Lecture 23

§1 Longitudinal waves

1. Sound wave

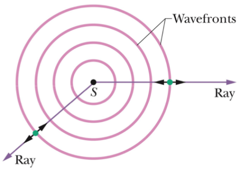
1° Point source S (点声源): tiny sound source

2° Wavefronts (波前): 振动的 radial displacement (径向位移) 相同的 surfaces

3° Rays: 垂直于波前, 表明传播方向的直线.

4° Double headed arrows: 表明介质的 elements 平行于 rays

- Applications: Sonar to detect the underwater obstacles; Heart beats to signal a medical problem
- To describe a sound wave
  - Point source S: tiny sound source.
  - **Wavefronts**: surfaces over which the oscillations due to the sound wave have the same radial displacement.
  - **Rays**: directed lines perpendicular to the wavefronts that indicate the direction of travel of the wavefronts



- Double Headed Arrows: indicates the elements of the medium **oscillate** parallel to the rays.

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2、声速

$$p = B\frac{\Delta V}{V}$$

$$v = \sqrt{\frac{B}{\rho}}$$

B: bulk modulus (Pa)

ρ: density

- For mechanical wave: the speed of wave depends on the medium

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{elastic\ property}{inertia\ property}}$$

- For sound wave through air, the potential energy is associated with the periodic compression and expansion of small volume elements of the air.

$$p = B\frac{\Delta V}{V}$$

- The speed of sound is

$$v = \sqrt{\frac{B}{\rho}}$$

- B is the bulk modulus (Pa)
- ρ is the density

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Table 17-1
The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

- For Bulk Modulus (Ch 12)

$$B_{solid} \gg B_{liquid} \gg B_{gas}$$

- Solid is more incompressible (less compressible) than liquid
- Liquid is more incompressible (less compressible) than gas
- Order of Sound Speed

$$v_{solid} \gg v_{liquid} \gg v_{gas}$$

- As shown in the table, the sound wave travels much faster in solid medium.

推导:

- 对于一个 moving air element:

$$\Delta m = \rho A \Delta x = \rho A v \Delta t$$

- 其加速度为:

$$a = \frac{\Delta v}{\Delta t}$$

- 作用于该物体上的力:

$$F = (p+\Delta p)A - pA = \Delta p A$$

- 利用牛二:

$$F = \Delta m a$$

$$\Delta p A = \rho A v \Delta t \times \frac{\Delta v}{\Delta t}$$

$$\Delta p = \rho v \Delta v$$

- 空气压缩 ΔV 后的压力

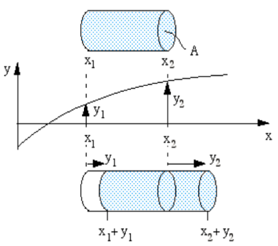
$$\Delta p = B\frac{\Delta V}{V} = B\frac{\Delta v t A}{v t A} = B\frac{\Delta v}{v}$$

- 由此,

$$\Delta p = \rho v \Delta v = B\frac{\Delta v}{v}$$

$$v = \sqrt{\frac{B}{\rho}}$$

Additional Information-Another way to find the sound speed

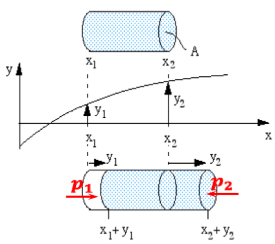


- Originally undisturbed element column:
  $p = p_0$, and $V = A(x_2 - x_1) = A\Delta x$
- Disturbed due to sound wave:
  $p = p_0 + \Delta p$
  $\Delta V = A(x_2 + y_2 - x_1 - y_1) - V = A\Delta y$

$$\Delta p = -B\frac{\Delta V}{V} = -B\frac{\Delta y}{\Delta x} \xRightarrow{\Delta x \to 0} -B\frac{\partial y}{\partial x}$$

$$p = p_0 - B\frac{\partial y}{\partial x}$$

Pressure of fluid element (initially at equilibrium position x) with average displacement y.



- Net force on the fluid element:

$$F_{net} = p_1 A - p_2 A = AB\left(\left.\frac{\partial y}{\partial x}\right|_2 - \left.\frac{\partial y}{\partial x}\right|_1\right) = AB\Delta\left(\frac{\partial y}{\partial x}\right)$$

- Mass: $\Delta m = \rho A(x_2 - x_1) = \rho A \Delta x$
- Acceleration: $a = \frac{\partial^2 y}{\partial t^2}$

Newton's 2nd law: $F_{net} = \Delta m a$

$$\lim_{\Delta x \to 0}\frac{\Delta\left(\frac{\partial y}{\partial x}\right)}{\Delta x} = \frac{\partial^2 y}{\partial x^2} = \frac{1}{(B/\rho)}\frac{\partial^2 y}{\partial t^2}$$

Traveling wave equation: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

$$v = \sqrt{\frac{B}{\rho}}$$

§2 Traveling sound wave

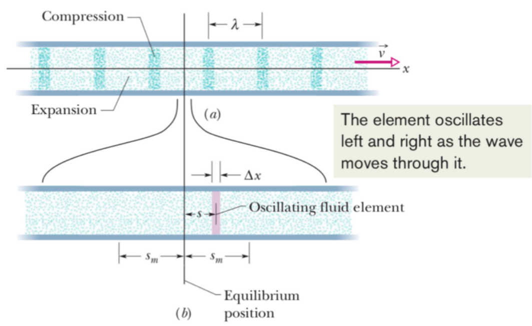
1. Sinusoidal function of displacement

$s(x,t) = s_m \cos(kx - \omega t)$

s_m：偏离平衡位置的最大位移.

$k, \omega, \lambda, f, T, v$ 与横波类似.

Figure 17-4 (*a*) A sound wave, traveling through a long air-filled tube with speed v, consists of a moving, periodic pattern of expansions and compressions of the air. The wave is shown at an arbitrary instant. (*b*) A horizontally expanded view of a short piece of the tube. As the wave passes, an air element of thickness Δx oscillates left and right in simple harmonic motion about its equilibrium position. At the instant shown in (*b*), the element happens to be displaced a distance s to the right of its equilibrium position. Its maximum displacement, either right or left, is s_m.



- For a longitudinal wave, the particle displacement is in the x-direction.
- In the following, we will replace y(x, t) by s(x, t). Here, s(x, t) points in the x-direction.
- The displacement amplitude of element at origin (x=0) is s_m.

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2. Pressure variation

$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t)$

Δp_m: Pressure amplitude：最大的 increase/decrease in pressure

$\Delta p_m = (v\rho\omega) s_m$

推导：

- 对于弹性形变：

  $$\Delta p = -B\frac{\Delta V}{V}$$

- 替换 ΔV 与 V

  $$V = A\Delta x$$

  $\Delta V = A\Delta s$ (Vol. change caused by oscillation)

  $$\Delta p = -B\frac{A\Delta s}{A\Delta x} = -B\frac{\partial s}{\partial x}$$

- 替换 $\frac{\partial s}{\partial x}$

  $$\frac{\partial s}{\partial x} = -k s_m \sin(kx - \omega t)$$

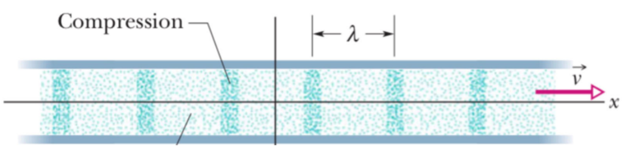
  $$\Delta p = B k s_m \sin(kx - \omega t)$$

- 由此，

  $$\Delta p_m = B k s_m = v^2 \rho k s_m$$

  $$v = \frac{\omega}{k}$$

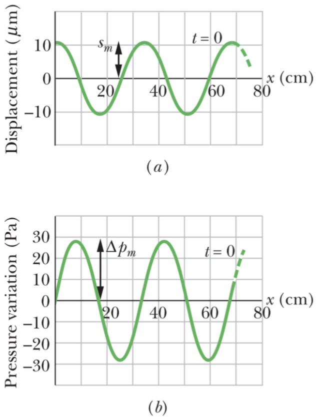
  $$\Delta p_m = v^2 \rho k s_m = v\rho\omega s_m$$



3、加速度与速度

$s(x,t) = s_m \cos(kx-\omega t)$

- D位移: compression / expansion
- $u(x,t) = \frac{\partial s}{\partial t} = \omega s_m \sin(kx-\omega t)$
- $a(x,t) = \frac{\partial u}{\partial t} = -\omega^2 s_m \cos(kx-\omega t)$



$s(x,t) = s_m\, cos(kx - \omega t)$

- Zero displacement → either Compression or Expansion
- $u(x,t) = \frac{\partial s}{\partial t} = \omega s_m \sin(kx - \omega t)$
- $a(x,t) = \frac{\partial u}{\partial t} = -\omega^2 s_m \cos(kx - \omega t)$

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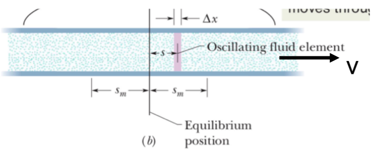
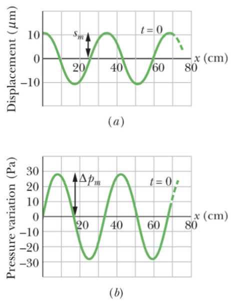
例: **Problem**

Checkpoint 1

When the oscillating air element in Fig. 17-4*b* is moving rightward through the point of zero displacement, is the pressure in the element at its equilibrium value, just beginning to increase, or just beginning to decrease?





Answer: Decrease
As x=42 cm in the fig.

例: **Problem**

The maximum pressure amplitude Δp_m that the human ear can tolerate in loud sounds is about **28 Pa** (which is very much less than the normal air pressure of about 10^5 Pa). What is the displacement amplitude s_m for such a sound in air of $\rho = 1.21\ kg/m^3$, at a frequency of **1000 Hz** and a speed of **343 m/s**?

Solution:

$$\Delta p_m = v^2 \rho k s_m = v\rho\omega s_m$$

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi f)} = 1.1 \times 10^{-5}\ m = 11\ \mu m$$

§3 Interference

1. Interference

两声波有相同的振幅，波长，向x轴正方向传播，相差为ϕ

Wave 1: $s_1(x,t) = s_m \cos(kx-\omega t)$

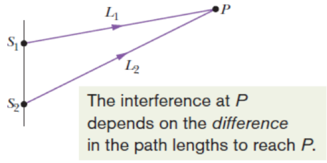
Wave 2: $s_2(x,t) = s_m \cos(kx-\omega t+\phi)$

Resultant wave: $s'(x,t) = 2s_m \cos(\frac{\phi}{2}) \cos(kx - \omega t + \frac{\phi}{2})$

$s_m' = |2s_m \cos(\frac{\phi}{2})|$

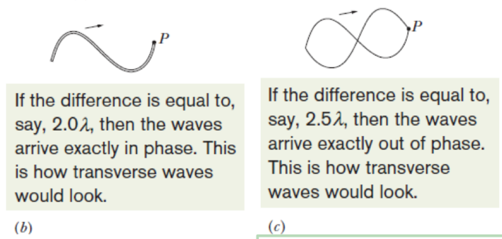
相差∅取决于 path length differences

$\Delta L = |L_2 - L_1|$



(a)

- Two point sources shown in (a)
- If in phase (b), fully constructive interference; if out of phase as (c), fully destructive interference.



$\Delta L = n\lambda$
$(n = 0,1,2\ldots)$
(Fully constructive)

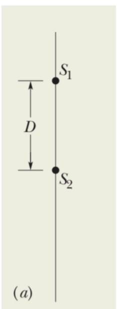
$\Delta L = \left(n + \frac{1}{2}\right)\lambda$
$(n = 0,1,2\ldots)$
(Fully destructive)

- To find the phase difference ∅:

$$\frac{\phi}{2\pi} = \frac{\Delta L}{\lambda}$$

$$\phi = 2\pi \frac{\Delta L}{\lambda}$$

例: **Problem**



In Fig. 17-8*a*, two point sources S_1 and S_2, which are in phase and separated by distance $D = 1.5\lambda$, emit identical sound waves of wavelength λ.

(a) What is the path length difference of the waves from S_1 and S_2 at point P_1, which lies on the perpendicular bisector of distance D, at a distance greater than D from the sources (Fig. 17-8*b*)? (That is, what is the difference in the distance from source S_1 to point P_1 and the distance from source S_2 to P_1?) What type of interference occurs at P_1?

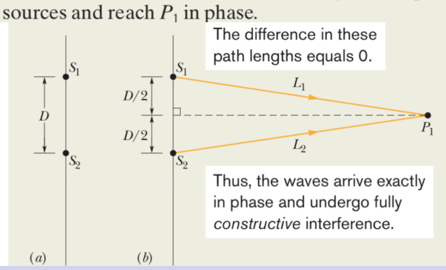
(b) What are the path length difference and type of interference at point P_2 in Fig. 17-8*c*?

Solution: (a)

Reasoning: Because the waves travel identical distances to reach P_1, their path length difference is

$$\Delta L = 0. \qquad \text{(Answer)}$$

From Eq. 17-23, this means that the waves undergo fully constructive interference at P_1 because they start in phase at the sources and reach P_1 in phase.

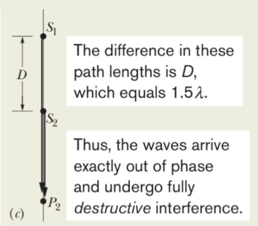


Solution: (b)

Reasoning: The wave from S_1 travels the extra distance D $(= 1.5\lambda)$ to reach P_2. Thus, the path length difference is

$$\Delta L = 1.5\lambda. \qquad \text{(Answer)}$$

From Eq. 17-25, this means that the waves are exactly out of phase at P_2 and undergo fully destructive interference there.



Summary

- Sound Waves

$$v = \sqrt{\frac{B}{\rho}}$$

(*a*) $$\overbrace{s(x,t)}^{\text{Displacement}} = \underbrace{s_m}_{\text{Displacement amplitude}} \underbrace{\cos(kx - \omega t)}_{\text{Oscillating term}}$$

(*b*) $$\underbrace{\Delta p(x,t)}_{\text{Pressure variation}} = \underbrace{\Delta p_m}_{\text{Pressure amplitude}} \overbrace{\sin(kx - \omega t)}^{\text{Oscillating term}}$$

- Pressure Variation Amplitude

$$\Delta p_m = (v \rho \omega) s_m$$

Summary

- Interference: two identical wave passing through one common point

$$\emptyset = 2\pi \frac{\Delta L}{\lambda}$$

- Fully Constructive

$$\frac{\Delta L}{\lambda} = 0, 1, 2 \ldots$$

- Fully Destructive

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5 \ldots$$