DATA 605 - Discussion 1

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Page 43, problem C31.

Row-reduce the matrix without the aid of a calculator, indicating the row operations you are using at each step using the notation of Definition RO.

I'll use Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & 2 & -4 \\ -3 & -1 & -3 \\ -2 & 1 & -7 \end{bmatrix}$$

$$R_2 = R_2 + 3R_1, \quad R_3 = R_3 + 2R_1 \begin{bmatrix} 1 & 2 & -4 \\ 0 & 5 & -15 \\ 0 & 5 & -15 \end{bmatrix}$$

$$R_2 = R_2 \times \frac{1}{5} \begin{bmatrix} 1 & 2 & -4 \\ 0 & 1 & -3 \\ 0 & 5 & -15 \end{bmatrix}$$

$$R_3 = R_3 - 5R_2 \begin{bmatrix} 1 & 2 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 = R_1 - 2R_2 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix is now in reduced row echelon form.

To verify, we can perform the operations in R.

```
# Set up our matrix
a <- matrix(c(1,2,-4,-3,-1,-3,-2,1,-7), ncol=3, nrow=3, byrow=T)
a
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -4
## [2,]   -3   -1   -3
## [3,]   -2    1   -7
```

I found this row-reducing function on github (https://gist.github.com/ZPears/0583aae73aa06d8abd9e).

```
convertToRREF <- function(matrix) {

  m <- nrow(matrix)
  n <- ncol(matrix)
  currCol <- 1
  nonZeroRowCount <- 0

  while ( (currCol < n+1) & (nonZeroRowCount+1 <= m) )  {
    if (sum(matrix[(nonZeroRowCount+1):m, currCol]) == 0) {

      currCol <- currCol + 1

    } else {

      rowIndex <- 0

      for (i in nonZeroRowCount+1:m) {

        if (matrix[i,currCol] != 0) {

          rowIndex <- i
          break

        }

      }

      nonZeroRowCount <- nonZeroRowCount + 1

      # switch rows

      row1 <- matrix[rowIndex,]
      row2 <- matrix[nonZeroRowCount,]
      matrix[rowIndex,] <- row2
      matrix[nonZeroRowCount,] <- row1

      # Use the second row operation (mult. by scalar)
      # to convert the entry in row
      # nonZeroRowCount and column currCol to a 1.

      matrix[nonZeroRowCount,] <- (1/matrix[nonZeroRowCount,currCol]) * matrix[nonZeroRowCount,]

      # Use the third row operation with row nonZeroRowCount
      # to convert every other entry of column j to zero.

      for (k in 1:m) {

        if ( (matrix[k, currCol] != 0) & (k != nonZeroRowCount) ) {

          # use row r (nonZeroRowCount) and row op 3 to make A[k, currCol] = 0

          scalar <- matrix[k, currCol] / matrix[nonZeroRowCount, currCol]

          matrix[k, ] <- -1 * scalar * matrix[nonZeroRowCount, ] + matrix[k, ]

        }

      }

      # increment and repeat
      currCol <- currCol + 1

    }

  }

  return(matrix)

}
```

Finally, let's verify that we get the same answer.

```
convertToRREF(a)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    2
## [2,]    0    1   -3
## [3,]    0    0    0
```