DATA_605_Discussion_14

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Section 8.1, Exercise 39, Page 394

Prove that if

$$\lim_{n\to\infty} |a_n| = 0, \text{ then } \lim_{n\to\infty} a_n = 0$$

We can solve this using a combination of the Triangle inequality, and the Squeeze Theorem.

Firstly, we note that the following is true:

$$-|a_n| \leq a_n \leq |a_n|$$

Then, if

$$\lim_{n\to\infty} -|a_n| = 0$$

this must be true as well

$$\lim_{n\to\infty} |a_n| = 0$$

And so, by the squeeze theorem, we get

$$\lim_{n\to\infty} a_n = 0$$

An alternative way to look at this using the definition of a limit:

If

$$\lim_{n\to\infty} |a_n| = 0$$

then $\exists N \in \mathbb{N} \ni n \geq N$, and $|a_n - 0| \leq \ \epsilon \ \forall \ \epsilon > 0$.

Similarly, if

$$\lim_{n\to\infty} a_n = 0$$

then $\exists N \in \mathbb{N} \ni n \geq N$, and $\Big||a_n| - 0\Big| \leq \ \epsilon \ \forall \ \epsilon > 0$.

But $\Big||a_n| - 0\Big| = \Big||a_n|\Big| = |a_n| = |a_n - 0|$.

Thus, they're essentially equal.