DATA_605_Assignment_14

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May 13, 2018

Task

This week, we'll work out some Taylor Series expansions of popular functions.

$$f(x) = \frac{1}{(1-x)}$$

$$f(x) = e^x$$

$$f(x) = \ln(1+x)$$

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as a R-Markdown document.

1.

The formula for a Taylor Series centered about c is

$$\begin{aligned} f(x) &= f(c) + f'(c)(x-c) + \frac{f''(c)}{2!} + \cdots \\ &= \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} x^n \end{aligned}$$

If this is centered about 0, aka a Maclaurin Series, we get:

$$\begin{aligned} &1 + x + x^2 + x^3 + \cdots \\ &= \sum_{n=0}^{\infty} x^n \quad x \in (-1, 1) \end{aligned}$$

This is basically a geometric series.

2.

Centered about 0:

$$\begin{aligned} &1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad x \in \mathbb{R} \end{aligned}$$

3.

Centered about 0:

$$
\begin{gathered}
0 + x - \frac{1}{2!}x^2 + \frac{2}{3!}x^3 - \frac{6}{4!}x^4 + \cdots \\
= \sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^n}{n} = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{x^n}{n} \quad x \in (-1, 1]
\end{gathered}
$$