A Small Collection Of Mathematical Formulas For The Prospective Statistician

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February 21, 2021

1 Arithmetic

Logarithms. In the following let a, $b > 0$ and $n \in \mathbb{R}$. Then

$$
\begin{aligned}
ln(a \cdot b) &= ln(a) + ln(b) \\
ln(\frac{a}{b}) &= ln(a) - ln(b) \\
ln(a^n) &= n \cdot ln(a) \\
e^{ln(a)} &= ln(e^a) = a.
\end{aligned}
$$

Binomial Coefficient. Suppose $0 < k \leq n$. Then

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

where $n! = 1 \cdot 2 \cdot \ldots \cdot n$.

2 Differential Calculus

Elementary Derivatives. In the following let f be a real-valued function of x and $a, b, r \in \mathbb{R}$. Then the derivative of f is given by f' as

$$
\begin{aligned}
f(x) = x^r \rightarrow f'(x) = r \cdot x^{r-1} \qquad & f(x) = e^x \rightarrow f'(x) = e^x \\
f(x) = r^x \rightarrow f'(x) = r^x \cdot ln(r) \qquad & f(x) = e^{a \cdot x + b} \rightarrow f'(x) = a \cdot e^{a \cdot x + b} \\
f(x) = \frac{1}{x^r} \rightarrow f'(x) = -r \cdot x^{-r-1} \qquad & f(x) = sin(x) \rightarrow f'(x) = cos(x) \\
f(x) = ln(x) \rightarrow f'(x) = \frac{1}{x} \qquad & f(x) = cos(x) \rightarrow f'(x) = -sin(x).
\end{aligned}
$$

Differentiation rules. In the following let h, u, v be real-valued functions of x.

- Product Rule: $h(x) = u(x) \cdot v(x) \to h'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$
- Chain Rule: $h(x) = u(v(x)) \to h'(x) = u'(v(x)) \cdot v'(x)$
- Reciprocal Rule: $h(x) = \frac{1}{v(x)} \to h'(x) = \frac{v'(x)}{(v(x))^2}$
- Quotient Rule: $h(x) = \frac{u(x)}{v(x)} \to h'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$

Gradient. Let f be a differentiable fuction of n variables. Then

$$\mathbf{grad}(f) = \nabla f = \left(\frac{\partial}{\partial x_1} f(x) \dots \frac{\partial}{\partial x_n} f(x) \right)^T .$$

Jacobian Matrix. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a differentiable function of n variables. Then the Jacobian matrix $\mathbf{J}$ of f is an $m \times n$ matrix whose (i, j)th entry is $\mathbf{J}_{ij} = \frac{\partial f_i}{\partial x_j}$. Which is

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial f}{\partial x_1} & \cdots & \dfrac{\partial f}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla^T f_1 \\ \vdots \\ \nabla^T f_m \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix}$$

where $\nabla^T f_i$ is the transpose (row vector) of the gradient of the ith component.

Symmetry Of Second Derivatives (Schwarz's theorem). The order of taking partial derivatives of a function $f(x) = f(x_1, x_2, \dots, x_n)$ of n variables is interchangeable

$$\frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} f(x) \right) = \frac{\partial}{\partial x_j} \left(\frac{\partial}{\partial x_i} f(x) \right).$$

3 Integral Calculus

Fundamental Theorem Of Calculus. Let f be a real-valued function on a closed interval $[a, b]$ and F an antiderivative of f in $[a, b]$: $F'(x) = f(x)$. If f is Riemann integrable on $[a, b]$ then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

Elementary Integrals.

$$\int x^r dx = \frac{x^{r+1}}{r+1} + C \text{ with } (r \neq -1) \qquad \int e^x dx = e^x + C$$

$$\int r^x dx = \frac{r^x}{ln(r)} + C \text{ with } (r > 1, r \neq 1) \qquad \int e^{a \cdot x + b} dx = \frac{e^{a \cdot x + b}}{a} + C$$

$$\int \frac{1}{x} dx = ln(x) + C \qquad \int ln(x) dx = x \cdot (ln(x) - 1) + C$$

$$\int \frac{1}{x^2} dx = -\frac{1}{x} + C \qquad \int sin(x) dx = -cos(x) + C$$

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2} + C \qquad \int cos(x) dx = sin(x) + C$$

Integration By Parts. Let u and v be two continuously differentiable functions of x in $[a, b]$. Then

$$\int_a^b u(x)v'(x)dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)dx.$$

Integration By Substitution.

References

[Bron] *Taschenbuch der Mathematik.* I. N. Bronstein, K. A. Semendjajew, G. Musiol, H. Mühlig, 11. Auflage, 2020.