

## LAB 5: MULTILoop CIRCUITS

### Purpose

The purpose of this lab is to understand Kirchhoff's law, parallel and series combinations of resistors and capacitors, and electric power.

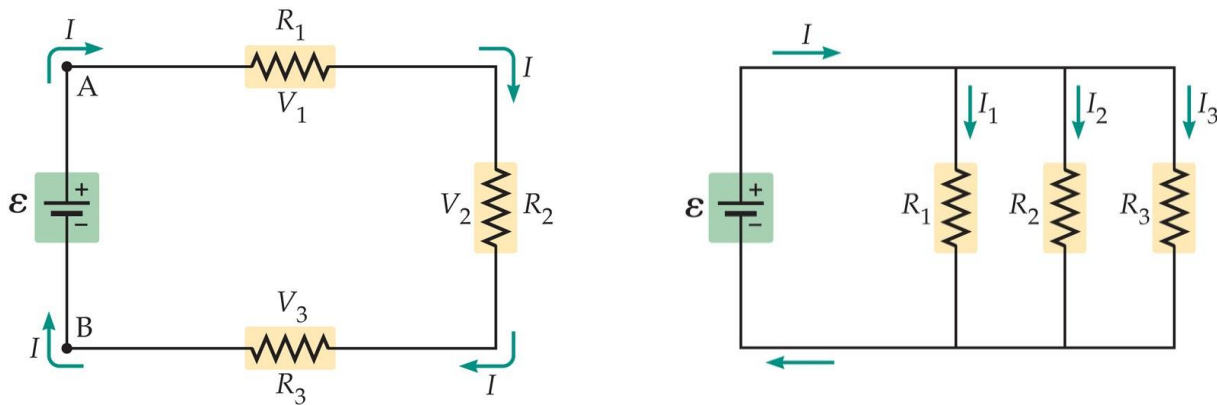
### Theory

Electric circuits consist of electric charge flowing through a closed path. An electric circuit contains a source of electromotive force (EMF) (e.g. a battery) and at least one circuit element. (e.g. resistor, capacitor, etc.). If there is more than one circuit element, then one can often construct a circuit which combines these elements either in series or in parallel.

Elements arranged in series are connected such that the current which flows through one element must flow through every other element. For example, each light bulb in your house is in "series" with a switch. Elements arranged in parallel are connected such that the same *voltage* is applied across each element in the circuit. For example, your home contains many circuits in parallel and each of these circuits is controlled by a single circuit breaker (or fuse).

### Resistors

Common circuit elements are resistors and capacitors. **Figure 1** shows a diagram of resistors in series and in parallel. In order to determine the equivalent resistance and (for capacitors) equivalent capacitance, one must consider whether the elements are in series or parallel.



**Figure 1:** The diagram on the left shows a circuit with a power supply and three resistors connected in a series while the diagram on the right shows resistors in parallel.

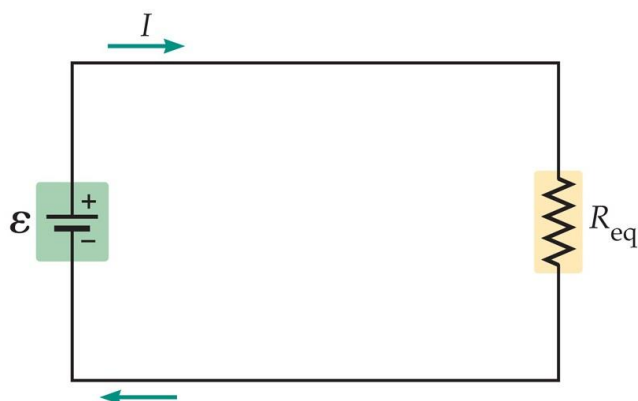
When resistors are connected in series, the same *current* flows through each resistor and that current acts as if it were flowing through a single resistor with resistance,  $R_{eq}$ . See **Figure 2**. To determine  $R_{eq}$ , we simply add the resistances:

$$R_{eq} = R_1 + R_2 + R_3 + \dots \sum R. \quad 1$$

Resistors in parallel are connected across the same *potential difference* (they have the same voltage). To determine the equivalent resistance,  $R_{eq}$ , we must add the reciprocals:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots = \sum \frac{1}{R}.$$

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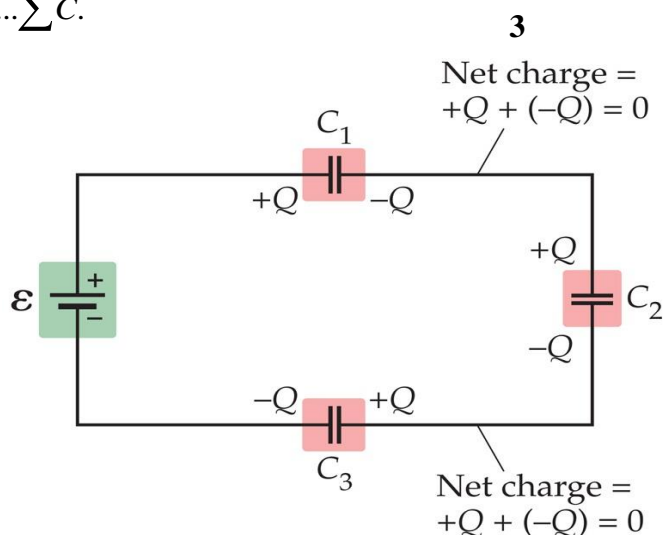
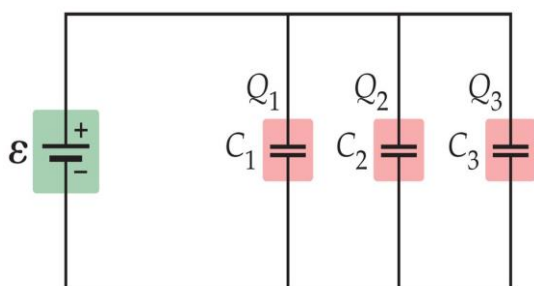


**Figure 2:** The equivalent circuit for resistors in series or in parallel.

## Capacitors

Capacitors may also be connected in series or in parallel. **Figure 3** shows several capacitors connected in parallel with a battery. As a result, each capacitor has the same potential difference across each plate. To determine the equivalent capacitance,  $C_{eq}$ , we simply sum the capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \dots = \sum C.$$



**Figure 3:** The diagram on the left shows a circuit with a power supply and three capacitors connected in parallel while the diagram on the right shows capacitors in series.

As you may have observed, capacitors connected in *parallel* combine in the same manner as resistors connected in *series*. Capacitors connected in series (**See Figure 3**) (where the *charge* is the same on each plate) have an equivalent capacitance computed by summing the reciprocals of each capacitance:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots = \sum \frac{1}{C}. \quad 4$$

## Power

The power,  $P$ , consumed by a circuit element is given by the product of the current,  $I$ , flowing through that element and the voltage drop,  $V$ , across it:

$$P = IV. \quad 5$$

In the special case of a resistor, the electrical power is dissipated in the form of heat. Using Ohm's law ( $V = IR$ ), one can compute the power as:

$$P = IV = I(IR) = I^2R. \quad 6$$

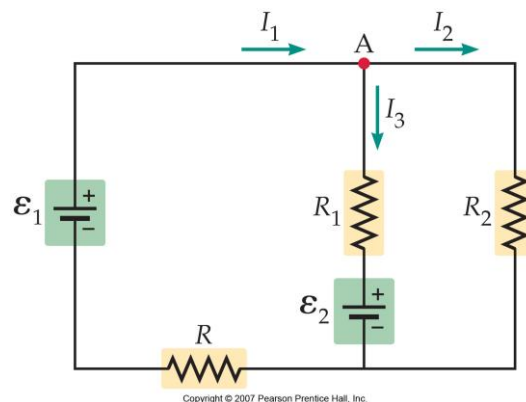
If you solve Ohm's law for current,  $P$  becomes:

$$P = IV = \left(\frac{V}{R}\right)(V) = \frac{V^2}{R}. \quad 7$$

## Kirchhoff's Laws

To find the current and voltages in a general electrical circuit, we use two rules established by the German physicist Gustav Kirchhoff (1824-1887). These rules are simply ways of expressing charge conservation (**junction rule**) and energy conservation (**loop rule**).

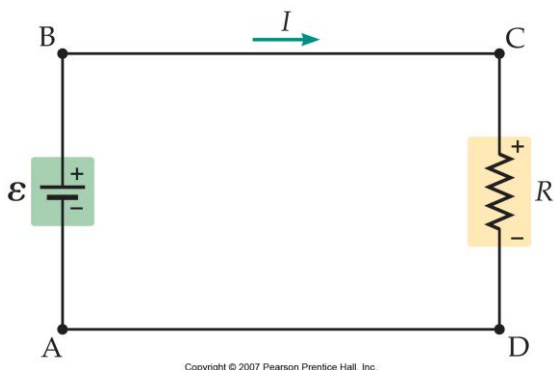
The junction rule (See **Figure 4.**) follows from the observation that the current entering any point in a circuit must equal the current leaving that point. A *branch* is a portion of a circuit which has one or more elements connected in series. A *junction* is a place where at branches of a circuit meet. In general, if we associate a (+) sign with currents entering a junction and a (-) sign with currents leaving a junction, then Kirchhoff's junction rule states that the algebraic sum of all currents meeting at any junction in a circuit must equal zero.



**Figure 4:** A circuit with a junction A. In this case,  $I_1 = I_2 + I_3$  or  $I_1 - I_2 - I_3 = 0$ .

Kirchhoff's loop rule (See **Figure 5.**) states that, as one moves around a closed loop in a circuit, the algebraic sum of all potential differences must be zero. The electric potential increases as one

moves from the (–) to the (+) plate of a battery. It decreases as one traverses a resistor in the direction of the current.



**Figure 5:** In this case of the loop rule, the electric potential increases by  $\varepsilon$  from point A to point B with no potential change from point B to point C. A potential drop of  $V=IR$  occurs from point C to D with no potential change from point D back to A.

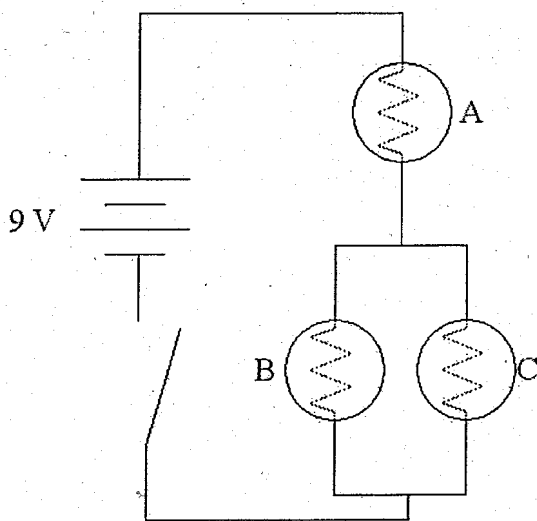
## Equipment

**Equipment needed at each station:** Pair of D-cell in a holder, single D-cell battery in a holder, lab-power supplies (odd-numbered posts=4.5 V and even-numbered posts = 9 V), protoboard, two knife-edge switches, plenty of wire (some with alligator clips and some for protoboard), three bulbs (6.3 V, 250 mA) in holders, one different bulb (2.5 V, 300 mA), two non-polar capacitors, numbered (soldered) network of three resistors, one or two digital multimeter(s) (w/leads), and computer. **Equipment needed in the front of the Room:** Capacitance meters, assortment of resistors

## Procedure

### Junction Rule

1. Construct the circuit diagramed below (which consists of three identical bulbs) but **do not close the switch yet**. Label the two junctions on the diagram.



2. Before closing the switch, make a guess as to which bulb will glow most brightly after the switch is closed. Enter your choice of A, B, or C: \_\_\_\_\_
3. Now close the switch: Which bulb was brightest and why?

### Loop Rule

Now you will use your digital voltmeter to confirm Kirchhoff's Loop Rule with the existing circuit.

1. Close the switch so that the bulbs are all glowing.
2. With your meter, determine the actual potential difference  $V_{ps}$  between the “-“ and “+” ends of your “power supply” (battery). Record your value: \_\_\_\_\_
3. Envision the direction that the current is flowing around your circuit (from “+” to “-“). In the subsequent measurements of potential differences, you will always want the meter's black probe to touch the “upstream” end of the circuit element and its red probe to touch the “downstream” end of the circuit element. (Be careful to note any minus signs which may appear!)
4. Determine and record the potential difference  $V_A$  across bulb “A”: \_\_\_\_\_
5. Repeat the previous step for bulb “B” and determine  $V_B$ : \_\_\_\_\_
6. Based on these three voltage measurements, calculate the sum of the potential differences around this loop of the circuit.

### Power:

The next few tasks will confirm the formulas (**Equations 5-7**) of power by using the light bulb's intensities as a qualitative indicator of their powers. (The non-ohmic characteristics of a light bulb will not affect your results.)

1. Consider the 3-bulb circuit left over from the previous activity. Explain its behavior in terms of **Equation 6**.

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Get your instructor's initials before proceeding.

## Equivalent Resistance

We can often “reduce” a circuit which contains multiple resistors into a circuit containing a single resistor with an equivalent resistance,  $R_{eq}$ . You will now confirm that a network constructed with ohmic circuit elements will itself also be ohmic. To confirm this experimentally, perform the following tasks with your soldered resistor-network.

1. Use your Digital Multimeter to set your power supply’s “A” terminals to 9 V. If the red LED is lit over the power supply outputs, then you need to allow more current through by turning the “current” knob slightly.
2. Place the resistor network across this voltage and determine the current,  $I$ , that flows through it. From this measurement, calculate a network resistance  $R$ . Enter your values below and show your calculation for  $R$ .

V = \_\_\_\_\_

I = \_\_\_\_\_

R = \_\_\_\_\_

3. Repeat the previous step for approximately half the voltage.

V = \_\_\_\_\_

I = \_\_\_\_\_

R = \_\_\_\_\_

4. Use your digital ohmmeter to measure the  $R_{\text{meter}}$  for the resistor network. Record your value:  
\_\_\_\_\_
5. Use the following algorithm to determine your nominal values of the individual resistances in your resistor network.

### Color Bands

The resistance of your resistor ( $\Omega$ ) is indicated by four color bands. The colors have the following values:

0 = black	1 = brown	2 = red	3 = orange	4 = yellow
5 = green	6 = blue	7 = violet	8 = grey	9 = white

1 <sup>st</sup> band	1 <sup>st</sup> digit
2 <sup>nd</sup> band	2 <sup>nd</sup> digit
3 <sup>rd</sup> band	Power to which ten is raised when multiplying this number
4 <sup>th</sup> band	Tolerance (gold = 5%; silver = 10%; no band = 20%)

Nominal value of resistor #1 = \_\_\_\_\_

Nominal value of resistor #2= \_\_\_\_\_

Nominal value of resistor #3= \_\_\_\_\_

Theoretical resistance of the network = \_\_\_\_\_

6. Use the percent error formula to calculate the discrepancy between your measured value and the nominal equivalent resistance. **Show your work.**

### Equivalent Capacitance

1. Using your DMM, measure the capacitance for your two capacitors (leave the black cable connected to COM and the red cable connected to  $V\Omega$  and set your DMM's dial to the capacitor symbol). Then measure the equivalent capacitance of a series and parallel combination of them. Complete the table below. **Show your work for the calculated  $C_{eq}$ .**

	C (F)	
	measured	calculated
$C_1$		N/A
$C_2$		N/A
series		
parallel		

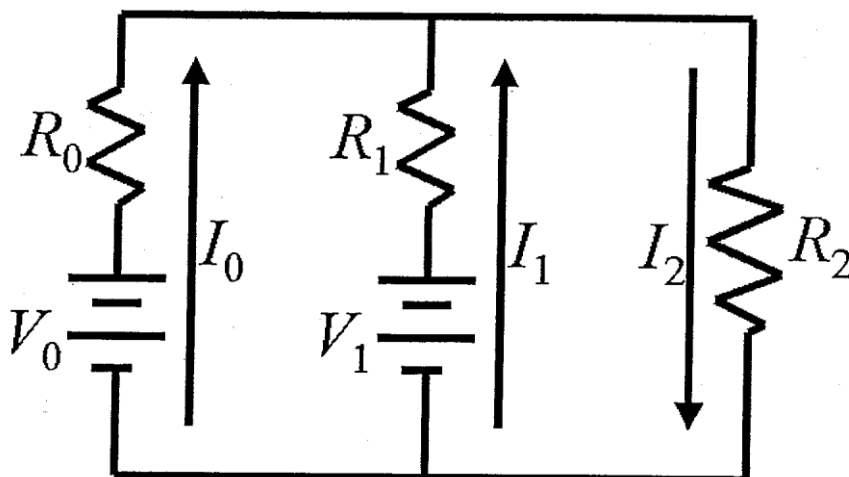


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## Multiple Power Supplies: Verifying Kirchhoff's Laws

Consider the multiloop circuit below. In such a circuit, the current is usually different within each branch. Therefore, it is important to remember to realize that, within each branch, there are **three different “directions”** to keep distinct in mind. (A) **The direction defined by you for positive current flow.** In an experiment, this direction is dictated by the orientation of your ammeter. (B) **The direction that the current actually flows.** If this direction is the same as that chosen by you, then your ammeter will display a positive number; if these two directions are opposite to each other, then the ammeter will display a negative number. (C) **The direction that you trace your finger when using the loop rule to analyze (theoretically) a particular loop of a circuit.** If, in a particular branch of the circuit, direction (A) coincides with direction (C) then the voltage drop across any resistor in that branch equals  $-IR$ . If, on the other hand, these two directions are opposite one another, then the resistors' voltage drops are  $+IR$ .



1. Set the Dual Channel Power Supply so that the two power supplies are in “independent” mode.
2. Use your Digital Multimeter (DMM) to measure the voltage coming out of the power supply labelled “A.” Set the voltage of the “A” power supply to 4.5 V.
3. Use your Digital Multimeter (DMM) to measure the voltage coming out of the power supply labelled “B.” Set the voltage of the “B” power supply to 3 V.

REMEMBER: If the red LED is lit over the power supply outputs, then you need to allow more current through by turning the “current” knob slightly.

4. Construct the diagram above for which three resistors  $R_0$ ,  $R_1$ , and  $R_2$  are obtained from the supply on the side of the room, your two power supplies  $V_0$  and  $V_1$ , which are set to be 4.5 V and 3 V respectively.
5. Record the values in the table below.

Quantity	Value
$V_0$	
$V_1$	
$R_0$	
$R_1$	
$R_2$	

6. Use your multimeter as an ammeter to measure the current ( $I_0$ ,  $I_1$ , and  $I_2$ ) in each branch. **Remember that the ammeter must be in series with the branch's resistor, not in parallel; otherwise, you *will* blow a fuse!** Make sure that your ammeter is measuring current in the direction specified by the arrows in the figure above.
7. Record your values in the table below.

Quantity	Value (A)
$I_0$	
$I_1$	
$I_2$	

8. Verify that the junction rule is satisfied for the top junction.

Sum of currents going into node: \_\_\_\_\_

Sum of currents leaving node: \_\_\_\_\_

9. Verify that the loop rule is satisfied for the outer loop (using the fact that  $\Delta V = \pm IR$  across each resistor). Start at the circuit's lower left corner and trace your finger clockwise when doing this.
10. Sum of voltage drops: \_\_\_\_\_ V + \_\_\_\_\_ V + \_\_\_\_\_ V = \_\_\_\_\_ V