DATA 605 - Assignment 10

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Question 1

Smith is in jail and has 1 dollar; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollars, he wins A dollars with probability .4 and loses A dollars with probability .6. Find the probability that he wins 8 dollars before losing all of his money if

(a) he bets 1 dollar each time (timid strategy).

(b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy).

(c) Which strategy gives Smith the better chance of getting out of jail?

Solutions

This is known as the Gambler's Ruin problem.

We are given:

Initial stake $z = k = 1$.

$M = 8$

$P = 0.4$

$q = 0.6$

$q_z = \frac{(\frac{q}{p})^z - 1}{(\frac{q}{p})^M - 1}$

(a)

$q_z = \frac{(\frac{0.6}{0.4})^1 - 1}{(\frac{0.6}{0.4})^8 - 1} = 0.0203013.$

There is a ~2% probability Smith will win using this strategy.

(b)

The quickest strategy is if he bets everything each time. That is, beginning from state $z = k = 1$, he can move fall to 0 with $q = 0.6$, or rise to 2 with $p = 0.4$. Similarly, suppose he moved to 2, he bets everything, and can fall to 0 with $q = 0.6$, or rise to 4 with $p = 0.4$.

Using the formula $q_k = p \cdot q_{k+1} + q \cdot qk - 1$:

$q_0 = 0$

$q_1 = (0.4)q_2 + (0.6)q_0$

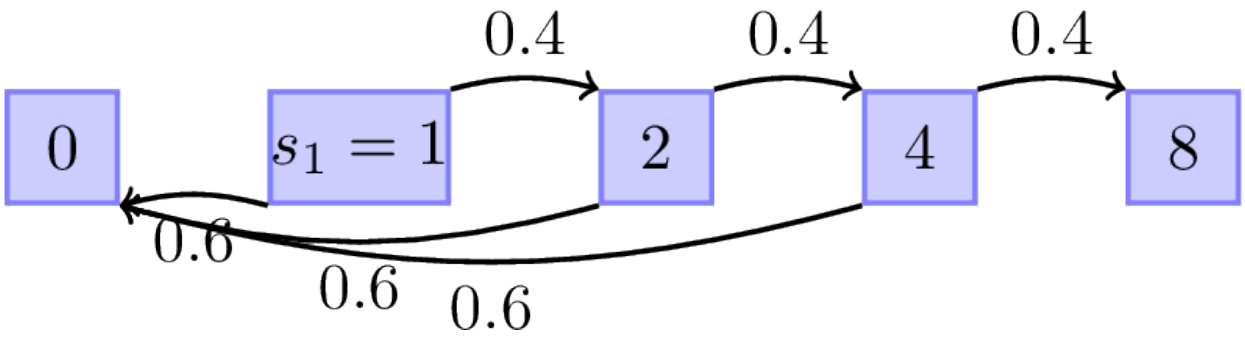

Figure 1:

$q_2 = (0.4)q_4 + (0.6)q_0$

$q_4 = (0.4)q_8 + (0.6)q_0$

$q_8 = 1$

$(0.4)^3 = 0.064$.

References

- http://people.math.umass.edu/~lr7q/ps_files/teaching/math456/Week4.pdf