单位时间流过截面的体积一定

$$\Delta V = A_1 V_1 \Delta t = A_2 V_2 \Delta t$$

- Equality of continuity:

$$A_1 V_1 = A_2 V_2$$

面积越小,速度越大

- Volume flow rate R_V

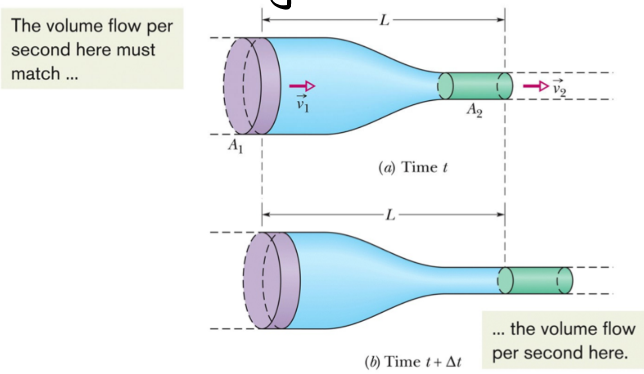
  $R_V = Av = a$ constant

  SI-unit : m^3/s

- Mass flow rate R_m

  $R_m = \rho A v = a$ constant

  SI-unit : kg/s

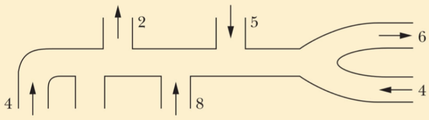


例:

Problem

Checkpoint 3

The figure shows a pipe and gives the volume flow rate (in cm^3/s) and the direction of flow for all but one section. What are the volume flow rate and the direction of flow for that section?



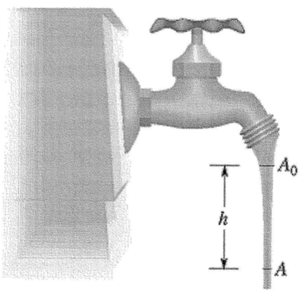
Answer:

4-2+8+5-6+4=13

13, out

例:

Problem



- Figure 15-18 shows how the stream of water emerging from a faucet "necks down" as it falls. The indicated cross-sectional areas are A_0 = 1.2 cm^2 and A = 0.35 cm^2. The two levels are separated by a vertical distance h = 45 mm. What is the volume flow rate from the tap?

Solution:

$$A_0 v_0 = Av$$

$$v^2 = v_0^2 + 2gh$$

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$= \sqrt{\frac{(2)(9.8\ m/s^2)(0.045\ m)(0.35\ cm^2)^2}{(1.2\ cm^2)^2 - (0.35\ cm^2)^2}}$$

$$= 0.286\ m/s = 28.6\ cm/s$$

The volume flow rate is :

$$R_v = A_0 v_0 = (1.2\ cm2)(28.6\ cm/s)$$

$$= 34\ cm^3/s$$

The water necks down because the velocity of water increases with decreasing height, but the volume flow rate (volume of water per sec) is constant!

4. Bernoulli's equation (伯努利方程)

1° · 流体动能

$K=\frac{1}{2}\rho v^2\cdot\Delta V$

· 压力势能

$p A\cdot\Delta s = p\Delta V$

· 重力势能

$\rho g y\cdot\Delta V$

2° 伯努利方程

· $p+\frac{1}{2}\rho v^2+\rho g y = \text{constant}$

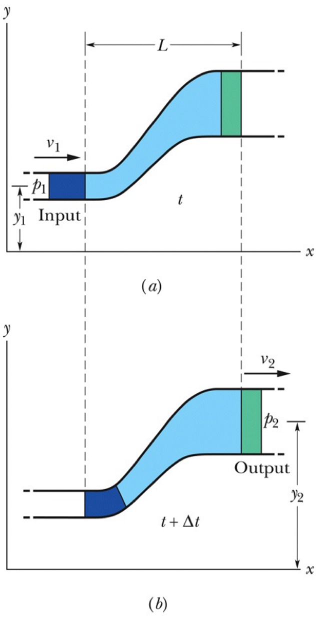
· 若 $v_1=v_2=0$，有

$p_2=p_1+\rho g(y_2-y_1)$

· 若 $y_1=y_2$，有

$p_1+\frac{1}{2}\rho v_1^2=p_2+\frac{1}{2}\rho v_2^2$

流速大，压强小



Proof

- Work done on the system

$$W = W_g + W_p = \Delta K$$

- Work done by Gravitational Force

$$W_g = -\Delta m g(y_2-y_1) = -\rho\Delta V g(y_2-y_1)$$

- Work done by the system

$$F\Delta x = pA\Delta x = p\Delta V$$

$$W_p = p_1\Delta V + (-p_2\Delta V) = -\Delta V(p_2-p_1)$$

- K.E gained:

$$\Delta K = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 = \frac{1}{2}\rho\Delta V(v_2^2-v_1^2)$$

- Thus,

$$-\rho\Delta V g(y_2-y_1) - \Delta V(p_2-p_1) = \frac{1}{2}\rho\Delta V(v_2^2-v_1^2)$$

$$p_1+\frac{1}{2}\rho v_1^2+\rho g y_1 = p_2+\frac{1}{2}\rho v_2^2+\rho g y_2$$

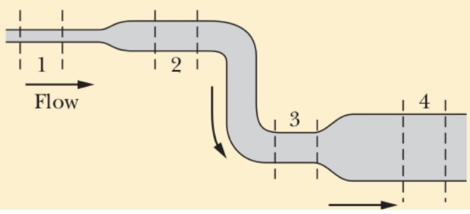
例:

Problem

Checkpoint 4

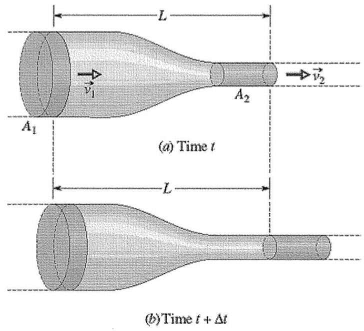
Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to (a) the volume flow rate R_V through them, (b) the flow speed v through them, and (c) the water pressure p within them, greatest first.

$$p_1+\frac{1}{2}\rho v_1^2+\rho g y_1 = p_2+\frac{1}{2}\rho v_2^2+\rho g y_2$$



Answer: (a) all the same volume flow rate
(b) 1, 2 & 3, 4
(c) 4, 3, 2, 1

例: **Problem**



- Ethanol of density ρ = 791 kg/m³ flows smoothly through a horizontal pipe that tapers in cross-sectional area from A_1 = 1.20 x 10⁻³ m² to $A_2 = A_1/2$. The pressure difference between the wide and narrow sections of pipe is 4120 Pa. What is the volume flow rate R_V of the ethanol?

SOLUTION:

$$R_v = v_1 A_1 = v_2 A_2$$

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y$$

$$v_1 = \frac{R_v}{A_1} \quad \text{and} \quad v_2 = \frac{R_v}{A_2} = \frac{2R_v}{A_1}$$

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2$$

$$\frac{2(p_1 - p_2)}{\rho} = v_2^2 - v_1^2 = \frac{3R_v^2}{A_1^2}$$

$$R_v = A_1 \sqrt{\frac{2(p_1 - p_2)}{3\rho}}$$

The lower speed v_1 means that p_1 is greater, we have :

$$R_v = 1.20 \text{ x } 10^{-3}\ \text{m}^2 \sqrt{\frac{(2)(4120\ \text{Pa})}{(3)(791\text{kg}/\text{m}^3)}}$$

$$= 2.24 \text{ x } 10^{-3}\ \text{m}^2 \times \frac{m}{s} = 2.24 \times 10^{-3} m^3/s$$

Summary

- Density and Pressure

$$\rho = \frac{m}{V} \ \&\ p = \frac{F}{A}$$

- Pressure Variation with Height and Depth

$$p = p_0 + \rho g h$$

- Pascal's Principle: change in pressure transferred undiminished to every portion of the fluid and to the walls of the containing vessel

- Archimedes' Principle

$$F_b = m_f g \ \&\ weight_{app} = weight - F_b$$

- Flow of Ideal Fluids

$$R_V = Av = a\ constant \ \&\ R_m = \rho A v = a\ constant$$

Summary

- Bernoulli's Equation

$$p + \frac{1}{2}\rho v^2 + \rho g y = constant$$