

A Small Collection Of Mathematical Formulas For The Prospective Statistician

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1 Arithmetic

Logarithms. In the following let $a, b > 0$ and $n \in \mathbb{R}$. Then

$$\ln(a \cdot b) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^n) = n \cdot \ln(a)$$

$$e^{\ln(a)} = \ln(e^a) = a.$$

Binomial Coefficient. Suppose $0 < k \leq n$. Then

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

where $n! = 1 \cdot 2 \cdot \dots \cdot n$.

2 Differential Calculus

Elementary Derivatives. In the following let f be a real-valued function of x and $a, b, r \in \mathbb{R}$. Then the derivative of f is given by f' as

$$f(x) = x^r \rightarrow f'(x) = r \cdot x^{r-1} \qquad f(x) = e^x \rightarrow f'(x) = e^x$$

$$f(x) = r^x \rightarrow f'(x) = r^x \cdot \ln(r) \qquad f(x) = e^{a \cdot x + b} \rightarrow f'(x) = a \cdot e^{a \cdot x + b}$$

$$f(x) = \frac{1}{x^r} \rightarrow f'(x) = -r \cdot x^{-r-1} \qquad f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$$

$$f(x) = \ln(x) \rightarrow f'(x) = \frac{1}{x} \qquad f(x) = \cos(x) \rightarrow f'(x) = -\sin(x).$$

Differentiation rules. In the following let h, u, v be real-valued functions of x .

- Product Rule: $h(x) = u(x) \cdot v(x) \rightarrow h'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$
- Chain Rule: $h(x) = u(v(x)) \rightarrow h'(x) = u'(v(x)) \cdot v'(x)$
- Reciprocal Rule: $h(x) = \frac{1}{v(x)} \rightarrow h'(x) = \frac{v'(x)}{(v(x))^2}$
- Quotient Rule: $h(x) = \frac{u(x)}{v(x)} \rightarrow h'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$

Gradient. Let f be a differentiable function of n variables. Then

$$\mathbf{grad}(f) = \nabla f = \left(\frac{\partial}{\partial x_1} f(x) \dots \frac{\partial}{\partial x_n} f(x) \right)^T.$$

Jacobian Matrix. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a differentiable function of n variables. Then the Jacobian matrix \mathbf{J} of f is an $m \times n$ matrix whose (i, j) th entry is $\mathbf{J}_{ij} = \frac{\partial f_i}{\partial x_j}$. Which is

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \dots & \frac{\partial f}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla^T f_1 \\ \vdots \\ \nabla^T f_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

where $\nabla^T f_i$ is the transpose (row vector) of the gradient of the i th component.

Symmetry Of Second Derivatives (Schwarz's theorem). The order of taking partial derivatives of a function $f(x) = f(x_1, x_2, \dots, x_n)$ of n variables is interchangeable

$$\frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} f(x) \right) = \frac{\partial}{\partial x_j} \left(\frac{\partial}{\partial x_i} f(x) \right).$$

3 Integral Calculus

Fundamental Theorem Of Calculus. Let f be a real-valued function on a closed interval $[a, b]$ and F an antiderivative of f in $[a, b]$: $F'(x) = f(x)$. If f is Riemann integrable on $[a, b]$ then

$$\int_a^b f(x) dx = F(b) - F(a).$$

Elementary Integrals.

$$\begin{array}{ll}\int x^r dx = \frac{x^{r+1}}{r+1} + C \text{ with } (r \neq -1) & \int e^x dx = e^x + C \\ \int r^x dx = \frac{r^x}{\ln(r)} + C \text{ with } (r > 1, r \neq 1) & \int e^{a \cdot x + b} dx = \frac{e^{a \cdot x + b}}{a} + C \\ \int \frac{1}{x} dx = \ln(x) + C & \int \ln(x) dx = x \cdot (\ln(x) - 1) + C \\ \int \frac{1}{x^2} dx = -\frac{1}{x} + C & \int \sin(x) dx = -\cos(x) + C \\ \int \frac{1}{x^3} dx = -\frac{1}{2x^2} + C & \int \cos(x) dx = \sin(x) + C\end{array}$$

Integration By Parts. Let u and v be two continuously differentiable functions of x in $[a, b]$. Then

$$\int_a^b u(x)v'(x)dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)dx.$$

Integration By Substitution. Let I be an real interval, $f : I \rightarrow \mathbb{R}$ a continuous function and $\varphi : [a, b] \rightarrow I$ continuously differentiable. Then

$$\int_a^b f(\varphi(x))\varphi'(x) dx = \int_{\varphi(a)}^{\varphi(b)} f(u) du.$$

The substitution $u = \varphi(x)$ yields $\frac{du}{dx} = \varphi'(x)$ which leads to $du = \varphi'(x) dx$.

References

[Bron] *Taschenbuch der Mathematik*. I. N. Bronstein, K. A. Semendjajew, G. Musiol, H. Mühlig, 11. Auflage, 2020.