Name: Section: 201L- Date:

Lab 8: Measurement of "g"

Objectives

- To learn to take good measurements
- To learn how to estimate and propagate errors
- To learn how to take measurements to verify a theory
- To learn how different experiments may yield different uncertainties

Equipment

- For each Air Track set-up:
  - Two (2) Photogates
  - One short "flag"
  - Two cart weights
  - Three 2x4 "shims"
  - One 1-m ruler
  - One 30 cm ruler
  - One scale
- For each Collision table set-up with Spark Timer and Carbon Mat:
  - A number of recording sheets
  - Two (2) Puck weights
  - Three 1x4 "shims"
  - One 30 cm ruler
- For each Pendulum set-up:
  - 3-m string
  - 100-g, 50-g, and 20-g hanging weights
  - 2-m ruler
  - 30-cm ruler
  - One (1) Stopwatch
  - One (1) Protractor

Safety

- Air Track Safety:
  - **Do not attempt to push the carts faster than 1 m/s.**
  - **Be careful placing the carts on the track. Do not damage the track.**
  - Please place paper underneath the carts when they are resting on the track without the air turned on. **Do not slide the carts on the track without the air turned on.**
- Air Table Safety:
  - **Electrical Hazard:** The Spark Timer generates dots on the recording sheet by passing high voltage from one puck, through the carbon mat, to the other puck.
  - The red and black cables must always be securely connected between the Spark Timer and the Table and they must be in good condition.
  - Do not depress the foot switch without both pucks on the table.
  - **Do not touch the metal pucks while the footswitch is depressed.**

- Alert your instructor if the cables are in poor condition, are not remaining seated, or if the sparking is clearly audible.
- Use the least possible force when sending a puck along the table.
- Do not touch the latex tubes connecting the compressed air distributor to the air pucks. Latex is damaged by oils and UV light (it is also possible---though unlikely---that you might have an allergy to latex).
- Do not depress the footswitch until you are ready to take data. Leaving a puck in one place for longer than 20 seconds while generating spark data can damage the carbon mat.

Introduction

Although apocryphal, it is reported that Galileo Galilee dropped objects off of the tower of Pisa to prove that all objects accelerate towards the Earth at the same rate: *"g."* But what is this acceleration? Most textbooks have decided on 9.81 m/s^2, but is this the correct value? And what is the uncertainty in this value? Your job in this lab will be to determine *"g"* for yourself, verify that it does *not* depend on the mass of the object, and find the uncertainty in this measurement. You will be using a pendulum and either: a "falling" air track cart or a sliding air puck (or both if you have time!)

It is always to your benefit to be slow and careful while performing this lab.

Theory

If all objects fall with the same acceleration, then gravity must act on them with differing forces. A simple re-arrangement of Newton's Second Law states:

$$a = F/m \tag{1}$$

But if the acceleration, *a*, is to equal the number, *"g,"* for *all* masses, then gravity must create a force of

$$F_g = mg \tag{2}$$

on an object of mass *m*.

In 1686, Isaac Newton published his law of Universal Gravitation which stated that the force of gravity between any two objects is given by:

$$F_g = \frac{GMm}{r^2} \tag{3}$$

where *"G"* is a universal constant, *"M"* is the mass of one object, *"m"* is the mass of the other, and *"r"* is the distance between them. For a mass, *m*, on the Earth's surface, equation (3) tells us that *"M"* is the mass of the Earth and *"r"* is the distance between the center of mass of the Earth and the center of mass of the object. If we compare equations (2) and (3) we find that

$$g = \frac{GM}{r^2} \tag{4}$$

This means that the value for *"g"* will change if we change the distance between that object and the center of mass of the Earth. Because the Earth is not perfectly round, *"g"* will vary from place to place. The textbook value of 9.81 m/s^2 is a convenient average for the entire Earth.

To accurately measure *"g,"* we must use some method that does *not* require us to know the mass by "weighing." When a mass is placed on a scale, the scale assumes it knows the acceleration due to gravity and, by measuring the force produced on the pan, it uses equation (2) to compute the mass (so you truly are weighing things on a scale---*not* "massing."). If we needed to weigh an object to determine its mass and then use that mass to determine *"g,"* we'd be using very circular logic. We must not do this.

Although there are other ways of determining mass, we will use three methods of determining *"g"* where the mass of the object shouldn't matter.

The Air Track

If we tilt the air track at some angle, θ, from the horizontal, you should be able to prove that the acceleration of a cart with mass, *m*, on that track should be:

$$F_g \sin\theta = ma$$

but if $F_g = mg$, then

$$g = \frac{a}{\sin\theta}$$

Just as in a previous lab, kinematics provides three ways for you to compute acceleration from the velocity data that the two photogates will give you.

The Air Table

Exactly like the air track, the air table can be tilted as well with the same results as the air track. However, the measurement of the acceleration will be different. How will you do this?

The Pendulum

You have not yet studied the pendulum. But by now you can analyze the *torques*. When you do, you will find that

$$\Gamma = I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$$

where *"Γ"* is the torque around the pivot, *"I"* is the moment of inertia of the plumb-bob (ml^2 in this case), and *"l"* is the distance from the pivot to the center-of-mass of the bob. The solution to this differential equation is difficult unless we restrict ourselves to small angles. If we do that, then we can make the approximation $\sin\theta \approx \theta$ (this equation only works for radians---not degrees) and the differential equation becomes

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0 \qquad (5)$$

where we have defined

$$\omega_0^2 = \frac{mgl}{I} = \frac{mgl}{ml^2} = \frac{g}{l}$$

The solution to (5) is

$$\theta(t) = A\cos(\omega_0 t + \phi)$$

which is periodic motion with an angular frequency of ω_0. Because the period and *angular* frequency are related by

$$\tau = \frac{2\pi}{\omega}$$

we find that the period of oscillation is given by

$$\tau = \frac{2\pi}{\sqrt{g/l}} = 2\pi\sqrt{\frac{l}{g}}$$

We can then solve for *g*:

$$g = 4\pi^2 \frac{l}{\tau^2} \tag{6}$$

Notice that we only need to know the length of the pendulum, *l*, and the period (the time it takes to swing back-and-forth once), τ, to compute *"g."*

Error Analysis

Your Lab Manual and the previous labs can guide you in estimating the errors in your measurements and propagating those errors into your computed energies and momenta. ***You may not use the naïve method for computing your uncertainties in this lab.*** The uncertainties in your velocities will come from how accurately you can measure distances and times. In some cases, it might be easier to use the standard deviation of a large number of measurements as your measurement uncertainty.

The uncertainty in *"g"* computed from the pendulum is dependent both on the uncertainty in *"l"* and the uncertainty in *"τ."* But you can make the uncertainty in *"τ"* very small by counting a large number, *n*, of periods, and then dividing the total time it took by *n*. We can see this by looking at the way we have to compute the period:

$$\tau = \frac{t}{n}$$

where *"t"* is the total time you let the pendulum swing for and *"n"* is the number of periods. Rule (2) says:

$$\left(\frac{\delta\tau}{\tau}\right)^2 = \left(\frac{\delta t}{t}\right)^2 + \left(\frac{\delta n}{n}\right)^2$$

But we know the number of periods, *n*, exactly. So we're left with

$$\frac{\delta\tau}{\tau} = \frac{\delta t}{t}$$

or

$$\delta\tau = \frac{\tau}{t}\delta t = \frac{t/n}{t}\delta t = \frac{\delta t}{n}$$

So the uncertainty in the period is your uncertainty in measuring *"n"* periods divided by *"n"*. Count many periods and you can get a very accurate value for the period. Although you get smaller uncertainties if you measure for longer times, you only have 2.75 hours to perform the lab….

In order to minimize random errors, it is extremely important that each of your measurements be performed a few times.

Experimental Procedure

The Air Track

Setting up the Air Track

You will need to level the airtrack. Look back to the second lab for instructions if you've forgotten how to do this. The more time you spend leveling the track (within reason), the better your results will be.

Plug the photogates into both Digital inputs.

Once you start up Data Studio and select "Create Experiment," you should left-click the jack on the diagram where you inserted the plug. Then select "Photogate" from the drop-down menu.

You should see a tab labeled "Constants." Under that tab will be a text box requesting the length of the "Flag" in meters. Measure the flag as best you can and enter that information in the box.

You must repeat this procedure for the second photogate.

Make sure that the photogate is triggered *only* by the flag. You can tell when the photogate is triggered by looking for the red LED to light when the infrared LED is blocked. Ensure that the red LED only lights when the proper portion of the cart passes through.

If you are having difficulties, check these four things before alerting your instructor:

1. Is the Workshop 500 connected to power?
2. Is the switch on the back of the Workshop 500 turned "on?"
3. Is the green light at the front of the Workshop 500 lit?
4. Is the Workshop 500 connected to the computer?

You may have to quit and restart Data Studio if you change anything related to the above.

The Air Track Experiment

The goal of this experiment is to find the acceleration of the cart for different angles of tilt. You should have three wooden shims to change the angle slightly for each run.

You will need to perform the experiment at each angle with three different cart masses. The reason is to verify that the acceleration is not affected by cart mass. Run each mass a few times at one angle. Then change angles and do it again for the new angle.

The Air Table

Setting up the Air Table

Before you begin, you will need to level the air table. Look back to the sixth lab if you've forgotten how to do this. The more time you spend leveling the table (within reason), the better your results will be.

Important points to remember:

- Keep the compressor as far from the table as possible. The compressor will vibrate the table which may influence your results. If possible, keep the compressor on the floor.
- Keep the compressor as far from the Spark Timer as possible. The electric motor will generate Electro-Magnetic Interference (EMI) which can interfere with the proper working of the Spark Timer.
- Be gentle with the carbon mat. This is the source of your data. If you damage it, you may not get clear marks on the recording paper.

When you are ready to take data, place a sheet of blank newsprint paper over the carbon mat and then place both pucks on the paper. You will need to hold one puck still while the other slides.

The Air Table Experiment

The goal of this experiment is to find the acceleration of the puck for different angles of tilt. You should have three wooden shims that you can place under the third leg to change the angle slightly for each run.

You will need to perform the experiment at each angle with three different puck masses. The reason is to verify that the acceleration is not affected by cart mass. Run each mass a few times at one angle. Then change angles and do it again for the new angle.

To change the puck mass:

1. ***Carefully*** unplug the air hose from the puck (**Do not let the chain jump up into the tube!**)
2. Gently slide a puck weight onto the plastic tube sticking up from the puck (it might take a bit of twisting and pushing).
3. Replace the air hose and make sure that the chain touches the screw.

4. Remove all puck masses when you're done.

Before taking data, you should turn on the air compressor and practice releasing the puck a few times *without* using the Spark Timer.

Once you are ready to take data, turn on the Spark Timer and pick a frequency (you will have to use a few trials to find the frequency that gives you the best data). The person releasing the puck should be the one to use the foot pedal. <u>Immediately after the puck is released, hold the foot pedal down until the puck stops at the bottom of the slope.</u> The data will appear on the <u>underside</u> of the paper sheet.

On what part of the path should you take your measurements to compute the acceleration of the puck? How many dots should you use? How should you determine the uncertainty in this measurement?

<u>The Pendulum</u>

Just as in the previous two experiments, we expect that the mass of the bob should make no difference in your results. But you are a thorough experimenter and want to be sure. So for each of the experiments, repeat the run with a different mass.

When you set the pendulum swinging, remember that equation (6) is only valid for small angles. How small? You should probably not pull the pendulum further than ten degrees away from the vertical.

You need to choose *at least* five lengths for the pendulum. To change the length of the pendulum, ***do not cut the string:***

1. Loosen the thumbscrews which clamp the string to the blade.
2. Remember that the string is doubled-up when used, so measure *twice* the desired length and mark the ends with a pencil.
3. Wrap the ends of the string around the thumbscrews so that the marks are at the edge of the blade.
4. Tighten the thumbscrews.

Remember that the length of the pendulum is from the bottom of the blade to the center of mass of the plum-bob. How will you find this center of mass?

There is no accompanying data sheet for this lab. All your data should be recorded in your lab notebook and this data must be signed by your instructor and included in your lab write-up.

Your Report

Introduction:

Write a few sentences about what you set out to measure and how you will compare the measured values with theory. (Hint: Write the Introduction ***last***. This way you'll have the whole lab in your head when you write it and you can properly foreshadow the results.)

Theory:

For a good theory section, you will need to write a few paragraphs and answer the following questions:

(1) What is your theoretical prediction for *"g"* and its uncertainty. How did you arrive at these numbers (do **not** quote someone else's value for *"g")*?
(2) How did you compute the acceleration of the mass for the air track and air table?
(3) How did you compute your uncertainties in those accelerations?
(4) How did you compute your uncertainty in *"g"* given your data?

Use equations where necessary.

Methods:

Detail carefully the experiments you performed and how you performed them. How did you find the center of mass of the plumb-bob for the pendulum? Talk about any problems you encountered and how you handled them.

Data:

Include your signed raw data sheet(s)

Include tables of cart and/or puck mass, tilt angle, and acceleration for every run.

Include a graph of the square of the period of oscillation vs. the length of the pendulum. Include error bars.

Results and Conclusion:

This will be your longest section. Use good paragraph structure to link the results together:

What values of "g" (**with uncertainties!**) did each of your measurements provide? Which had the lowest uncertainty? How does this value compare with the theoretical value you provided (use the %Error comparison)?

What is the slope of your "Square-period vs. Pendulum length" graph and what does it represent?

How did changing the mass of the cart or puck affect your result? Is this expected? If it changed your results, what would account for this?

Question: You need to make a “seconds” pendulum: this is a pendulum which has a period of exactly one second. How long should this pendulum be?

As always, talk about your errors and how you might be able to reduce them. Make sure that you discuss all possible sources of error in your experiments.