DATA 605 - Discussion 6

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Problem 19

In a poker hand, John has a very strong hand and bets 5 dollars. The probability that Mary has a better hand is .04. If Mary had a better hand she would raise with probability .9, but with a poorer hand she would only raise with probability .1. If Mary raises, what is the probability that she has a better hand than John does?

Solution

Let M be the event that Mary has a better hand. Then the event that Mary doesn't have a better hand is simply the complement, i.e. $\sim P(M) = P(M^c)$.

Let R be the event that Mary raises.

We have $P(M) = 0.04$, so $P(M^c) = 1 - 0.04 = 0.96$.

We are also given $P(R|M) = 0.9$, and $P(R|M^c) = 0.1$.

We're looking for $P(M|R)$.

The formula for conditional probability is: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Our problem becomes $P(M|R) = \frac{P(M \cap R)}{P(R)}$.

We're missing $P(R)$. We can use the alternate form of Bayes' Theorem, rewriting it as:

$P(M|R) = \frac{P(R|M)P(M)}{P(R|M)P(M)+P(R|M^c)P(M^c)}$

This simplifies to: $\frac{0.9*0.04}{(0.9*0.04)+(0.1*0.96)} = \frac{0.036}{0.132} = 0.\overline{2727}$.