JSturm_Assignment_13

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1.

Use integration by substitution to solve the integral $\int 4e^{-7x}dx$

1. Solution

Let $u = -7x$. Then $du = -7dx \ \rightarrow \ dx = \frac{du}{-7}$.

Our integral is now $\int \frac{4e^{u}du}{-7}$. Taking out the constants: $\frac{4}{-7}\int e^{u}du$.

Replacing u with our original substitution: $\frac{-4}{7}e^{-7x} + C$.

2.

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after day 1 was 6530 per cubic centimeter.

2. Solution

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{3150}{t^4} - 220 \ \rightarrow \ \mathrm{d}N = (-\frac{3150}{t^4} - 220)\mathrm{d}t$$

To find N, we can take the antiderivative, i.e. the integral.

$N = \int(-\frac{3150}{t^4} - 220)\mathrm{d}t \quad = \int -3150(t^{-4})\mathrm{d}t - \int 220\mathrm{d}t$

Using the power rule for integration: $N = \frac{-3150}{-3}(t^{-3}) - 220t + C$.

Solving for $N(1) = 6530$:

$N(1)\frac{-3150}{-3}(1^{-3}) - 220(1) + C = 6530$

$C = 6530 - 1050 + 220 = 5700$.

$N(t) = -1050(t^{-3}) - 200(t) + 5700$.

3.

Find the total area of the red rectangles in the figure below, where the equation of the lines is $f(x) = 2x - 9$.

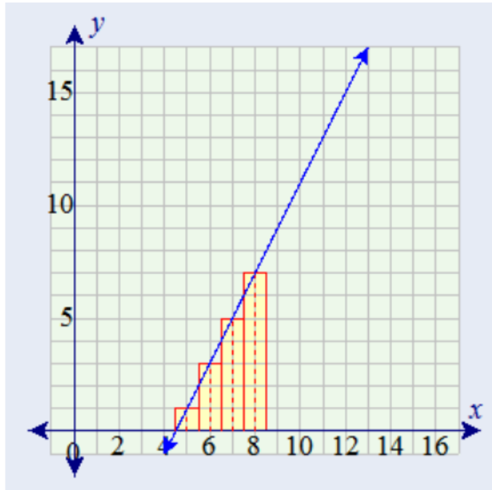


Figure 1:

3. Solution

The equation is given as $2x - 9$, and the ends of the rectangles look to be 4.5 and 8.5. Since we're looking for the area, we can integrate this function over these boundaries.

$$\int_{4.5}^{8.5} (2x - 9)dx$$

Using the power rule for integration:

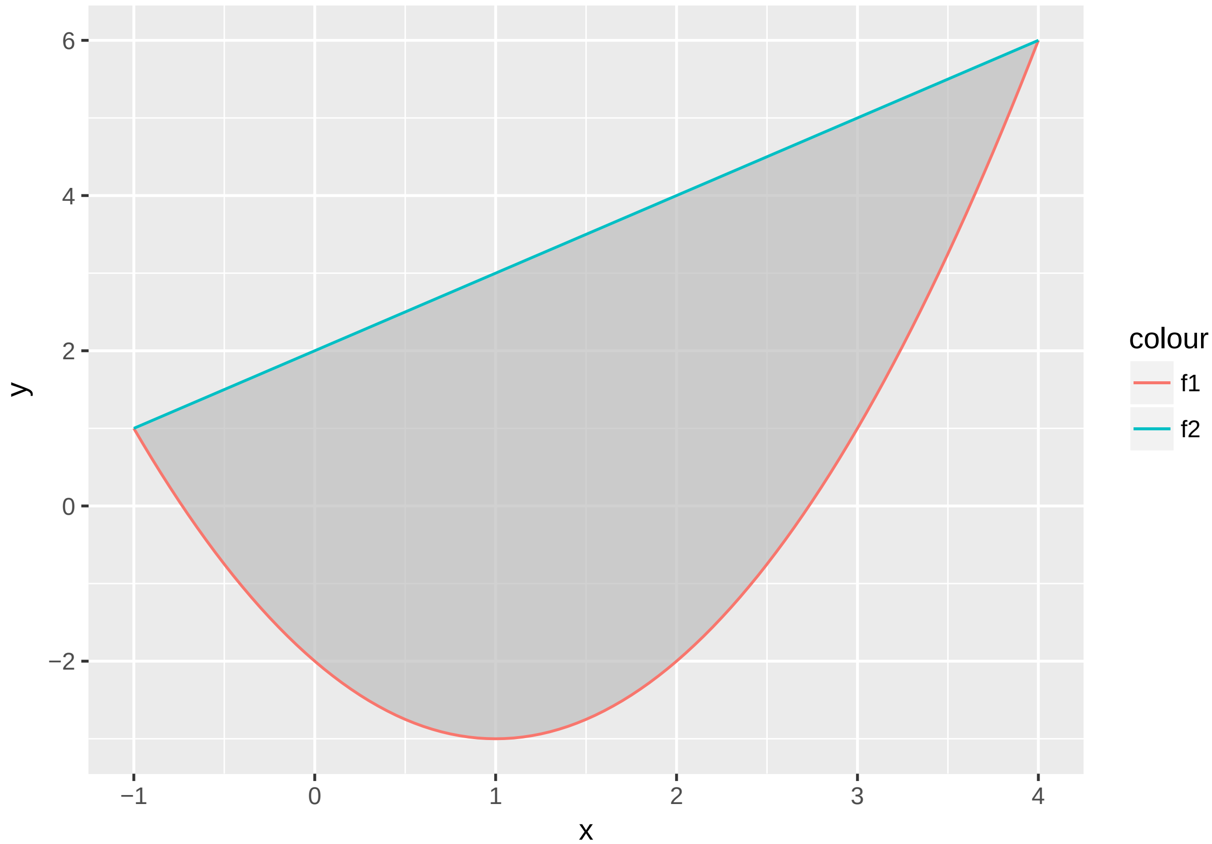
$$(x^2 - 9x)\Big|_{4.5}^{8.5} = \Big[(8.5)^2 - 9(8.5)\Big] - \Big[(4.5)^2 - 9(4.5)\Big] = [72.25 - 76.5] - [20.25 - 40.5] = 16$$

4.

Find the area of the region bounded by the graphs of the given equations

$$y = x^2 - 2x - 2, \quad y = x + 2$$

4. Solution



We're looking for the area between the two functions. The intersection points are (-1, 1) and (4, 6), which will serve as the boundaries.

$$A = \int_{-1}^{4} x + 2 \ dx \ - \ \int_{-1}^{4} x^2 - 2x - 2dx A = \left[\frac{1}{2}x^2 + 2x\right]_{-1}^{4} - \left[\frac{1}{3}x^3 - x^2 - 2x\right]_{-1}^{4} = -\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x\Big|_{-1}^{4}$$

≈ 20.8333333

5.

A beauty supply store expects to sell 110 flat irons during the next year. It costs \$3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

5. Solution

I was unsure how to solve this problem, so I googled how to optimize inventory, and stumbled upon economic order quantity (EOQ).

Quoting the article:

- P = purchase unit price, unit production cost

- Q = order quantity.
- Q^* = optimal order quantity.
- D = annual demand quantity.
- K = fixed cost per order, setup cost (not per unit, typically cost of ordering and shipping and handling. This is not the cost of goods)
- h = annual holding cost per unit, also known as carrying cost or storage cost (capital cost, warehouse space, refrigeration, insurance, etc. usually not related to the unit production cost)

The single-item EOQ formula finds the minimum point of the following cost function:

Total Cost = purchase cost or production cost + ordering cost + holding cost

Where:

- Purchase cost: This is the variable cost of goods: purchase unit price × annual demand quantity. This is P × D
- Ordering cost: This is the cost of placing orders: each order has a fixed cost K, and we need to order D/Q times per year. This is K × D/Q
- Holding cost: the average quantity in stock (between fully replenished and empty) is Q/2, so this cost is h × Q/2

$TC = PD + \frac{DK}{Q} + \frac{hQ}{2}$.

$\frac{\mathrm{d}}{\mathrm{d}Q} = -\frac{DK}{Q^2} + \frac{h}{2}$.

We next set this equal to zero, and solve for Q in order to find the function minimum.

$-\frac{DK}{Q^2} + \frac{h}{2} = 0$.

$Q^{*2} = \frac{2DK}{h} \ \rightarrow \ Q^* = \sqrt{\frac{2DK}{h}}$

We are given these variables, so we just need to plug them into the formula.

$D = 110$.

$K = 8.25$.

$h = 3.75$.

$$Q^* = \sqrt{\frac{2 \cdot 110 \cdot 8.25}{3.75}} = \sqrt{\frac{1815}{3.75}} = \sqrt{484} = 22.$$

We found 22 to be the lot size per order.

We are given that the store expects to sell $n = 110$ flat irons. If there are x number of irons in each order, our equation is $22 \cdot x = 110 \ \rightarrow \ x = \frac{110}{22} = 5$.

So to minimize inventory costs, the store should make 5 orders of 22 irons per year.

6.

Use integration by parts to solve the integral below:

$$\int \ln(9x) \cdot x^6 \ dx$$

6. Solution

The formula for integration by parts is:

$$\int u\ dv = uv - \int v\ du$$

Let $u = \ln(9x)$. Using the chain rule $du = \frac{1}{9x} \cdot 9\ dx = \frac{1}{x}\ dx$.

Let $dv = x^6$, then $v = \int x^6 = \frac{x^7}{7}$.

Plugging this into the formula:

$$\ln(9x) \cdot \frac{x^7}{7} - \int \frac{x^7}{7} \cdot \frac{1}{x}\ dx$$

We can pull out the constant:

$$\ln(9x) \cdot \frac{x^7}{7} - \frac{1}{7} \int \frac{x^7}{x}\ dx = \ln(9x) \cdot \frac{x^7}{7} - \frac{1}{7} \int x^6\ dx$$

Using the power rule for integration:

$$\ln(9x) \cdot \frac{x^7}{7} - \frac{1}{7}\left(\frac{x^7}{7}\right) + C$$

$$= \ln(9x) \cdot \frac{x^7}{7} - \frac{x^7}{49} + C$$

7.

Determine whether $f(x)$ is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral.

$$f(x) = \frac{1}{6x}$$

7. Solution

There are two conditions for a probability density function:

- $f(x) \geq 0 \quad \forall x$
- $\int_{-\infty}^{\infty} f(x)\ dx = 1$

$$\int_1^{e^6} \frac{1}{6x}\ dx = \frac{1}{6} \int_1^{e^6} \frac{1}{x}\ dx = \frac{1}{6} [\ln(x)]_1^{e^6} = \frac{\ln(e^6) - \ln(1)}{6} = \frac{6 \cdot \ln(e) - 0}{6} = \frac{6}{6} = 1$$

Since the area sums up to 1, we conclude that $f(x)$ is a pdf on the interval $[1, e^6]$.