A Small Collection Of Mathematical Formulas For The Prospective Statistician

Joshua Simon
Otto-Friedrich-University Bamberg

February 21, 2021

1 Basic Arithmetic

Logarithms. In the following let a, $b > 0$ and $n \in \mathbb{R}$. Then

$$ln(a \cdot b) = ln(a) + ln(b)$$
$$ln(\frac{a}{b}) = ln(a) - ln(b)$$
$$ln(a^n) = n \cdot ln(a)$$
$$e^{ln(a)} = ln(e^a) = a.$$

Binomial Coefficient. Suppose $0 < k \leq n$. Then

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

where $n! = 1 \cdot 2 \cdot \ldots \cdot n$.

2 Differential Calculus

Elementary Derivatives. In the following let f be a real-valued function of x and $a, b, r \in \mathbb{R}$. Then the derivative of f is given by f' as

$$f(x) = x^r \rightarrow f'(x) = r \cdot x^{r-1}$$
$$f(x) = e^x \rightarrow f'(x) = e^x$$
$$f(x) = e^{a \cdot x + b} \rightarrow f'(x) = a \cdot e^{a \cdot x + b}$$
$$f(x) = ln(x) \rightarrow f'(x) = \frac{1}{x}$$
$$f(x) = r^x \rightarrow f'(x) = r^x \cdot ln(r)$$
$$f(x) = \frac{1}{x^r} \rightarrow f'(x) = -r \cdot x^{-r-1}$$
$$f(x) = sin(x) \rightarrow f'(x) = cos(x)$$
$$f(x) = cos(x) \rightarrow f'(x) = -sin(x).$$

Differentiation rules. In the following let h, u, v be real-valued functions of x.

- Product Rule: $h(x) = u(x) \cdot v(x) \rightarrow h'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$

- Chain Rule: $h(x) = u(v(x)) \rightarrow h'(x) = u'(v(x)) \cdot v'(x)$

- Reciprocal Rule: $h(x) = \frac{1}{v(x)} \rightarrow h'(x) = \frac{v'(x)}{(v(x))^2}$

- Quotient Rule: $h(x) = \frac{u(x)}{v(x)} \rightarrow h'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$

Gradient. Let f be a differentiable fuction of n variables. Then

$$\mathbf{grad}(f) = \nabla f = \left(\frac{\partial}{\partial x_1} f(x) \ldots \frac{\partial}{\partial x_n} f(x) \right)^T.$$

Jacobian Matrix. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a differentiable function of n variables. Then the Jacobian matrix $\mathbf{J}$ of f is an $m \times n$ matrix whose (i, j)th entry is $\mathbf{J}_{ij} = \frac{\partial f_i}{\partial x_j}$. Which is

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial \mathbf{f}}{\partial x_1} & \cdots & \dfrac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla^T f_1 \\ \vdots \\ \nabla^T f_m \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix}$$

where $\nabla^T f_i$ is the transpose (row vector) of the gradient of the ith component.

Symmetry Of Second Derivatives (Schwarz's theorem). The order of taking partial derivatives of a function $f(x) = f(x_1, x_2, \ldots, x_n)$ of n variables is interchangeable

$$\frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} f(x) \right) = \frac{\partial}{\partial x_j} \left(\frac{\partial}{\partial x_i} f(x) \right).$$

3 Integral Calculus

References

[Bron] *Taschenbuch der Mathematik.* I. N. Bronstein, K. A. Semendjajew, G. Musiol, H. Mühlig, 11. Auflage, 2020.