DATA 605 - Assignment 1

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02/01/2018

Problem Set 1

1. Calculate the dot product of u,v where u=[0.5;0.5] and v=[3;-4]

Solution

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{u}_1 \cdot \mathbf{v}_1 + \mathbf{u}_2 \cdot \mathbf{v}_2 0.5 \cdot 3 + 0.5 \cdot -4 \rightarrow 1.5 - 2 = -0.5$$

```
u <- c(0.5, 0.5)
v <- c(3,-4)
udotv <- drop(u %*% v) # The drop function is used to remove dimensions > 1
udotv
```

```
## [1] -0.5
```

2. What are the lengths of u and v?

Solution

$$||\mathbf{u}|| = \sqrt{\mathbf{u}_1^2 + \mathbf{u}_2^2 + ... + \mathbf{u}_n^2} \sqrt{0.5^2 + 0.5^2} = \sqrt{0.25 + 0.25} = \sqrt{0.5} = 0.7071068$$

.

$$||\mathbf{v}|| = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

.

```
normu <- sqrt(u[1]^2 + u[2]^2)
normv <- sqrt(v[1]^2 + v[2]^2)
normu
```

```
## [1] 0.7071068
```

```
normv
```

```
## [1] 5
```

3. What is the linear combination 3u-2v?

Solution

$$a\mathbf{u} + b\mathbf{v} = [a \cdot \mathbf{u}_1, a \cdot \mathbf{u}_2, ..., a \cdot \mathbf{u}_n] + [b \cdot \mathbf{v}_1, b \cdot \mathbf{v}_2, ..., b \cdot \mathbf{v}_n]$$

$$[3 \cdot 0.5, 3 \cdot 0.5] - [2 \cdot 3, 2 \cdot -4] = [1.5, 1.5] - [6, -8] = [-4.5, 9.5]$$

.

```r
c(3*u - 2*v)
```

```
## [1] -4.5  9.5
```

4. What is the angle between u and v?

Solution

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||\ ||\mathbf{v}||} \quad \rightarrow \quad \theta = cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||\ ||\mathbf{v}||}\right)$$

$$\theta = cos^{-1}\left(\frac{-0.5}{0.7071 * 5}\right) = cos^{-1}(-0.1414) \approx 1.7127 \text{ rad} \approx 98.13°.$$

Problem Set 2

```r
matsolver <- function(a, b){
  # Assume a is the matrix, and b is the constrain.
  # Merge the two into an augmented matrix
  augmm <- cbind(a, b)

  # Need a 1 in position [1,1]
  if (augmm[1,1] == 0){
    if (augmm[2,1] != 0){
      augmm <- augmm[c(2,1,3),]
    }
    else {
      augmm <- augmm[c(3,2,1),]
    }
  }
  else if (augmm[1,1] != 1){
    augmm[1,] <- augmm[1,] / augmm[1,1]
  }

  # All other rows in column 1 should be zero
  if (augmm[2,1] != 0){
    augmm[2,] <- (augmm[2,1] * augmm[1,]) - augmm[2,]
  }
  if (augmm[3,1] != 0){
    augmm[3,] <- (augmm[3,1] * augmm[1,]) - augmm[3,]
  }

  #Need a non-zero number in the middle
  if (augmm[2,2] == 0){
    augmm <- augmm[c(1,3,2),]
  }

  augmm[2,] <- augmm[2,] / augmm[2,2]
  augmm[3,] <- (augmm[3,2] * augmm[2,]) - augmm[3,]

  # Use back substitution
```

```
  solvedm <- numeric(3)
  solvedm[3] <- augmm[3,4] / augmm[3,3]
  solvedm[2] <- (augmm[2,4] - augmm[2,3] * solvedm[3]) / augmm[2,2]
  solvedm[1] <- (augmm[1,4] - augmm[1,2] * solvedm[2] - augmm[1,3] * solvedm[3]) / augmm[1,1]

  return(cbind(solvedm))
}
```