DATA 605 - Assignment 3

Joshua Sturm

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```r
library(pracma)
```

Problem Set 1

1. What is the rank of matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

```r
A <- matrix(c(1, 2, 3, 4,
              -1, 0, 1, 3,
              0, 1, -2, 1,
              5, 4, -2, -3),
            nrow = 4, ncol = 4, byrow = T)

rref(A)
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

There are 4 pivot columns in the reduced row echelon form of the matrix, so it has a rank of 4.

2. Given an $m \times n$ matrix where $m > n$, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

Since the rank is the number of linearly independent columns, the maximum rank can't be larger than the number of columns, i.e. n.

If the matrix is non-zero, the minimum rank would be 1.

3. What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

Using the same process as question 1:

```r
B <- matrix(c(1, 2, 1,
              3, 6, 3,
              2, 4, 2),
            nrow = 3, ncol = 3, byrow = T)

rref(B)
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    0    0    0
## [3,]    0    0    0
```

Since the reduced row echelon form of B only has one pivot column, its rank is 1.

Problem Set 2

1. Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$|A - \lambda I| = \left| \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right|$$

$= \left| \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix} \right|$

$= (1-\lambda)(4-\lambda)(6-\lambda)$

$(1-\lambda)\left[24 - 10\lambda + \lambda^2\right] = 24 - 10\lambda + \lambda^2 - 24\lambda + 10\lambda^2 - \lambda^3 = -\lambda^3 + 11\lambda^2 - 34\lambda + 24$

After factoring, we get $(\lambda - 1)(\lambda - 4)(\lambda - 6)$.

So the eigenvalues are 1, 4, 6.

For $\lambda = 1$:

```r
A <- matrix(c(1, 2, 3,
              0, 4, 5,
              0, 0, 6),
            nrow = 3, ncol = 3, byrow = T)

I <- diag(3)

rref(A-I)
##      [,1] [,2] [,3]
## [1,]    0    1    0
## [2,]    0    0    1
## [3,]    0    0    0
```

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

For $\lambda = 4$:

```
rref(A - 4*I)
##      [,1]       [,2] [,3]
## [1,]    1 -0.6666667    0
## [2,]    0  0.0000000    1
## [3,]    0  0.0000000    0
```

$x_1 - \frac{2}{3}x_2 = 0$

$v_4 = \begin{bmatrix} \frac{2}{3} \\ 1 \\ 0 \end{bmatrix}$

For $\lambda = 6$:

```
rref(A - 6*I)
##      [,1] [,2] [,3]
## [1,]    1    0 -1.6
## [2,]    0    1 -2.5
## [3,]    0    0  0.0
```

$x_1 - 1.6x_3 = 0 \qquad x_2 - 2.5x_3 = 0 \quad x_3 \neq 0.$

$v_6 = \begin{bmatrix} 1.6 \\ 2.5 \\ 1 \end{bmatrix}$