DATA 605 - Discussion 4 response

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Page 460, Exercise C25

Define the linear transformation

$T : \mathbb{C}^3 \to \mathbb{C}^2, \; T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 + 5x_3 \\ -4x_1 + 2x_2 - 10x_3 \end{bmatrix}.$

Find a basis for the kernel of T, $\mathcal{K}(T)$. Is T injective?

Set $T(x) = 0$, and then row reduce.

$$\left[\begin{array}{ccc|c} 2 & -1 & 5 & 0 \\ -4 & 2 & -10 & 0 \end{array}\right]$$

```r
library(pracma)

A <- matrix(c(2, -1, 5, 0,
              -4, 2, -10, 0),
             nrow = 2, ncol = 4, byrow = T)

a <- rref(A)
a
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1 -0.5  2.5    0
## [2,]    0  0.0  0.0    0
```

Which gives us the system of equations:

$$\begin{aligned} x_1 - \frac{1}{2}x_2 + \frac{5}{2}x_3 &= 0 \\ 0 &= 0 \end{aligned}$$

So $x_1 = \frac{1}{2}x_2 - \frac{5}{2}x_3$, with x_2 and x_3 being free variables.

Thus, the basis set for the null space is $\left\{ \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{-5}{2} \\ 0 \\ 1 \end{bmatrix} \right\}$

Theorem KILT says T is injective iff $\mathcal{K}(T) = \{0\}$.

Since the kernel is not trivial, i.e. $\mathcal{K}(T) \neq \{0\}$, T is not injective.