DATA 609 - Homework 5

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Chapter 6 problems

1 (Page 228, exercise #1)

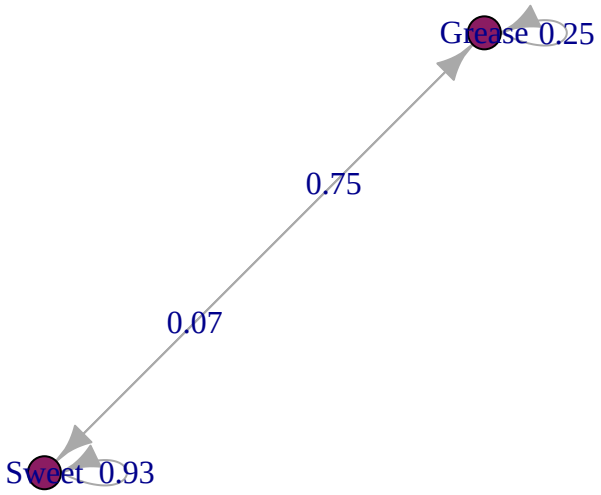
Consider a model for the long-term dining behavior of the students at College USA. It is found that 25% of the students who eat at the college's Grease Dining Hall return to eat there again, whereas those who eat at Sweet Dining Hall have a 93% return rate. These are the only two dining halls available on campus, and assume that all students eat at one of these halls. Formulate a model to solve for the long-term percentage of students eating at each hall.

1 Solution

The transitional matrix is

```
## Markov Chain 1
##  A  2 - dimensional discrete Markov Chain defined by the following states:
##  Grease, Sweet
##  The transition matrix  (by rows)  is defined as follows:
##        Grease Sweet
## Grease   0.25  0.75
## Sweet    0.07  0.93
```

And the markov chain is show plotted below:



Let G_n = the percentage of students who eat at Grease Dining Hall at the end of period n. \ Let S_n = the percentage of students who eat at Sweet DininG Hall at the end of period n.

Then,

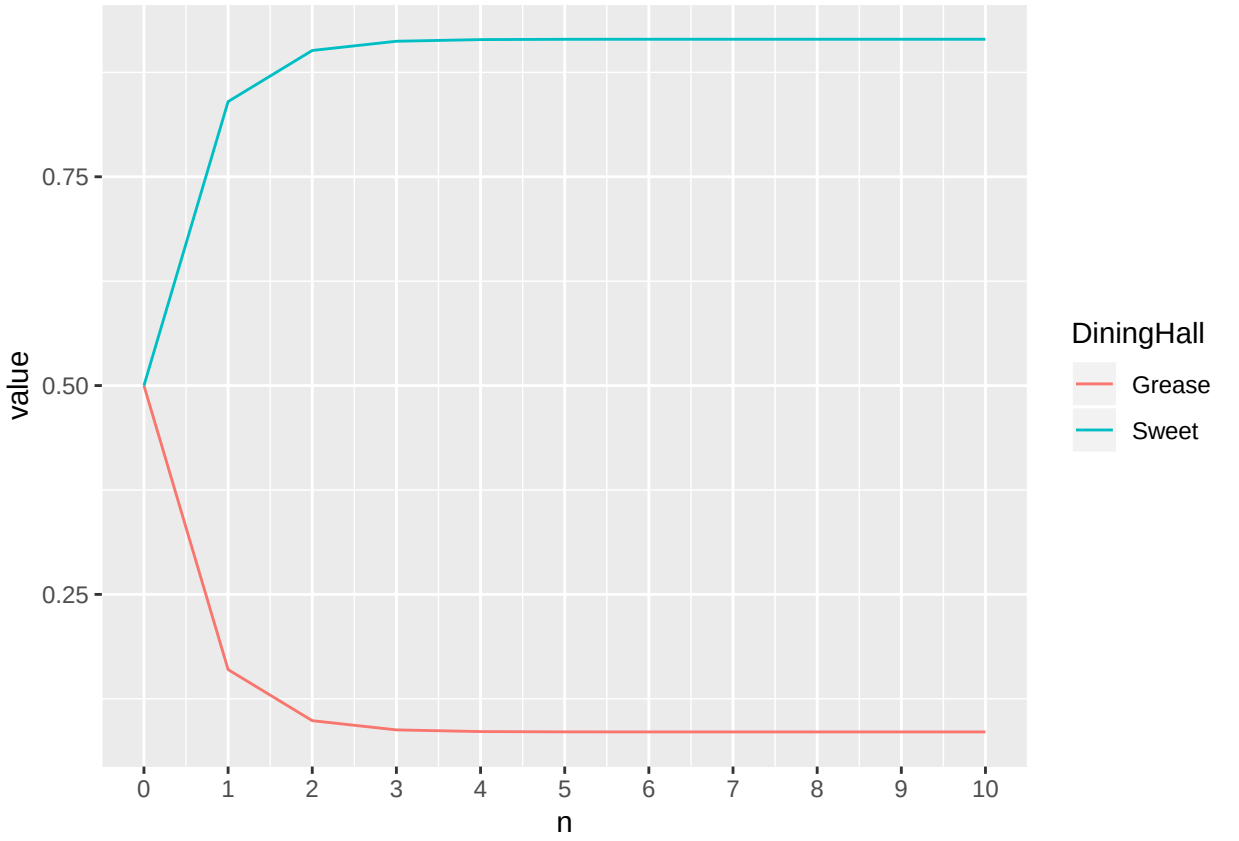
$G_{n+1} = 0.25G_n + 0.07S_n \ \backslash \ S_{n+1} = 0.75G_n + 0.93S_n$

We can visualize the model's prediction over time.

```
##      n    Grease     Sweet
## 1    0 0.5000000 0.5000000
## 2    1 0.1600000 0.8400000
## 3    2 0.0988000 0.9012000
## 4    3 0.0877840 0.9122160
## 5    4 0.0858011 0.9141989
## 6    5 0.0854442 0.9145558
## 7    6 0.0853800 0.9146200
## 8    7 0.0853684 0.9146316
## 9    8 0.0853663 0.9146337
## 10   9 0.0853659 0.9146341
## 11  10 0.0853659 0.9146341
```

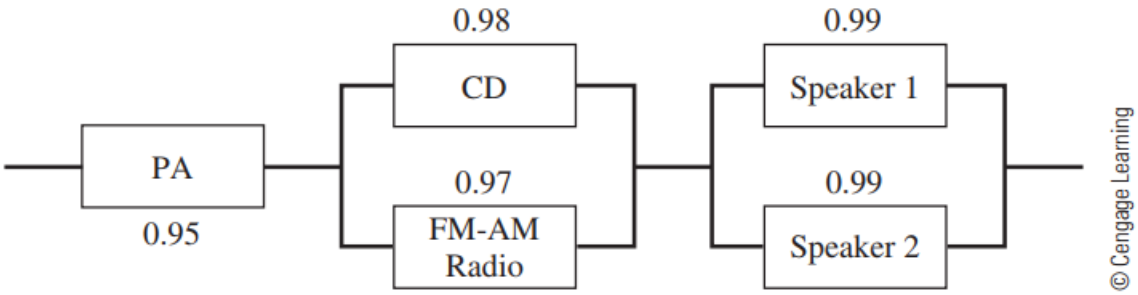
We can see the model converges after 10 timesteps.

To view this graphically:



2 (Page 232, exercise #1)

Consider a stereo with CD player, FM-AM radio tuner, speakers (dual), and power amplifier (PA) components, as displayed with the reliabilities shown in Figure 6.11. Determine the system's reliability. What assumptions are required in your model?



2 Solution

We can divide it into three parts: - The PA system - The CD and FM-AM radio - Speaker 1 and speaker 2

Let R_1, R_2, and R_3 represent the above three parts, respectively.

Then,

$$R_1 = 0.95$$
$$R_2 = R_2(1) + R_2(2) - (R_2(1) \times R_2(2)) = 0.98 + 0.97 - (0.98 \times 0.97) = 0.9994$$
$$R_3 = R_3(1) + R_3(2) - (R_3(1) \times R_3(2)) = 0.99 + 0.99 - (0.99 \times 0.99) = 0.9999$$

The reliability of the system as whole is $R_1 \times R_2 \times R_3$.

$R = 0.95 \times 0.9994 \times 0.9999 = 0.9493351$.

References: - https://rpubs.com/JanpuHou/326048 - https://www.datacamp.com/community/tutorials/markov-chain-analysis-r