Name: Section: 201L- Date:

Experiment 3: Projectile Motion

Objectives

- To learn to take good measurements
- To learn how to estimate and propagate uncertainties
- To learn how to apply the kinematic equations to projectile motion

Equipment

- One PASCO Projectile Launcher with plastic projectile
- Two (2) C-Clamps
- One 30 cm ruler
- One two-meter stick
- One sheet of carbon paper
- Plain white or ruled paper
- Tape

Safety

1. **DO NOT STAND ON LAB CHAIRS**
2. **Treat the launcher like a weapon:**
   - **NEVER fire the launcher at anyone---even in jest.**
   - **DO NOT “arm” the launcher until you’re ready to fire it.**
3. When “arming” the launcher, place the ball on the pin and press down on the ball. Trying to press down on the bare pin can hurt your hand.

Introduction

Projectile motion is two-dimensional motion where one direction is under constant acceleration and the other direction is unaccelerated. In this lab, your job will be to compare the prediction for the range of a projectile given by the kinematic equations with the experimentally determined ranges.

Theory

Kinematics

Let us recall the four important equations in kinematics. First, the definitions of velocity and acceleration in the *x*-direction:

$$v_x \equiv \frac{\Delta x}{\Delta t} \tag{1}$$

$$a_x \equiv \frac{\Delta v_x}{\Delta t} \tag{2}$$

Second, the two equations useful for objects moving under constant accelerations:

$$x(t) = \frac{1}{2} a_x t^2 + v_{0,x} t + x_0 \tag{3}$$

$$v_{f,x}^2 - v_{0,x}^2 = 2a_x \ \Delta x \tag{4}$$

The same equations can be used in the *y*-direction by replacing all the *x*'s with *y*'s.

In the horizontal direction (the *x*-direction), there is no acceleration, so that $a_x = 0$ m/s^2. However, gravity contributes a constant acceleration *downwards* (the –*y* direction) such that $a_y = -g$ where $g \equiv 9.81 \pm 0.005$ m/s^2 so that $\delta g =$ 0.005 m/s^2.

When you want to predict the distance that the projectile will go, you will need to combine two of the above equations. Because there is no acceleration in the *x*-direction, you can use equation (1) to start. But you will need to know how long the ball is in flight for. This requires solving for time in equation (3) for the *y*-direction. You may need to use the quadratic formula.

Computing the "Muzzle Velocity"

One of the most important values you will need for this experiment will be the "muzzle velocity," v_0, of the launcher. One way to compute that velocity is to fire the launcher vertically and utilize the *y* version of equation (4). *Remember that, in our coordinate system,* a_y *is a negative value.* By measuring the height that the ball attains and knowing the value of a_y, we can compute the "muzzle velocity" of the launcher.

Theoretical Range Prediction

Once the muzzle velocity has been determined, the theoretical range of the projectile can be predicted. Equation (1) states that the range is completely determined by the *x*-component of the launch velocity and the time in the air:

$$\Delta x = v_x(\Delta t) = v_0 \cos\theta (\Delta t) \tag{5}$$

What remains is to determine the time, Δt, that the ball is in the air. This is completely determined by the vertical motion governed by equation (3). This equation can be rearranged as a quadratic equation in standard form:

$$A(\Delta t)^2 + B(\Delta t) + C = 0 \tag{6}$$

which has a solution given by the quadratic formula:

$$\Delta t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{7}$$

The positive root yeilds the predicted range for the projectile.

Uncertainty Analysis

Uncertainty in Launch Velocity

It will be important to know what the uncertainties are in our computed positions and velocities. First, recall that a difference in position is a subtraction of two quantities:

$$\Delta y = y_2 - y_1 \tag{8}$$

Therefore, we can use the addition/subtraction rule to compute the uncertainty in the difference:

$$(\delta(\Delta y))^2 = (\delta y_2)^2 + (\delta y_1)^2 \tag{9}$$

This time, the uncertainties δy_1 and δy_2 may *not* be the same, so we can't further reduce this uncertainty.

The uncertainty in "muzzle velocity," δv_0, depends on how it is computed. If one uses a horizontal shot and measures the distance and time it travels, then this is a straightforward application of the product/quotient rule. Looking at equation (1) we have

$$\left(\frac{\delta v}{v}\right)^2=\left(\frac{\delta(\Delta x)}{\Delta x}\right)^2+\left(\frac{\delta(\Delta t)}{\Delta t}\right)^2 \tag{10}$$

This time, the uncertainty in time, δt, will be mostly caused by reaction time. This uncertainty is at least 0.1 s, but could be more. Use your best judgment to estimate this. The uncertainty in position will be the standard deviation of a number of trials.

When using equation (4) to determine "muzzle velocity," we arrive at the following equation:

$$v_0=\sqrt{2g\Delta y} \tag{11}$$

To compute the uncertainty, we will have to use both the product/quotient and power rules. First, we rewrite the equation as

$$v_0=\sqrt{q} \quad \text{where} \quad q=2g\Delta y$$

Then the power rule says

$$\frac{\delta v_0}{v_0}=\frac{1}{2}\frac{\delta q}{q}$$

So now we work on δq. The product/quotient rule says

$$\left(\frac{\delta q}{q}\right)^2=\left(\frac{\delta g}{g}\right)^2+\left(\frac{\delta(\Delta y)}{\Delta y}\right)^2$$

Combining them we find

$$\frac{\delta v_0}{v_0}=\frac{1}{2}\sqrt{\left(\frac{\delta g}{g}\right)^2+\left(\frac{\delta(\Delta y)}{\Delta y}\right)^2} \tag{12}$$

Uncertainty in Calculated Range

The uncertainty in the calculated range is one of the more difficult computations in this lab---especially because our standard three rules will actually underestimate this uncertainty. For this lab you may use the following relationship:

$$\left(\frac{\delta R}{R}\right)^2\approx 2\left(\frac{\delta v_0}{v_0}\right)^2 \tag{13}$$

where v_0 is your launch velocity.

Using Standard Deviations

Oftentimes, computing uncertainties can be tedious. Another method that is useful is to collect enough data so that one can trust the standard deviations in the collected data. For the purposes of this course---wherever you are allowed to do so---if you have taken at least five data points, you may use the standard deviation of those data points as your measurement uncertainty. Be aware that this number may be greater than your computed uncertainty; especially if you have few data points.

Experimental Procedure

Measuring the “muzzle velocity”

For this part of the experiment, you will determine the muzzle velocity of the launcher by firing the projectile vertically and measuring the maximum height that the ball attains. Equation (4) can then be used to find the (initial) muzzle velocity.

Figure 1: Vertical Launch Configuration

1. **1.** Mount the projectile launcher to the lab table using a C-clamp (see Figure 1). Set the launcher so the protractor measures an angle of 90°. This ensures that the ball will fire vertically. To adjust the launcher angle, loosen the bolts on the back of launcher. Be careful not to remove screws completely.
2. **2.** Clamp the two-meter stick to the side of the launcher using the second C-clamp. Ensure that the metric side faces you. Align the side of the two-meter stick to the edge of the launcher frame so that the two-meter stick is as vertical as possible.
3. **3.** Note the initial height of the ball, y_1, (indicated on the front of the launcher). Enter this into Table 1.
4. **4.** Set the ball into the launcher and use the ramrod to push ball into launcher. **Stop after the second click.**
5. **5.** Fire the ball vertically by pulling the string on the release lever. Determine the final height, y_2**,** by locating the center of the ball when it stops rising. (Taking a short video with your cell phone might help with this step). Enter your result in Table 1.
6. **6.** Repeat step 5 at least five times and enter your data in Table 1 on your data sheet.
7. **7.** Use Equation (4) to determine the initial velocity of the ball, v_o when it left the launcher. This is called the ‘**muzzle velocity**’ of the launcher**.**

<u>Where does the ball begin its flight?</u> Is the uncertainty in y_1 the same as the uncertainty in y_2? (Translation: Do you know y_2 as well as you know y_1? Try it a few times before answering.)

The Horizontal Shot: Range Prediction

Using your value for the muzzle velocity, you should now be able to predict the range of the projectile when it is fired horizontally. Use the following procedure to set up the horizontal shot:

1. **1.** Remove the two-meter stick.
2. **2.** Loosen the two screws and rotate the launcher so that the protractor reads $\theta = 0^\circ$ (See Figure 2). Re-tighten the two screws.
3. **3.** Measure the vertical distance, Δy, from the bottom of the ball as labeled on your launcher to the floor. Record this value in the Raw Data section.
4. **4.** Use Equation (3) to determine the time, Δt, for a ball released from rest, (v_{0y} = 0 m/s), to fall the vertical distance, Δy, to the floor. **Include this calculation in your Sample Calculations section.**

Figure 2: Horizontal Launch Configuration

5. Because the horizontal and vertical components for projectile motion are independent, the time for a ball fired horizontally from the table to reach the floor is the same as that calculated in step 4. Use Equation (5) to predict the horizontal distance, Δx, that the ball will travel in this time. Note that the initial horizontal velocity, v_{0x}, will be the "muzzle velocity" found earlier. This distance is called the range, R. Show your work in your Sample Calculations section.

Compute your predicted range and stop here for your instructor's initials:

6. Place the center of the wooden "catcher" at your predicted range and tape a piece of white paper to the bottom. At the center of the paper, you might make a fiducial mark and label it with the horizontal distance from the initial firing location to the mark. This allows you to measure the distance to the actual landing point by adding (or subtracting) the distance from the mark that the ball makes to the fiducial mark.
7. Cover the white paper with a sheet of carbon paper (carbon side down). **When you fire the ball, it should land on the carbon paper and make a mark on the white paper.**
8. Fire the ball five times, **time each shot,** and complete Table 2.

The Angled Shot: Range Prediction and Verification

1. Loosen the screws and adjust the launcher to an angle between 55° and 80° (Your lab instructor may suggest an angle). Retighten the screws and record your chosen angle on the Data Sheet.
2. Measure the new vertical distance, Δy, from the bottom of the ball in the launcher to the floor. Record this value on the Raw Data Sheet.
3. Using your measured muzzle velocity and chosen angle, predict the new range, Δx, for the projectile. Place this value in the Raw Data Sheet.

Compute your predicted range and stop here for your instructor's initials:

4. Place the center of the wooden "catcher" at your predicted range and tape a piece of white paper to the bottom. At the center of the paper, you might make a fiducial mark and label it with the horizontal distance from the initial firing location to the mark. This allows you to measure the distance to the actual landing point by adding (or subtracting) the distance from the mark that the ball makes to the fiducial mark.

Figure 3: Launcher at 30°

5. Cover the white paper with a sheet of carbon paper (carbon side down) **When you fire the ball, it should land on the carbon paper and make a mark on the white paper.**
6. Fire the ball five times and complete Table 3.

Raw Data

First method for measuring "Muzzle Velocity"

Table 1. Muzzle Velocity Data

Trial	Initial Height (m) (y_1)	Final Height (m) (y_2)	Vertical Displacement (m) $\Delta y = y_2 - y_1$
1			
2			
3			
4			
5			

$\delta y_1 =$ $\delta y_2 =$ Mean $\Delta y =$ $\delta(\Delta y) =$

With this data, you should now be able to compute: $v_0 =$ $\delta v_0 =$

The Horizontal Shot: Range Prediction and Verification and Second Measurement of the "Muzzle Velocity"

Table 2. Horizontal Range Data

Trial	Range Δx (m)
1	
2	
3	
4	
5	
Mean	
Std Dev.	

$\Delta y =$ $\delta(\Delta y) =$

Predicted Range and uncertainty:

The Angled Shot: Range Prediction and Verification

Your launch angle: $\theta =$ $\delta\theta =$

$\Delta y =$ $\delta(\Delta y) =$

Predicted range and uncertainty:

Table 3. Angled Shot Range

Trial	Range Δx (m)
1	
2	
3	
4	
5	

Average Range:

Standard Deviation:

Your Report

Experimental Procedure

You will need to write a few paragraphs to *carefully* describe the following things:

- Carefully describe how you set up your experiments
- For the vertical shot:
  - Explain your procedure for verifying that the ball was fired purely vertically.
  - Explain your procedure for measuring the height that the ball reached.
- For the horizontal and angled shots:
  - Explain your procedure for measuring the height of the ball.
  - Explain your procedure for measuring the horizontal travel of each shot.
- How did you make each of these measurements as accurate as possible?

Write this section so that someone else could perform the experiment by just reading this section (but do not write instructions). Talk about any problems you encountered and how you handled them.

Data

Report the following values and their uncertainties:

- What was your final determination of your "muzzle velocity?" (Hint: Look at the result for your vertical shot.)
- What was the angle at which you fired the "angled" shot?
- Create the following table in your lab report and fill in the values:

Table 1. Theoretical and Experimental Projectile Ranges

	Theoretical (m)	**Mean (m)**	**Std. Deviation (m)**
Horizontal			
Angled			

Results and Conclusion

This should be your longest section. The primary goal of this section is to compare your theoretical predictions with your experimental results. You must also explain the uncertainties and errors you encountered and their origins. Be brief, but be complete.

This is a technical report. Do not use the words: "about," "almost," "close to," "kind of," "roughly," or "sort of." Instead, use the proper numerical comparison (percent error or percent difference).

Use good paragraph structure to link these results together:

- Compare the theoretical range of the horizontal shot with the experimentally measured range. Use the % error formula and cite your errors. Is your experimental range "inside" your theoretical range plus/minus your uncertainty? If not, what does this indicate?
- Do the same for the angled shot.

- Talk about your uncertainties and errors: what caused them and how you might be able to reduce them.

Appendix

Include your signed raw data sheet and sample calculations.