DATA 609 - Homework 6

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Chapter 7 Problems

1 (Page 251, exercise #2)

Nutritional Requirements-A rancher has determined that the minimum weekly nutritional requirements for an average-sized horse include 40 lb of protein, 20 lb of carbohydrates, and 45 lb of roughage. These are obtained from the following sources in varying amounts at the prices indicated:

	Protein	Carbohydrates	Roughage	Cost
Hay	0.5	2.0	5.0	1.8
Oats	1.0	4.0	2.0	3.5
Feeding blocks	2.0	0.5	1.0	0.4
High-protein concentrate	6.0	1.0	2.5	1
Requirements per horse	40.0	20.0	45.0	

Formulate a mathematical model to determine how to meet the minimum nutritional requirements at minimum cost.

1 Solution

We are given four variables:

$$
\begin{aligned}
x_1 &= \text{Hay} \\
x_2 &= \text{Oats} \\
x_3 &= \text{Feeding blocks} \\
x_4 &= \text{High-Protein concentrate}
\end{aligned}
$$

Our cost function is:

$$1.80x_1 + 3.5x_2 + 0.4x_3 + 1.0x_4$$

And we want to minimize it, subject to the constraints:

$$
\begin{aligned}
0.5x_1 + x_2 + 2.0x_3 + 6.0x_4 &\leq 40.0 \quad (Protein) \\
2.0x_1 + 4.0x_2 + 0.5x_3 + 1.0x_4 &\leq 20.0 \quad (Carbohydrates) \\
5.0x_1 + 2.0x_2 + 1.0x_3 + 2.5x_4 &\leq 45.0 \quad (Roughage)
\end{aligned}
$$

2 (Page 284, exercise #1)

For the example problem in this section, determine the sensitivity of the optimal solution to a change in c_2 using the objective function $25x_1 + c_2x_2$.

2 Solution

Let $Z = 25x_1 + c_2x_2$.

Then $x_1 = \frac{-c_2}{25}$.

The slope of the lumber constraint is $\frac{-2}{3}$, and the slope of the labour constraint is $\frac{-5}{4}$.

Thus, the range of values for which the current extreme point remains optimal is given by the inequality

$$\frac{-5}{4} \leq \frac{-c_2}{25} \leq \frac{-2}{3}$$

$$\frac{50}{3} \leq c_2 \leq \frac{125}{4}$$

Resources used:

- http://sites.fas.harvard.edu/~apm121/lectures/lec8-hq.pdf