Lecture 15

§1 Gravitation

1. Newton's law of gravitation

1° 对于两个质点:

$$F = G\frac{m_1 m_2}{r^2}$$

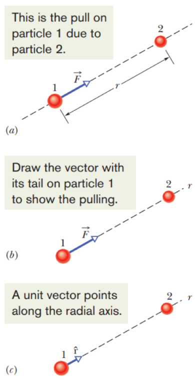
G 为 gravitational constant (引力常量):

$$G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$$
$$= 6.67\times10^{-11}\ m^3/kg\cdot s^2$$

2° 用 radial unit vector $\hat{r}$ 表示:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r}$$

Newton's Law of Gravitation



Newton's force law between two particles

$$F = G\frac{m_1 m_2}{r^2} \quad \text{(Newton's law of gravitation).}$$

G is the **gravitational constant:**

$$G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$
$$= 6.67\times10^{-11}\ \mathrm{m^3/kg\cdot s^2}.$$

We can describe $\vec{F}$ by using a radial unit vector $\hat{r}$

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r}.$$

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2. Shell Theorem (壳层定理)

一个均匀的 shell，吸引其外部的一个质点，等效于将该 shell 的质量集中于球心，该球心吸引另一质点。

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

3. Third-Law force pair

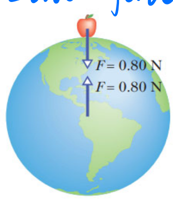


Figure 13-3 The apple pulls up on Earth just as hard as Earth pulls down on the apple.

These two forces form a force pair in Newton's third law.

例:

Checkpoint 1

A particle is to be placed, in turn, outside four objects, each of mass m: (1) a large uniform solid sphere, (2) a large uniform spherical shell, (3) a small uniform solid sphere, and (4) a small uniform shell. In each situation, the distance between the particle and the center of the object is d. Rank the objects according to the magnitude of the gravitational force they exert on the particle, greatest first.

Answer: The four forces are the same.

4. Principle of superposition

对于n个互相作用的质点，对于质点1的gravitational force为

$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13} + \cdots + \vec{F}_{1n}$$

或写作

$$\vec{F}_{net} = \sum_{i=2}^{n} \vec{F}_{1i}$$

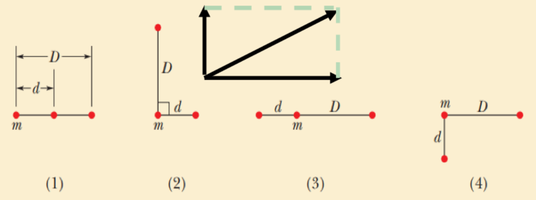
对于一个包含质点2, 3, ……n的 continuous system:

$$\vec{F}_1 = \int d\vec{F}$$

例 **Problem**

Checkpoint 2

The figure shows four arrangements of three particles of equal masses. (a) Rank the arrangements according to the magnitude of the net gravitational force on the particle labeled m, greatest first. (b) In arrangement 2, is the direction of the net force closer to the line of length d or to the line of length D?



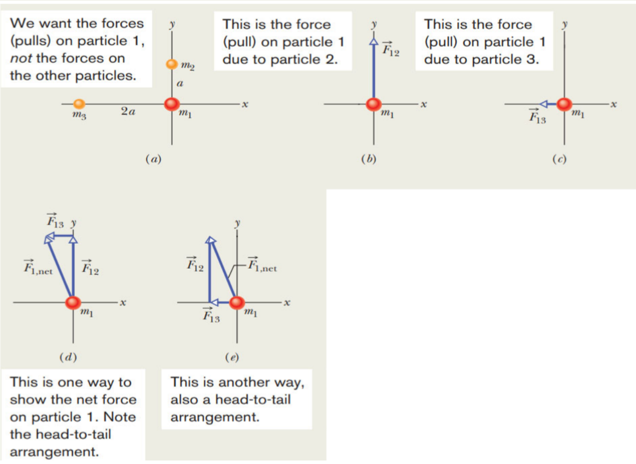
(1): $Gm^2\left(\frac{1}{d^2}+\frac{1}{D^2}\right)$; direction to right

(3): $Gm^2\left(\frac{1}{d^2}-\frac{1}{D^2}\right)$; direction to left

Answer: (a): (1), (2) and (4), (3); (b) close to the line of length d.

例: **Problem**

Figure 13-4a shows an arrangement of three particles, particle 1 of mass $m_1 = 6.0$ kg and particles 2 and 3 of mass $m_2 = m_3 = 4.0$ kg, and distance $a = 2.0$ cm. What is the net gravitational force $\vec{F}_{1,net}$ on particle 1 due to the other particles?



5. Gravitation near Earth's surface

- 对于地球外的一质点，其受地球的gravitational force大小为

$$F = G\frac{Mm}{r^2}$$

- gravitational force提供的向心加速度为

$$a_g = \frac{GM}{r^2}$$

Three Reasons for Difference of Acceleration

- Earth's mass is not uniformly distributed.
- Earth is not a sphere.
- Earth is rotating.

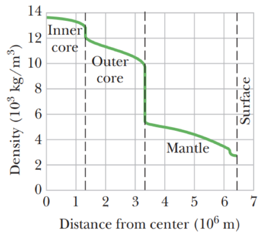


Figure 13-5 The density of Earth as a function of distance from the center. The limits of the solid inner core, the largely liquid outer core, and the solid mantle are shown, but the crust of Earth is too thin to show clearly on this plot.

6. 考虑地球自转的影响

Earth is Rotating

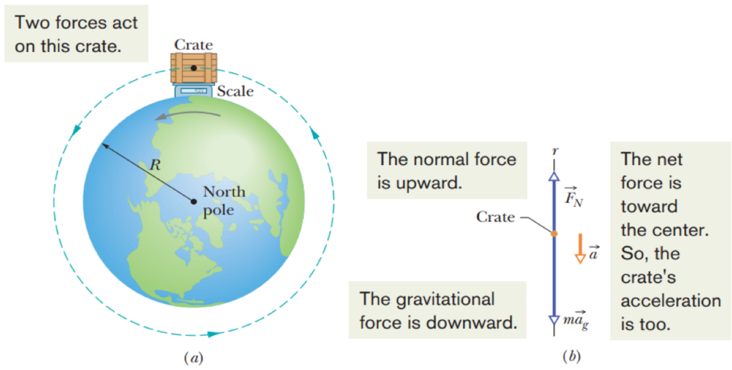


Figure 13-6 (*a*) A crate sitting on a scale at Earth's equator, as seen by an observer positioned on Earth's rotation axis at some point above the north pole. (*b*) A free-body diagram for the crate, with a radial *r* axis extending from Earth's center. The gravitational force on the crate is represented with its equivalent $m\vec{a}_g$. The normal force on the crate from the scale is $\vec{F}_N$. Because of Earth's rotation, the crate has a centripetal acceleration $\vec{a}$ that is directed toward Earth's center.

- 由牛顿第二定理：

$$ma_g - F_N = m\omega^2 R$$

$F_N (= mg)$ 为 measured weight

g 为 measured acceleration

- 有

$$ma_g - mg = m\omega^2 R$$

$$g = a_g - \omega^2 R$$

例

Problem

(a) An astronaut whose height h is 1.70 m floats "feet down" in an orbiting space shuttle at distance $r = 6.77 \times 10^6$ m away from the center of Earth. What is the difference between the gravitational acceleration at her feet and at her head?

The gravitational acceleration is

$$a_g = \frac{GM_E}{r^2}.$$

Taking the derivative of a_g with respect to r gives

$$da_g = -2\frac{GM_E}{r^3}\,dr,$$

We have

$$da_g = -2\frac{(6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2})(5.98 \times 10^{24}\ \mathrm{kg})}{(6.77 \times 10^6\ \mathrm{m})^3}(1.70\ \mathrm{m})$$

$$= -4.37 \times 10^{-6}\ \mathrm{m/s^2}, \qquad \text{(Answer)}$$

Summary

- Any particle in the universe attracts any other particle with a gravitational force whose magnitude is

$$F = G\frac{m_1 m_2}{r^2} \qquad \text{(Newton's law of gravitation)},$$

where m_1 and m_2 are the masses of the particles, r is their separation, and $G\,(= 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})$ is the gravitational constant.

- Gravitational forces obey the principle of superposition; that is, if n particles interact, the net force $\vec{F}_{1,\mathrm{net}}$ on a particle labeled particle 1 is the sum of the forces on it from all the other particles taken one at a time:

$$\vec{F}_{1,\mathrm{net}} = \sum_{i=2}^{n} \vec{F}_{1i},$$

in which the sum is a vector sum of the forces $\vec{F}_{1i}$ on particle 1 from particles 2, 3, . . . , n.

Summary

- The gravitational force $\vec{F}_1$ on a particle from an extended body is found by first dividing the body into units of differential mass dm, each of which produces a differential force $d\vec{F}$ on the particle, and then integrating over all those units to find the sum of those forces:

$$\vec{F}_1 = \int d\vec{F}.$$

- The gravitational acceleration a_g of a particle (of mass m) is due solely to the gravitational force acting on it. When the particle is at distance r from the center of a uniform, spherical body of mass M, the magnitude F of the gravitational force on the particle is given by Eq. 13-1. Thus, by Newton's second law,

$$F = ma_g,$$

which gives

$$a_g = \frac{GM}{r^2}.$$