DATA 605 - Final Exam

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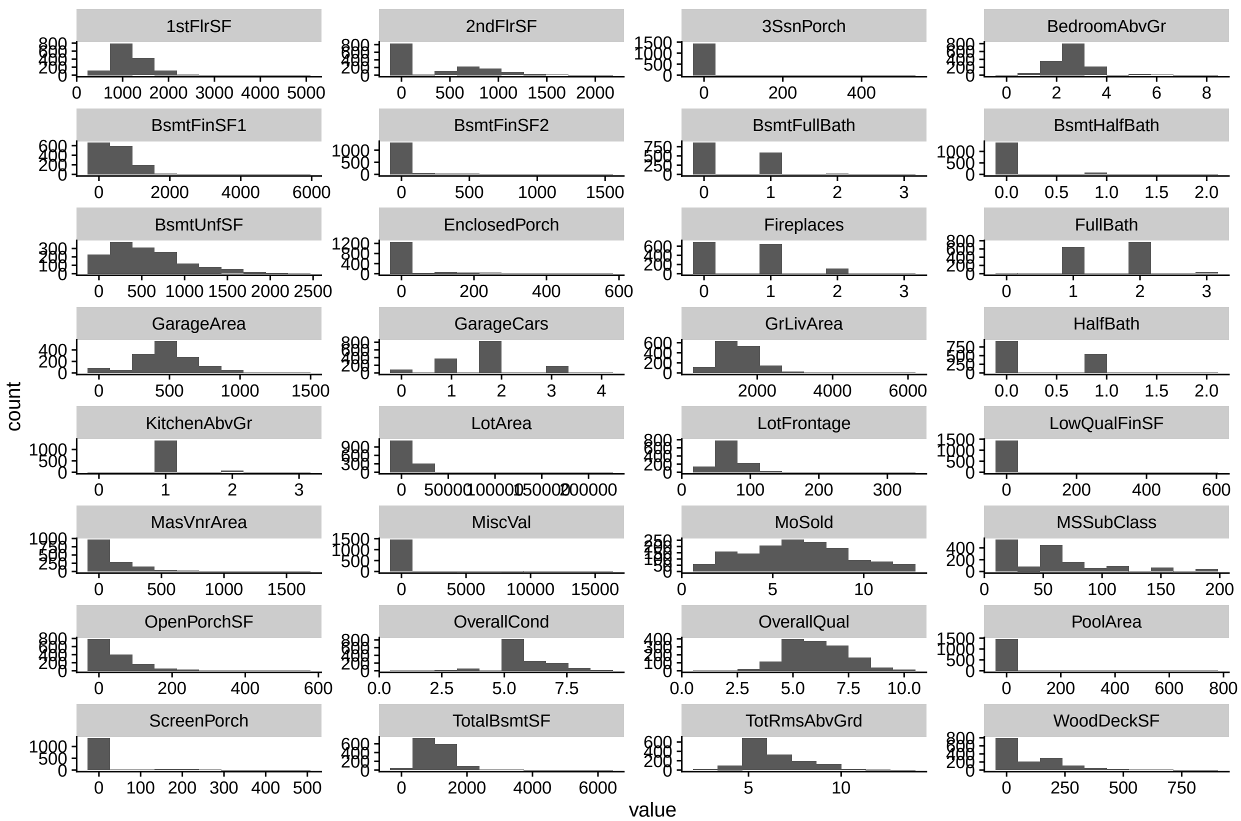
1. Kaggle Data

You are to register for Kaggle.com (free) and compete in the House Prices: Advanced Regression Techniques competition. https://www.kaggle.com/c/house-prices-advanced-regression-techniques. I want you to do the following

- Pick one of the quantitative independent variables from the training data set (train.csv), and define that variable as X. Make sure this variable is skewed to the right!
- Pick the dependent variable and define it as Y.

Since the requirement is to pick an independent variables that is skewed to the right, I'll begin by plotting a histogram for each (numeric) variable.



I'm going to use `OpenPorchSF` as my independent variable. Of course, since we're trying to predict the house's sale price, `SalePrice` will be the dependent variable.

2. Probability

Calculate as a minimum the below probabilities a through c. Assume the small letter "x" is estimated as the 1st quartile of the X variable, and the small letter "y" is estimated as the 1st quartile of the Y variable. Interpret the meaning of all probabilities. In addition, make a table of counts as shown below.

a. $P(X > x|Y > y)$ b. $P(X > x, Y > Y)$ c. $P(X < x|Y > y)$

a.

$$P(X > x|Y > y) = \frac{P(X > x \cap Y > y)}{P(Y > y)}$$

In words, we're looking for the conditional probability that the `OpenPorchSF` variable is greater than its first quantile, **given** that `SalePrice` is greather than its first quantile.

```
## [1] "Probability = 0.647488584474886"
```

b.

$$P(X > x, Y > Y) = P(X > x \cap Y > y)$$

```
## [1] "Probability = 0.485616438356164"
```

c.

$$P(X < x|Y > y) = \frac{P(X < x \cap Y > y)}{P(Y > y)}$$

```
## [1] "Probability = 0"
```

x/y	$\leq$ 1st quartile	$>$ 1st quartile	Total
$\leq$ 1st quartile	270	386	656
$>$ 1st quartile	95	709	804
Total	365	1095	1460

2.2 Independence

Does splitting the training data in this fashion make them independent? Let A be the new variable counting those observations above the 1st quartile for X, and let B be the new variable counting those observations above the 1st quartile for Y. Does P(AB)=P(A)P(B)? Check mathematically, and then evaluate by running a Chi Square test for association.

```
## [1] "A = 804"
## [1] "B = 1095"
## [1] "P(A) = 0.550684931506849"
## [1] "P(B) = 0.75"
## [1] "P(A)P(B) = 0.413013698630137"
## [1] "P(AB) = 0.485616438356164"
## [1] "Mean relative difference: 0.1757877"
```

We can see that the two are not equal, that is, $P(AB) \neq P(A) \times P(B)$. So we've shown mathematically that they're not independent.

```
##
##  Pearson's Chi-squared test
##
## data:  train$OpenPorchSF and train$SalePrice
## X-squared = 144030, df = 133060, p-value < 2.2e-16
```

With a p-value < 0.05, we can reject the null hypothesis that the variables are independent, and conclude that they are dependent.

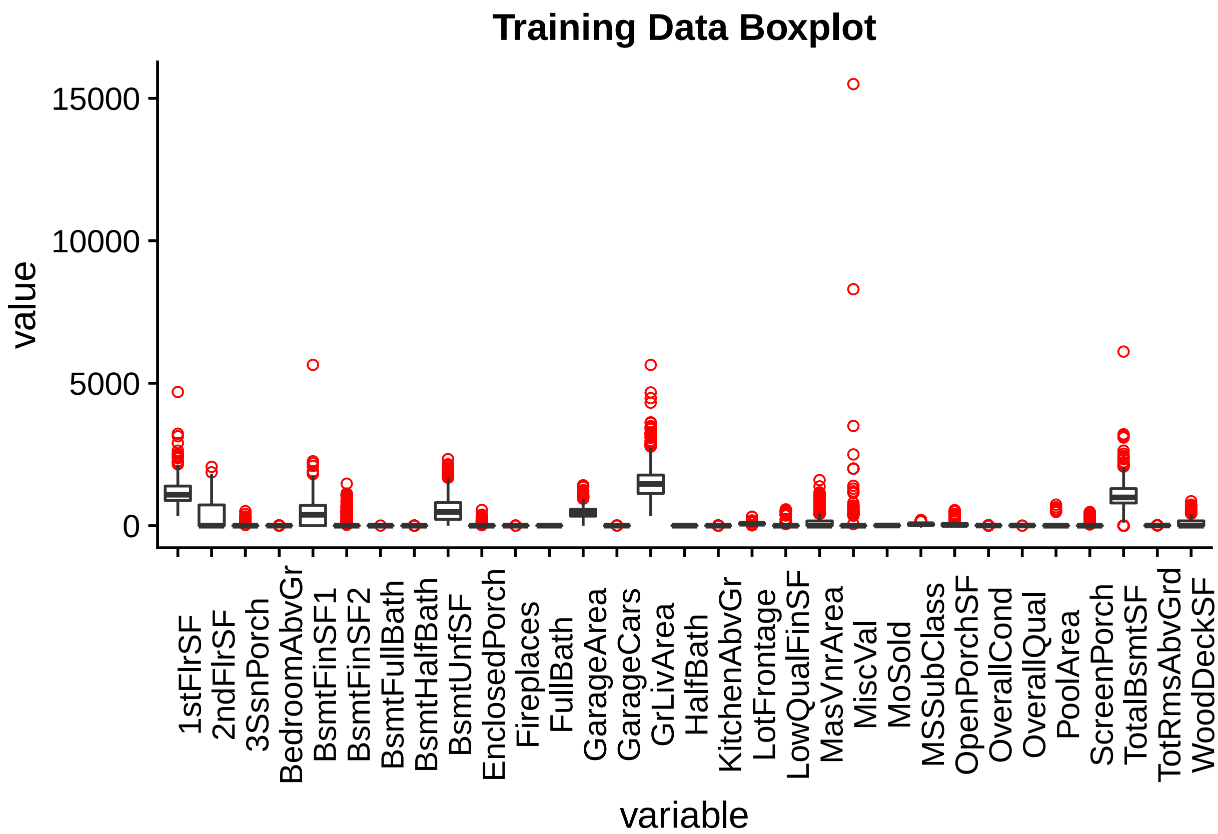
3. Descriptive and Inferential Statistics

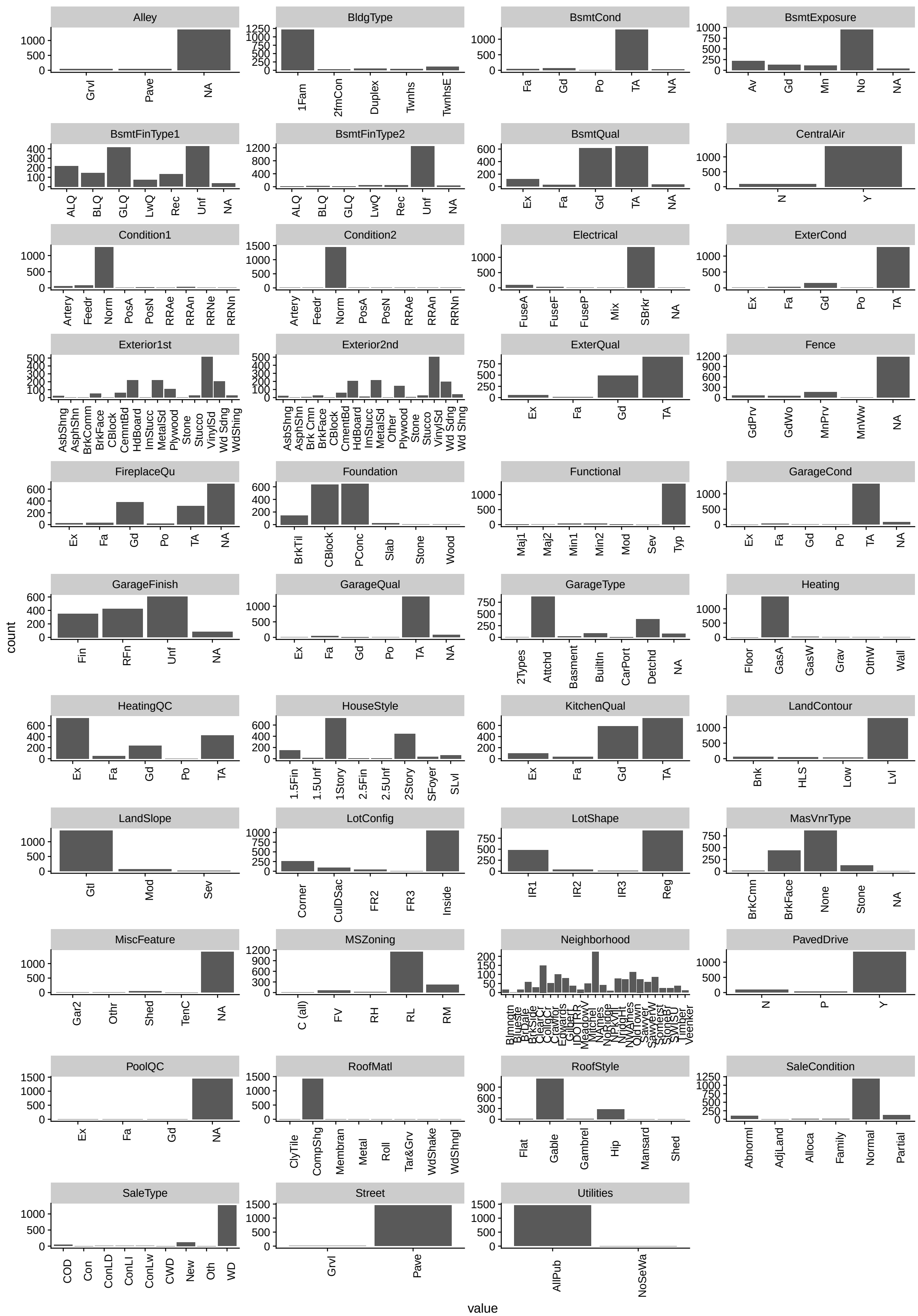
Provide univariate descriptive statistics and appropriate plots for the training data set. Provide a scatterplot of X and Y. Derive a correlation matrix for any THREE quantitative variables in the dataset. Test the hypotheses that the correlations between each pairwise set of variables is 0 and provide a 92% confidence interval. Discuss the meaning of your analysis. Would you be worried about familywise error? Why or why not?

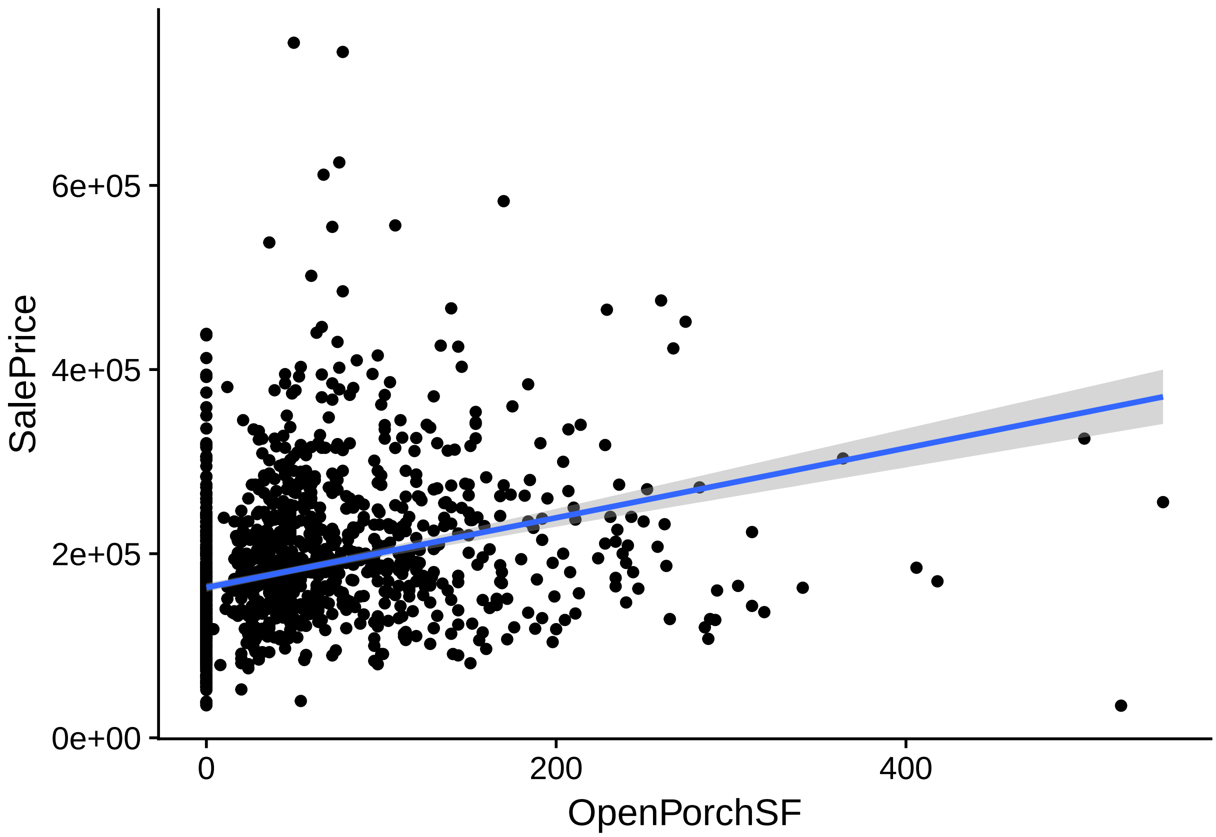
For brevity's sake, I'll only summarize the last 10 (numeric) columns.

```
##               vars    n      mean       sd median   trimmed      mad   min
## WoodDeckSF      29 1460     94.24   125.34      0     71.76     0.00     0
## OpenPorchSF     30 1460     46.66    66.26     25     33.23    37.06     0
## EnclosedPorch   31 1460     21.95    61.12      0      3.87     0.00     0
## 3SsnPorch       32 1460      3.41    29.32      0      0.00     0.00     0
## ScreenPorch     33 1460     15.06    55.76      0      0.00     0.00     0
## PoolArea        34 1460      2.76    40.18      0      0.00     0.00     0
## MiscVal         35 1460     43.49   496.12      0      0.00     0.00     0
## MoSold          36 1460      6.32     2.70      6      6.25     2.97     1
## YrSold          37 1460   2007.82     1.33   2008   2007.77     1.48  2006
## SalePrice       38 1460 180921.20 79442.50 163000 170783.29 56338.80 34900
##                  max  range  skew kurtosis      se
## WoodDeckSF       857    857  1.54     2.97    3.28
## OpenPorchSF      547    547  2.36     8.44    1.73
## EnclosedPorch    552    552  3.08    10.37    1.60
## 3SsnPorch        508    508 10.28   123.06    0.77
## ScreenPorch      480    480  4.11    18.34    1.46
## PoolArea         738    738 14.80   222.19    1.05
## MiscVal        15500  15500 24.43   697.64   12.98
## MoSold            12     11  0.21    -0.41    0.07
## YrSold          2010      4  0.10    -1.19    0.03
## SalePrice     755000 720100  1.88     6.50 2079.11
```

I've already plotted histograms in part 1. I will do boxplots for numeric variables, and bar charts for categorical data.


Training Data Boxplot
15000
10000
5000
0
value
1stFlrSF
2ndFlrSF
3SsnPorch
BedroomAbvGr
BsmtFinSF1
BsmtFinSF2
BsmtFullBath
BsmtHalfBath
BsmtUnfSF
EnclosedPorch
Fireplaces
FullBath
GarageArea
GarageCars
GrLivArea
HalfBath
KitchenAbvGr
LotFrontage
LowQualFinSF
MasVnrArea
MiscVal
MoSold
MSSubClass
OpenPorchSF
OverallCond
OverallQual
PoolArea
ScreenPorch
TotalBsmtSF
TotRmsAbvGrd
WoodDeckSF
variable


Alley
Grvl Pave NA
BldgType
1Fam 2fmCon Duplex Twnhs TwnhsE
BsmtCond
Fa Gd Po TA NA
BsmtExposure
Av Gd Mn No NA
BsmtFinType1
ALQ BLQ GLQ LwQ Rec Unf NA
BsmtFinType2
ALQ BLQ GLQ LwQ Rec Unf NA
BsmtQual
Ex Fa Gd TA NA
CentralAir
N Y
Condition1
Artery Feedr Norm PosA PosN RRAe RRAn RRNe RRNn
Condition2
Artery Feedr Norm PosA PosN RRAe RRAn RRNn
Electrical
FuseA FuseF FuseP Mix SBrkr NA
ExterCond
Ex Fa Gd Po TA
Exterior1st
AsbShng AsphShn BrkComm BrkFace CBlock CemntBd HdBoard ImStucc MetalSd Plywood Stone Stucco VinylSd Wd Sdng WdShing
Exterior2nd
AsbShng AsphShn Brk Cmn BrkFace CBlock CmentBd HdBoard ImStucc MetalSd Other Plywood Stone Stucco VinylSd Wd Sdng Wd Shng
ExterQual
Ex Fa Gd TA
Fence
GdPrv GdWo MnPrv MnWw NA
FireplaceQu
Ex Fa Gd Po TA NA
Foundation
BrkTil CBlock PConc Slab Stone Wood
Functional
Maj1 Maj2 Min1 Min2 Mod Sev Typ
GarageCond
Ex Fa Gd Po TA NA
GarageFinish
Fin RFn Unf NA
GarageQual
Ex Fa Gd Po TA NA
GarageType
2Types Attchd Basment BuiltIn CarPort Detchd NA
Heating
Floor GasA GasW Grav OthW Wall
HeatingQC
Ex Fa Gd Po TA
HouseStyle
1.5Fin 1.5Unf 1Story 2.5Fin 2.5Unf 2Story SFoyer SLvl
KitchenQual
Ex Fa Gd TA
LandContour
Bnk HLS Low Lvl
LandSlope
Gtl Mod Sev
LotConfig
Corner CulDSac FR2 FR3 Inside
LotShape
IR1 IR2 IR3 Reg
MasVnrType
BrkCmn BrkFace None Stone NA
MiscFeature
Gar2 Othr Shed TenC NA
MSZoning
C (all) FV RH RL RM
Neighborhood
PavedDrive
N P Y
PoolQC
Ex Fa Gd NA
RoofMatl
ClyTile CompShg Membran Metal Roll Tar&Grv WdShake WdShngl
RoofStyle
Flat Gable Gambrel Hip Mansard Shed
SaleCondition
Abnorml AdjLand Alloca Family Normal Partial
SaleType
COD Con ConLD ConLI ConLw CWD New Oth WD
Street
Grvl Pave
Utilities
AllPub NoSeWa
count
value



H_0 : The correlation coefficient is 0, which is to say that these variables aren't related. H_1 : The correlation coefficient is not 0.

```
##
##  Pearson's product-moment correlation
##
## data:  train$MSSubClass and train$firstFlrSF
## t = -9.933, df = 1458, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 92 percent confidence interval:
##  -0.2941963 -0.2083297
## sample estimates:
##        cor
## -0.2517584
##
##  Pearson's product-moment correlation
##
## data:  train$MSSubClass and train$GrLivArea
## t = 2.8662, df = 1458, p-value = 0.004214
## alternative hypothesis: true correlation is not equal to 0
## 92 percent confidence interval:
##  0.02912052 0.12027312
## sample estimates:
##        cor
## 0.07485318
##
##  Pearson's product-moment correlation
```

```
##
## data:  train$firstFlrSF and train$GrLivArea
## t = 26.217, df = 1458, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 92 percent confidence interval:
##  0.5340458 0.5963849
## sample estimates:
##      cor
## 0.566024
```

Since the p-value for all three tests is much smaller than $\alpha = 0.08$, we can reject the null hypothesis, and conclude that there is some correlation between the variables.

According to this wikipedia article, familywise error is a type 1 error, which occurs when one rejects a true null hypothesis (aka a false positive). The probability of making one or more of these can be controlled by ensuring

$$FWER = P(V \geq 1) = 1 - P(v = 0) \leq \alpha$$

$FWER = 1 - (1 - 0.08)^3 = 0.221312$

So we should proceed with caution, since there is a ~22% chance of making a type 1 error.

4. Linear Algebra and Correlation

Invert your 3×3 correlation matrix from above. (This is known as the precision matrix and contains variance inflation factors on the diagonal.) Multiply the correlation matrix by the precision matrix, and then multiply the precision matrix by the correlation matrix. Conduct LU decomposition on the matrix.

Invert our correlation matrix

Multiplying the correlation matrix by the precision (inverted) matrix

```
##            MSSubClass firstFlrSF GrLivArea
## MSSubClass          1          0         0
## firstFlrSF          0          1         0
## GrLivArea           0          0         1
##            MSSubClass firstFlrSF GrLivArea
## MSSubClass          1          0         0
## firstFlrSF          0          1         0
## GrLivArea           0          0         1
```

The result of the matrix multiplcation is the identity matrix.

To perform the LU decomposition, I'll recycle the function I created for assignment 2.

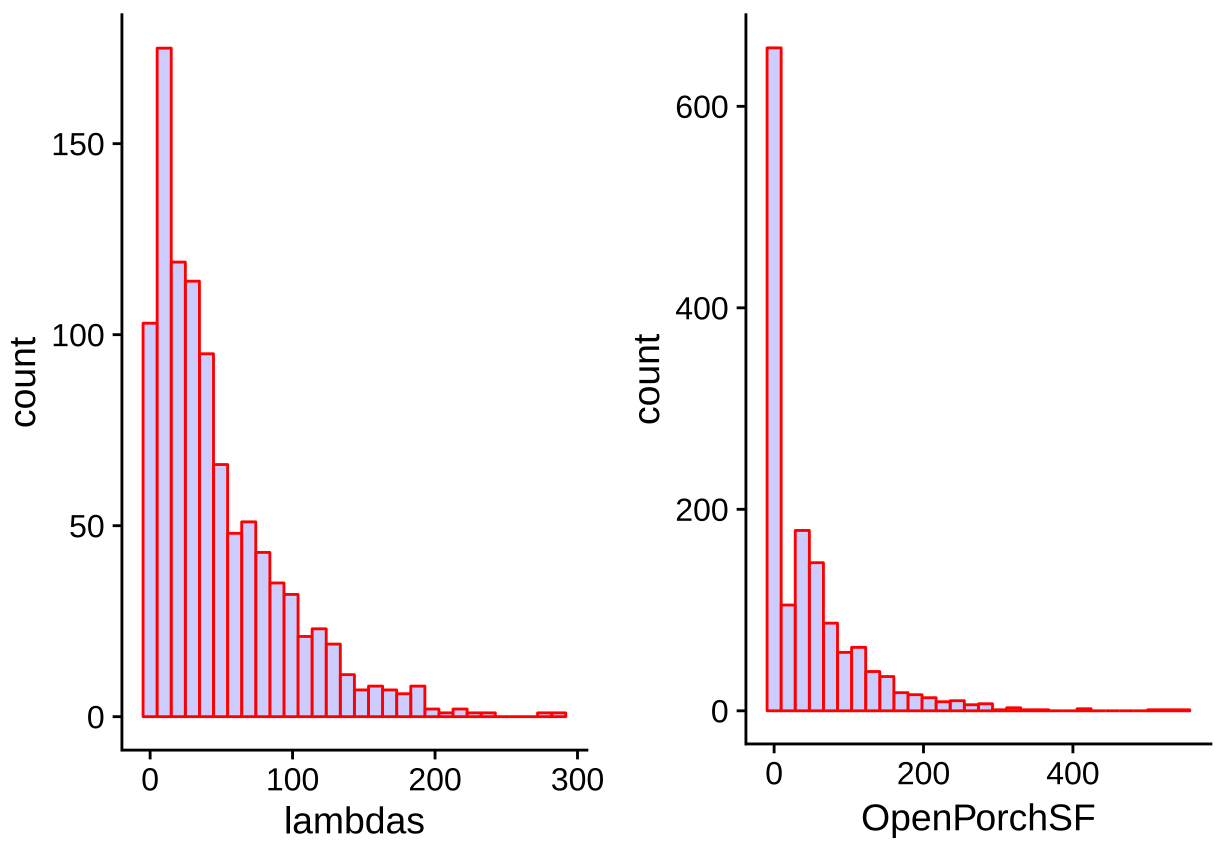
```
## [[1]]
##             [,1]      [,2] [,3]
## [1,]  1.00000000 0.0000000    0
## [2,] -0.25175835 1.0000000    0
## [3,]  0.07485318 0.6244478    1
##
## [[2]]
##            MSSubClass firstFlrSF  GrLivArea
## MSSubClass          1 -0.2517584 0.07485318
## firstFlrSF          0  0.9366177 0.58486888
## GrLivArea           0  0.0000000 0.62917691
```

5. Calculus-Based Probability & Statistics

Many times, it makes sense to fit a closed form distribution to data. For the first variable that you selected which is skewed to the right, shift it so that the minimum value is above zero as necessary. Then load the MASS package and run fitdistr to fit an exponential probability density function. (See https://stat.ethz.ch/R-manual/R-devel/library/MASS/html/fitdistr.html). Find the optimal value of λ for this distribution, and then take 1000 samples from this exponential distribution using this value (e.g., rexp(1000, λ)). Plot a histogram and compare it with a histogram of your original variable. Using the exponential pdf, find the 5th and 95th percentiles using the cumulative distribution function (CDF). Also generate a 95% confidence interval from the empirical data, assuming normality. Finally, provide the empirical 5th percentile and 95th percentile of the data. Discuss.

Since the minimum `OpenPorchSF` value is 0, I'll shift it to the right by 0.0001.

The optimal lambda value is $\lambda = 0.0214315$.



From Wikipedia, the Cumulative Distribution Function for the Exponential Distribution is:

$$F(x; \lambda) = 1 - e^{-\lambda x}$$

The quantile function is thus

$$F^{-1}(p; \lambda) = \frac{-\ln(1-p)}{\lambda}, \quad 0 \le p < 1$$

```
## [1] "5th Percentile of simulated data = 2.39336429840992"
## [1] "95th Percentile of simulated data = 139.781988205825"
```

```
## [1] "5th Percentile of empirical data = 1e-04"
## [1] "95th Percentile of empirical data = 175.0501"
```

The formula for a confidence interval on normally distributed data is

$$x \pm t_{\frac{\alpha}{2},n-1} \cdot \frac{s}{\sqrt{n}}$$

The Z-value for a 95% confidence interval is 1.96.

```
## [1] "95% Confidence interval for empirical data = (43.2617349622305, 50.059012982975)"
```

5% of houses in the sample have an `OpenPorchSF` value of less than 0.0001$, while 95% are below 175.0501. The 95% confidence interval tells us that with each sampling of the population (house prices), we are 95% sure the mean will be between ~43.26 and ~50.06. Due to the large difference in percentile values between the sampled and empirical data, the exponential distribution may not be the best fit for our dataset.

6. Modeling.

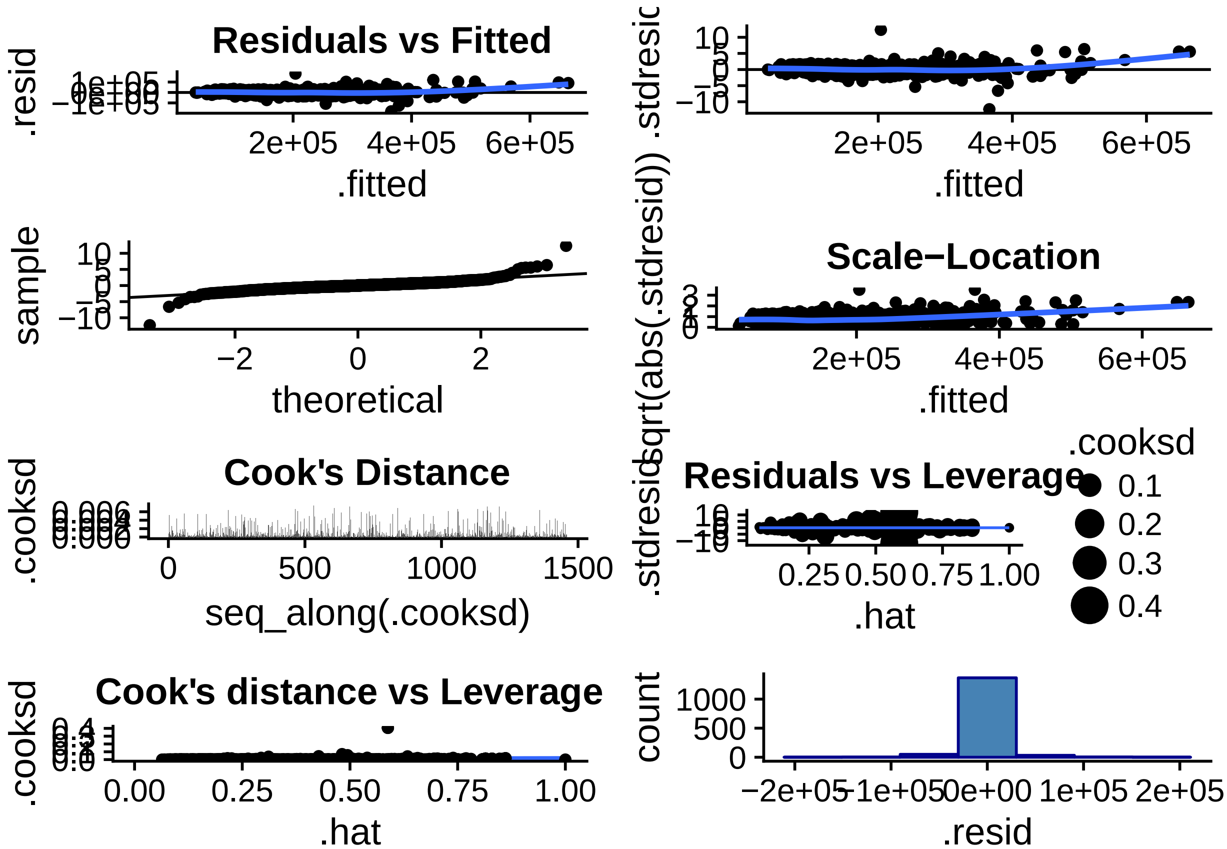
Build some type of multiple regression model and submit your model to the competition board. Provide your complete model summary and results with analysis. Report your Kaggle.com user name and score.

```
##            Id    MSSubClass      MSZoning   LotFrontage       LotArea
##             0             0             0           259             0
##        Street         Alley      LotShape   LandContour     Utilities
##             0          1369             0             0             0
##     LotConfig     LandSlope  Neighborhood    Condition1    Condition2
##             0             0             0             0             0
##      BldgType    HouseStyle   OverallQual   OverallCond     YearBuilt
##             0             0             0             0             0
##  YearRemodAdd     RoofStyle      RoofMatl   Exterior1st   Exterior2nd
##             0             0             0             0             0
##    MasVnrType    MasVnrArea     ExterQual     ExterCond    Foundation
##             8             8             0             0             0
##      BsmtQual      BsmtCond  BsmtExposure  BsmtFinType1    BsmtFinSF1
##            37            37            38            37             0
##  BsmtFinType2    BsmtFinSF2     BsmtUnfSF   TotalBsmtSF       Heating
##            38             0             0             0             0
##     HeatingQC    CentralAir    Electrical    firstFlrSF      secFlrSF
##             0             0             1             0             0
##  LowQualFinSF     GrLivArea  BsmtFullBath  BsmtHalfBath      FullBath
##             0             0             0             0             0
##      HalfBath  BedroomAbvGr  KitchenAbvGr   KitchenQual  TotRmsAbvGrd
##             0             0             0             0             0
##    Functional    Fireplaces   FireplaceQu    GarageType   GarageYrBlt
##             0             0           690            81            81
##  GarageFinish    GarageCars    GarageArea    GarageQual    GarageCond
##            81             0             0            81            81
##    PavedDrive    WoodDeckSF   OpenPorchSF EnclosedPorch threeSsnPorch
##             0             0             0             0             0
##   ScreenPorch      PoolArea        PoolQC         Fence   MiscFeature
##             0             0          1453          1179          1406
##       MiscVal        MoSold        YrSold      SaleType SaleCondition
##             0             0             0             0             0
##     SalePrice
```

```
##                 0
```

There are many variables that can be recoded. Many columns can be converted from numeric to categorical (factor). Some columns, especially those with a high number of missing values, can be converted to factors with two levels - those with the property, and those without it (1 and 0). This will allow us to still use the variable, and not have to discard it. Lastly, we can convert four columns containing quality information to ordered factors.

I won't be imputing any data in this model, since I believe the NAs are not due to missing data, but rather indicate that the property does not have that quality. For example, an NA in the `PoolQC` column probably means that the house doesn't have a pool.




```
##    ID Predicted
## 1   1  112738.1
## 2   2  159855.4
## 3   3  199412.6
## 4   4  200603.6
## 5   5  194311.2
## 6   6  162449.4
## 7   7  173211.2
## 8   8  155506.3
## 9   9  185514.1
## 10 10  119749.0
```