Homework #2

COSC 5110
Analysis of Algorithms
Fall 2019

Due: Thursday, October 31, 2:45pm

1. Exercise 2.23.

2. Let
$$T(n) = T(an) + T(bn) + cn,$$
where $1 > a \geq b > 0$ and $c > 0$ are constants.

   (a) Prove that the total work performed across the k^{th} level of the recursion tree is at most $c(a+b)^k n$, for every $k \geq 0$.

   (b) For which values of k is the total work performed across the k^{th} level of the recursion tree equal to $c(a+b)^k n$?

   (c) What is the depth of the recursion tree?

   (d) Solve for $T(n)$ in each of the following cases:

      i. $a + b < 1$.

      ii. $a + b = 1$.

      iii. $a + b > 1$.

3. The median-of-medians algorithm we covered in class begins by dividing the array into blocks of 5. What happen if we modify the algorithm to use a different block size? Bound the number of comparisons in each of the following cases. (You may use your results from the previous problem.)

   (a) Block size 3.

   (b) Block size 5.

   (c) Block size 7.

   (d) Block size 9.

   (e) *Extra credit:* Block size $2k + 1$, for a general $k \geq 1$.

   (f) Compare your above results [(a)-(d), or (a)-(e) if you did (e)] and determine which block size minimizes the number of comparisons.