DATA 605 - Discussion 10

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Section 11.1, Page 414, Exercise 8

Exercise 8

A certain calculating machine uses only the digits 0 and 1. It is supposed to transmit one of these digits through several stages. However, at every stage, there is a probability p that the digit that enters this stage will be changed when it leaves and a probability $q = 1 - p$ that it won't. Form a Markov chain to represent the process of transmission by taking as states the digits 0 and 1. What is the matrix of transition probabilities?

Solution

If we wanted to visualize how the machine works:

$X_1 \rightarrow$ STAGE 1 $\rightarrow X_2 \rightarrow$ STAGE 2 $\rightarrow X_3 \rightarrow$ STAGE 3 $\rightarrow x_4 \rightarrow \cdots$

At each stage (or step), X_i can take on one of two values (or states): $S = \{0, 1\}$.

The transition matrix will be

$$P = \begin{pmatrix} q & p \\ p & q \end{pmatrix} = \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}$$