Lab 6: Collisions in 2D

Objectives

- To learn to take good measurements
- To learn how to estimate and propagate uncertainties
- To learn how to take measurements to verify a theory
- To experiment with collisions

Equipment

- One Collision table set up with Spark Timer and Carbon Mat
- A number of recording sheets
- One protractor
- One 30 cm ruler
- One scale

Safety

- **<u>Electrical Hazard:</u>** The Spark Timer generates dots on the recording sheet by passing high voltage from one puck, through the carbon mat, to the other puck.
  - The red and black cables <u>must always be securely connected</u> between the Spark Timer and the Table and they <u>must be in good condition</u>.
  - <u>Do not depress the foot switch without both pucks on the table.</u>
  - **<u>Do not touch the metal pucks while the footswitch is depressed</u>.**
  - Alert your instructor if the cables are in poor condition, are not remaining seated, or if the <u>sparking is clearly audible</u>.
- Use the <u>least possible force</u> when sending a puck along the table. Ask your instructor to help you assemble a rubber-band launcher if you prefer---they will give you more reproducible results as well.
- Do not touch the latex tubes connecting the compressed air distributor to the air pucks. Latex is damaged by oils and UV light (it is also possible---though unlikely---that you might have an allergy to latex).
- <u>Do not depress the footswitch until you are ready to take data.</u> Leaving a puck in one place for longer than 20 seconds while generating spark data can damage the carbon mat.
- Do not write on the paper while it is on the carbon mat.

Introduction

One of the primary results of Newtonian Mechanics is that, if there are no external forces, the momentum of a system is conserved---even if mechanical energy is not. This experiment will allow you to test that theory by sending two pucks towards each other and recording their speeds and trajectories both before and after a collision.

The collision table has a couple of features to help you analyze collisions: The pucks use compressed air to act like hovercraft---greatly reducing friction. And a Spark Timer will generate dots at a frequency of your choosing so that you can locate each puck accurately and use simple kinematics to determine the puck's velocity and direction.

Your experiments should be performed so that it is clear whether or not momentum is conserved during collisions which both conserve and do not conserve kinetic energy.

It will be to your benefit to be slow and careful while performing this lab.

Theory

Conservation of Momentum

The momentum of an object is the product of the mass and the velocity of that object. Because velocity is a vector, momentum must also be a vector:

$$\vec{p} \equiv m\vec{v} \tag{1}$$

One must keep track of both the *x* and *y* components of the momentum at all times.

The momentum of a system of particles is simply the sum of the momenta of each particle:

$$\vec{p}_{sys} \equiv \sum_i \vec{p}_i \tag{2}$$

Remember that these are *vectors* and must be added using vector addition. Consult chapter three of your textbook for more details.

Because Newton's second law actually reads

$$\vec{F} \equiv \frac{d\vec{p}}{dt} \tag{3}$$

it can be interpreted to say "an external force on a system induces a *change* in momentum of that system." Therefore, if there is no external force, there is no change in momentum. (If you assume the mass of the system is constant and you know how to work with derivatives, you should be able to re-derive the old version: $\vec{F} \equiv m\vec{a}$.)

In a collision, the most important forces will be *internal* to the system. In other words, every force will be one of an "action-reaction" pair which comes from within the system itself (*e.g.* one puck striking the other). Because there will be no *external* forces (forces that have their origin *outside* the system), by Newton's second law, we expect the momentum of the system to remain constant.

To verify this, you will need to measure the masses of the pucks and their velocities both before and after each collision.

Elastic and Inelastic Collisions

Although *momentum* is conserved in *all* collisions (when they are free from external forces), it is not always true that *kinetic energy* is conserved. When two cars collide on an icy street, they may bounce off of one another (for low-speed collisions), or they may stick to one another (for high-speed collisions).

If the cars bounce off each other, we expect the *total* kinetic energy to be conserved (so that the sum of the kinetic energies before the collision equals the sum afterwards, even if the individual speeds changed). This kind of collision is called a **totally elastic collision**.

If the cars stick to each other, we expect that some (or all) of the kinetic energy was lost (where did it go?) These kinds of collisions are called **inelastic collisions**.

To determine if the collision was totally elastic or inelastic, you will need to compute the kinetic energy of each puck

$$K = \frac{1}{2} m v^2 \quad \text{where} \quad v = \sqrt{v_x^2 + v_y^2} \tag{4}$$

and add them together. Then compare the sum before the collision to the sum following the collision.

Kinematics

Clearly, you will need to compute the velocity of each puck. The information you will have, however, are dots generated by the Spark Timer. The dots will be generated at a rate you set (either with a knob or with the digital controller) and that rate will allow you to compute the puck's velocity using basic kinematics:

$$v \equiv \frac{\Delta s}{\Delta t} \tag{5}$$

where Δs is the distance *between* dots and Δt is the time between sparks. The Spark Timer has *frequency* settings (measured in Hz or 1/s). Remember that the frequency is inversely related to the period:

$$f \equiv \frac{1}{\tau} \tag{6}$$

(You can remember this by thinking about tapping your finger on the desk: If you tap your finger 8 times per second, then the time between taps is $1/8 = 0.125$ second. If you tap your finger more quickly, the frequency goes *up*, but the time between taps must go *down*.)

Velocity of the Center of Mass

If the two pucks were connected with a thin rod, the center of mass of the system would be the place where you could balance that rod. We find the center of mass of two objects by a weighted average:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \text{and} \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \tag{7}$$

By taking derivatives, we can find the *x*- and *y*-velocities of the center of mass:

$$v_{x,cm} = \frac{m_1 v_{x,1} + m_2 v_{x,2}}{m_1 + m_2} \quad \text{and} \quad v_{y,cm} = \frac{m_1 v_{y,1} + m_2 v_{y,2}}{m_1 + m_2} \tag{8}$$

Uncertainty Analysis

Your Lab Manual and the previous labs can guide you in estimating the uncertainties in your measurements and propagating those errors into your computed energies and momenta. The uncertainty in the masses of the air puck will be determined by the accuracy of the scale.

The uncertainties in your velocities will come from how accurately you can measure distances and times. The uncertainty in the Spark Timer's frequency is 1% of the frequency you chose. To reduce the uncertainty in your distance between dots, you can measure the distance, L, between n dots. The average distance between dots is then $\Delta s = L/(n-1)$. Your uncertainty in that distance is then $\delta(\Delta s) = \delta s/(n-1)$.

In some cases, it might be easier to use the standard deviation of a large number of measurements as your measurement uncertainty.

Experimental Procedure

Setting up the Air Table

Before you begin, you will need to level the air table. To make this process easier, you should have all three feet on the same surface (do not have two feet on one table and one on another).

1. Remove one puck from the table and place the second puck in the center of the table directly under the air distributor.
2. Plug in/turn on the air compressor.
3. Hold the puck steady under the distributor and, carefully, release the puck without disturbing it.
4. If the puck tends to move to one side, you will need to raise (unscrew) the foot nearest that side of the table. In some cases, you will need to adjust two feet simultaneously.
5. When the table is level, the puck will meander slowly in no particular direction.

The more time you spend leveling the table (within reason), the better your results will be.

Important points to remember:

- Keep the compressor as far from the table as possible. The compressor will vibrate the table which may influence your results. If possible, keep the compressor on the floor.
- Keep the compressor as far from the Spark Timer as possible. The electric motor will generate Electro-Magnetic Interference (EMI) which can interfere with the proper working of the Spark Timer.
- Be gentle with the carbon mat. This is the source of your data. If you damage it, you may not get clear marks on the recording paper.
- Be very careful if you must detach the pucks from their tubes. Doing this improperly may cause the conductive chain to retract into the tubing---making it especially difficult to retrieve.

When you are ready to take data, place a sheet of blank newsprint paper over the carbon mat and then place both pucks on the paper.

Totally Elastic Collisions

One puck will be launched from one edge of the table towards a stationary puck at the center of the table so that they collide near the center of the table. Before taking data, you should turn on the air compressor and practice launching the pucks a few times *without* using the Spark Timer.

Once you are ready to take data, turn on the Spark Timer and pick a frequency (you will have to use a few trials to find the frequency that gives you a good line of dots). The person launching the puck should be the one to use the foot pedal. <u>Immediately after the puck is launched, hold the foot pedal down until a second or so after the collision.</u> **Your data will appear on the <u>underside</u> of the paper sheet.**

Some things to consider before analyzing your data:

- Try aligning your coordinate system so that the *x*-axis points along the path of one of the pucks.

- On what part of the path should you take your measurements to compute the velocity of the puck both before and after the collision? How many dots should you use? How should you determine the uncertainty in this measurement?

Inelastic Collisions

The procedure for creating an inelastic collision is exactly the same as for creating an elastic collision except that you must modify the pucks slightly by making them "sticky." This is accomplished by wrapping Velcro strips around each puck. It is not necessary that the Velcro actually adheres to the puck, but as long as it is wrapped tightly enough, it won't come off. Don't allow the velcro to drag on the table.

You should re-weigh the pucks.

The inelastic collision will have a complication: after the two pucks stick together, they may "orbit" each other as they move away from the collision location. You will need to determine the <u>center-of-mass</u> velocity of these pucks as they move away from the collision.

Raw Data

The Elastic Collision

Spark Timer Frequency: $f_1=$ $\delta f_1=$

Incoming Mass: $m_1=$ $\delta m_1=$

Dot Spacing: $\Delta s=$ $\delta \Delta s=$

Outgoing Mass (I): $m_1=$ $\delta m_1=$

Dot Spacing: $\Delta s_1=$ $\delta \Delta s_1=$ Angle: $\theta_1 =$

Outgoing Mass (II): $m_2=$ $\delta m_2=$

Dot Spacing: $\Delta s_2=$ $\delta \Delta s_2=$ Angle: $\theta_2 =$

	v_x (m/s)	v_y (m/s)	p_x (kg m/s)	p_y (kg m/s)	Kinetic (J)
Initial ("Incoming")					
Puck A ("outgoing")					
Puck B ("outgoing")					

The Inelastic Collision

Spark Timer Frequency: $f_1=$ $\delta f_1=$

Incoming Mass: $m_1=$ $\delta m_1=$

Dot Spacing: $\Delta s=$ $\delta \Delta s=$

Outgoing Mass (I): $m_1=$ $\delta m_1=$

Dot Spacing: $\Delta s_1=$ $\delta \Delta s_1=$ Angle: $\theta_1 =$

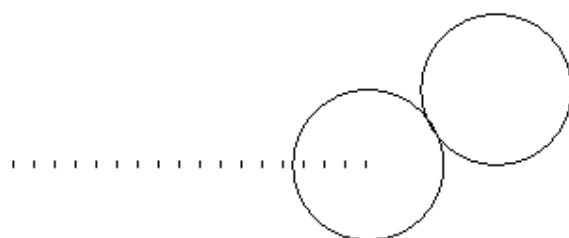
Outgoing Mass (II): $m_2=$ $\delta m_2=$

Dot Spacing: $\Delta s_2=$ $\delta \Delta s_2=$ Angle: $\theta_2 =$

	v_x (m/s)	v_y (m/s)	p_x (kg m/s)	p_y (kg m/s)	Kinetic (J)
Initial ("incoming")					
Puck A ("outgoing")					
Puck B ("outgoing")					
Center of Mass					

Before

Contact
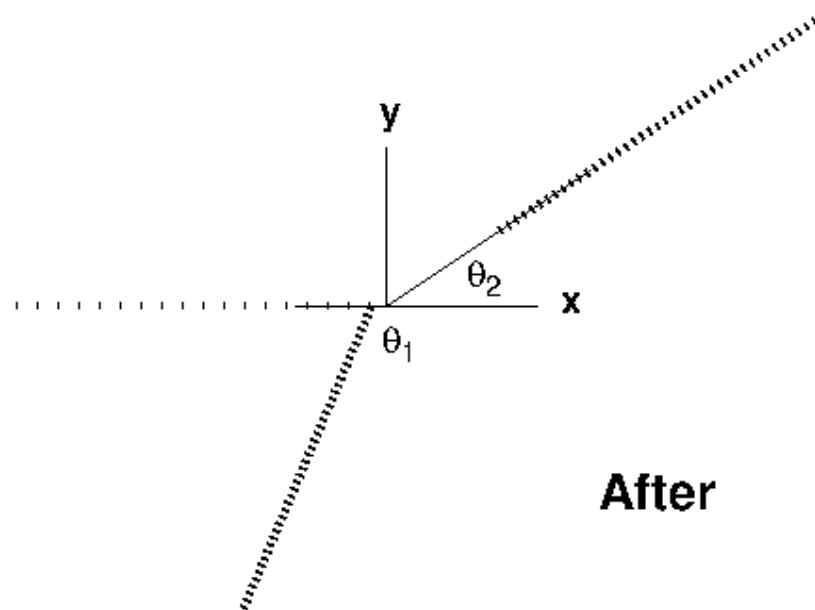
After

Your Report

Introduction

Write a few sentences about what you set out to measure and how you will compare the measured values with theory.

Theory

Write few paragraphs about how you computed the momenta and kinetic energies for the pucks and what you expect them to be for your experiments. Use equations where necessary. As always, talk about how you computed your errors.

[**Extra Credit**: define "coefficient of restitution" *AND*….]

Experimental Procedure

Write a paragraph or two about how you took your data. Write it so that someone else could repeat your experiment by just reading this section. What experiments did you perform? How did you measure the velocities of the pucks? Talk about any problems you encountered and how you handled them.

Data

Include four tables: Initial and final x and y-momenta and initial and final kinetic energies for each of the experiments you performed *along with your uncertainties*. (Remember, this is just the data section, not the discussion section.) One column in each table should be for percent difference.

Results and Conclusion

This will be your longest section. Use good paragraph structure to link the results together:

How do your initial and final momenta compare for your experiments?
How do your initial and final kinetic energies compare for your experiments?
Does your data support or refute Conservation of Momentum? What, if anything, might have interfered with perfect agreement?
If conservation of momentum is correct, what was your percent error and why was it not zero?
Did your "elastic collisions" truly conserve kinetic energy? What was your percent error and why was it not zero?

[**Extra Credit**: …*AND* Compute the coefficient of restitution for your elastic and inelastic collisions]

As always, talk about your uncertainties and how you might be able to reduce them. Make sure that you discuss all possible sources of *experimental* error in your experiments.

Appendix

Include your signed raw data sheet(s) and Sample Calculations.