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COSC 4780 HW #3

1.1 0.) $(\lambda y.y)\,z \longrightarrow_\beta y[y:=z]$
$= \boxed{z}$

1.) $w((\lambda y.y)\,z) \longrightarrow_\beta w\,y[y:=z]$
$= \boxed{w\,z}$

2.) $((\lambda y.y)\,z)\,w \longrightarrow_\beta y[y:=z]\,w$
$= \boxed{z\,w}$

3.) $(\lambda x.\lambda y.x\ y)z \longrightarrow_\beta (\lambda x.x[y:=y])\,z$
$= (\lambda x.x)\,z \longrightarrow_\beta x[x:=z]$
$= \boxed{z}$

4.) $((\lambda x.(\lambda y.(y\,x)))\,y)$
$= (\lambda x.(\lambda y.(yx)))y \longrightarrow_\beta \lambda y.(yx)[x:=y]$
$= \lambda z.(zx)[x:=y]$
$= \boxed{\lambda z.\,zy}$

5.) $(\lambda x.((\lambda y.(y\,x))\,z)) \longrightarrow_\beta \lambda x.(yx[y:=z])$
$= \boxed{\lambda x.\,zx}$

1.2 Prove: $ZM \equiv_{\beta\eta} M(ZM)$

$\forall M.\quad ZM = M(ZM)$

(By def. of Z)

$ZM = \lambda f.(\lambda x.f(\lambda y.xxy))(\lambda x.f(\lambda y.xxy))M$
$\longrightarrow_\beta (\lambda x.f(\lambda y.xxy))(\lambda x.f(\lambda y.xxy))[f:=M]$
$= (\lambda x.M(\lambda y.xxy))(\lambda x.M(\lambda y.xxy))$
$\longrightarrow_\eta (\lambda x.M(xx))(\lambda x.M(xx))$

$\longleftarrow_\beta M((\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))M$
$\longleftarrow_\eta M((\lambda f.(\lambda x.f(\lambda y.xxy))(\lambda x.f(\lambda y.xxy)))M$
$= MZM$ $\blacksquare$ (by def. of Z)