Example Lab Report: Specific Heat Capacities of Metals

Introduction

The specific heat capacity of a material is a measure of how much energy is required to raise the temperature of a unit of mass of that substance by one degree. This is an intrinsic property of the material and can be used to identify the substance.

A common way of determining the specific heat capacity of a material is by the "method of mixtures." This method mixes the material with the unknown heat capacity with a substance with a known heat capacity. The two materials start at different temperatures and are then allowed to reach thermal equilibrium where they both have the same final temperature. The energy lost by the hot substance is gained by the cold substance. By measuring the temperature changes, one can determine the specific heat capacity of the unknown material.

We chose four materials and measured their specific heat capacities: Aluminum, Copper, Lead, and steel. Each of these metals was heated and then put into a calorimeter filled with water because we know the heat capacity of water. By measuring the temperature change of the metal and the water, we calculated the specific heat capacities of the metals.

Theory

As long as there are no phase changes, the change in internal energy of a substance can be determined from the definition of heat capacity:

$$\Delta U = mc\Delta T \tag{1}$$

where m is the mass of the substance, c is its specific heat capacity (heat capacity per unit mass), and ΔT is the change in the object's temperature.

From conservation of energy, the energy lost by the hot object is gained by the cold object in contact with it. Using equation (1) we can write

$$m_1 c_1 \Delta T_1 + m_2 c_2 \Delta T_2 = 0 \tag{2}$$

where the subscript is for either the hot object or cold object.

We need to take into account that the inner container of the calorimeter will also absorb some of the heat and start at the temperature of the cold object (in this case, water) and end at the same temperature:

$$m_1 c_1 \Delta T_1 + m_w c_w \Delta T_2 + m_c c_c \Delta T_2 = 0 \tag{3}$$

where the w subscript refers to the water and the c subscript refers to the calorimeter, itself.

If we know the heat capacities of the water and calorimeter, we can solve for the heat capacity of the hot object:

$$c_1 = -\frac{(m_w c_w + m_c c_c)\Delta T_2}{m_1 \Delta T_1} \tag{4}$$

The minus sign makes sense because the hot object will cool off which makes ΔT_1 a negative number. Therefore, c_1 will be positive.

Error Analysis

Because we are not performing many trials of the same experiment, we cannot use a standard deviation of results to compute our uncertainties. Therefore, we need to use Propagation of Error to compute our uncertainties.

Equation (4) is a sum of two products:

$$c_1 = -\left[\frac{m_w c_w \Delta T_2}{m_1 \Delta T_1} + \frac{m_c c_c \Delta T_2}{m_1 \Delta T_1}\right] \equiv -[A + B] \tag{5}$$

Therefore, by the sum/difference rule, we can write

$$(\delta c_1)^2 = (\delta A)^2 + (\delta B)^2 \tag{6}$$

But the values A and B are purely multiplicative, so we can use the product/quotient rule:

$$\left(\frac{\delta A}{A}\right)^2 = \left(\frac{\delta m_w}{m_w}\right)^2 + \left(\frac{\delta m_1}{m_1}\right)^2 + \left(\frac{\delta c_w}{c_w}\right)^2 + \left(\frac{\delta \Delta T_2}{\Delta T_2}\right)^2 + \left(\frac{\delta \Delta T_1}{\Delta T_1}\right)^2 \tag{7}$$

$$\left(\frac{\delta B}{B}\right)^2 = \left(\frac{\delta m_c}{m_c}\right)^2 + \left(\frac{\delta m_1}{m_1}\right)^2 + \left(\frac{\delta c_c}{c_c}\right)^2 + \left(\frac{\delta \Delta T_2}{\Delta T_2}\right)^2 + \left(\frac{\delta \Delta T_1}{\Delta T_1}\right)^2 \tag{8}$$

After some algebra, we find, from equations 5–8 that

$$\begin{aligned}(\delta c_1)^2 &= A^2\left[\left(\frac{\delta m_w}{m_w}\right)^2 + \left(\frac{\delta c_w}{c_w}\right)^2\right] + B^2\left[\left(\frac{\delta m_c}{m_c}\right)^2 + \left(\frac{\delta c_c}{c_c}\right)^2\right] \\ &\qquad + (A^2+B^2)\left[\left(\frac{\delta m_1}{m_1}\right)^2 + \left(\frac{\delta \Delta T_2}{\Delta T_2}\right)^2 + \left(\frac{\delta \Delta T_1}{\Delta T_1}\right)^2\right] \\ &= \left(\frac{\Delta T_2}{m_1 \Delta T_1}\right)^2 \left\{(m_w c_w)^2\left[\left(\frac{\delta m_w}{m_w}\right)^2 + \left(\frac{\delta c_w}{c_w}\right)^2\right] + (m_c c_c)^2\left[\left(\frac{\delta m_c}{m_c}\right)^2 + \left(\frac{\delta c_c}{c_c}\right)^2\right]\right. \\ &\qquad \left. + [(m_w c_w)^2 + (m_c c_c^2)]\left[\left(\frac{\delta m_1}{m_1}\right)^2 + \left(\frac{\delta \Delta T_2}{\Delta T_2}\right)^2 + \left(\frac{\delta \Delta T_1}{\Delta T_1}\right)^2\right]\right\}\end{aligned} \tag{9}$$

This is fairly ugly, but can be computed by defining equation (9) as a function in Python, Mathematica, or Excel (we used Python and confirmed the value with Excel).

Our uncertainties in all of our masses were approximately 0.1 g (0.0001 kg), which was the accuracy of the scale we used. Our ability to read the mercury thermometer was about $\pm 0.2^\circ$. Because we are interested in differences in temperature, we need to use the sum/difference rule:

$$\delta(\Delta T) = \sqrt{(\delta T_f)^2 + (\delta T_i)^2} = \sqrt{(0.2^\circ)^2 + (0.2^\circ)^2} \approx 0.3^\circ$$

The uncertainty in the specific heat capacity of water is more difficult. It turns out that, over the range of temperatures we were interested in, the specific heat capacity of water runs from 4178–4182 J/kg K. So we chose the value (4180$\pm$2) J/kg K. We could not find the heat capacity of aluminum (the material that the calorimeter is made of) over the same temperature range, so we assumed the uncertainty to be the same: (900$\pm$2) J/kg K.

Finally, where we averaged two specific heat capacities together, the uncertainty in the average was calculated using

$$\delta c = \frac{\sqrt{(\delta c_1)^2 + (\delta c_2)^2}}{2} \tag{10}$$

from the Laboratory Handbook.

Methods

To determine the heat capacity of the metals, we heated them up and then utilized the "method of mixtures" with water which has a specific heat capacity of $c_w = 4180$ J/kg K. For the calorimeter, we assumed it was made of aluminum $c_{Al} \approx 900$ J/kg K and included the mass of the stirrer. Because we only wanted to change the temperature of the water and not change its phase, we needed to make sure the metal did not get hot enough to boil the water. To accomplish this, for each of the metals, we heated a measured amount of the metal pellets in a double-boiler placed over a Bunsen burner. The double-boiler keeps the metal from reaching temperatures above the boiling point of water.

While we waited for the metal to heat up, we poured a measured amount of water into a calorimeter. The calorimeter is basically an insulated mixing chamber so that when the hot metal is poured into the water, little heat is lost through the walls of the calorimeter. A second thermometer measures the temperature of the water in the calorimeter.

Once the metal pellets reached a temperature between 90–95° C, we recorded the initial temperature of the metal and the initial temperature of the water. We removed the rubber stopper and thermometer from the calorimeter. Using a heat resistant glove, we removed the boiler cup from the double-boiler and poured the metal pellets into the water in the calorimeter. We replaced the rubber stopper and thermometer into the calorimeter and began stirring the water with the stirrer until equilibrium was reached. We determined the equilibrium temperature to be the maximum temperature reached before the temperature began falling due to heat loss through the calorimeter wall.

Data

Mass of calorimeter inner cup and stirrer: (0.0897±0.0001) kg

Table 1. Experimental Results

Trial	m_1 (kg)	ΔT_1 (°C)	m_w (kg)	ΔT_2 (°C)	c_1 (J/kg K)	δc_1 (J/kg K)
Aluminum	0.0500	-53.9	0.0533	9.1	1025	27
Aluminum	0.0999	-55.0	0.1000	11.0	998	24
Copper	0.0500	-56.8	0.0520	4.5	472	25
Copper	0.1000	-59.0	0.1000	5.0	422	22
Lead	0.0500	-61.9	0.0522	1.2	116	23
Steel	0.0502	-62.8	0.0527	6.1	582	22
Steel	0.0998	-59.0	0.1000	5.0	423	22

Where the specific heat capacity, c_1, was computed using equation (4) and the uncertainty in that heat capacity, δc_1, was computed using equation (9).

Discussion and Conclusions

Table 2 summarizes our findings in this experiment. Where the experimental uncertainty for Aluminum, Copper, and steel was computed using equation (10). In general, we found reasonable values for the specific heat capacities of the four metals with our experimental value for lead being the closest and that of steel being the furthest.

Table 2. Comparison of Heat Capacities

Substance	**Accepted**[1] (J/kg K)	**Experimental** (J/kg K)	δc (J/kg K)	**% Error**
Aluminum	900	1012	18	12.4
Copper	386	447	16	15.8
Lead	128	116	23	-9.4
Steel	449[2]	533	16	18.7

[1] Accepted values (except for steel) taken from HyperPhysics, Georgia State University website.
[2] Value is for iron from CRC Handbook of Chemistry and Physics, 2001.

We did not get as close as we would have liked (within 5%) to the accepted values. This could be due to a few factors. First, we notice that three of our four heat capacities were higher than expected. This leads us to believe that there may be a systematic error. The most reasonable possibility is that we mis-identified the material for the calorimeter cup. If the actual material was something that had a lower specific heat capacity, then, from equation (4), we would have computed heat capacities that would have been too large. In fact, when playing with the values in Excel, if the calorimeter cup was made of iron ($c \approx 445$ J/kg K) we would have gotten much closer to the accepted values.

Another systematic error that may have played a role in our error is that the material for steel and lead were actually composed of alloys with other components and were not pure iron or lead. Lead shot is often mixed with tin and antimony and steel is often alloyed with carbon and other metals. We did not know the exact composition of these materials, so attempting to compare our values to the pure metals was inappropriate.

Finally, in examining our Excel spreadsheet, our uncertainties from equation (9) were dominated by a single term: $\delta(\Delta T)/\Delta T_2$. If we could have reduced our uncertainty in our temperature measurement or been able to increase the temperature rise in the water, our uncertainty, δc_1, could have been much lower. A digital thermometer would have helped reduce our uncertainty in temperature, but we could also have chosen a liquid with a lower heat capacity than water. Alcohol would work, but it evaporates quickly, but mercury would be ideal if it weren't so hazardous.