DATA 605 - Discussion 15

Joshua Sturm

May 16, 2018

Section 12.3, Exercise 28, Page 679

28.

Form a function $z = f(x, y)$ such that $f_x = x + y$ and $f_y = x + y$.

Solution

Firstly, since the second partials are equal to each other (0 in this case), by Schwarz's Theorem, $f(x, y)$ exists.

If $f_x = x + y$, then $f(x, y) = \frac{1}{2}x^2 + xy + c(y) + A$.

If $f_y = x + y$, then $f(x, y) = xy + \frac{1}{2}y^2 + d(x) + B$.

Combining these, we get $f(x, y) = \frac{1}{2}x^2 + xy + c(y) + A = xy + \frac{1}{2}y^2 + d(x) + B$.

So our original function is $z = f(x, y) = \frac{1}{2}x^2 + xy + \frac{1}{2}y^2$.

Reference: https://math.stackexchange.com/questions/2439726/determine-whether-fx-y-exists-given-the-partial-derivatives