Lecture 6

# §1 Potential Energy

## 1. Conservative forces（保守力）

1° 保守力：gravitational force，spring force

非保守力：kinetic friction force，drag force

2° 保守力做功与路径无关，与始末位置有关

保守力在闭合路径上做功为 0

## 2. Potential energy（势能）

1° 在 gravitational force 或 spring force etc. 作用下的物体有势能

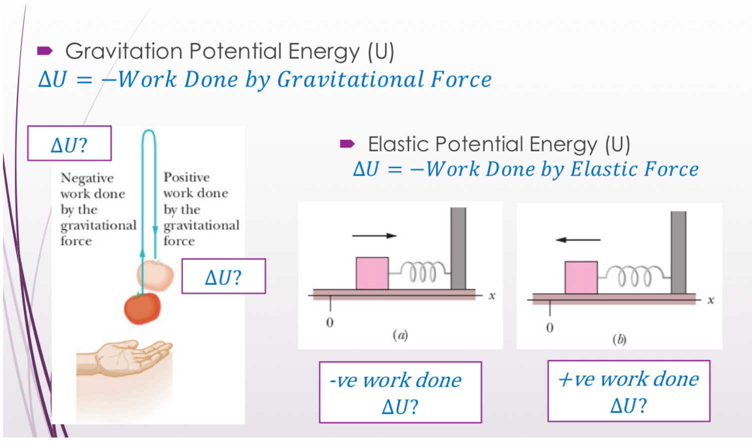
2° 定义：

> The energy that is associated with the configuration (arrangement/position) of a system in which a conservative force acts.

例：gravitational potential force（重力势能），elastic potential energy（弹性势能）

3° 符号：$U$

4° 势能大小与位置有关




## 3. Gravitational potential energy

1° 对于 partical – Earth system

2° 仅与 vertical position $y$（相对于 reference position $y=0$）有关

3° $\Delta U = -W_g$

$$= -\int_{y_i}^{y_f} (-mg)\,dy$$

$$= mg\,y\Big|_{y_i}^{y_f}$$

$$= mg\Delta y$$

+ve direction：up

$$U_f - U_i = mg(y_f - y_i)$$

$U_i$: reference configuration　　$y_i$: reference point

## Gravitational Potential Energy

$$\Delta U = -W$$
$$W = \int_{x_i}^{x_f} F(x)dx$$
$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$

- Gravitational Potential Energy
$$\Delta U = -\int_{y_i}^{y_f}(-mg)dy = mg\int_{y_i}^{y_f}dy$$
$$= mg[y]_{y_i}^{y_f}$$
$$\Delta U = mg(y_f - y_i) = mg\Delta y$$
$$U_f - U_i = mg(y_f - y_i)$$
- $U_i$: Reference Configuration
- $y_i$: Reference point
$$U(y) = mgy$$
- $U_i = 0$, and $y_i = 0$
- +ve direction: up

# 4. Elastic potential energy

1° $\Delta U = -W_s$
$$= -\int_{x_i}^{x_f}(-kx)\,dx$$
$$= \frac{1}{2}k\,x^2\Big|_{x_i}^{x_f}$$
$$= \frac{1}{2}k(x_f^2 - x_i^2)$$

## Elastic Potential Energy

$$\Delta U = -W$$
$$W = \int_{x_i}^{x_f} F(x)dx$$
$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$

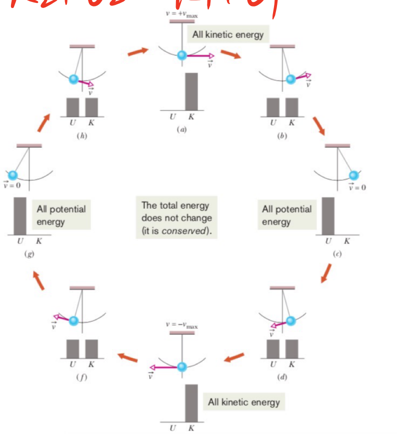
- Elastic Potential Energy
$$F(x) = -kx$$
$$\Delta U = -\int_{x_i}^{x_f}(-kx)dx = k\int_{x_i}^{x_f}xdx = \frac{1}{2}k[x^2]_{x_i}^{x_f}$$
$$\Delta U = U_f - U_i = \frac{1}{2}k(x_f^2 - x_i^2)$$
- $U_i$: Reference Configuration
- $x_i$: Reference point
$$U(x) = \frac{1}{2}kx^2$$
- $U_i = 0, x_i = 0$, at its relaxation position

# §2 Conservation of Mechanical Energy

## 1、Conservation of Mechanical Energy (机械能守恒)

对于一个 isolated system (没有影响系统内能量变化的外力作用)
仅有保守力做功，则

$$E_{mec} = K + U$$
$$\Delta K = -\Delta U$$
$$K_2 + U_2 = K_1 + U_1$$

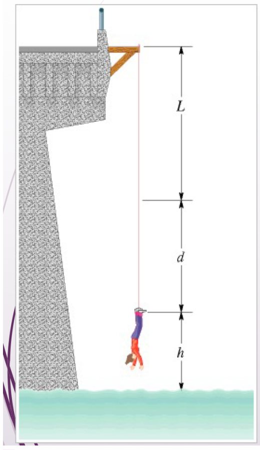



In an isolated system where only conservation forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy $E_{mec}$ of the system, cannot change

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

例:

## Sample Problem




A 61.0 kg bungee-cord jumper is on a bridge 45.0 m above a river. The elastic bungee cord has a relaxed length of $L$ = 25.0 m. Assume that the cord obeys Hooke's law, with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height $h$ of her feet above the water at her lowest point?

### SOLUTION:

$$\Delta K + \Delta U_e + \Delta U_g = 0$$

$$\Delta U_g = mg\Delta y = -mg(L+d)$$

$$\Delta U_e = \frac{1}{2}kd^2$$

$$0 + \frac{1}{2}kd^2 - mg\,(L+d) = 0$$

$$\frac{1}{2}kd^2 - mgL - mgd = 0$$

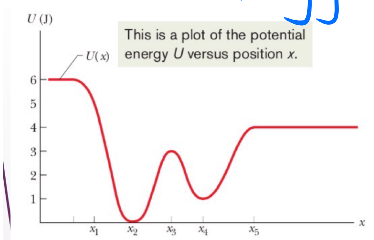
$$\frac{1}{2}(160\ \mathrm{N/m})d^2 - (61.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(25.0\ \mathrm{m}) - (61.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})d = 0$$

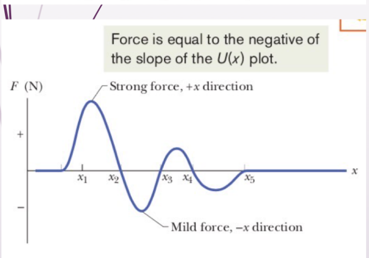
$$d = 17.9\ \mathrm{m}$$

$$h = 45.0\ \mathrm{m} - 42.9\ \mathrm{m} = 2.1\ \mathrm{m}$$

# §3 Potential energy curve

## 1. Potential energy curve




$$\Delta U = -W$$
$$W = \int_{x_i}^{x_f} F(x)dx$$
$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$
$$F(x) = -\frac{dU(x)}{dx}$$

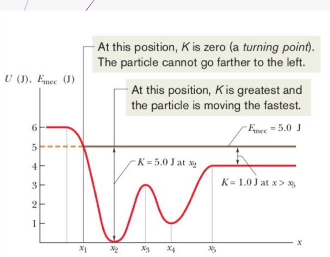
Restoring Force: the negative tangent of the potential curve

$$F(x) = -\frac{dU(x)}{dx}$$

## 2. Turning points (拐点)

动能为0的点

- In an isolated system, only conservative forces involved
$$E_{mec} = K(x) + U(x)$$
- The kinetic energy as a function of x
$$K(x) = E_{mec} - U(x)$$




- Turning Point:
  $K = 0$, at position $x_1$
- K→ Max, at $x_2$

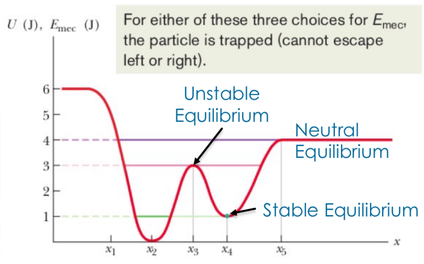
3. Equilibrium

1° neutral equilibrium (隨遇平衡)

2° unstable equilibrium (不穩定平衡)

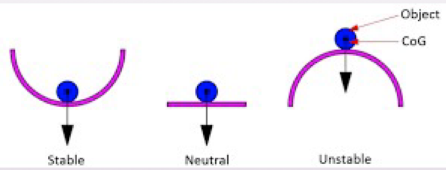
3° stable equilibrium (穩定平衡)

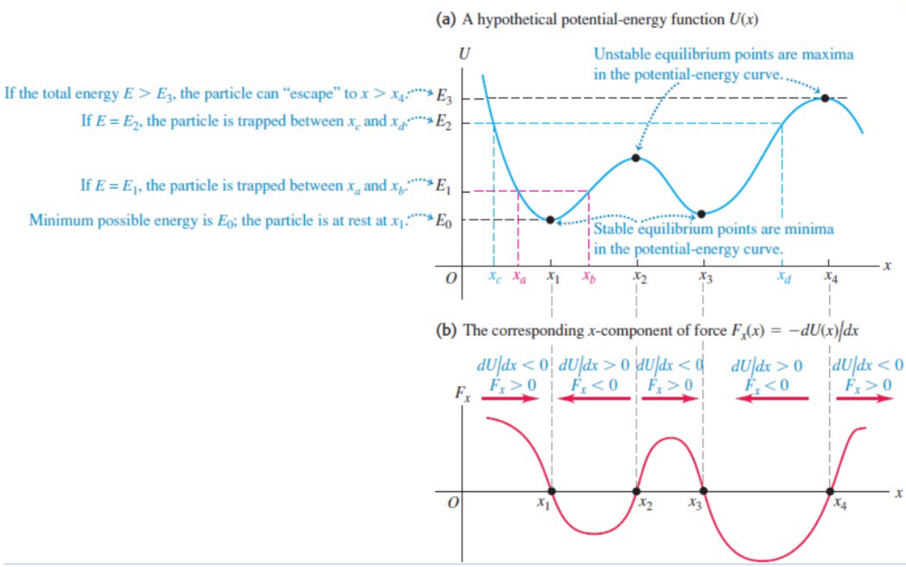
## Potential Energy Curve




If the $E_{mec}$ is

- Purple Line: after $x_5$, $K = 0$, Neutral Equilibrium
- Pink Line: at $x_3$, $K = 0$, Unstable Equilibrium
- Green Line: at $x_4$, $K = 0$, Stable Equilibrium




例

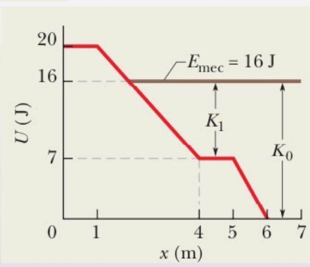
## Problem

A 2.00 kg particle moves along an $x$ axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10$a$. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of $U$. At $x = 6.5$ m, the particle has velocity $\vec{v}_0 = (-4.00\text{ m/s})\hat{\mathrm{i}}$.

(a) From Fig. 8-10$a$, determine the particle's speed at $x_1 = 4.5$ m.

(b) Where is the particle's turning point located?

(c) Evaluate the force acting on the particle when it is in region 1.9 m $< x <$ 4.0 m.



## SOLUTION

**Calculations:** At $x = 6.5$ m, the particle has kinetic energy

$$K_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(2.00\text{ kg})(4.00\text{ m/s})^2 = 16.0\text{ J}.$$

Because the potential energy there is $U = 0$, the mechanical energy is

$$E_{mec} = K_0 + U_0 = 16.0\text{ J} + 0 = 16.0\text{ J}.$$

This value for $E_{mec}$ is plotted as a horizontal line in Fig. 8-10*a*. From that figure we see that at $x = 4.5$ m, the potential energy is $U_1 = 7.0$ J. The kinetic energy $K_1$ is the difference between $E_{mec}$ and $U_1$:

$$K_1 = E_{mec} - U_1 = 16.0\text{ J} - 7.0\text{ J} = 9.0\text{ J}.$$

Because $K_1 = \frac{1}{2}mv_1^2$, we find

$$v_1 = 3.0\text{ m/s}. \qquad \text{(Answer)}$$

The turning point is where the force momentarily stops and then reverses the particle's motion. That is, it is where the particle momentarily has $v = 0$ and thus $K = 0$.

**Calculations:** Because $K$ is the difference between $E_{mec}$ and $U$, we want the point in Fig. 8-10*a* where the plot of $U$ rises to meet the horizontal line of $E_{mec}$, as shown in Fig. 8-10*b*. Because the plot of $U$ is a straight line in Fig. 8-10*b*, we can draw nested right triangles as shown and then write the proportionality of distances

$$\frac{16 - 7.0}{d} = \frac{20 - 7.0}{4.0 - 1.0},$$

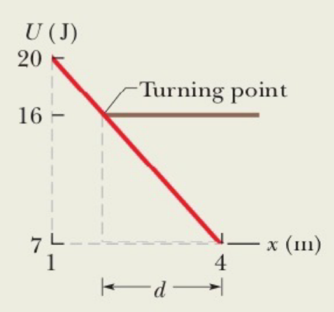
which gives us $d = 2.08$ m. Thus, the turning point is at

$$x = 4.0\text{ m} - d = 1.9\text{ m}. \qquad \text{(Answer)}$$

The force is given by Eq. 8-22 ($F(x) = -dU(x)/dx$): The force is equal to the negative of the slope on a graph of $U(x)$.

**Calculations:** For the graph of Fig. 8-10*b*, we see that for the range $1.0\text{ m} < x < 4.0\text{ m}$ the force is

$$F = -\frac{20\text{ J} - 7.0\text{ J}}{1.0\text{ m} - 4.0\text{ m}} = 4.3\text{ N}. \qquad \text{(Answer)}$$




# §4 Conservation of energy

## 1. Work done on a system by an external force

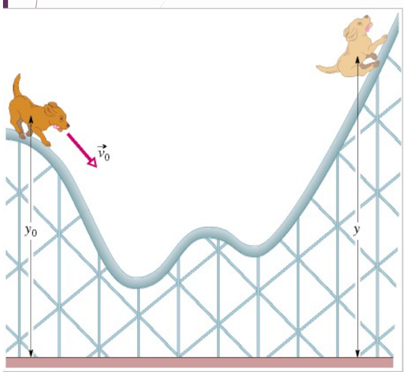
1° 不考虑 friction force

$$W = \Delta E_{mec} = \Delta K + \Delta U$$

2° 考虑 friction force

$$W = \Delta E_{mec} + \Delta E_{th} = \Delta K + \Delta U + \Delta E_{th}$$

例: **Problem**




A circus beagle of mass $m$ = 6.0 kg runs onto the left end of a curved ramp with speed $v_0$ = 7.8 m/s at height $y_0$ = 8.5 m above the floor. It then slides to the right and comes to a momentary stop when it reaches a height $y$ = 11.1 m from the floor. The ramp is not frictionless. What is the increase $\Delta E_{th}$ in the thermal energy of the beagle and ramp because of the sliding?

**SOLUTION:**

$$\Delta E_{mec} + \Delta E_{th} = 0$$

$$K_2 + U_2 = K_1 + U_1 - E_{th}$$

$$E_{th} = K_1 - K_2 + U_1 - U_2$$

$$\Delta E_{th} = \frac{1}{2}mv_0^2 - mg(y - y_0)$$

$$= \frac{1}{2}(6.0\text{ kg})(7.8\text{ m/s})^2 - (6.0\text{ kg})(9.8\text{ m/s}^2)(11.1\text{ m} - 8.5\text{ m})$$

$$\approx 30\text{ J}$$